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6. *A Long Wave around a Breakwater (Case of Perpendicular Incidence) [III].*

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Abstract

In the present work, the calculations are carried out for the waves of large wave-length in the windward waters around the gap of the breakwater in order to examine the nature of the trapezoid of the crest lines, which was ascertained for the case of the long wave around an estuary in the previous work. The result of the computation reveals that the feature of the trapezoid appears in front of the gap of the breakwater in a form a little different from the trapezoid for the case of the long wave around the estuary and that the waves advancing to the entrance in the nearby waters of the terminus of the breakwater are retarded as the result of the collision of the waves with the breakwater wing. In the last part of this study, a brief note on the generation of the high waves along the windward part of the breakwater is made referring to other authors' works, which show the appearance of the high waves near the terminus of the breakwater.

1. Introduction

Succeeding the previous two works[1),2)] with the same title, numerical calculations of long waves around the entrance of the breakwater are carried out with special attention focussed upon the generation of the trapezoid nature of the crest lines, the feature of which has already been discovered in the study with respect to long waves around an estuary[3)].

1) T. MOMOI, "A Long Wave around a Breakwater (Case of Perpendicular Incidence) [I]," *Bull. Earthq. Res. Inst.*, **45** (1967), 91.

2) T. MOMOI, "A Long Wave around a Breakwater (Case of Perpendicular Incidence) [II]," *Bull. Earthq. Res. Inst.*, **45** (1967), 749.

3) T. MOMOI, "A Long Wave in the Vicinity of an Estuary [III]," *Bull. Earthq. Res. Inst.*, **44** (1966), 1009.

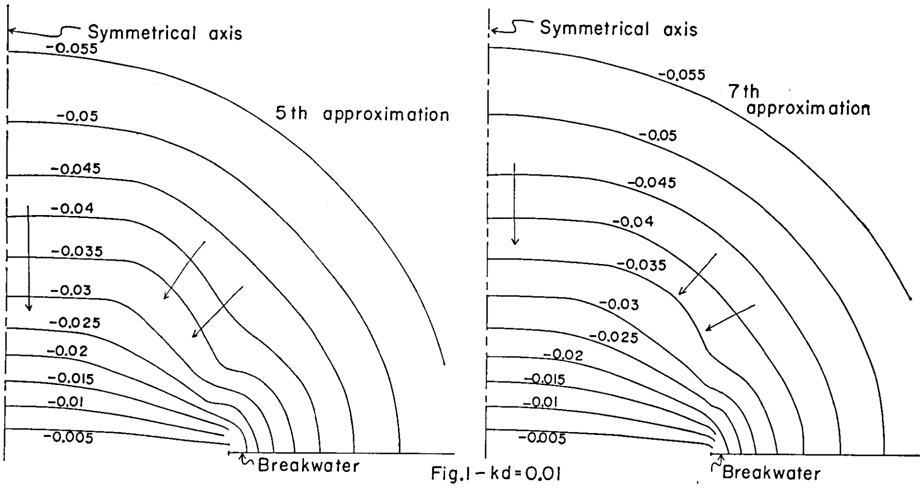

Fig. 1. Phase variation of the wave in front of the breakwater gap for $kd=0.01$.

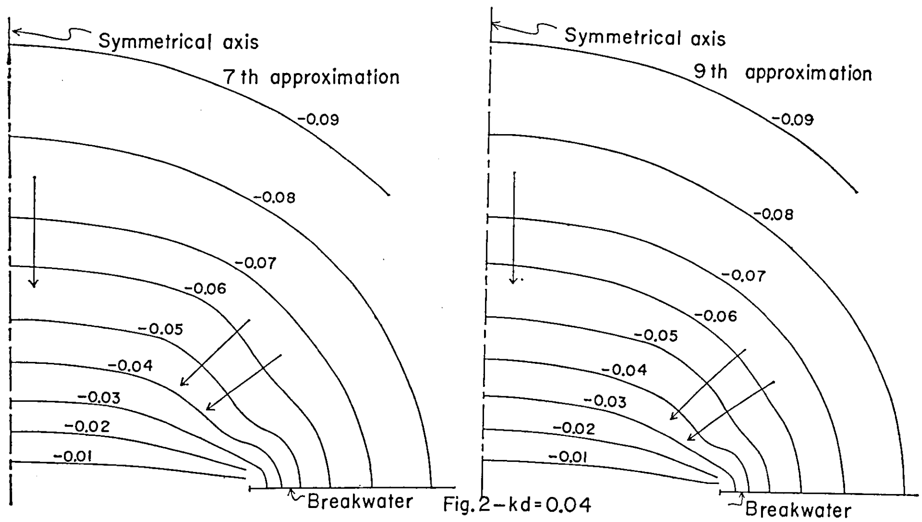

Fig. 2. Phase variation of the wave in front of the breakwater gap for $kd=0.04$.

