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48. *Derivation of the Utsu-Seki Formula for the Aftershock Area and the Magnitude of the Principal Shock from Deformation~Fracture Relation in the Earthquake Genesis.*

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1. Introduction

The estimation of the maximum size of an earthquake is one of the most important problems in the theory of earthquake genesis. In the preceding paper[1], deformation~fracture relations in earthquake genesis was newly introduced and the frequency distributions of earthquakes are theoretically derived from that relation. The frequency distribution of earthquakes will be used to estimate the maximum size of the earthquake in a given focal region.

If this is done, the maximum size of earthquake will also be related to various parameters of the deformation in the focal region, since the frequency distribution of earthquakes is derived from the deformation in in the focal region.

As regards the magnitude of the principal shock and the area of aftershocks, there is a famous formula obtained by Utsu and Seki[2]. Even though the Utsu-Seki formula was originally presented as a formula which expressed the aftershock area as a function of the magnitude of the principal shock, it is also the formula which gives the magnitude of the principal shock from the area of aftershocks, which are nothing but the deformation in the focal region.

Thus it is expected that the Utsu-Seki formula will be derived form the deformation~fracture relation in the earthquake genesis. This paper will show its derivation and will discuss some physical meaning of the parameters which appear in the Utsu-Seki formula.

2. Estimation of the maximum energy and magnitude from the frequency distribution of earthquakes.

The maximum energy E_{max} and the maximum magnitude M_{max} of a

1) S. NAGUMO, *Bull. Earthq. Res. Inst.*, **47** (1969), 1015-1027.
2) T. UTSU and A. SEKI, *Zisin* [ii], **7** (1954), 233-240.

series of earthquakes will be obtained from the frequency distribution of earthquakes from the condition

$$N(E_{max})=1 \quad (1)$$

$$N(M_{max})=1 \quad (2)$$

where $N(E_{max})$, $N(M_{max})$ denote the number of earthquakes of which energy or magnitude is E_{max}, M_{max} or greater respectively. $N(E)$ and $N(M)$ are defined by the formulas

$$N(E)=\int_E^{\infty} n(E)dE \quad (3)$$

$$N(M)=\int_M^{\infty} n(M)dM \quad (4)$$

where $n(E)dE$ and $n(M)dM$ are frequency distribution density functions with respect to energy and magnitude respectively. As shown in the preceding paper,[3] the expressions of $N(E)$ and $N(M)$ are obtained from the deformation $\sim$ fracture relations in earthquake genesis, and are

$$N(E)=K_E/(n-1)\cdot(E/E_0)^{-n+1} \quad (5)$$

$$N(M)=(1/(b\log_e 10))10^a\cdot 10^{-bM} . \quad (6)$$

Thus, from equation (1)(2) and (5)(6), we have

$$E_{max}=E_0\{K_E/(n-1)\}^{1/(n-1)} , \quad (7)$$

$$M_{max}=(a/b)-(1/b)\log_{10}(b\log_e 10) , \quad (8)$$

where

$$K_E=(1/3)(Sc/b_0)(A_0/k_0)_0^3k \quad (9)$$

$$10^a=K_E\cdot 10^{-\alpha(n-1)}\cdot\beta\log_e 10(e_0/E_0)^{-n+1} \quad (10)$$

$$b=-\beta(1-n) , \quad (11)$$

and n is a certain constant which represents the sharpness of the spectrum of the deformation, α, β are the coefficients in the formula $\log_{10}E=\alpha+\beta M$, and e_0 unit of energy, E_0 the energy of seismic wave radiation per unit volume, S area of the focal region, A_0 the spectrum strength of the deformation, c/b_0 coefficient of crack production, k_0 unit of wave number.

3) S. NAGUMO, *loc. cit.*, 1).

