

24. *On the Radiative Heat Transfer and the Thermal Conductivity in the Upper Mantle.**

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Abstract

In § 1 and 2, the effect of radiation on the heat transfer in solids is described in terms of radiative thermal conductivity. In § 3, radiative thermal conductivity of olivine in the temperature range from 300° to 1300°K is obtained from the results of the measurement of its optical absorption coefficient in the range of wave length from 0.3 to 10 μ (Fukao et al., 1968). The sum of the ordinary lattice conductivity and the radiative conductivity of olivine is approximately constant and its value is about 0.012 cal/cm. sec. deg. from 300° to 1300°K. This conclusion is different from the simple T^3 law which assumes temperature independent absorption. The conductivity deduced from the ordinary experiment will depend not only on the material but also on the thickness of the specimen. Quantitative discussion about the thickness effect is made (§ 4) for the case of the relatively large thickness of the specimen. § 5 deals with some geophysical implications of the present results. If the upper mantle is mainly composed of peridotite and the crystal size in it is larger than about 1 cm, the sum of the lattice conductivity and radiative conductivity is estimated to be approximately constant and its value is about 0.010 or 0.011 cal/cm. sec. deg. up to 1300°K. This result suggests that the possibility of mechanisms other than the above two modes of the heat transfer, such as solid state convection, should also be taken into account especially in the oceanic upper mantle.

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1. Introduction

The suggestion that the radiation mode may play an important part

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in the heat transfer in the earth's mantle was first proposed by Birch and Clark (1940). Since 1950, the problems of the radiative heat transfer within semi-transparent solids have been treated mainly by glass technologists. Stimulated by these studies, some earth scientists have paid attention to this mode of heat transfer (Preston, 1956; Clark, 1956, 1957; Lawson and Jamieson, 1958). Especially Clark (1957b) measured the optical spectra of various important minerals. He found the existence of a pass-band near the infrared region for most of these minerals. This experiment is important because it showed directly the possible effectiveness of radiative transfer in the earth's mantle.

Effect of the radiation on the heat transfer can be represented by the "radiative thermal conductivity, K_r ". After Clark (1957a) and others, K_r is expressed as,

$$K_r = \frac{4}{3} \int_0^\infty \frac{n_\nu^2}{\epsilon_\nu} \frac{\partial B_\nu}{\partial T} d\nu, \quad (1)$$

where T is the absolute temperature, ν the frequency of radiation, n_ν the refractive index, $\epsilon_\nu = \alpha_\nu + \sigma_\nu$ the extinction coefficient or opacity defined by the sum of the absorption coefficient and scattering coefficient and B_ν the spectral black body radiation function.

Now if we define the mean extinction coefficient $\bar{\epsilon}$ as

$$\frac{1}{\bar{\epsilon}} = \int_0^\infty \frac{1}{\epsilon_\nu} \frac{\partial B_\nu}{\partial T} d\nu \bigg/ \int_0^\infty \frac{\partial B_\nu}{\partial T} d\nu, \quad (2)$$

(1) becomes

$$K_r = \frac{16n^2sT^3}{3\bar{\epsilon}} \quad (3)$$

(s =Stefan Boltzmann constant)

provided that $n_\nu = n$ (=constant). The above formula has been used in the thermal history calculations of the earth while there has been a serious ambiguity regarding the actual value of $\bar{\epsilon}$ in the earth.

Since MacDonald (1964) and Clark and Ringwood (1964), the constant mean opacity, such as $\bar{\epsilon} = 10 \text{ cm}^{-1}$ or 5 cm^{-1} , has been widely used in discussing the thermal state of the upper mantle. The most characteristic feature of the constant opacity is that it makes the temperature-depth curves convex because of the rapid increase of the radiative conductivity with increasing temperature. Recently, however Aronson et al. (1967a, b) and Fukao et al. (1968) have found a large temperature effect on the optical absorption spectra of olivine and other minerals. The direct measurements of thermal conductivity at high temperatures

of various crystals (Kanamori et al., 1968) and rocks (Kawada, 1966) also seem to reject such a large increase of thermal conductivity with increasing temperature as assumed in Clark and Ringwood (1964). These experimental results suggest that we must re-examine the temperature distribution in the mantle and the thermal history of the earth having a lower thermal conductivity. This paper aims to help us in advancing such a course of investigation.

2. A Brief Review of Radiant Heat "Conduction"

Thermal conduction in solids is described in terms of the propagation of energy by elastic waves caused by thermal vibration of atoms. Solids in which the atoms undergo harmonic motion were called ideal solids by Debye (1914). In a plane sheet of an ideal solid bounded by two walls at different fixed temperatures, thermal elastic waves generated at the boundaries propagate independently with each other from right to left and from left to right, so that any unit volume element in the sheet has the same vibration energy. Finite net flow of energy and zero gradient of energy density mean infinite thermal conductivity of ideal solids. The reason why the thermal conductivity is finite even in a structurally perfect crystal is that when an elastic wave creates instantaneous volume change at some point, other waves encountering this region are scattered due to the change of sound velocity at that point, that is, due to the anharmonic effect. Such a mutual scattering of elastic waves assures the redistribution of energy among waves with different frequencies of vibration and then the possibility of attaining thermal equilibrium.

Debye called the intensity of the energy transported by thermal elastic waves "radiation intensity" as in the case of the theory of heat radiation. The amount of energy passing through a small element of area in solid is the sum of "radiations" scattered in the different volume elements. Those "radiations" would reach this area after weakening their intensities due to scattering on the way. Following such an idea, Debye (1914) derived the formula of the thermal conductivity as follows

$$K_p = \frac{1}{4} vl \frac{dE}{dT}, \quad (4)$$

where l is the mean free path which is the distance that the intensity of "radiation" weakens to $1/e$ of the original, E the internal energy of thermal elastic waves per unit volume at equilibrium and v the velocity of sound. dE/dT is, of course, the heat capacity per unit volume.