2. Trapezoid Nature of Crest Lines

In the study[3] entitled "A Long Wave in the Vicinity of an Estuary [III]", it is found that the crest lines near the entrance of the estuary take a trapezoid form for waves with $kd<1.0$ (k is the wave number of the incident wave and d the half width of the canal). A similar

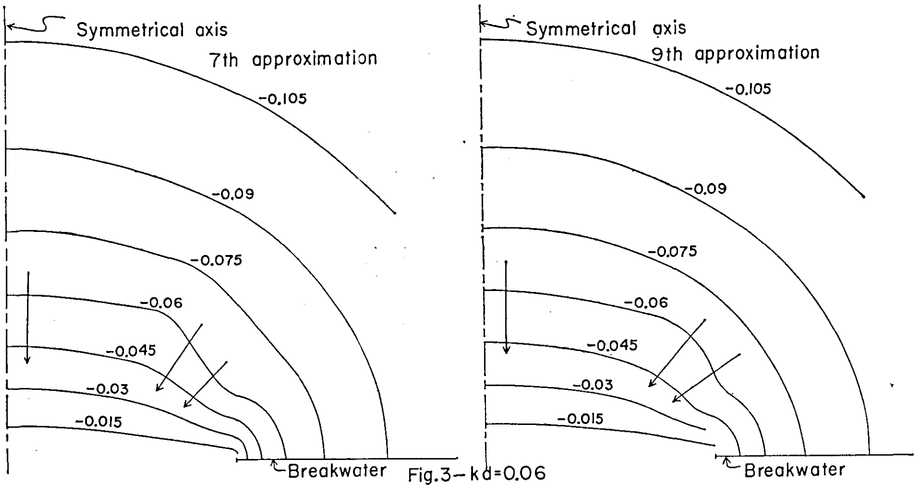

Fig. 3. Phase variation of the wave in front of the breakwater gap for $kd=0.06$.

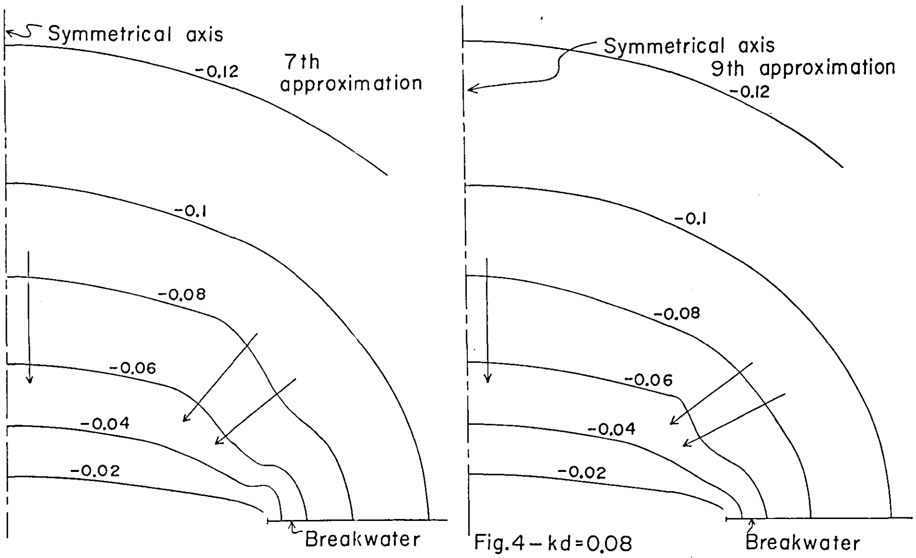

Fig. 4. Phase variation of the wave in front of the breakwater gap for $kd=0.08$.

