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18. *On the Secular Variation of Magnitude-Frequency Relation of Earthquakes.**

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1. Introduction

It is well known that the following formula is used for expressing the number N of earthquakes in relation to their magnitude M:

$$\log N = a_1 + b(8 - M) \tag{1}$$

or

$$\log N = a_2 - bM \,, \tag{2}$$

where

$$a_2 = a_1 + 8b \,.$$

As a result of some of the recent studies[1], it is obvious that the value of coefficient b differs from area to area and this difference is related to the mechanical behaviour of the part where the earthquakes occur.

Supposing b has a value peculiar to a certain area, there would arise a question whether or not the value b varies with the length of time. Because, it is highly possible that the occurrence of earthquakes would somewhat change the previous conditions and consequently would influence succeeding earthquakes. For example, in the case of the earthquake-swarm near Matsushiro, northern Chubu, Japan, the value of b, or the coefficient m in the Ishimoto-Iida's relation, did not necessarily hold a definite value.[2] (The Ishimoto-Iida's formula is given by

* Communicated by Y. Satô.

1) S. MIYAMURA, *Proc. Jap. Acad.*, **38** (1962), 27.
S. MIYAMURA, *J. Seism. Soc. Jap.* (*Zisin*), [ii], **15** (1962), 23, (in Japanese).
K. MOGI, *Bull. Earthq. Res. Inst.*, **40** (1962), 125.

2) K. HAMADA and T. HAGIWARA, *Bull. Earthq. Res. Inst.*, **44** (1966), 1213, 1239 and 1665.
The Party for Seismographic Observation of Matsushiro Earthquakes and the Seismometrical Section, *Bull. Earthq. Res. Inst.*, **44** (1966), 309 and 1689.

$N(a)\delta a = ka^{-m}\delta a$,[3] and $N(a)\delta a$ is the frequency of the earthquakes of of which the maximum trace amplitude, a, lies between a and $a+\delta a$. k and m are the constants.)

Therefore, in the present study the writer examined whether there are any secular variations in the value of b in a given area around Japan. This paper is the first report of the study.

2. Data and the treatment

When the relationship between M and N is studied, using the data supplied by the Japan Meteorological Agency, we experience difficulty in obtaining values of M. Because the data, at least those before 1960, are lacking the magnitude of earthquakes of $M<6$. This has been also pointed out by several other authers (e.g. Tsuboi[4]).

If we were to obtain the mean value of b during a long period of time, one of the relevant methods would be to deal only with the earthquakes of which M is larger than 6, as was done by many investigators in the past. However, for the purpose of finding secular variations in the value of b, it is necessary to use a number of earthquakes of a relatively short period, say several years. Therefore, should the data be limited to earthquakes of $M \geqq 6.0$, the frequency of earthquakes would be too low to do a statistical treatment.

Under these circumstances, the writer decided to use, in place of M, the value $\log_{10}\Delta$, a common logarithm of the maximum felt distance of the earthquakes in question. The earthquakes treated here are limited to those occurring at depths shallower than 60 km.

It was confirmed experimentally by Ichikawa[5] that the following relation is established between M and $\log \Delta$:

$$M = \gamma \log \Delta - \delta\ , \qquad (3)$$

where γ and δ are constants.

Substituting (3) into (2)

$$\log N = \alpha - \beta \log \Delta\ , \qquad (4)$$

where

3) M. ISHIMOTO and K. IIDA, *Bull. Earthq. Res. Inst.*, **17** (1939), 443.
4) C. TSUBOI, *J. Phys. Earth*, **1** (1952), 47.
5) M. ICHIKAWA, *Quart. J. Seism.* (*Kensin-Zihô*), **25** (1960), 83, (in Japanese).

$$\alpha = a_2 + b\delta\ ,\quad \beta = b\gamma\ . \qquad (5)$$

According to Ichikawa, the values of γ and δ for the Pacific side of Northeast Japan are 2.7 and 1.0, respectively.