By Dulong-Petit law, the internal energy of a crystal composed of

N single atoms per unit volume at sufficiently high temperatures is

$$E = 3NkT, \quad (5)$$

where k is Boltzman constant. Substitution of eq. (5) into eq. (4) yields

$$K_p = \frac{3}{4}Nk \cdot vl. \quad (6)$$

Radiant heat conduction in solids is described in terms of the transfer of energy by the light waves caused by thermal emission of material particles. It is apparent that Debye's method is completely useful to derive the radiative thermal conductivity of solids. In this case no mutual scattering of waves exists. Redistribution of energy among waves with different frequencies and the thermal equilibrium are realized through the absorption of radiation by material particles and simultaneous emission of radiation by the same particles. As inferred from eq. (4), the radiative conductivity K_r should be represented by

$$K_r = \frac{1}{4} \frac{q}{\alpha} \frac{dU}{dT}, \quad (7)$$

where U is the volume density of radiation at the equilibrium state in a medium of which refractive index is n and α is the absorption coefficient which is the inverse of the mean free path of radiation. $q = c/n$ is the light velocity in the medium, while c is the velocity of light in vacuum.

If the volume density of black body radiation in a pure vacuum is denoted by U_0 , then

$$U = n^3 U_0 = n^3 \frac{4}{c} s T^4, \quad (8)$$

(e.g. Planck, 1959). Substitution of eq. (8) into eq. (7) yields

$$K_r = \frac{4n^2 s T^3}{\alpha}. \quad (9)$$

Equation (9) is essentially equal to eq. (3) except a slight difference in constant factor, which is due to the roughness of the treatment of Debye. In truth a more exact three dimensional treatment leads to a constant factor $1/3$ instead of $1/4$ in eqs. (4) and (7). Here the scattering process of radiation has been ignored for simplicity. Thus the radiative conductivity is proportional to T^3 while the lattice conductivity is constant so long as the mean free path is assumed to be constant.

When a solid has a size comparable with the mean free path, a

quantity relating to the mean free path is seriously affected by the boundary of the body. This is the so-called boundary effect or size effect. Roughly speaking, the mean free path of a lattice wave is of the order of Å and that of radiation is of the order of cm in ordinary dielectrics. This situation often makes it difficult to use the concept of radiative thermal conductivity in actual cases while the concept of lattice conductivity is entirely valid.

3. Optical Absorption Spectra at High Temperatures and Radiative Thermal Conductivity of Olivines.

Fukao et al. (1968) measured the absorption coefficient of olivine crystals in the range of wave length from 0.3 to 10 μ at every 100°K from 300° to 1300°K. Based on these measurements, they calculated K_r and $\bar{\epsilon}$ using eqs. (1) and (2) in which $\sigma_v=0$ was assumed and obtained the following results.

1. In general, α increases with increasing temperature except in a region near 1 μ in which an Fe^{2+} absorption peak exists. Especially α in the pass-band (from 1.8 to 3.0 μ) increases by a factor of about 100 (from 0.05 to 5 cm^{-1}) in going from 300° to 1300°K.
2. Because of the rise of α , K_r becomes smaller than the one expected from Clark's measurements (1957b) of α at the room temperature by more than an order of magnitude. This small value of K_r is in a good agreement with the radiative component of the thermal conductivity K estimated from the actual measurement of thermal diffusivity on olivine crystals (Kanamori et al., 1968).

Fukao et al. (1968) in their paper left the confirmation of the reproducibility of their optical results above 900°K for further experiment. The present author practised this experiment. The specimen used is olivine from Arizona, U.S.A. and its Fe/Mg ratio seemed to be the same as the one used in the previous experiments because the height of the Fe^{2+} absorption peak which is sensitive to the amounts of Fe^{2+} ions was nearly equal. That is, the composition of the present olivine may be fayalite 12% and forsterite 88%. Thickness of the specimen was 2.603 \pm 0.002 mm. In the experiment by Fukao et al. (1968) it was found that the spectrum in the near-infrared was connected smoothly with the one in the infrared at every temperature up to 1300°K, therefore it was thought sufficient to make the confirmation of reproducibility only in the visible and near-infrared regions.

The experimental method was entirely the same as before; the optical system is shown in Fig. 1. Measurements were made at the

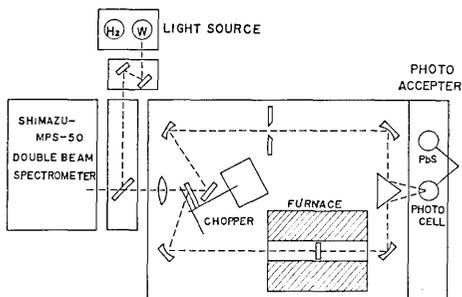


Fig. 1. Optical system for visible and near-infrared measurement.

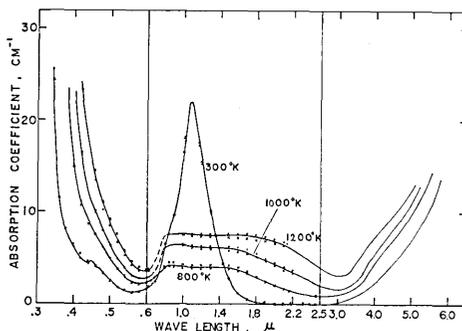


Fig. 2. Absorption spectrum of olivine at high temperatures.

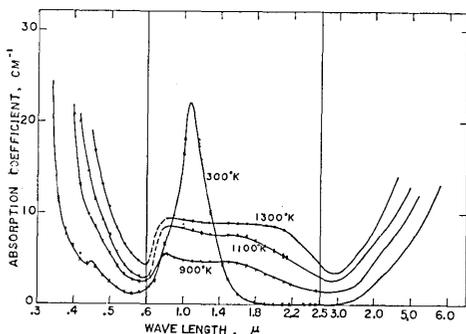


Fig. 3. Absorption spectrum of olivine at high temperatures.