phenomenon might be expected in the present case of a long wave around the gap of the breakwater for the waves of very long wave-length in the windward waters. With this idea in mind, numerical calculations of the waves in the range $kd=0.01$ to 1.0 (k is the wave number and d the half width of the gap of the breakwater) are carried out by use of

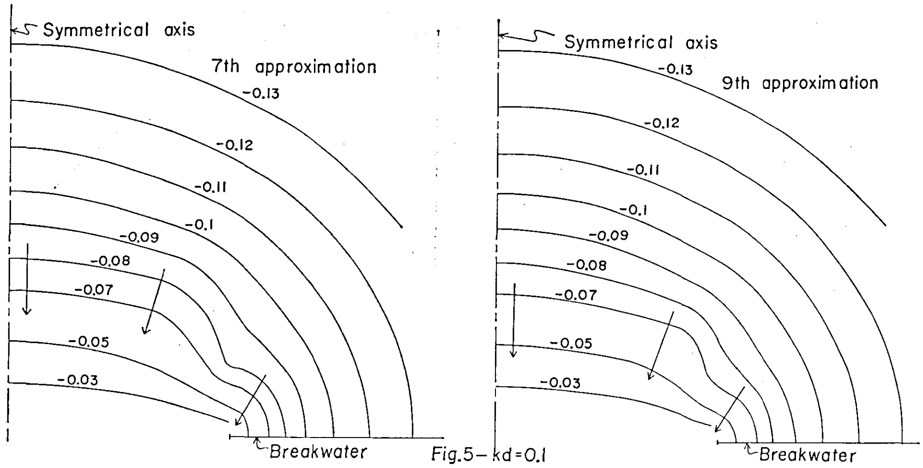

Fig. 5. Phase variation of the wave in front of the breakwater gap for $kd=0.1$.

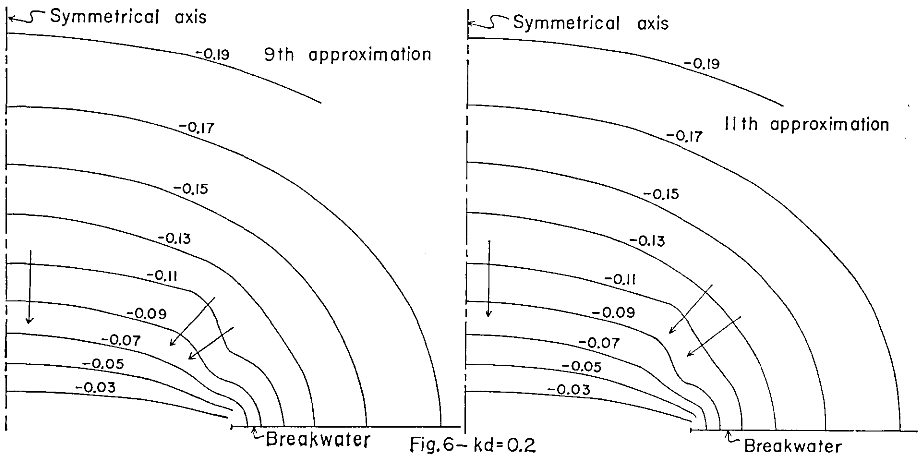

Fig. 6. Phase variation of the wave in front of the breakwater gap for $kd=0.2$.

the program constructed in the previous studies[1),2)]. These results are presented in Figs. 1 to 10 which show the variation of the phase of the wave in front of the gap of the breakwater*. In order to avoid the misunderstanding of the result due to the approximation, two figures with different approximations are arranged for each value of kd. According

*) The stated values in Figs. 1 to 10 denote those of $\arg \zeta$ (ζ : the wave height of the *RST*, i.e. resultant wave), where the unit is a radian.

