BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 46 (1968), pp. 889-899

43. *A Long Wave around a Breakwater (Case of Perpendicular Incidence) [V].*

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(Read May 28, 1968.—Received May 30, 1968.)

Abstract

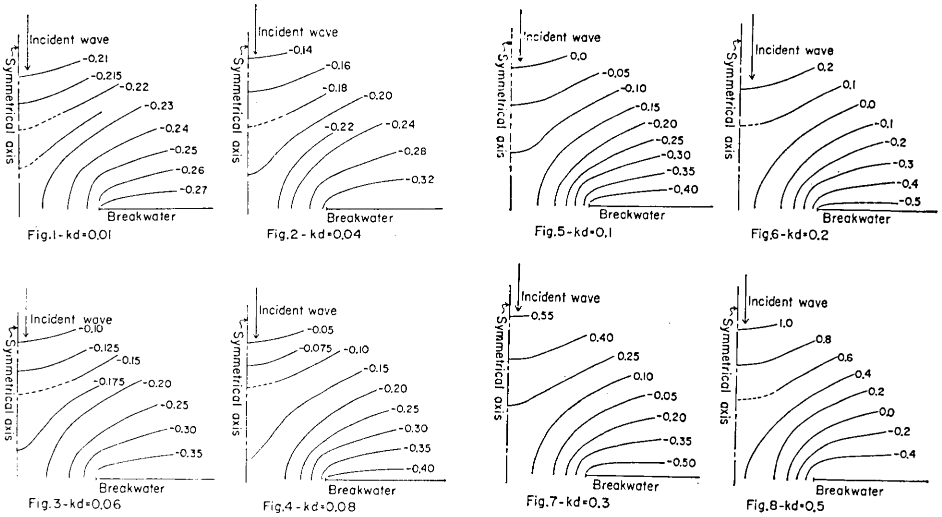
In the present paper, the difference of generation mechanism of the kinking crest lines of the RST (resultant) wave near the breakwater terminus and of the RD (reflected and diffracted) wave near the corner of the mouth of the estuary is discussed through the numerical computation of the RD wave around the breakwater, and further the comparison of our theory is made with the other authors' approximated theories.

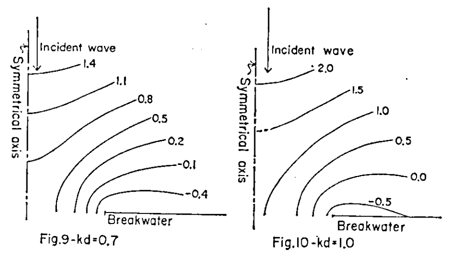
1. Introduction

Suceeding the previous works (*Momoi*, 1967a, 1967b, 1968a and 1968b), the long wave around the breakwater is discussed numerically for the case of normal incidence of the wave to the breakwater wing.

2. Note of Kinking Crest Line

In the third report (*Momoi*, 1968a) concerning the long wave around the breakwater, we have already noted the appearance of the kinking crest line of the RST (resultant) wave near the terminus of the breakwater wing. A similar kinking crest line appears near the corner of the mouth of an estuary for the RD (reflected and diffracted) wave, which is reported in the fourth paper (*Momoi*, 1968b) concerning the long wave in the vicinity of the estuary. The above two kinking crest lines might be considered as being very different in generation mechanism. In order to ascertain the difference, the numerical calculation is made for the RD wave in the windward waters of the breakwater in the range $kd=0.01$ to 1.0. The calculated results are presented in Figs. 1 to 10. The computation is made by use of the theory described in the second report (*Momoi*, 1967b). The numerals stated in the figures denote the values of $\arg \phi_{rd}$ (ϕ_{rd}: the RD wave). Inspection of these





Figs. 1–10. Phase variations of the RD wave in the windward waters, respectively, for $kd=0.01$, 0.04, 0.06, 0.08, 0.1, 0.2, 0.3, 0.5, 0.7 and 1.0.

