

58. *Energy Density and the Development  
of the Source Region of the Matsushiro  
Earthquake Swarm.\**

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In this study an attempt was made to determine whether or not there is an equivalence between the Matsushiro earthquake swarm and a single earthquake more significant than merely the total energy released. From one viewpoint, that of the seismic energy density, a relation between the swarm and some properties of earthquakes developed by other seismologists was sought in order to determine if there is a physical basis for considering the swarm as an alternate mechanism for the release of strain energy that, under other geologic conditions, might have been released as one large earthquake, followed by a normal aftershock sequence.

The approach taken was to compare the Richter magnitude equivalent to the total energy release and the volume of the source region at various stages of the swarm with empirical relations between aftershock volume and magnitude developed for ordinary earthquakes by others. The relations used are:

For energy-magnitude, the Gutenberg-Richter<sup>1)</sup> relation

$$\log E = 1.5M + 11.8 \text{ (ergs)} \quad (1)$$

and for aftershock volume ( $V_a$ )—magnitude, the relation of Iida<sup>2)</sup>

$$\log V_a = 1.06M + 12.22 \text{ (cm}^3\text{)} \quad (2)$$

\* Communicated by T. Hagiwara.

1) B. GUTENBERG and C. F. RICHTER, "Magnitude and energy of earthquakes," *Annali di Geofisica*, **9** (1956), 1-15. It should be noted that the form of this equation given in Professor Richter's book *Elementary Seismology*, W.H. Freeman and Company (1958), Equation (10) on page 366 contains an error. The additive constant is given as 11.4, a value that does not follow from the preceding equations. Unfortunately, this error has been reproduced in many places.

2) K. IIDA, "Earthquake magnitude, earthquake fault, and source dimensions," *Journal of Earth Sciences, Nagoya University*, **13** (1965), 115-132.

and of Båth and Duda<sup>3)</sup>

$$\log V_a = 1.47M + 9.58 \text{ (cm}^3\text{)} \quad (3)$$

It was recognized from the beginning that the volume-magnitude relations had been based on data from strong events, magnitude 6 to 8.5 for the Iida equation and 5.3 to 8.7 for that of Båth and Duda. The extent to which agreement with these relations has been found for the equivalent values derived for the Matsushiro swarm is better than was expected at the outset.

Two kinds of equivalent magnitudes were considered: a magnitude equivalent to all the energy released during each individual month ( $M_i$ ) and a magnitude equivalent to all of the the accumulated energy released from November, 1965 through the month under consideration ( $M_\Sigma$ ). The energy values were obtained by the Seismometrical Section of Earthquake Research Institute, by applying equation (1) to individual earthquakes after measuring the magnitudes and summing the energies. The same relation could then be used to get the equivalent magnitude corresponding to the total energy.

The energy release as a function of time, one part of the input data,

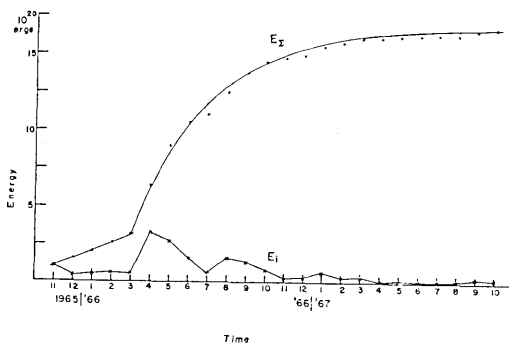


Fig. 1. Seismic energy release as a function of time.  $E_i$  is the energy released during a single month,  $E_\Sigma$  is the cumulative energy released from November 1, 1965, through the month for which the value is plotted.

is shown in Figure 1. Only the total energy released during each month,  $E_i$  and the cumulative energy released from November, 1966, through each month,  $E_\Sigma$ , are shown. More detailed data showing daily values of released energy and curves similar to those in Figure 1 are given in the various reports of the Seismometrical Section<sup>4)</sup>. Energy was released at an almost constant rate of  $5 \times 10^{19}$  ergs per month during the four

3) M. BÅTH and S. J. DUDA, "Earthquake volume, fault plane area, seismic energy, strain, deformation and related quantities," *Annali di Geofisica*, **17** (1964), 353-368.

4) The Party for Seismographic Observation of Matsushiro Earthquake and the Seismometrical Section, Earthquake Research Institute, "Matsushiro Earthquake Observed with a Temporary Seismographic Network. Part 1," *Bull. Earthq. Res. Inst.*, **44** (1966), 303-333, Part 2, *Bull. Earthq. Res. Inst.*, **44** (1966), 1689-1714, Part 3, *Bull. Earthq. Res. Inst.*, **45** (1967), 197-233, Part 4, *Bull. Earthq. Res. Inst.*, **45** (1967), 887-917.

months following November, 1965. The character of the energy release changed completely during April and May, 1966, the peak of the swarm activity. The data are fitted quite well by an exponential curve. The curve for  $E_{\Sigma}$  is represented approximately by

$$E_{\Sigma} = (0.50 + 0.51 t) \times 10^{20} \text{ erg} \quad 1 \leq t \leq 5$$

$$= [16.75 - 13.69 e^{-0.25(t-5)}] \times 10^{20} \quad t \geq 5$$

where  $t$  is measured in months, with  $t=1$  corresponding to November, 1965. The monthly release data,  $E_i$ , showing peaks in April, May and August, September, 1966, have been discussed in earlier papers by other investigators<sup>5)</sup>.