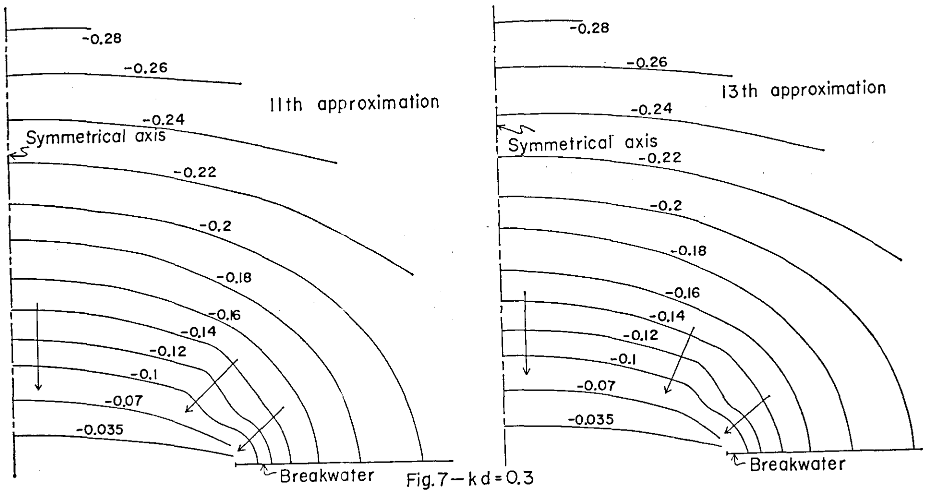

Fig. 7. Phase variation of the wave in front of the breakwater gap for $kd=0.3$.

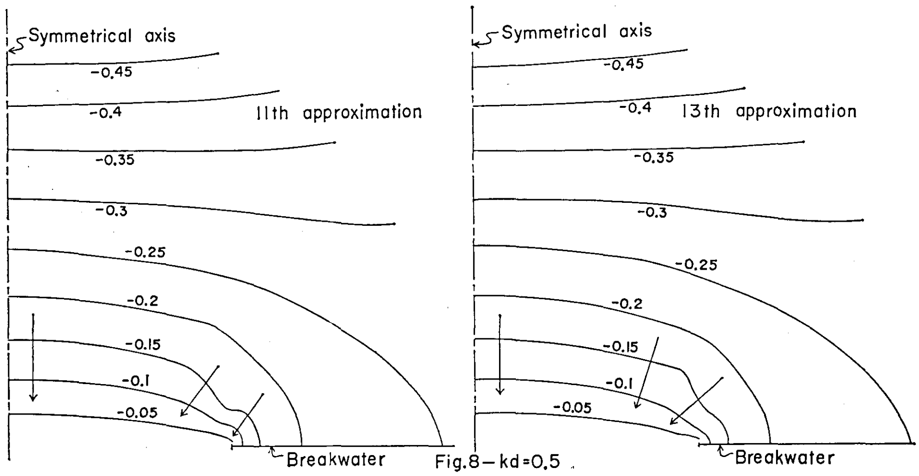

Fig. 8. Phase variation of the wave in front of the breakwater gap for $kd=0.5$.

to these figures, the following facts are exposed.

Passing through all the figures, it is found that the waves advance towards the entrance of the breakwater with the crest lines of trapezoid form (refer to Fig.11). When kd is small (the case of a very long wave, say $kd\approx0.01$) the crest line is relatively in a wedge form, while, as kd increases ($kd\approx0.1$ to 1.0), the above crest line degenerates to a flat

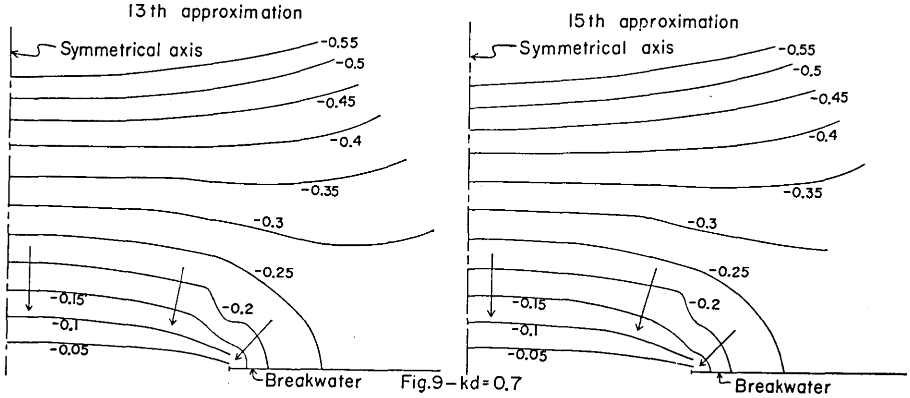

Fig. 9. Phase variation of the wave in front of the breakwater gap for $kd=0.7$.