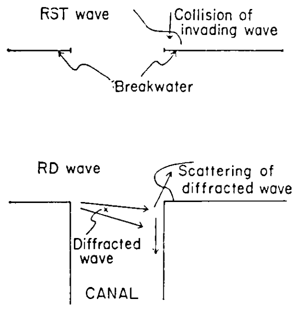
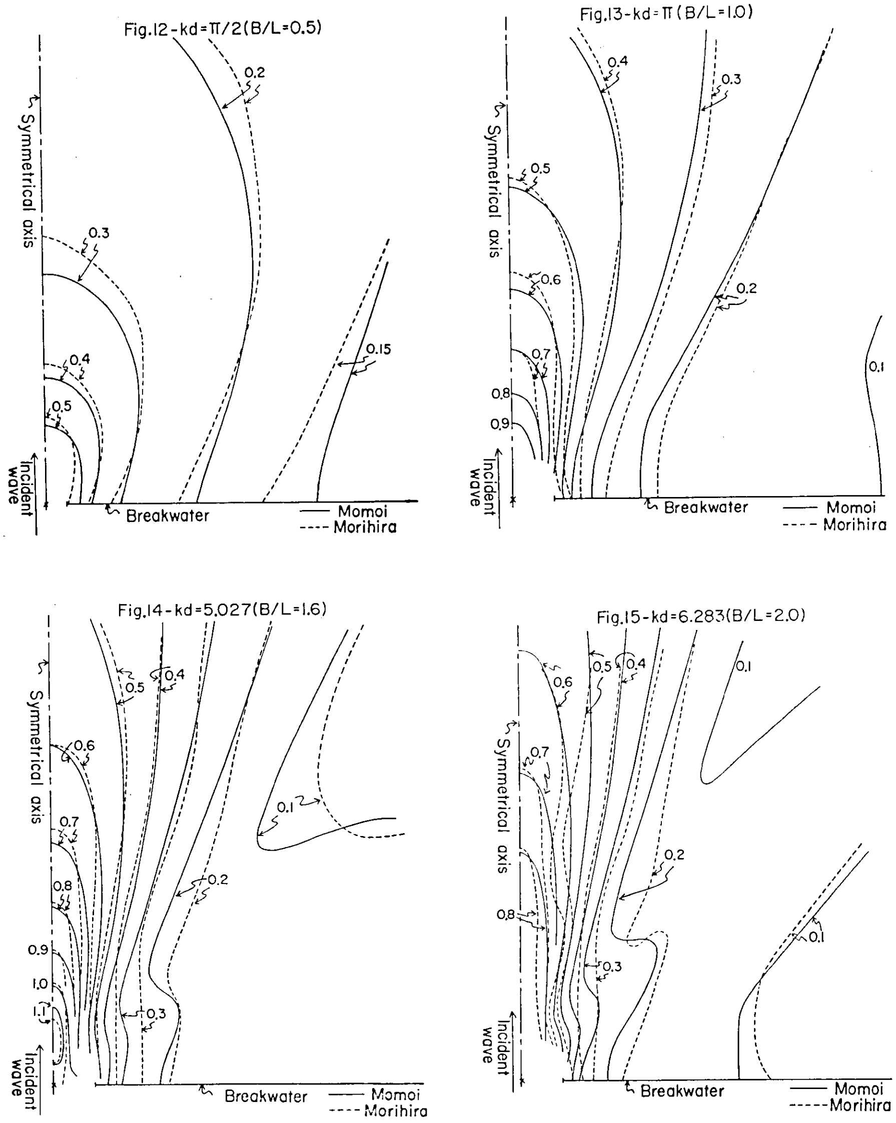


Fig. 11. Generation mechanism of the kinking crest lines of the RST wave around the breakwater and of the RD wave around the estuary.

figures reveals that no kinking crest lines appear, for the RD wave, in the waters near the terminus of the breakwater wing, while the kinking lines are definitely found, for the RD wave in the same range $kd=0.01$ to 1.2, in the nearby waters of the corner of the estuary (refer to Figs. 2b to 13b of the paper (*Momoi*, 1968b)). From the above results, we have arrived at the conclusion that the kinking crest lines of the RST wave near the terminus of the breakwater are produced definitely by the collision of the invading wave with the nearby wall of the terminus of the breakwater, while those of the RD wave near the corner of the estuary are caused by the scattering of the diffracted wave to the outer sea which advances from the corner toward the wall of the canal near the other corner to collide therewith (refer to Fig. 11).