The volume occupied by the hypocenters was the other part of the input data. Initially, the volume defined by the distribution of hypocenters during each month, called the currently active volume,  $V_i$ , was determined. It was found, however, that the data took on a more organized and systematic aspect when the concept of the cumulative active volume,  $V_{\Sigma}$ , was introduced. This volume is defined as the total volume at the end of any month that had been active at some time during the swarm (since October, 1965) whether or not it still was during the month under consideration.

The volumes were derived from maps prepared by the Seismometrical Section and published as part of the reports on research on the Matsushiro swarm<sup>5)</sup>. These maps show the geographic position of the epicenters and the depths of the hypocenters of the stronger earthquakes in the swarm. It was assumed that the currently active volume for each month was defined adequately by the distribution of the larger earthquakes plotted on these maps, that is, the smaller events not plotted occurred within the limits of the distribution of the larger ones. The active region was outlined by straight line segments going through the outermost epicenters or hypocenters, divided into rectangular prisms one kilometer wide, oriented northwest-southeast, for which the length along the surface and the depth range were measured at the midplane. The orientation of the prisms was chosen to be perpendicular to the vertical plane onto which the hypocenters were projected to show the depth distribution. The currently active volume for each month,  $V_i$ , was calculated as the sum of the volumes of these prisms. It was further assumed that seismic activity was uniformly distributed through this volume during the month. This assumption appears to be sound

5) *loc. cit.*

during the early and middle part of the swarm, but not so good in the latter part, when the volume is spread out and the number of events reduced. During some months  $V_i$  consisted of separate sub-volumes, with inactive regions between.

A very detailed analysis of energy released within square cells 2 km

on a side is presented in a recent publication of Earthquake Research Institute<sup>6)</sup>. This analysis shows the specific regions from which the largest amounts of energy have been released and is a valuable aid to understanding the character of the swarm. For the purpose of the present study it was considered appropriate to average the energy through the entire active volume. The only disagreement between that study and this is that where the investigators conclude that all the energy released in any cell can be explained as originally stored in the cell, it is here concluded, as will be seen, that the energy released in the most active zone is too great to account for this way, and the growth of the source region with time is a necessary consequence.

The cumulative active volume,  $V_{\Sigma}$ , was found by superimposing the monthly maps and noting that region which became active during each month. In this process, no attempt was made to keep track of the volume which ceased to be active during the month in question. Not all of the

cumulative active volume is necessarily active at any one time.

Figures 2, 3, and 4 show the growth of the cumulative active region during the period under investigation. The source region for October, 1965 was taken as the starting volume, even though it had to be ex-

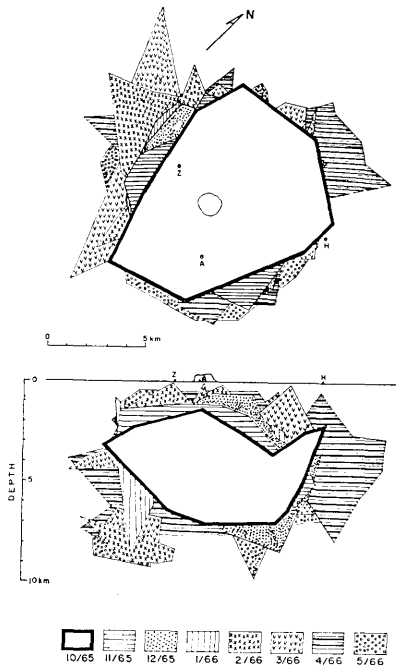


Fig. 2. Growth of the cumulative active region, October, 1965, through May, 1966. Top is the map of epicenter distribution, bottom the depth distribution projected onto a vertical plane oriented NE-SW. Stations Akashiba (A), Zozan (Z), and Hoshina (H) and the outlines of the volcano Minakami are shown for reference.

6) *loc. cit.*, Part 4, pp. 902-912.