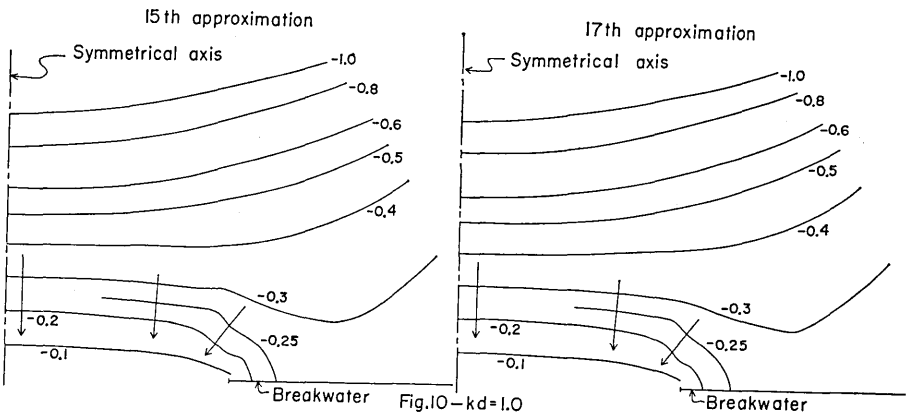

Fig. 10. Phase variation of the wave in front of the breakwater gap for $kd=1.0$.

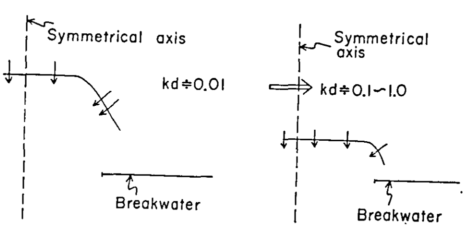

Fig. 11. Shape of crest lines (the arrows stand for the direction of the wave travel).

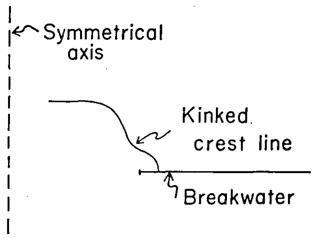

Fig. 12. Kink of the crest line near the terminus of the breakwater.

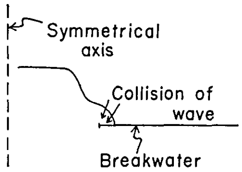

Fig. 13. Generation mechanism of kinked crest line.

trapezoid form (see Fig. 11). The above results suggest that a strong diffraction due to the breakwater wing takes place for the wave of very long wave-length (say $kd \approx 0.01$), while a relatively weak diffraction is found for the waves of relatively short wave-length (say $kd \approx 0.1$ to 1.0).

Through Figs. 1 to 10, another conspicuous feature is the appearance of a violent kink in the crest lines near the terminus of the breakwater wing, the form of which is illustrated in Fig. 12. A physical interpretation of this phenomenon is that the waves advancing towards the entrance near the terminus of the breakwater wing are retarded as the result of the collision of the waves with the nearby part of the terminus of the breakwater (see Fig. 13). In the previous study[3] on the long wave around the estuary, the above mentioned kink of the crest line is not definitely found. The reason might be ascribed to the defect of the approximation used. If the approximation is improved, a similar kink such as described for the present case is likely to appear near the corner of the estuary.

3. Maximum Wave Height along the Windward Part of the Breakwarer

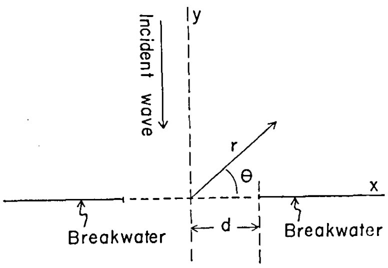

Fig. 14. Nomenclature of the model used by Mitsui.

In the papers entitled "A Long Wave around a Breakwater (Case of Perpendicular Incidence) [I], [II]"[1),2)] and "A Long Wave in the Vicinity of an Estuary [III]"[3)], we have already mentioned the generation of high waves (maximum wave heights) in the nearby part of the entrance of the breakwater (for the former paper) and in the adjacent coast of the estuary facing the open sea (for the latter paper).

Recently, I have received papers which seem to verify the existence of maximum wave height mentioned above. The first work[4)] is a paper entitled "Distribution of Wave Height in the nearby Region of the

4) H. MITSUI, *Collected Papers of the Thirteenth Coastal Engineering Conference* (1966) 80.