The equations (7) and (8) are the ones which give the E_{max} and M_{max} as functions of various source parameters. In usual cases, where $W(k)\propto 1/k$ and n is $n=5/2$, the equation (7) becomes

$$E_{max}=E_0\left(\frac{1}{2}\right)^{3/2}S^{3/2}\left(\frac{cA_0}{b_0}\right)^{3/2}k_0^3 . \tag{12}$$

This means that the maximum energy of seismic wave radiation is proportional to the $S^{3/2}$, and $A^{3/2}$.

3. Derivation of the Utsu-Seki formula

Utsu and Seki[4] have shown that there is a relation between the area S of aftershocks and the magnitude M of the principal shock

$$\log_{10} S=cM+d , \tag{13}$$

where c, d are non-dimensional constants and are obtained empirically as

$c=0.9$	$d=6.8$	(for submarine earthquakes)
$c=0.8_5$	$d=6.9_5$	(for land earthquakes)
$c=1.0$	$d=6.0$.	(for both earthquakes)

Since the magnitude of the principal shock is the maximum magnitude of a series of earthquakes, namely foreshock, the principal shock and aftershocks, the equation (13) is also the formula which gives M_{max} as a function of S.

M_{max} is given as a function of a by the equation (8), and a is a function K_E as given by the equation (10), and K_E is a function of S as given by the equation (9). Therefore, M_{max} will be expressed as a function S.

From equations (9) and (10), we have

$$a=\log_{10} S+\text{constant} . \tag{14}$$

Substituting equation (14) into (8) for a, we have the equation

$$bM_{max}=\log_{10} S-H , \tag{15}$$

where

$$-H=\log_{10}\{(1/2)(A_0/b_0)k_0^2 10^{-\alpha(n-1)}(e_0/E_0)^{-n+1}\} .$$

The equation (15) is the expression of the Utsu-Seki formula. That

4) T. Utsu and A. Seki, *loc. cit*, 2).

is, the expression M_{max}, which is derived from the frequency distribution of earthquakes, is none other than the Utsu-Seki formula for the relation between the magnitude of the principal shock and the area of aftershocks.

4. Remarks

Comparing the equation (13) and (15), we have the relations

$$c=b\,, \qquad d=H\,. \tag{16}$$

This is a theoretical prediction. This prediction is strikingly well in accordance with the observational facts that the empirical value of c in the Utsu-Seki formula is $c\fallingdotseq 1$ and the empirical value b in Gutenberg-Richter's formula is $b=1$.

The difference of d in the Utsu-Seki formula for submarine earthquakes and land earthquakes, may be the differences in the strength of the spectrum deformation and energy density between submarine and land.

5. Summary and Conclusion

The Utsu-Seki formula for the aftershock area and the magnitude of the principal shock is derived from the deformation~fracture relation in the earthquake genesis.

The Utsu-Seki formula is equal to the expression of the magnitude of the maximum earthquake which will be derived from the frequency distribution of earthquakes with respect to magnitude.

The Utsu-Seki formula can be interpreted as the formula which gives the magnitude of the maximum earthquakes in a series of earthquakes as a function of the area of focal region.

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48. 地震発生における変形～破壊の関係式から余震面積と主震のマグニチュードに関する宇津・関公式を導くこと

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前論文に引続き，表題の問題を取扱つた．地震のマグニチュード別頻度分布から期待される最大の地震の表現を求めてみると，宇津・関公式が得られることが分つた．

最大の地震の M を規定する条件としては，ある M 以上の地震の累積頻度が 1 になるという M をもつて最大の M とする，という条件を採用した．この条件を使用すると，地震の頻度分布から

M の最大値, M_{max}, が求められ, M_{max} が震源域の面積の函数として求められる. すなわち, 宇津・関公式が求められる.

従来宇津・関公式は余震域面積が主震の M によつて規定される公式として提出され, 理解されてきたが, 同公式は逆に主震の M が震源域の面積によつて規定される公式としても解釈される.

このことは, ある地域, ある時期に発生する地震の最大の M を推定するためには, 震源域の構造およびその活動をあらかじめ知る必要があることを示している.