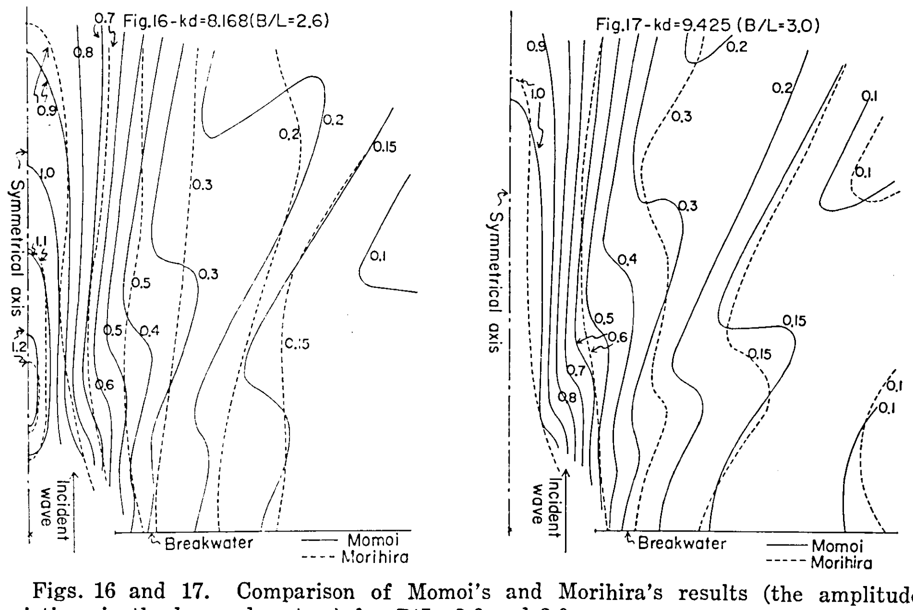


Figs. 12-15. Comparison of Momoi's and Morihira's results (the amplitude variations in the leeward waters) for $B/L=0.5$, 1.0, 1.6 and 2.0.

3. Comparison with Morihira-Okuyama's Result

In 1949, Blue and Johnson published a paper concerning the diffraction of water waves by breakwaters (*Blue* and *Johnson*, 1949), in which they developed approximated theory of the wave around two breakwater wings (the case of the normal incidence of the invading wave). Making use of the above theory, Morihira and Okuyama calculated the wave height* in the leeward waters of the breakwater (*Morihira* and *Okuyama*, 1966). Some of their figures are reproduced in Figs. 12 to 17 in order to compare with the results of the rigorous theory which is developed by the author in the previous papers (*Momoi*, 1967a and 1967b).

According to Blue-Johnson's work, the applicability of their approximated theory is more preferable in large ratio B/L (B: the breadth of the breakwater gap, L: the wave-length of the incident wave) than in small ratio. When B/L becomes small, the coupling effect of the two breakwater wings begins to be so strong that their theory cannot be applied. Figs. 12 to 17 show that Blue-Johnson's theory may be used down to the range $B/L=0.5$ at least to explain qualitatively the behaviors of the wave around the breakwater gap.



Figs. 16 and 17. Comparison of Momoi's and Morihira's results (the amplitude variations in the leeward waters) for $B/L=2.6$ and 3.0.

* The wave height is normalized by that of the incident wave.

4. Comparison with Lamb's Theory

When the width of the breakwater gap is very narrow as compared with the wave-length of the incident wave, Lamb established an approximated theory of the long wave around the breakwater with small aperture (*Lamb*, 1932). He then obtained the velocity potential

$$\left.\begin{aligned} &\phi=e^{+ikx}+e^{-ikx}+\chi \\ &\quad (k\text{: the wave number, } x\text{: the cartesian coordinate}) \\ &\text{in the windward waters and} \\ &\phi=-\chi \end{aligned}\right\} \qquad (1)$$

in the leeward waters, where

$$\chi=\frac{\frac{1}{2}\pi}{\log\frac{1}{4}kb+\gamma+\frac{1}{2}i\pi}D_0(kr)$$

($2b$: the breadth of the breakwater gap).