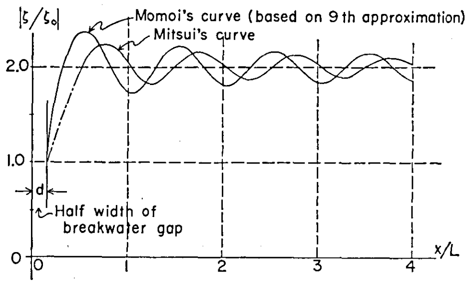

Fig. 15. Amplitude variation of the wave along the windward wall of the breakwater for $kd=1.0$, where L is the wave-length of the incident wave and $|\zeta/\zeta_0|$ the wave height divided by that of the incident wave.

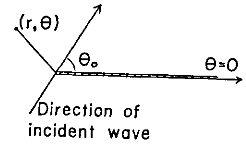

Fig. 16. Nomenclature of the semi-infinite breakwater wing.

Discontinuous Part of Coastal Structures" (in Japanese) by Mitsui. Since this paper is written in Japanese, the discussions are carried out recapitulating a part of the contents of the above paper in English.

The coordinates are taken as shown in Fig. 14. Under the nomenclature described in Fig. 14,

$$\phi = \exp(iky) + \exp(-iky) + \chi, \tag{1}$$

where ϕ : velocity potential;
$\exp(iky)$: incident wave;
$\exp(-iky)$: reflected wave;
χ : scattered wave;
k : wave number of the incident wave.

In the above equation, the approximate solution is given as follows by Lamb[5)]

$$\chi = \frac{1}{\log\frac{1}{4}kd+\gamma+\frac{1}{2}i\pi}\left(\frac{\pi}{2kr}\right)^{1/2}\exp\left\{-i\left(kr+\frac{1}{4}\pi\right)\right\}, \tag{2}$$

where γ is the Euler constant. By use of (1) and (2), Mitsui computed the wave height along the windward part of the breakwater wing for a specified parameter $kd=1.0$. The computed result is reproduced in Fig. 15, where the curve calculated by the author's method is also inserted (refer to the paper of reference number 1). In Mitsui's work, the curve in the neighborhood of the terminus of the breakwater wing is drawn by a broken line which tends to a certain value different

5) H. LAMB, *Hydrodynamics* (Cambridge, 1932), 6th edition, p. 532.

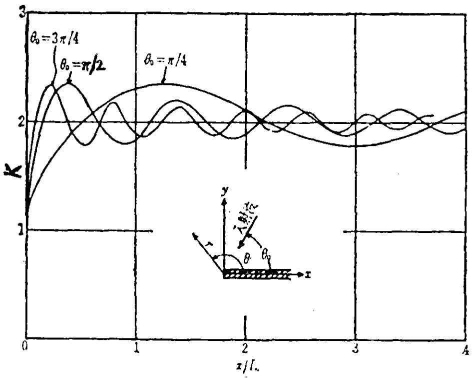

Fig. 17. Amplitude variation of the wave along the windward wall of the semi-infinite breakwater, where K stands for the relative wave height $|F(r, \theta)|_{\theta=0}$ and L the wave-length of the incident wave (the abscissa denotes x/L).

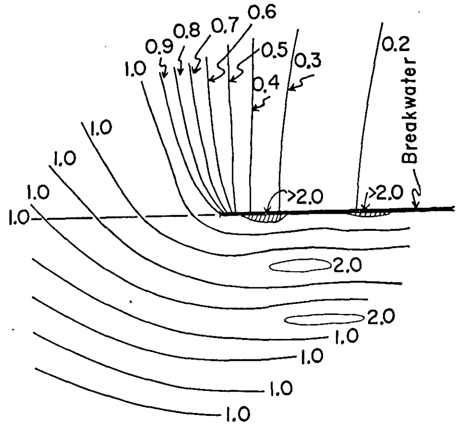

Fig. 18. Variation of the wave height ($|F(r, \theta)|$) near the terminus of the single breakwater (case of normal incidence of the wave), which is a simplified figure after Morihira and Okuyama's work.