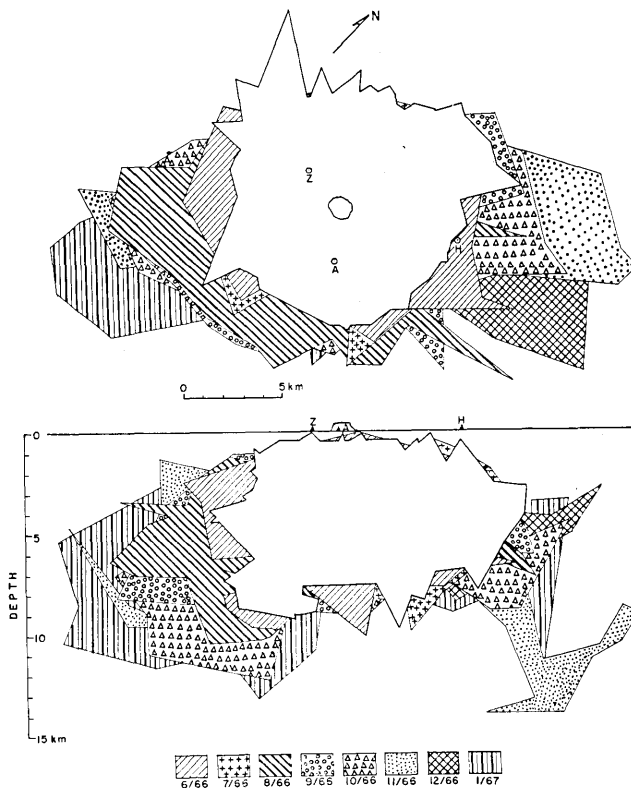


Fig. 3. Growth of the cumulative active region, June, 1966, through January, 1967.

cluded from the analysis because the data are incomplete for that month. The three periods of growth shown on the three figures correspond to three segments of the growth curve given below (Figures 5 and 7). A detailed examination of these maps reveals interesting features about the pattern of growth that cannot be easily reduced to numbers. For example, an extension of the active volume into new territory in the form of a narrow spike (usually corresponding to a single event) was often a forerunner of substantial growth later into that new region.

Figure 5 summarizes the time rate of growth of the two kinds of active volumes considered. It should be recalled that  $V_{\Sigma}$  is not the summation of  $V_i$ .  $V_i$  builds up in a somewhat erratic way to its maximum value in January and February, 1967. After that, even though the active region is spread out much more than during the initial part of the swarm, large portions near the center are no longer active, and

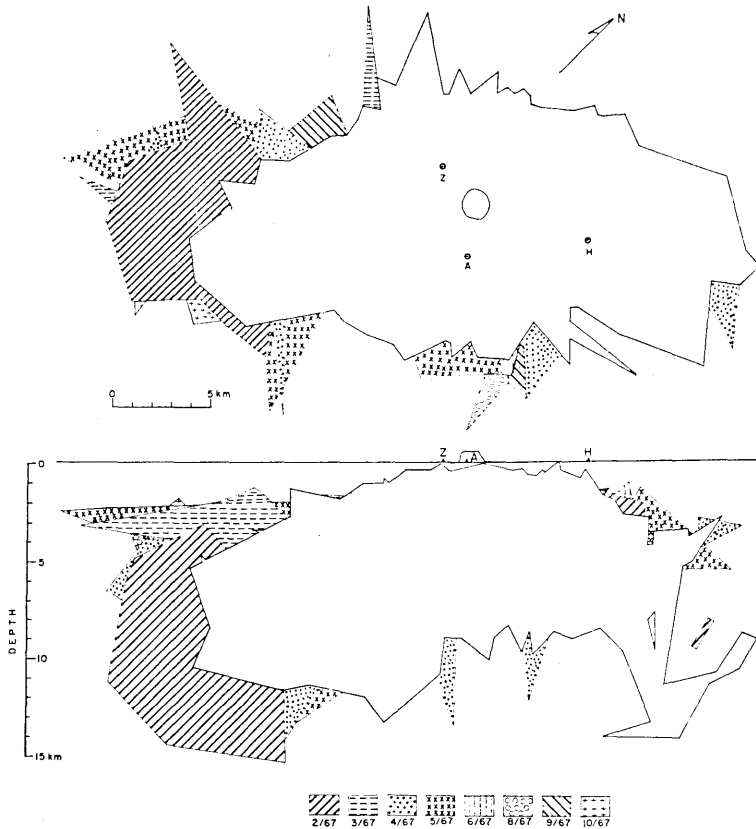


Fig. 4. Growth of the cumulative region, January, 1967, through October, 1967.

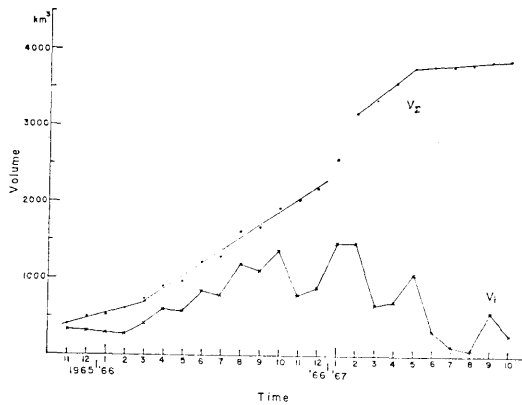


Fig. 5. Active volumes as a function of time.  $V_i$  is the currently active volume for each month,  $V_\Sigma$  is the cumulative volume through the month for which the value is plotted.

the currently active volume, made up of two or more separate active zones, is smaller.

$V_\Sigma$ , on the other hand, grows in a more regular way. For four months it increases at  $68 \text{ km}^3$  per month, then the rate increases to  $170 \text{ km}^3$ /month. A big jump is seen in January and February, 1967, during which  $V_i$  is not only big, but also much of it is in new material (see Figures 3 and 4). After this, the growth

at about 200 km<sup>3</sup>/month resumes. Since May, 1967, the volume has grown very little, levelling off at about 4000 km<sup>3</sup>.