In the above expression, e^{+ikx} denotes the incident wave. In order to make Lamb's notation the same as the author's one, the form of the incident wave is changed to e^{-iky} (y: the cartesian coordinate). Expressions (1) become

$$\left.\begin{aligned} &\phi=e^{-iky}+e^{+iky}+\bar{\chi} \\ &\text{in the windward waters and} \\ &\phi=-\bar{\chi} \end{aligned}\right\} \qquad (2)$$

in the leeward waters, where $\bar{\chi}$ is conjugate function of χ.

Using expression (2), the numerical calculations were made for the amplitude and phase of the RST wave around the breakwater gap in the range $kd=0.1$ to 2.0, the results of which are presented in Figs. 18 to 33. In order to examine the applicability of Lamb's theory to the problem, the calculated results based on the author's theory (*Momoi*, 1967a and 1967b) are also arranged.** Though the variations based on Lamb's and the author's theories are shown in the same figure in the leeward waters, those in the windward waters are given in different

** The amplitude and phase are calculated by $|\phi|$ and $\arg\phi$, where ϕ is normalized by the amplitude of the incident wave.

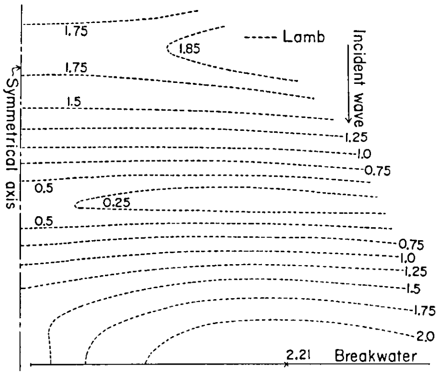


Fig.18l-kd=0.1 (Amplitude in the windward)

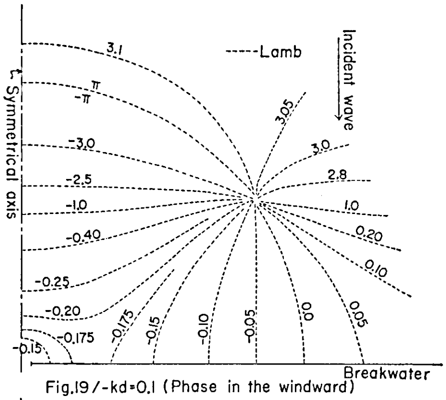


Fig.19l-kd=0.1 (Phase in the windward)

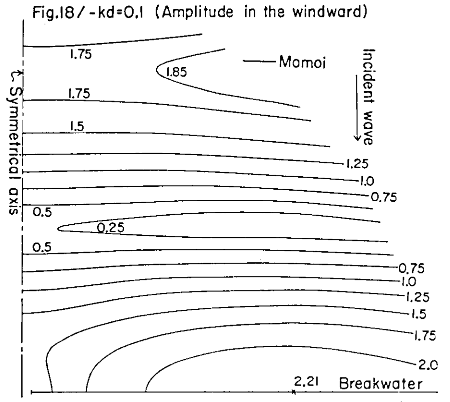


Fig.18m-kd=0.1 (Amplitude in the windward)

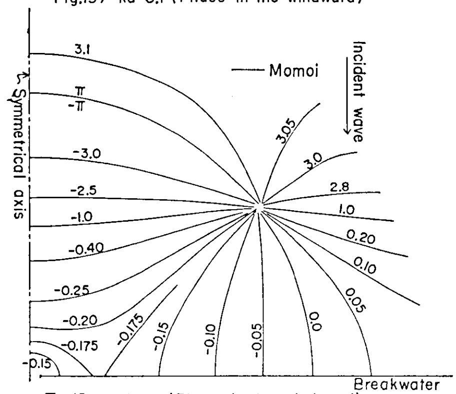


Fig.19m-kd=0.1 (Phase in the windward)

Figs. 18$l(m)$ and 19$l(m)$. Comparison of Lamb's and Momoi's results for kd=0.1 in the windward waters.