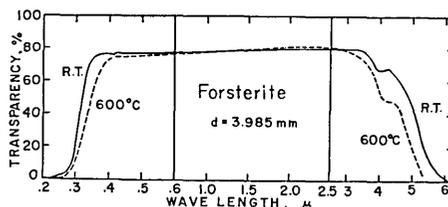


Fig. 4. Transmission spectrum of forsterite at high temperatures.

room temperature and at every 100°K from 800°K up to 1300°K. Experimental results are shown in Figs. 2 and 3, in which the infrared spectra are reproduced from the earlier results. Spectrum at every temperature in these figures includes the data in both the heating and cooling processes and the coincidence between them is very good. Comparison with the previous data shows also a good coincidence of the absolute values of the absorption coefficient. Especially the near-infrared spectra are remarkably similar to each other, while the present olivine is slightly more transparent in the visible region at every temperature and its spectrum at the room temperature is closer to Clark's result. This slight discrepancy may be reasonable when we consider the possible difference of the crystallographic orientation and of the chemical composition. The value of $\bar{\epsilon}$ and K , calculated from the present data give only negligibly small differences from those calculated from the earlier data. In conclusion, the results of Fukao et al. (1968) are reliable through the entire temperature range.

The author measured, though preliminarily, the transmission spectrum

of a single crystal of forsterite, which was kindly offered by Dr. T. J. Shankland, The University of Newcastle-upon-Tyne, at high temperatures. Some of the results are shown in Fig. 4. No distinct decrease of transparency was found in the pass band with the increase of temperature. A large decrease in transparency can be seen but only in the tails of the long and short wavelength absorption bands. This

experiment may support the explanation that the remarkable increase of absorption coefficient of olivine in the pass band is mainly attributed to the broadening of Fe^{2+} absorption band. It does not mean that there exists no increase of the absorption coefficient in the transparent region of forsterite but that $\alpha=0.001\text{ cm}^{-1}$, for example, cannot be distinguished from $\alpha=0.0001\text{ cm}^{-1}$. It is notable that in fact Grybnack and Burch (1965) observed an increase of the coefficient of this magnitude for sapphire (single crystal of Al_2O_3) at high temperatures.

Figure 6* shows the relations between K , K_p , $K_p + K_r$ and other quantities with the absolute temperature in which lattice conductivity K_r is estimated from the actual measurement of thermal diffusivity of olivine as in Fig. 5 (Kanamori et al., 1968). Here K_{r0} means the radiative conductivity calculated from the room temperature spectral data. It may be noticed that $K_p + K_r$ is in a fairly good agreement with the observed conductivity K , while $K_p + K_{r0}$ is larger than K by more than an order of magnitude. When we, however, examine the curves in Fig. 6 in more detail, K is found to be smaller than $K_p + K_r$ at most temperatures. As will be stated in the next section, this fact may be interpreted as due to the size effect that takes place in the measurement of the thermal conductivity.

The dotted line in Fig. 6 shows the thermal conductivity of the powdered forsterite measured by Kingery et al. (1954). In this result, the porosity of the sample is reduced to zero by a simple correction. No deviations from the proportionality between this line and $1/T$ is found at least in the concerned temperature range and this line is very close to K_p curve. Again this phenomenon may be interpreted as due to the

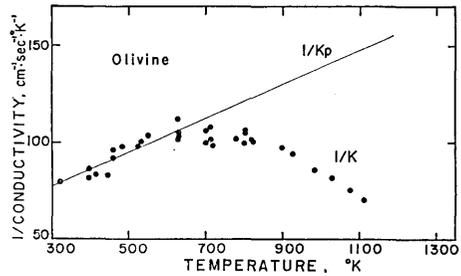


Fig. 5. Reciprocal of thermal conductivity of olivine (from Kanamori et al., 1968).

* The main part of this figure is essentially the same as Fig. 4 in Fukao et al. (1968) except for a slight improvement of the estimation of K_p . The older estimation was based on the values of Table 3 in Kanamori et al. (1968), which are the average values of original data.

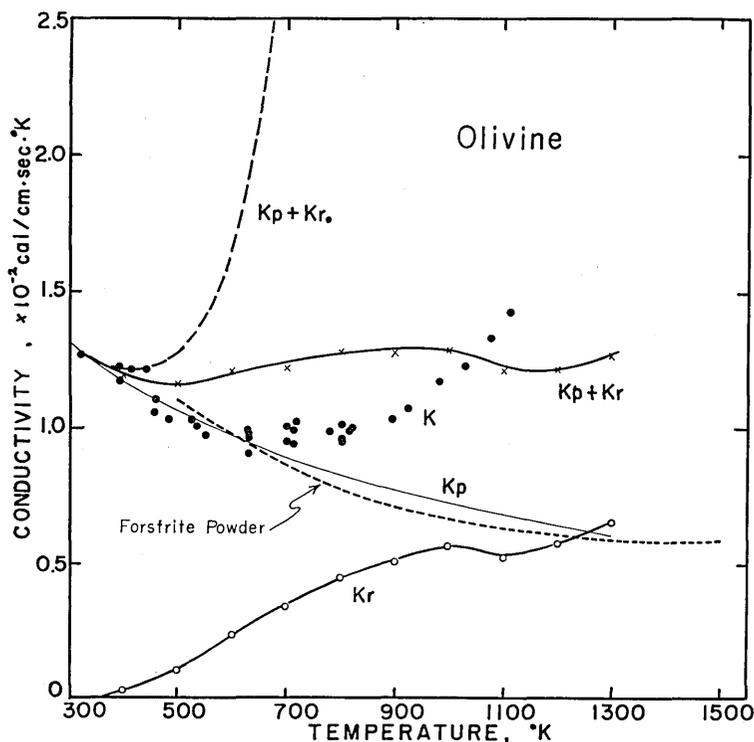


Fig. 6. Thermal conductivity of olivine.

- K: thermal conductivity deduced from the diffusivity measurement (Kanamori et al., 1968).
 Kp: lattice conductivity estimated from the diffusivity measurement as in Fig. 5.
 Kr: radiative conductivity calculated from room temperature spectral data.
 Kr: radiative conductivity calculated from high temperature spectral data.

boundary effect for the radiative transfer. In powdered material radiation cannot penetrate so far because of the almost entire scattering by the grain boundaries of particles. Recently Fujisawa et al. (1968) measured the thermal diffusivity of powdered forsterite at high pressures and temperatures. They also found no deviation from the proportionality between the diffusivity and $1/T$ up to 1200°K.