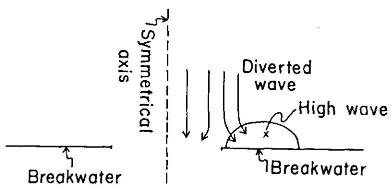

Fig. 19. Generation mechanism of the high wave near the terminus of the breakwater.

from 1.0 at the terminus. The above broken line tends to 1.0 at the terminus from the theory described in Lamb's Hydrodynamics[5]. This nature is also ascertained by the author's theory[1]. Therefore, the correction for the above broken line is given in Fig. 15 on the occasion of reproducing Mitsui's curve. Though a few differences between the two curves are found, both curves yield a maximum point of the wave height in the neighbouring point of the terminus of the breakwater wing.

A similar phenomenon is also exposed in diffraction of waves around a semi-infinite breakwater wing, the rigorous solution of which is given by Putnam and Arthur[6] using Sommerfeld's solution[7]. That is to say, the surface elevation is of the form (under the nomenclature in Fig. 16)

$$\eta=(Aikc/g)\exp(ikct)\cosh kh\cdot F(r, \theta) \tag{3}$$

6) J. A. PUTNAM and R. S. ARTHUR, "Diffraction of water waves by breakwaters," *Trans. Amer. Geophys. Union*, **29** (1948), 481.

7) G. WOLFSOHN, *Handbuch der Physik XX, Kapitel 7.*

where $c=\sqrt{(g/k)\tanh kh}$;
g: gravity constant;
k: wave number of the incident wave;
h: depth of water (uniform depth);
amplitude $=(Akc/g)\cdot\cosh kh$;

$$F(r,\theta)=(1/\sqrt{2})\exp[i\{\pi/4-kr\cos(\theta_0-\theta)\}]\cdot\int_{-\infty}^{u_1}e^{-i\pi v^2/2}dv$$
$$+(1/\sqrt{2})\exp[i\{\pi/4-kr\cos(\theta_0+\theta)\}]\cdot\int_{-\infty}^{u_2}e^{-i\pi v^2/2}dv \qquad (4)$$

with

$$u_1=-\sqrt{4kr/\pi}\sin\{(\theta_0-\theta)/2\}$$

and

$$u_2=-\sqrt{4kr/\pi}\sin\{(\theta_0+\theta)/2\}.$$

$|F(r,\theta)|$ then denotes the ratio of the wave height at the point (r,θ) to that of the incident wave. Using the expression (4), the relative wave height $|F(r,\theta)|$ along the single breakwater wing is computed by Mitsui[4], which shows the appearance of a maximum point of the wave height at a point a little apart from the terminus of the breakwater wing (his result is reproduced in Fig. 17). A similar calculation has also been made by Morihira and Okuyama[8]. This computation is carried out over the nearby region of the terminus of the breakwater, which is reproduced in Fig. 18. According to Fig. 18, regions of high waves exceeding twice the amplitude of the incident wave are found along the breakwater (the shaded regions along the breakwater). In the previous study[1] concerning a long wave around the breakwater with the gap, a generation of high waves near the entrance of the breakwater is inferred as the result of coupling of the waves reflected and diffracted at the two breakwater wings, which is described in Section *4* (*Complementary Remarks*) of the paper of reference number 1. Judging from the results computed by Mitsui or by Morihira and Okuyama, the above inference is considered as being wrong. Thus we have arrived at the conclusion that the high wave near the entrance is produced by the collision of the diverted wave with the breakwater (refer to Fig. 19), instead of as the result of coupling of the waves.

8) R. Morihira and I. Okuyama, "Computation Method of Diffraction of Sea Waves and the Diffraction Diagram", *Report of Port and Harbour Research Institute*, No. 21 (1966).

6. 防波堤のまわりにおける長波 (垂直入射の場合) [III]

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筆者は河口における長波に関する前報告において，すでに河口前面において波の峰線が梯形をなして水路に進入することを確めた．この現象が防波堤の前面においてもおこることを予想して，防波堤前面における長波の峰線を計算したのが本報告である．その結果によると，波の峰線の梯形的性質は河口前面同様防波堤前面においても現われる．そのほか知られた著しい事実は防波堤端近くの波の峰線に一つのゆがみがおこることである．これは防波堤開口部に向って進む波が防波堤の端にぶつかり停滞をおこした結果生ずるものと考えられる．

また本報告の後半においては，防波堤前面部に，最大波高が進入波の波高の2倍をこえる高波が現われることについて，他の著者の研究結果（その結果はいずれも前述の高波の発生を述べている）を参照しながら，簡単な註釈がおこなわれている．