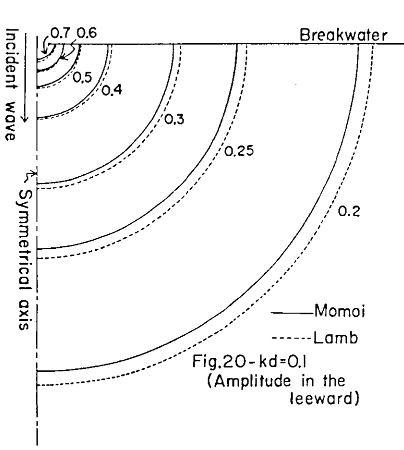


Fig.20-kd=0.1 (Amplitude in the leeward)

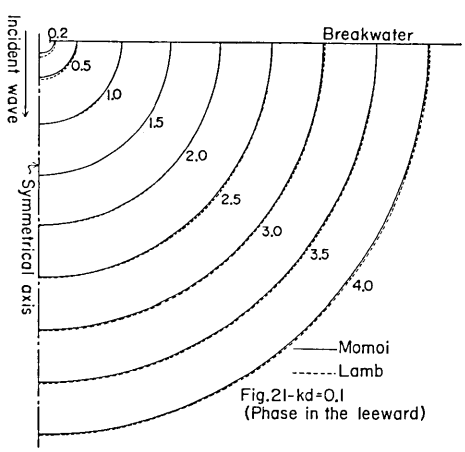


Fig.21-kd=0.1 (Phase in the leeward)

Figs. 20 and 21. Comparison of Lamb's and Momoi's results for kd=0.1 in the leeward waters.

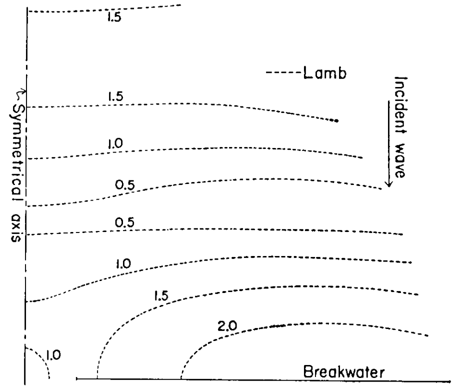


Fig.22*l*-kd=0.5 (Amplitude in the windward)

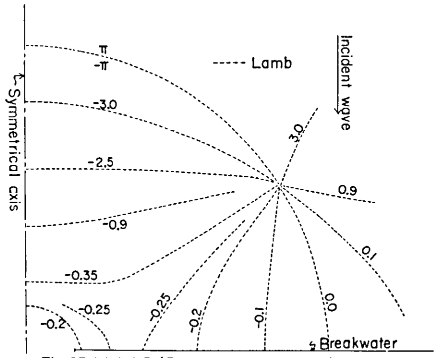


Fig.23*l*-kd=0.5 (Phase in the windward)

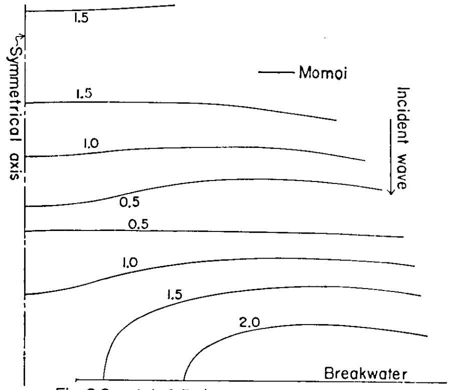


Fig.22m-kd=0.5 (Amplitude in the windward)