Aronson et al. (1967a, b) measured the absorption spectra of olivine, diopside and oligoclase at elevated temperatures. They obtained $K_r = 0.0050$ cal/cm. sec. $^{\circ}$ K at 1513°K for olivine. They find further that K_r decreases beyond 1500°K (Aronson, private communication). Thus it may be inferred that the magnitude of K_r of olivine would be about 0.005 at most in the whole temperature range.

4. Size Effect in the Measurement of Thermal Conductivity

Thermal conductivities are determined usually by measuring the rate of heat flow through a specimen and the corresponding temperature drop across it under the steady state condition. In Fig. 7 two opaque walls are separated by an infinite planar sheet with thickness d . Surfaces of the sheet are in good contact with the walls so that their temperatures are maintained at those of the walls, that is, T_0 and T_1 ($T_0 < T_1$) respectively. As stated in section 3, the mean free path of a lattice wave is so small that the specimen can be assumed to have infinite thickness for the lattice conduction. On the contrary the mean free path of radiation is often comparable with the thickness of the specimen. In this case some of the radiation reaching a layer inside the sheet may be directly attributed to the radiation emitted by the walls or reflected at the walls. Now let the sheet be so thin that it can be assumed to be a nearly ideal solid for the radiant transfer. Then, through the sheet, the radiation emitted by one wall penetrates with negligibly small attenuation before reaching the other wall and the radiation reflected at the latter also reaches the former without attenuation. Under such a condition the temperature gradient within the sheet increases with the reduction of the thickness, d , of the sheet when the temperature of both surfaces is fixed at T_0 and T_1 , while the rate of heat flow by radiation through it is held nearly constant. Thus the thinner the sheet is, the more the usual lattice conduction contributes to the total heat transfer. To summarize, the observed thermal conductivity should be the sum of the lattice and radiative conductivity for a sheet of infinite thickness, while it approaches the lattice conductivity for an infinitesimally thin sheet. Here the terms "infinite" and "infinitesimal" are used in comparison with the mean free path of radiation.

The problem of radiation between parallel plates separated by semitransparent material whose thickness is neither "infinite" nor "infinitesimal" is a difficult one, including many tedious numerical or graphical calculations (e.g. Hottel and Sarofin, 1967). Existence of the usual lattice conduction in solid gives farther complex aspects (Gardon, 1961). Analytical solution can be obtained only for the case when the optical thickness,

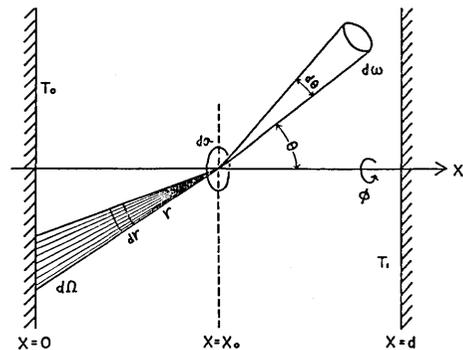


Fig. 7. Model for static measurement of thermal conductivity.

which is the thickness of the sheet multiplied by the extinction coefficient or divided by the mean free path of radiation, is relatively large. Only such a case will be discussed in the remainder of this section. Kellet (1952, 1953), who first reached the idea of radiative thermal conductivity, treated the problem along this line, though his paper is concerned with the unidirectional radiation only (Gardon 1956). It is easy to amend this point, however, as will be explained from now on, by the introduction of the two flux method.

The spectral intensity, at a distance x from the surface, of a beam of radiation travelling in a direction specified by polar angle θ with the positive x axis is denoted by $I_\nu(x, \theta)$ which is assumed to be independent of azimuthal angle ϕ (see Fig. 7). Then the energy radiated per unit time through the element of area $d\sigma$, which is perpendicular to x axis, in the direction of the cone $d\omega = \sin\theta d\theta d\phi$ is

$$I_\nu \cos\theta \cdot d\omega \cdot d\sigma.$$

Denote the radiant spectral flux across unit area of a plane perpendicular to x axis at a distance x from the left by $I_{\nu+}(x)$ and that from the right by $I_{\nu-}(x)$. Then,

$$\left. \begin{aligned} I_{\nu+}(x) &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_\nu(x, \theta) \cos\theta d\omega, \\ -I_{\nu-}(x) &= \int_{\phi=0}^{2\pi} \int_{\theta=\pi/2}^{\pi} I_\nu(x, \theta) \cos\theta d\omega. \end{aligned} \right\} \quad (10)$$

Spectral net flux of radiation through this unit area is then

$$H_\nu(x) = I_{\nu+}(x) - I_{\nu-}(x) = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\nu(x, \theta) \cos\theta d\omega. \quad (11)$$

General equation of radiative transfer in a medium with isotropic scattering is

$$\cos\theta \frac{dI_\nu}{dx} = -\epsilon_\nu I_\nu + \sigma_\nu \frac{qE_\nu}{4\pi} + n_\nu^2 \alpha_\nu \frac{B_\nu}{\pi}, \quad (12)$$

where E_ν is the volume density of radiation and q the light velocity in the solid, that is,

$$qE_\nu = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\nu d\omega. \quad (13)$$

The first term in the right hand side of eq. (12) means the loss of beam intensity due to absorption and scattering. The second and third terms express the gains of intensity by the re-emission due to scattering

and by the thermal emission respectively. Multiplication of eq. (12) by $d\omega$ and subsequent integration from $\phi=0$ to 2π and then from $\theta=0$ to $\pi/2$ or from $\theta=\pi/2$ to π yields respectively

$$\left. \begin{aligned} \frac{dI_{\nu+}}{dx} &= -\varepsilon_{\nu} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_{\nu} d\omega + \sigma_{\nu} \frac{qE_{\nu}}{2} + 2n_{\nu}^2 \alpha_{\nu} B_{\nu}, \\ -\frac{dI_{\nu-}}{dx} &= -\varepsilon_{\nu} \int_{\phi=0}^{2\pi} \int_{\theta=\pi/2}^{\pi} I_{\nu} d\omega + \sigma_{\nu} \frac{qE_{\nu}}{2} + 2n_{\nu}^2 \alpha_{\nu} B_{\nu}. \end{aligned} \right\} \quad (14)$$