If, instead of time, equivalent magnitude is used as the independent variable, direct comparison with the empirical volume-magnitude relations for ordinary large earthquakes, Equations 2 and 3, is possible. In Figure 6 the current active volume is plotted on a logarithmic scale as a function of monthly equivalent magnitude, with the Iida and Bâth-Duda curves plotted for comparison. The numbers are the months, with "60" deleted from the year (*e. g.* 11/5 denotes November, 1965). The data are within a factor of two of the Bâth-Duda value for most of the first 11 months shown (July and September, 1966, are considerably farther off than this), and the points for the five months of almost constant rate of energy release at the beginning of the sequence

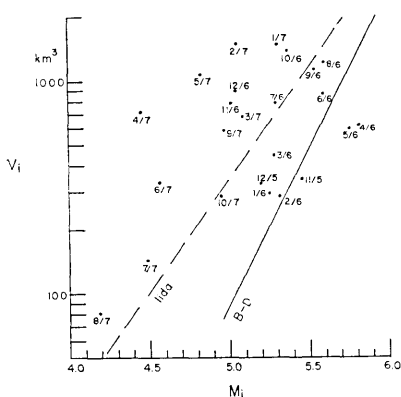


Fig. 6. Currently active volume as a function of the equivalent magnitude for the month. The curves are the graphs of Equations (2) and (3), the volume vs. magnitude relations of Iida and of Bâth and Duda.

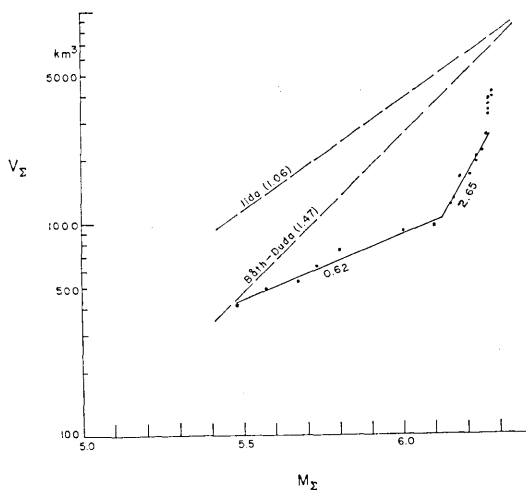


Fig. 7. Cumulative active volume as a function of the cumulative equivalent magnitude. Numerical values are the slopes of the straight lines relating  $\log V_{\Sigma}$  to  $M_{\Sigma}$ .

cluster closely about this line. This curve is, however, far below most of the data, that is, the volumes are too larger for the amount of energy released after the first year of the swarm, at least. Iida's curve is seen to fit the data even better in the sense of falling through them, but it, too, falls below for the latter part of the activity. Perhaps the most remarkable thing is that these curves fall anywhere near the

data points. The data points do not refer to individual earthquakes but are equivalent values that lump together many earthquakes, the magnitudes are at the low end of the values for which these curves were derived, and, of course, the curves refer to single large events.

Figure 7, in which  $\log V_{\Sigma}$  is plotted against  $M_{\Sigma}$ , shows why  $V_{\Sigma}$  was adopted as a useful concept. Whereas the  $V_i$  vs  $M_i$  data, Figure 6, were widely scattered, these points distribute along three linear trends. The numbers 0.62 and 2.65 are the coefficients of  $M$  in the assumed linear relation

$$\log V = CM + D.$$

The first point, for November, 1955, is almost exactly that predicted by the Bath-Duda relation. However, the growth rate of the cumulative active volume is too slow for the rate of increase of cumulative magnitude, until May, 1966. This segment corresponds to the first of the maps of cumulative active volume, Figure 2. Starting in June, 1966, well after the maximum monthly energy release occurred in April and May, the volume begins to increase at a more rapid rate that either of the empirical curves, until January, 1967, corresponding to Figure 3. After that, the volume increases even more sharply, with almost no increase in equivalent magnitude. The source volume has spread out, the central part of the region, in which all the activity was concentrated in the months before, becomes almost quiet, and the rate of energy release becomes very low. The last volume shown here, as of October, 1967, is sixty percent of the value predicted by the B-D curve, and

about fifty percent of the Iida value. The agreement with the empirical curve is quite remarkable. Data since October, 1967, are needed to complete the picture.

The data presented above can be recast in a form that is more closely related to the physics of the swarm mechanism. In Figure 8, the energy density for both the current and cumulative values is shown as a function of the time.

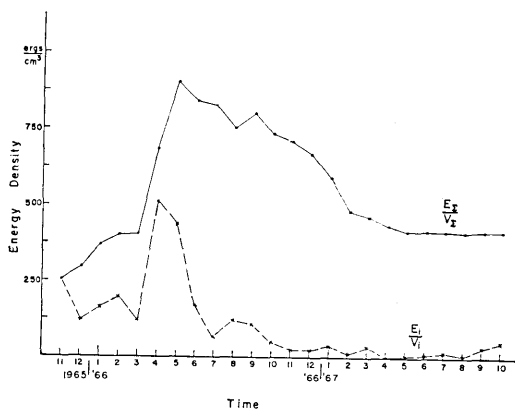


Fig. 8. Energy density as a function of time.