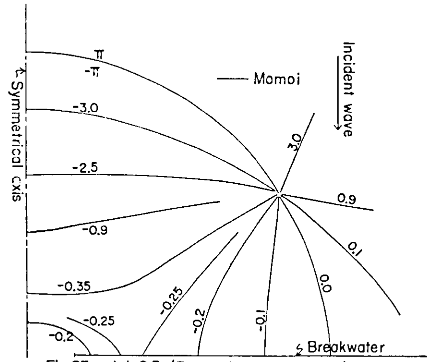
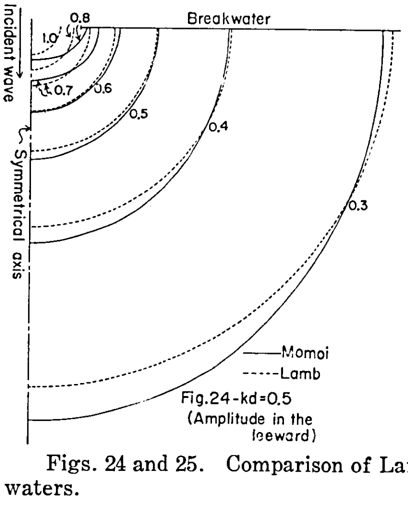
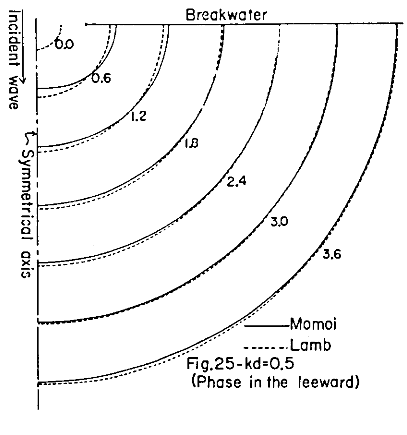


Fig.23m-kd=0.5 (Phase in the windward)

Figs. 22*l*(*m*) and 23*l*(*m*). Comparison of Lamb's and Momoi's results for $kd=0.5$ in the windward waters.





Figs. 24 and 25. Comparison of Lamb's and Momoi's results for $kd=0.5$ in the leeward waters.

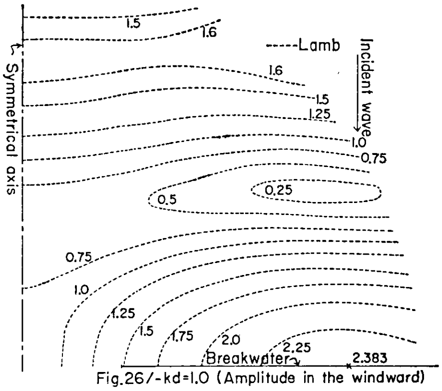


Fig.26l-kd=1.0 (Amplitude in the windward)

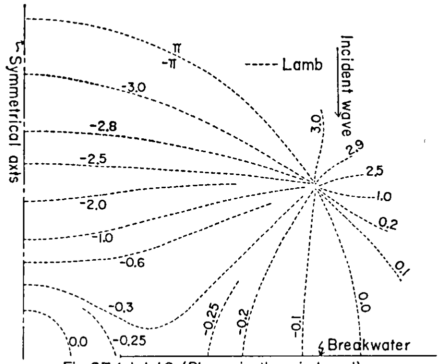


Fig.27l-kd=1.0 (Phase in the windward)

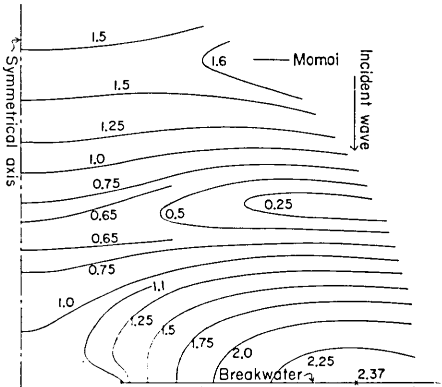


Fig.26m-kd=1.0 (Amplitude in the windward)

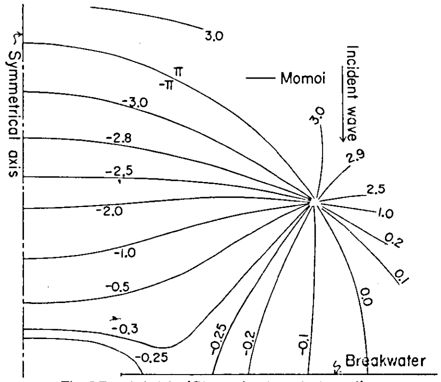


Fig.27m-kd=1.0 (Phase in the windward)

Figs. 26$l(m)$ and 27$l(m)$. Comparison of Lamb's and Momoi's results for $kd=1.0$ in the windward waters.