We then expand I_{ν} by a Legendre polynomial

$$I_{\nu} = A + BP_1(\cos \theta) + CP_2(\cos \theta) + \dots \quad (15)$$

According to eqs. (11) and (13), A and B are represented respectively by qE_{ν} and H_{ν} as

$$\left. \begin{aligned} A &= \frac{1}{4\pi} qE_{\nu}, \\ B &= \frac{3}{4\pi} H_{\nu}. \end{aligned} \right\} \quad (16)$$

From now on the terms higher than the third in the expansion (15) will be ignored. This approximation may not be exactly valid near the boundary. We shall now consider the conical element on $d\sigma$ specified by the solid angle $d\Omega$ and cut by the boundary (Fig. 7). First we shall pay attention to the case of uniform temperature of the sheet. All volume elements of this conical element produce by their emission and scattering a certain amount of energy reaching $d\sigma$. The closer this conical element lies to x axis, the less amount of energy reaching $d\sigma$ is produced because of the smaller volume of the conical element. In other words, in a sheet of finite thickness, the intensity of radiation is higher in oblique directions when the direct radiation from the boundary is ignored. Obviously, the thinner sheets radiate relatively more strongly in oblique directions.

Now let us consider the case of such nonuniform temperature that the right side is hotter than the left in Fig. 7. Let a certain volume-element of the conical element in the colder half space, as in Fig. 7, be bounded by two cross-sections at distance equal to r and $r+dr$ respectively from the vertex. The closer the conical element lies to x axis, this volume emits the less amount of energy reaching $d\sigma$ because of the present temperature distribution. Then the oblateness of the intensity distribution is relatively more emphasized than in the case of uniform temperature for the radiation propagating from the left to the right. For the radiation in the direction of decreasing x , the effect due to the

finite thickness and the effect due to the nonuniform temperature distribution act in opposite direction and it gives rather complex intensity distribution of radiation. Until now the direct radiation from the boundary has been ignored, which is due to the emission and the reflection by the wall. The wall is assumed now to emit and reflect radiation uniformly in all directions. It cancels the effect due to the finite thickness in the case of uniform temperature.*

This effect, however, remains at least to some degree when it is accompanied by nonuniform temperature distribution and I_ν cannot be adequately described by the first two terms alone in eq. (15). The third or higher terms become relatively more significant in thinner sheet.

Then, our approximation is exactly valid only for relatively thick sheets and under such limitations it leads to the following equations for hemispherical fluxes, $I_{\nu+}(x)$ and $I_{\nu-}(x)$, as a result of eliminating qE_ν using eqs. (10)~(16).

$$\left. \begin{aligned} \frac{dI_{\nu+}}{dx} &= - \left\{ 2\alpha_\nu + \left(\frac{3}{4}\varepsilon_\nu - \alpha_\nu \right) \right\} I_{\nu+} + \left(\frac{3}{4}\varepsilon_\nu - \alpha_\nu \right) I_{\nu-} + 2\alpha_\nu n_\nu^2 B_\nu, \\ -\frac{dI_{\nu-}}{dx} &= - \left\{ 2\alpha_\nu + \left(\frac{3}{4}\varepsilon_\nu - \alpha_\nu \right) \right\} I_{\nu-} + \left(\frac{3}{4}\varepsilon_\nu - \alpha_\nu \right) I_{\nu+} + 2\alpha_\nu n_\nu^2 B_\nu. \end{aligned} \right\} \quad (17)$$

Readers might notice that eq. (17) is the modification of Kubelka's formula for diffuse radiation (Kubelka, 1948).

$$\left. \begin{aligned} \frac{dI_{\nu+}}{dx} &= -(2\alpha_\nu + 2\sigma_\nu) I_{\nu+} + 2\sigma_\nu I_{\nu-}, \\ -\frac{dI_{\nu-}}{dx} &= -(2\alpha_\nu + 2\sigma_\nu) I_{\nu-} + 2\sigma_\nu I_{\nu+}, \end{aligned} \right\} \quad (18)$$

Let us calculate the rate of heat flow Q through unit area of the sheet, which is the sum of the one due to the ordinary conduction and that due to the radiant transfer.

$$Q = -K \frac{dT}{dx} + \int H_\nu(x) d\nu. \quad (19)$$

The condition of steady state demands

$$\frac{dQ}{dx} = 0. \quad (20)**$$

* If the reflection takes place according to Snell's law, the direct radiation from the boundary does not cancel this effect even in the case of uniform temperature because the reflected ray in the oblique direction is relatively stronger than in the normal direction.

** In the case of practically linear distribution of temperature within a distance of mean free path of radiation, substitution of eq. (19) into (20) yields

To get an analytical solution, all the quantities should be independent of frequency of radiation ν ,

$$\alpha_\nu = \alpha, \quad \varepsilon_\nu = \varepsilon, \quad n_\nu = n, \tag{21}$$

and

$$I_+ = \int I_{\nu+} d\nu, \quad I_- = \int I_{\nu-} d\nu, \quad B = \int B_\nu d\nu = sT^4, \tag{22}$$

Under such simplifications the necessary equations to be solved are therefore

(i) rate of net heat flow

$$Q = -K \frac{dT}{dx} + I_+(x) - I_-(x), \tag{23}$$

(ii) steady state condition

$$\frac{dQ}{dx} = 0, \tag{20}$$

(iii) equations of radiative transfer

$$\left. \begin{aligned} \frac{dI_+}{dx} &= - \left\{ 2\alpha + \left(\frac{3}{4}\varepsilon - \alpha \right) \right\} I_+(x) + \left(\frac{3}{4}\varepsilon - \alpha \right) I_-(x) + 2n^2\alpha B(x), \\ -\frac{dI_-}{dx} &= - \left\{ 2\alpha + \left(\frac{3}{4}\varepsilon - \alpha \right) \right\} I_-(x) + \left(\frac{3}{4}\varepsilon - \alpha \right) I_+(x) + 2n^2\alpha B(x). \end{aligned} \right\} \tag{24}$$