The peak value for a single month is 518 ergs per  $\text{cm}^3$  in April, 1966, the month in which the greatest energy release took place and before the growth of the active volume occurred. The peak of cumulative energy per unit cumulative volume, 910 ergs per  $\text{cm}^3$ , was reached in May 1966. It is also noteworthy that the values of  $E_{\Sigma}/V_{\Sigma}$  were leveling off at about 400 ergs  $\text{cm}^3$  in March before the big outbreak occurred, and then after another year the values again level off, at almost the same value, about 425 ergs per  $\text{cm}^3$ . As mentioned above, the recent results from the Earthquake Research Institute show much higher values in localized parts of the region.

If the Gutenberg-Richter energy-magnitude relation is combined with the Bath-Duda volume-magnitude relation, the energy per unit volume is almost independent of the magnitude,

$$\log \frac{E}{V_a} = 2.22 + 0.03M, \quad (4)$$

and for a magnitude 6 earthquake is about 250 ergs per  $\text{cm}^3$ . The value approached by the data in this swarm is close to this. The use of the Iida relation with the same energy-magnitude equation seems to require that the energy density be a rapidly varying function of the magnitude, in contradiction to the assumption made by most investigators that energy density is independent of magnitude,

$$\log \frac{E}{V_a} = 0.44M - 0.42. \quad (5)$$

The abandonment of this physically acceptable assumption can be avoided only by accepting some rather remarkable changes in either the efficiency of energy conversion or the ratio of the volume involved in the original earthquake to the aftershock volume with magnitude (see Appendix). None of these is an attractive choice. For a magnitude 6 earthquake, the Iida equation combined with the Gutenberg-Richter relation calls for an energy density of about 165 ergs per  $\text{cm}^3$ .

Thus, the observed cumulative energy density in the swarm approaches a value of the same order of magnitude as that derived from previous investigations of single earthquakes. The values predicted by application of the relations (1), (2), (3), as well as the final value of  $E_{\Sigma}/V_{\Sigma}$ , are definitely lower than the largest values of energy density

found for individual cells in the same region,  $5 \times 10^3$  ergs/cm<sup>3</sup> 7). Therefore, where the previous investigators conclude that all the energy that might have been originally stored as strain in the rocks of the source region has been released, these data indicate that more energy has been released than one would expect to find originally stored and that energy must have been replenished by some mechanism.

The value of seismic strain energy density released from a strained volume may be calculated from

$$\frac{E}{V_e} = \frac{1}{2} \mu \varepsilon^2 f, \quad (6)$$

where  $\mu$  is an elastic constant,  $\varepsilon$  the strain, and  $f$  the fraction of strain energy released that appears as radiated seismic waves (the efficiency of the source as a seismic wave generator). Here  $V_e$  is the volume from which energy is released in the earthquake, which has usually been taken as equal to  $V_a$ , the aftershock volume. None of the numbers on the right hand side of (6) are well determined for any natural seismic source, but some reasonable values can be assumed. The value of 425 ergs per cm<sup>3</sup> found for the final value of  $E_\Sigma/V_\Sigma$  corresponds to a combination of  $\mu = 5 \times 10^{11}$  dynes/cm<sup>2</sup>,  $\varepsilon = 5 \times 10^{-5}$ , with  $f = 0.68$ . The value of the peak energy density in any cell found previously,  $5.1 \times 10^3$  ergs/cm<sup>3</sup>, is accounted for by making  $f = 1$  and  $\varepsilon = 1.4 \times 10^{-4}$ . Clearly, there is no way of distinguishing these alternatives at the present state of knowledge. However, the agreement with independently derived empirical relations lends some weight to the lower values.

If magnitude is used as the independent variable rather than time, Figure 9, it is seen that the monthly values of energy density do vary systematically and strongly with the monthly equivalent magnitude. The line is a least squares fit, with slope 0.89. If the energy density of individual earthquakes is independent of their magnitude, this result indicates that more than one earthquake occurred in the same volume, or the volumes from which energy was extracted for the separate events overlap. This conclusion would lead to the further inference that energy within the active region was being replenished.

It is noteworthy that the data for November, 1965, through June, 1966, fall above the best-fitting line, along an almost parallel trend. There is a tendency for the monthly values of energy density to level

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7) *loc. cit.*

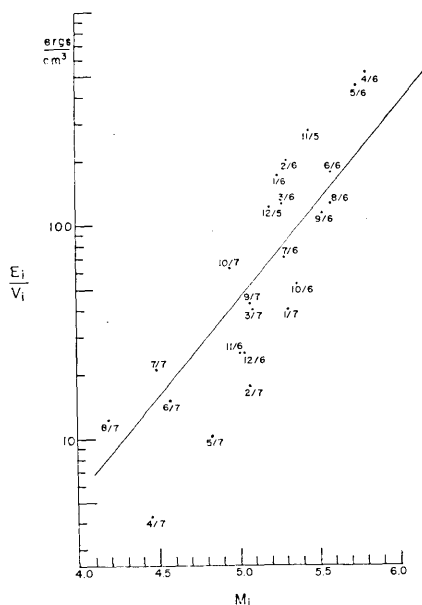


Fig. 9. Energy density for each month as a function of the equivalent magnitude for the month.