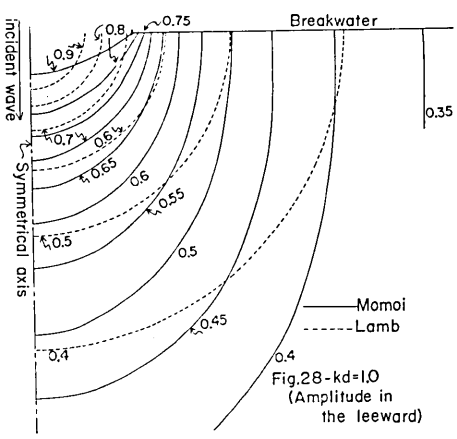


Fig.28-kd=1.0 (Amplitude in the leeward)

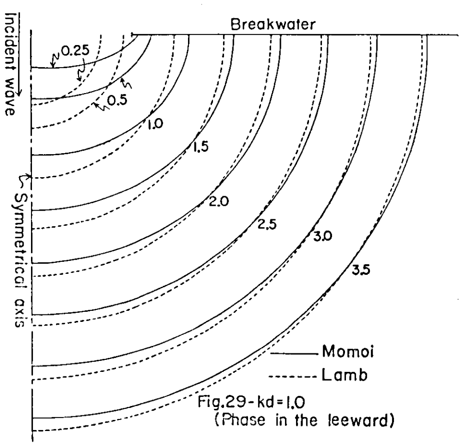


Fig.29-kd=1.0 (Phase in the leeward)

Figs. 28 and 29. Comparison of Lamb's and Momoi's results for $kd=1.0$ in the leeward waters.

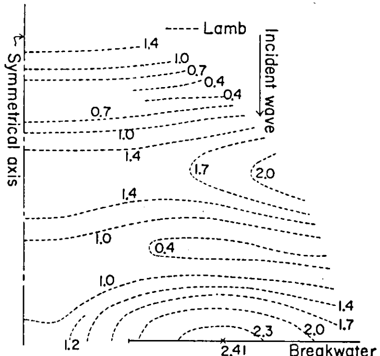


Fig.30l-kd=2.0
(Amplitude in the windward)

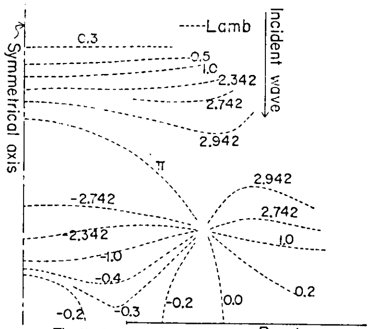


Fig.31l-kd=2.0
(Phase in the windward)

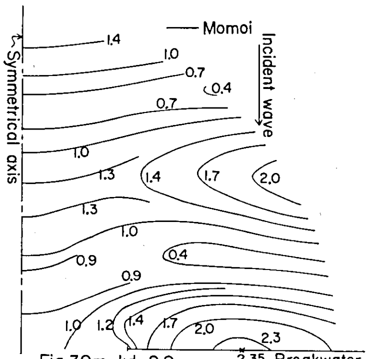


Fig.30m-kd=2.0
(Amplitude in the windward)

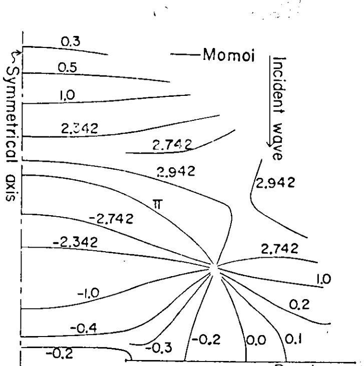


Fig.31m-kd=2.0
(Phase in the windward)

Figs. 30*l*(*m*) and 31*l*(*m*). Comparison of Lamb's and Momoi's results for $kd=2.0$ in the windward waters.

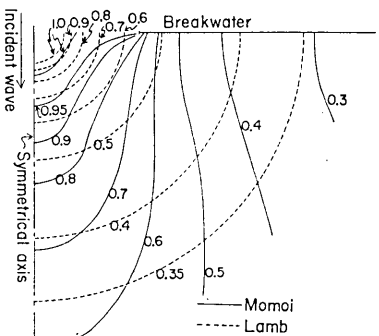


Fig.32-kd=2.0
(Amplitude in the leeward)