Hemispherical flux I_+ at the boundary $x=0$ is attributed to the emission by the surface of which hemispherical emissivity is e and the

$$\frac{dH_\nu}{dx} = \frac{dI_{\nu+}}{dx} - \frac{dI_{\nu-}}{dx} = 0, \tag{i}$$

Substituting eq. (17) into eq. (i), then

$$I_{\nu+} + I_{\nu-} = 2n_\nu^2 B_\nu, \tag{ii}$$

It follows from eqs. (17) and (19) that

$$Q = -K \frac{dT}{dx} - \frac{2}{3} \int_{\varepsilon_\nu} \frac{1}{dx} (I_{\nu+} + I_{\nu-}) d\nu, \tag{iii}$$

and from eq. (ii) that

$$Q = - \left\{ K + \frac{4}{3} \int_{\varepsilon_\nu} \frac{1}{\partial T} (n_\nu^2 B_\nu) d\nu \right\} \frac{dT}{dx}, \tag{iv}$$

The second term in parenthesis in eq. (iv) represents the "radiative conductivity" and coincides with eq. (1). Use of the two flux method for derivation of eq. (1) and for discussion of scattering process had already been tried by Aronson et al. (1967b).

reflection of I_- reaching this surface. Thus the boundary conditions at $x=0$ are

$$\left. \begin{aligned} T(0) &= T_0, \\ I_+(0) &= en^2B(0) + (1-e)I_-(0), \end{aligned} \right\} \quad (25)$$

Similarly at $x=d$

$$\left. \begin{aligned} T(d) &= T_1, \\ I_-(d) &= en^2B(d) + (1-e)I_+(d). \end{aligned} \right\} \quad (26)$$

Equations (23), (20) and (24) correspond to Kellet's eqs. (1), (2) and (3) and the solution is easily obtained as follows after his way of calculation.

$$Q = - \frac{n^2B(T_0) - n^2B(T_1) + \frac{3}{4}\varepsilon K(T_0 - T_1) + \frac{2-e}{e} \frac{Km}{2} (T_0 - T_1) \frac{1+e^{-md}}{1-e^{-md}}}{\frac{3}{4}\varepsilon d + \frac{2-e}{e} \left(1 - \frac{3\alpha\varepsilon}{m^2}\right) + \frac{2-e}{e} \frac{3\alpha\varepsilon d}{2m} \frac{1+e^{-md}}{1-e^{-md}}}, \quad (27)$$

where

$$m^2 = \frac{4n^2\alpha\{B(T_0) - B(T_1)\}}{K(T_0 - T_1)} + 3\alpha\varepsilon. \quad (28)$$

If $K=0$, $e=1$ and $\varepsilon=\alpha$ (no scattering), eq. (27) yields

$$Q = - \frac{n^2B(T_0) - n^2B(T_1)}{\frac{3}{4}\varepsilon d + 1}, \quad (29)$$

This corresponds to eq. (30) in Genzel (1953).

Observed value of thermal conductivity K_{eff} is given by

$$Q = K_{\text{eff}} \frac{T_1 - T_0}{d}, \quad (30)$$

If the average temperature of the sheet is denoted by \bar{T} , $B(T_0) - B(T_1)$ is approximated by

$$B(T_0) - B(T_1) = 4s\bar{T}^3(T_0 - T_1), \quad (31)$$

Under this approximation, substitution of eq. (27) into (30) gives

$$\frac{K_{\text{eff}}}{K_{\infty}} = \frac{\frac{e}{2-e} + \frac{2}{\sqrt{3}} \varphi P \coth\left(\frac{\sqrt{3}\varphi D}{2P}\right)}{\frac{4}{3D}(1-P^2) + \frac{e}{2-e} + \frac{2}{\sqrt{3}} \varphi P \coth\left(\frac{\sqrt{3}\varphi D}{2P}\right)}, \quad (32)$$

where

$$\left. \begin{aligned} D &= \epsilon d, \\ \varphi^2 &= \alpha/\epsilon, \\ K_\infty &= K + K_r = K + \frac{16n^2s\bar{T}^3}{3\epsilon}, \\ P^2 &= K/K_\infty, \end{aligned} \right\} \quad (33)$$

In the case of $\varphi^2=0$ (no absorption) eq. (32) becomes

$$\frac{K_{\text{eff}}}{K_\infty} = \frac{4P^2}{3D} + \frac{e}{2-e} \quad (34)$$

Equation (34) is equivalent to eq. (5d) in Van der Held (1952). Notice that

$$\begin{aligned} K_{\text{eff}} &\rightarrow K \quad \text{for } d \rightarrow 0 \\ K_{\text{eff}} &\rightarrow K_\infty = K + K_r \quad \text{for } d \rightarrow \infty \end{aligned}$$

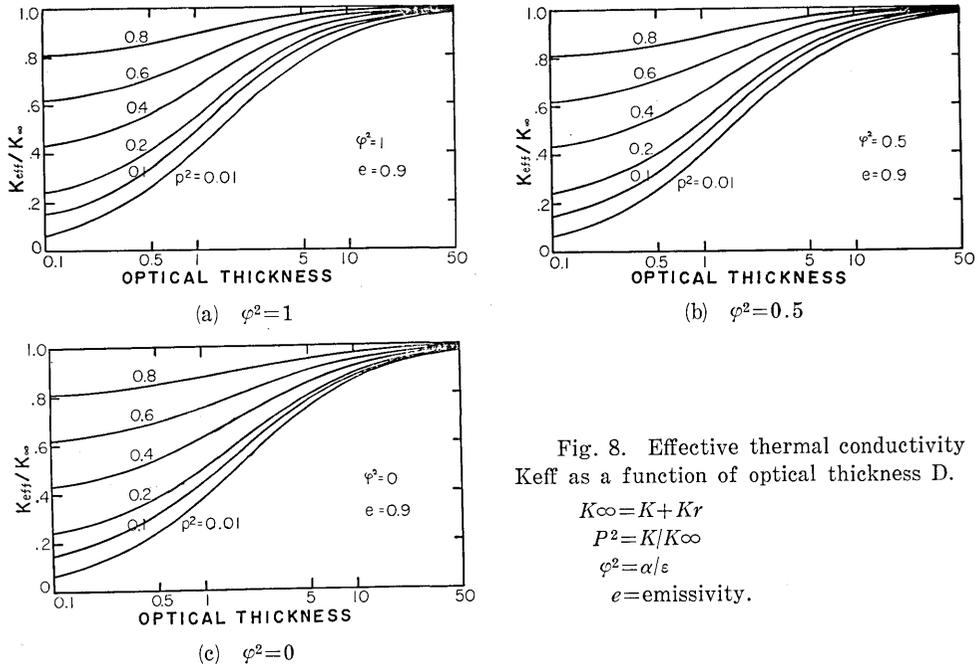


Fig. 8. Effective thermal conductivity K_{eff} as a function of optical thickness D .