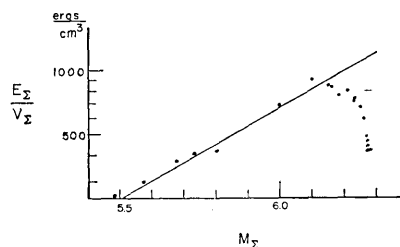


Fig. 10. Cumulative energy density through each month as a function of the cumulative equivalent magnitude through that month.

off at low values of 10-20 ergs/cm<sup>3</sup> toward the end of the period covered. In Figure 10, cumulative value of energy density are plotted against cumulative magnitude. Because  $M_{\Sigma}$  increases monotonically with time, this curve is equivalent to the curve in Figure 8, with the early part stretched out as the equivalent magnitude increased rapidly and the latter part condensed. The slope of the linear portion is 0.89. The sharp decrease in the cumulative energy density corresponds to the rapid growth of the source region after May, 1966, and especially after January, 1967. The same effect is seen in Figure 11, in which the rapid increase in cumulative energy density until May, 1966, is seen as a function of the cumulative active source volume, followed by the decline to a final value, of 425 ergs per cm<sup>3</sup>.

The following is offered as tenta-

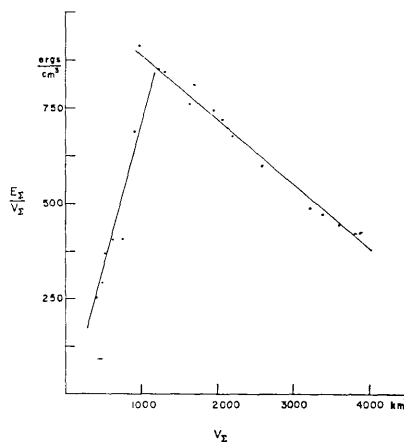


Fig. 11. Cumulative energy density as a function of cumulative active volume.

tive hypothesis for the growth of the swarm source region. Evidence assembled by other investigators indicates that the earthquakes are associated with a northwest-southeast trending fault<sup>8)</sup>. The activity is originally localized under the ancient volcano, Minakamiyama, probably because a zone of material weaker than that of the surrounding rock exists below it. From the beginning of the swarm until May, 1966, the activity was concentrated in a small volume around the point of origination. During this time the total energy released reached such a large value relative to the volume involved that it is concluded that not all this energy was originally stored in the active volume. Rather, the energy was being replenished by additional strain that occurred at a high rate. This is supported by the geodetic measurements of Kasahara, *et al.*<sup>9)</sup>, which show rapid extensional strain in the north-south direction from March through October, 1966. During this period the stronger surrounding rocks were strained, and eventually the activity spread out in the direction perpendicular to the fault. After January, 1967, most of the activity occurred away from the central region, and though the number of earthquakes diminished greatly, some of the strongest individual shocks of the swarm occurred, especially in the newly active southwestern sector. After October, 1966, at least through March, 1967, Kasahara's geodetic observations show that deformations associated with the main fault had almost ceased, and there is even a suggestion of some slight rebound. The rapid growth of the cumulative active volume, accompanied by small energy release, then represents a period of "catching up." The end result, as far as the present data take us, is that the total energy density approaches a value close to that expected for a single major earthquake with magnitude corresponding to the total energy release of the swarm.

A critical test of this model of the swarm development would be afforded by a detailed investigation of the focal mechanisms of the individual earthquakes based on directions of first motion. If this model is correct, the earthquakes in the regions active after May 19 should have occurred along *en echelon* faults parallel to the postulated main

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8) K. NAKAMURA and Y. TSUNEISHI, "Ground Cracks at Matsushiro Probably of Underlying Strike-slip Fault Origin, I—Preliminary Report," *Bull. Earthq. Res. Inst.*, **44** (1966), 1371-1384, "II—The Matsushiro Earthquake Fault," *Bull. Earthq. Res. Inst.*, **45** (1967), 417-471.

9) K. KASAHARA, A. OKADA, M. SHIBANO, K. SASAKI and S. MATSUMOTO, "Electro-optical Measurements of Horizontal Strains Accumulating in the Swarm Earthquake Area (3)," *Bull. Earthq. Res. Inst.*, **45** (1967), 225-239, and personal communication.

fault, with sense of motion appropriate to a drag effect of displacement along the original fault on the surrounding medium.

#### Acknowledgements.

The authors wishes to express his gratitude to the staff of the Earthquake Research Institute in making available to him their data, much of it in advance of publication, and especially to Dr. T. Hagiwara, Dr. K. Kasahara, and Mr. T. Iwata of the Seismometrical Section for their assistance.