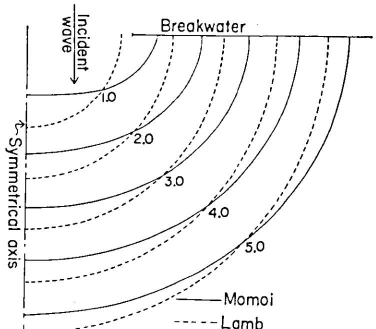
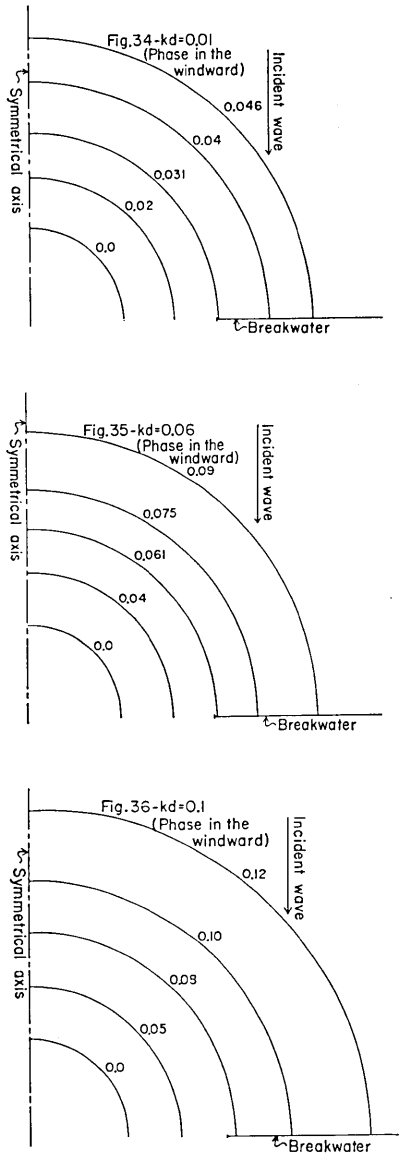


Fig.33-kd=2.0
(Phase in the leeward)

Figs. 32 and 33. Comparison of Lamb's and Momoi's results for $kd=2.0$ in the leeward waters.

figures. Inspection of these figures reveals that (i), in the windward waters, the variation of the amplitude and phase of Lamb's theory is in fairly good agreement with that of the author's theory up to $kd=2.0$ at least excepting the variation near the breakwater gap (the derivation of Lamb's theory (*Lamb*, 1932) denotes that his theory is inappropriate to explain the behavior of the wave near the breakwater gap) (refer to Figs. 18 l, 18 m, 19 l, 19 m, 22 l, 22 m, 23 l, 23 m, 26 l, 26 m, 27 l, 27 m, 30 l, 30 m, 31 l and 31 m), (ii) the phase variation of the two authors in the leeward waters is in good agreement up to $kd=$about 1.0, beyond which the difference in the two authors' curves becomes so great that use of Lamb's theory is inappropriate for discussing the phase variation (refer to Figs. 21, 25, 29 and 33), and (iii), as far as the amplitude variation in the leeward waters, the two authors' variation is in agreement with each other up to $kd=$about 0.5 (refer to Figs. 20 and 24), beyond which Lamb's theory cannot be used to explain the amplitude variation in the leeward waters (refer to Figs. 28 and 32).



Figs. 34-36. Phase variation of the RST wave near the breakwater gap in the windward waters (based on Lamb's theory).

In order to examine generation of the kinking phase line of the RST wave near the terminus of the breakwater, the numerical calculation is carried out for very small kd, i.e., 0.01, 0.06 and 0.1 by use of Lamb's theory. As already mentioned, Lamb's theory cannot be applied to the problem of the wave near the breakwater gap as the result of the approximation used in the derivation, but I have ventured to use his theory to inspect the behavior of the wave near the breakwater terminus. The results are shown in Figs. 34 to 36. According to these figures, no kinking crest line appears as would be expected.

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43. 防波堤のまわりにおける長波 (垂直入射の場合) [V]

地震研究所 桃 井 高 夫

本報告においては，防波堤端近傍における RST 波の峰線のゆがみの発生機構と河口の端点近傍における RD 波の峰線のゆがみの発生機構との相異が論じられている．また更に筆者の厳密解と他の近似解との比較がなされている．