$$\begin{aligned} K_\infty &= K + K_r \\ P^2 &= K/K_\infty \\ \varphi^2 &= \alpha/\epsilon \\ e &= \text{emissivity.} \end{aligned}$$

as have been stated at the beginning of this section. Of course, eq. (27) and then eq. (32) are good approximations only for a relatively thick sheet, say $D \geq 3$ or 4. More sophisticated calculations, however,

seem to be meaningless in practical use so long as we take such approximations as eqs. (21) and (31). Figures 8a, b and c illustrate $K_{\text{eff}}/K_{\infty}$ versus $D=\epsilon d$ for various $P^2=K/K_{\infty}$ when $\varphi^2=\alpha/\epsilon=1, 0.5$ and 0 respectively ($e=0.9$ for all cases). As can be seen in the figure, the value of $K_{\text{eff}}/K_{\infty}$ does not depend significantly on the value of φ^2 for given D and P^2 .

5. Discussions and Geophysical Implications

Recently Fujisawa (1968) estimated the temperature in the upper mantle by comparing the phase diagrams of the olivine-spinel transition in the $\text{Mg}_2\text{SiO}_4\text{-Fe}_2\text{SiO}_4$ system obtained by Akimoto and Fujisawa (1967) with the latest P-wave velocity distribution (Johnson, 1967; Kanamori, 1967). His fundamental assumptions based on various observations and experiments are:

1. The upper mantle is composed mainly of olivine and pyroxene, and the average content of olivine possibly amounts to about 80%.
2. The plausible values for the Fe/Mg ration of mantle minerals lie between 1:9 and 2:8, and from the petrological evidence the magnesium-rich side in this range seems more probable.

Under these assumptions, according to Fujisawa, the olivine-spinel transition accounts for the sharp discontinuity found by Johnson (1967) and Kanamori (1967) that starts at the depth of about 370 km in the mantle and whose thickness is of the order of several tens of kilometers. The temperature necessary for starting transition at the pressure corresponding to 370 km depth was found to lie in the range between 1150° and 1530°C, using the stability relation of olivin-spinel transition in 10% fayalite 90% forsterite. To interpret the observed thickness of the transition layer which ranges from 50 to 70 km, in terms of the width of the two-phase region in the olivine-spinel transition, the temperature at the termination of this region must be only 0° to 100°C higher than that at its beginning.

This estimation appears to be based on fewer and more sound assumptions than others so that the proposed temperature range and temperature gradient at 370 km depth may be considered as one of the important boundary conditions in estimating the temperature distribution in the upper mantle. Clark-Ringwood's geotherm (1964) or Ringwood's geotherm (1966) were derived from a constant mean opacity $\bar{\epsilon}$ and, therefore, from the assumption that the radiative conductivity K_r increases as T^3 . This is a wrong approximation as mentioned before. Now it seems to be possible to inquire into the more probable value of the thermal conductivity in the upper mantle. The author adopts Fujisawa's two assumptions. According to the first assumption, thermal

conductivity K in the upper mantle would be described as

$$\frac{1}{K} = \frac{0.8}{K_{ol}} + \frac{0.2}{K_{py}}, \quad (35)$$

in which K_{ol} and K_{py} are the thermal conductivities of olivine and pyroxene (mainly enstatite) respectively. According to the second assumption Fe/Mg ratio in those minerals is 1:9 or a little more. Olivines used in our optical experiments are of 12% fayalite and 88% forsterite and may be taken as the representative of the Fe/Mg ratio in the upper mantle. Figure 7 shows that K_{∞} of olivine is approximately constant and about 0.012 cal/cm. sec. deg. in the temperature range between 300°K and 1300°K if the crystal size is infinite. Is this assumption justified in the upper mantle? The peridotite nodules or olivine nodules have been thought to be the fragments of rocks in the upper mantle brought to the surface by the basaltic melts (e.g. Harris et al., 1967). It is probable that these nodules have been crushed during the upward movements to the surface. The size of the olivine crystal in these nodules lies in the range of from less than 1 mm to 1 or 2 cm. Therefore it does not seem unnatural to consider that olivine crystal has the size of at least about 1 cm in the upper mantle.

Figure 9 shows the size effect on the thermal conductivity of olivine at various temperatures. Calculations were made from eq. (32) in which the values of K and K_r were taken from Fig. 6 and extinction coefficient ϵ was assumed to be equal to $\bar{\epsilon}$ at each temperature. According to this figure, K_{eff} is more than 90% of K_{∞} at all temperatures if the

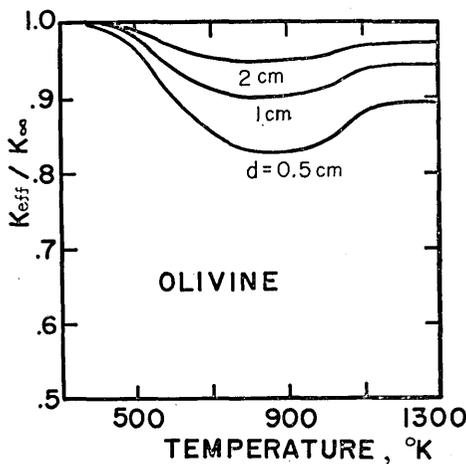


Fig. 9. Size effect on thermal conductivity of olivine.

thickness, d , of the plate of olivine is equal to 1 cm. Even if $d=0.5$ cm, K_{eff} is more than 85% of K_{∞} at most temperatures. Judging from these considerations, K_{ol} has a constant value of 0.011 or 0.012 cal/cm. sec. deg. up to 1300°K and very large increase of K_{ol} cannot be expected in the upper mantle.