#### Appendix. Discussion of Relations between Energy, Volume, and Magnitude.

Empirical relations like (1), (2), (3) are quite uncertain at this time. In fact, several other relations between energy and magnitude have been suggested by other investigators as replacements for (1). It is not even well established that a simple linear relation between  $\log E$  or  $\log V_a$  and magnitude is the best fit to the data. If it is assumed that the basic form of these relations is correct, simple considerations place some constraints on the relation between the various coefficients that may be value in future studies.

$$\text{Let} \quad \log E = A + BM \quad (\text{A-1})$$

$$\log V_a = D + CM \quad (\text{A-2})$$

$$E = \frac{1}{2} \mu \varepsilon^2 f V_e \quad (\text{A-3})$$

where the notation is the same as that used above.

$$\text{From A-1) and (A-2)} \quad \log \frac{E}{V_a} = (A - D) + (B - C)M \quad (\text{A-4})$$

$$\text{From (A-3)} \quad \log \frac{E}{V_e} = \log \left( \frac{1}{2} \mu \varepsilon^2 f \right) \quad (\text{A-5})$$

$$\text{So that} \quad \log \frac{V_e}{V_a} = (A - D) + (B - C)M - \log \left( \frac{1}{2} \mu \varepsilon^2 f \right) \quad (\text{A-6})$$

It is generally assumed, but not supported by evidence, that  $V_e =$

$V_e$  for all earthquakes. It is not known how the efficiency factor  $f$  varies with magnitude. Assume that  $1/2\mu\varepsilon^2$  is constant. Then, if  $B \neq C$ ,  $V_e/V_a$  and/or  $f$  must vary with magnitude. A possible hypothesis is that  $V_e/V_a=1$  for all magnitude. Then  $f$  is a function of  $M$

$$\log f = (A - D) + (B - C)M - \log \frac{1}{2}(\mu\varepsilon^2)$$

An alternate hypothesis is that  $f$  is independent of magnitude, that is, the efficiency of conversion of strain energy to seismic waves is the same for all earthquakes. Then  $V_e/V_a$  is a function of magnitude, that is, the volume occupied by the aftershocks is not the same as that from which energy in the original earthquake was drawn. In any case, it seems that  $B$  and  $C$  should be close to the same value, as strong variations in either  $f$  or  $V_e/V_a$  would seem to be unlikely.

Values for  $A$ ,  $B$ ,  $C$ ,  $D$  are given in the references (1), (2), (3) cited above:

	A	B	C	D
Gutenberg and Richter	11.8	1.5	—	—
Iida	—	—	1.06	12.22
Båth and Duda	12.24	1.44	1.47	9.58

The values of  $A$  and  $B$  given by other authors could be used, but these are adequate to illustrate the point.

Using Gutenberg's and Iida's values (A-6) becomes

$$\log \frac{V_e}{V_a} = -0.42 + 0.44M - \log \left( \frac{1}{2} \mu \varepsilon^2 f \right) \quad (\text{A-7})$$

For Gutenberg and Båth, (A-6) becomes

$$\log \frac{V_e}{V_a} = 2.22 + 0.03M - \log \left( \frac{1}{2} \mu \varepsilon^2 f \right) \quad (\text{A-8})$$

Using the values of Båth and Duda,

$$\log \frac{V_e}{V_a} = 2.66 - 0.03M - \log \left( \frac{1}{2} \mu \varepsilon^2 f \right) \quad (\text{A-9})$$

If we assume that  $V_e = V_a$ , with  $\mu = 5 \times 10^{11}$  and  $\varepsilon = 5 \times 10^{-5}$ , the three combinations yield, respectively,

$$\log f = 0.44M - 3.22$$

$$\log f = 0.03M - 0.58$$

$$\log f = -0.03M - 0.14$$

Thus, the Gutenberg-Iida combination requires that an earthquake of magnitude 7.3 be perfectly efficient as a wave generator, and stronger earthquakes are revealed as more than 100 percent efficient. The Gutenberg-Båth values are such that the largest earthquake, say  $M=9$ , has an efficiency of about 60 percent. And using the values of Båth and Duda, the efficiency for the largest earthquake is about 40 percent, and the efficiency increases as the magnitude decreases.

Some of the difficulty in using Iida's equation can be removed by a slight adjustment of  $\mu$  or  $\epsilon$ . For example, if  $\epsilon=10^4$ , then  $f=1$  for  $M=8.7$ , and earthquakes with  $f$  greater than 1 are no longer called for. However, the stronger dependence of  $f$  on  $M$ , already pointed out by Iida himself<sup>10)</sup>, is not an attractive hypothesis. Either of the other combinations seems acceptable, especially with regard to the slow variation of  $f$  with  $M$ .