Measurements of neither the thermal conductivity nor the absorption coefficient of enstatite crystal have been done because it is difficult to obtain a flawless sample with adequate size. Measurements of the thermal

conductivity of peridotite and basalt (Kawada, 1964), the thermal diffusivity of olivine and jadeite (Kanamori et al., 1968) and the absorption coefficient of olivine (18% fayalite) and diopside (12% hedenbergite) (Clark, 1957b; see also Fig. 3 in Clark and Ringwood (1964)) suggest that the lattice conductivity and the radiative conductivity of enstatite are both smaller than those of olivine. Putting the values ranging from 0.006 to 0.009 cal/cm. sec. deg. for K_{py} into eq. (35), the following conclusions are obtained: the thermal conductivity K is approximately constant and about 0.010 or 0.011 cal/cm. sec. deg. up to about 1000°C in the upper mantle and that the rapid increase of K as in the case of constant opacity cannot be expected.

These conclusions revive the classical difficulties in explaining the heat flow through the oceanic mantle. To avoid them, Clark and Ringwood (1964) have proposed the oceanic pyrolite model in which large upward concentrations of radiogenic heat sources were assumed. Even if their model were adopted, however, the oceanic geotherm, based on the surface heat flow value of $1.2 \sim 1.5 \times 10^{-6}$ cal/cm². sec. under the steady state condition with $K=0.010$ or 0.011 cal/cm. sec. deg. tends to exceed by far the temperature range and the temperature gradient at about 370 km depth estimated by Fujisawa (1968) (see Fig. 10). It suggests that the more effective mechanisms for heat transfer exist in the upper mantle. Excitonic energy transport proposed by Lubimova (1963) is of negligible importance for the thermal conductivity of mantle material because the energy of formation of excitons in mantle materials is too high to create a large number of excitons at the temperature prevailing in the earth's mantle (Shankland, 1968).

In Fig. 10 the curves of the beginning of melting of peridotite are shown for the cases of no water in host materials and of water pressure \doteq total pressure (Kushiro et al., 1968). The present geotherm intersects with the curves for solidus with $P_{H_2O} \doteq P_{total}$ at 100 km depth. If a small

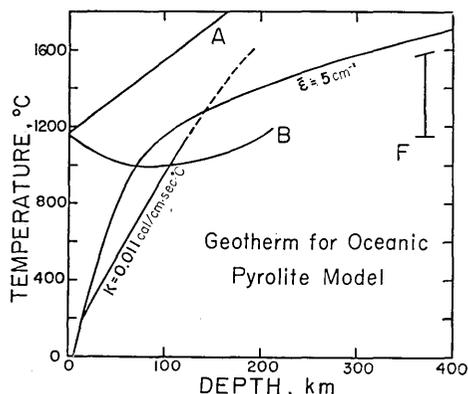


Fig. 10. Geotherm based on oceanic pyrolite model with the surface heat flow of 1.2×10^{-6} cal/cm². sec. under the steady state condition in cases of $\epsilon=5$ cm (Clark and Ringwood, 1964) and $K=0.011$ cal/cm. sec. deg. A: Solidus of peridotite for unhydrated conditions. B: Solidus of peridotite for $P_{H_2O} \doteq P_{total}$. F: Olivine-spinel transition in peridotite whose Fe/Mg ratio is 1:9.

volume fraction of liquid exists in the intergranular channels of coherent solid matter, upward current by liquid is capable of carrying heat many orders of magnitude larger than that due to ordinary heat conduction even with a melt fraction less than 10^{-3} (Frank, 1968). It is noteworthy that this mechanism inevitably accompanies a solid plastic flow essentially downward.

Tozer (1967) considered that there exists a low viscosity region in the upper mantle in which the decrease of viscosity due to the temperature rise overcomes the pressure effect, and that the convection was probably confined to this region which extended from the depth of about 50 km for the oceanic regions to the depth of about 600 km. The temperature distribution is nearly equal to an adiabatic distribution in the convecting layer. In any case it now appears fairly certain that mechanisms other than ordinary conduction and radiative conduction should be taken into consideration to account for the heat transfer in the oceanic upper mantle.

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24. マントル上部における輻射による熱輸送と熱伝導率

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不完全に光を吸収する無限固体中の輻射による熱輸送は普通の熱伝導と同じ拡散現象であり、従って輻射熱伝導率 K_r を定義することができる。 K_r は温度 T の 3 乗に比例し、光の吸収係数 α の波長に関するある種の平均 $\bar{\alpha}$ に反比例する。先に我々はオリビンの α を波長 0.3μ から約 10μ にわたって温度 1300°K までの範囲で測定した。 $\bar{\alpha}$ の温度依存性は非常に大きく、実験結果から計算された K_r は、よく仮定されるような簡単な T^3 則には従わない。熱拡散率の実測から推定された格子熱伝導率 K_p と今の K_r との和は 1300°K まで大体一定で、約 $0.012 \text{ cal/cm. sec. deg.}$ となる。この結果は熱拡散率の測定結果と比較的良く一致するが、殆どの温度範囲で後の方が低目となる。一般に半透明物質の熱伝導率の測定に際して、輻射波の自由行程はサンプルの大きさと同程度でありこのとき測定値はサンプルの大きさ、その他に依存する。その依存性を比較的簡単な場合について考察した。もしマントル上部がおもにカンラン岩からなり、その中の結晶の大きさが少くとも 1 cm 程度あるものとする、マントル上部における実質的な熱伝導率は約 1000°C までほぼ一定で 0.010 ないし $0.011 \text{ cal/cm. sec. deg.}$ と推定される。それ以上の温度においても α の大きな T 依存性からみて、 T^3 則のような急激な熱伝導率の増加は望めない。この結果は少なくとも海洋下のマントルにおいては格子熱伝導および輻射熱伝導以外の熱輸送形式一例えば熱対流が存在していることを示唆する。