An alternate hypothesis that has not been explored because of lack of data is that  $V_e$  and  $V_a$  are not necessarily the same. If  $f$  is arbitrarily fixed at some value, (A-6) can be used to determine  $V_e/V_a$  as a function of magnitude. For example, with  $f=0.5$ ,  $\mu=5 \times 10^{11}$  dynes/cm<sup>2</sup>,  $\epsilon=5 \times 10^{-5}$ , (A-7) yields

$$\log \frac{V_e}{V_a} = 0.44M - 2.91$$

$$\text{from (A-8)} \quad \log \frac{V_e}{V_a} = 0.03M - 0.27$$

$$\text{and from (A-9)} \quad \log \frac{V_e}{V_a} = -0.03M - 0.17$$

Thus, the Gutenberg-Iida combination predicts, for this value of efficiency, that  $V_e=V_a$  for an earthquake with magnitude 6.6, and  $V_e=0.2 V_a$  for  $M=5$ .  $V_e$  can be made equal to  $V_a$  for  $M=8$  by increasing  $\epsilon$  to  $10^{-4}$ . Present data are insufficient to enable a choice between  $10^{-4}$  and  $5 \times 10^{-5}$  for the strain released to be made. In considering this hypothesis, however, it seems reasonable to require that  $V_e \approx V_a$  for the largest earthquakes. However, the small value of  $V_e/V_a$  called for by using Iida's equation for small magnitude does not seem acceptable.

10) IIDA, *op. cit.*

The Gutenberg-Båth values on the other hand, yield  $V_e = V_a$  for  $M=9$ , and  $V_e = 0.75 V_a$  for  $M=5$ , a much more plausible result. The negative coefficient of  $M$  in (A-9) seems to rule out this combination under the present hypothesis.

The determination of the volume from which energy is released in the major event by means other than the distribution of aftershocks is a formidable task. These results indicate that the volume occupied by the aftershocks is greater than that involved in the main event for earthquakes of moderate magnitude.

If it turns out that in fact  $B=C$ , then these questions cannot be approached this way, as  $M$  drops out of (A-6). For this case, using the Gutenberg-Båth values and assuming  $V_e = V_a$  for all  $M$ , the value of  $f$  for the selected values of  $\mu$  and  $\epsilon$  is 0.26.

It is clear that much more observational research is required before the questions raised by this discussion can be resolved.

## 58. 松代群発地震のエネルギー 密度と震源域の拡大

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松代群発地震の各活動期間における放出エネルギーと活動体積(震源分布の占める体積)を調べた。これは群発地震と同じエネルギーを放出する単一の地震との間に有意な相違があるか否かを知ることを目的としたものである。

1965年11月～1967年10月の期間につき、各月の放出エネルギーに相当する1個の地震のマグニチュード( $M_i$ )と、1965年11月から各月に至る積算エネルギーに相当する1個の地震のマグニチュード( $M_\Sigma$ )を、グーテンベルグ-リヒターの式を使って求めた。一方、各月の地震体積( $V_i$ )とその月までに地震発生を経験した体積(積算活動体積)とを地震研究所が作った震源分布図から求めた。

積算活動体積は直線的に増す傾向を見せ、最初は  $68 \text{ km}^3/\text{month}$  のゆるい速度であつたが、 $170 \sim 200 \text{ km}^3/\text{month}$  に増し、1967年1月と2月に不連続的に増した。しかし、1967年5月以後は積算活動体積はほとんど一定となり、約  $4000 \text{ km}^3$  の値を保っている。

この活動体積と相当マグニチュードとの関係を、飯田や Båth-Duda が一般の地震の余震体積とマグニチュードにつき求めた式と比較してみた。各月の  $V_i$  と  $M_i$  の関係はかなり大きくばらついているが、前記2つの経験式のまわりに集つて見える。  $V_\Sigma$  と  $M_\Sigma$  の関係は、3つの直線に乗る。最初(1965年11月)は Båth-Duda の曲線に近い。しかし、1966年6月までは活動体積の成長が放出エネルギーに比べて遅すぎるので、図の点が曲線の下に離れる。1966年6月～1967年1月の期間になると、活動体積は放出エネルギーに比べて、急激に増大し、1967年1月以後の活動体積増加の割合はエネルギーに比べて著しく大きくなつたので、図の点は曲線に再び近づく。1967年10月における積算活動体積は Båth-Duda の式および飯田の式から予測される体積のそれぞれ60%および50%であつた。



次に、エネルギー密度を計算した。毎月のエネルギー密度は相当マグニチュードと共に大きく変化した。積算値から求めたエネルギー密度 ( $E_{\Sigma}/V_{\Sigma}$ ) は、1966年5月まで増加したが、活動体積が大きくなるに及んで減小した。その最終値は  $425 \text{ ergs/cm}^3$  となつた。この値はグーテンベルグ-リヒターのエネルギー-マグニチュードの式と Bath-Duda の地震体積-マグニチュードの式の組合せから予測される値にかなり良く一致する。

決論としては、次のことが言える。この群発地震の初期(1966年5月まで)に放出された全エネルギーは、最初から活動体積中に蓄積されていたと考えるには、余りに大きすぎる。エネルギーは最初の震源域に接した岩石中の歪から供給されたものである。最初の地震活動体積より強固であつた隣接区域の岩石は、次の期間に破壊を開始し活動体積が増大し最終値に達した。そして、最後に全放出エネルギーに相当するマグニチュードを持つ 1 個の地震の場合のエネルギー密度に近づいたのである。