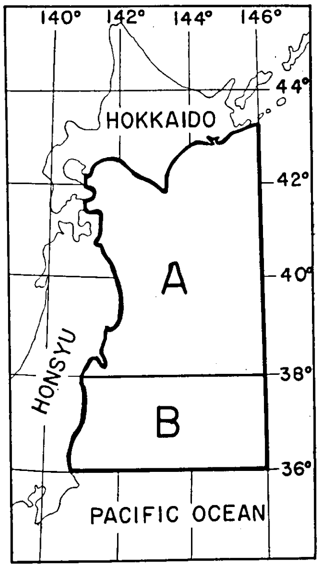

Fig. 1. The regions A and B.

The areas investigated for this study are Regions A, B and Region (A+B) indicated in Fig. 1. The data covers 41 years, from 1924 to 1964. The maximum felt distances of the earthquakes which occurred in these regions were sorted out from the Geophysical Review (Kishô-Yôran) for the years 1924–1950 and from the Seismological Bulletin of the Japan Meteorological Agency (Zisin-Geppô) for the years 1951–1964. Both journals failed to record the values of maximum felt distances for some of the years. To make up for the lack of information for those years, the writer measured on the map the maximum distances between the points where the seismic intensity was I according to the intensity scale of Japan Meteorological Agency. However, when the stations are within the abnormal seismic intensity zone they were excluded, and the maximum felt distances within the zone of normal seismic intensity were used. Then, among the maximum felt distances thus obtained, the values from 150 km to 540 km were picked out and were divided into the following four groups:

$\Delta = 200$ km . . . . . . 150 km–240 km ,

$\Delta = 300$ km . . . . . . 250 km–340 km ,

$\Delta = 400$ km . . . . . . 350 km–440 km ,

$\Delta = 500$ km . . . . . . 450 km–540 km .

Table 1 shows the numbers of earthquakes corresponding to the above groups in Regions A, B and (A+B) during the years from 1924 to 1964. The numbers are cumulative figures obtained by consecutively adding the number of earthquakes in the order of larger Δ to smaller Δ.

Using these figures, the writer tried to find out whether any secular variation occurs in the value of β. In the first place, with regard to

Table 1. The cumulative numbers of earthquakes during the years from 1924 to 1964 in Regions A, B and (A+B).

Year \ Δ(km)	Region A				Region B				Region (A+B)			
	200	300	400	500	200	300	400	500	200	300	400	500
1924	20	4	2	1	31	5	1	1	51	9	3	2
25	25	6	3	3	24	6	3	1	49	12	6	4
26	35	11	8	5	22	5	0	0	57	16	8	5
27	30	12	3	1	31	4	2	1	61	16	5	2
28	50	12	5	2	16	4	1	0	66	16	6	2
29	27	6	1	1	21	8	0	0	48	14	1	1
30	19	5	2	2	24	2	1	1	43	7	3	3
31	32	9	3	1	23	7	1	0	55	16	4	1
32	29	15	4	2	7	0	0	0	36	15	4	2
33	85	49	29	17	20	1	0	0	105	50	29	17
34	19	6	4	2	19	5	2	2	38	11	6	4
35	30	15	4	1	16	7	2	0	46	22	6	1
36	21	7	2	1	11	1	0	0	32	8	2	1
37	18	7	4	1	10	3	1	0	28	10	5	1
38	12	10	6	1	104	41	16	5	116	51	22	6
39	17	12	4	1	28	9	3	0	45	21	7	1
40	14	8	1	1	15	5	0	0	29	13	1	1
41	16	6	1	0	13	4	1	0	29	10	2	0
42	15	6	5	4	15	7	6	3	30	13	11	7
43	20	6	6	3	32	7	7	4	52	13	13	7
44	24	13	7	5	10	0	0	0	34	13	7	5
45	24	8	4	1	18	7	2	1	42	15	6	2
46	16	4	2	0	20	3	2	2	36	7	4	2
47	23	8	3	3	14	4	0	0	37	12	3	3
48	15	2	0	0	29	8	4	1	44	10	4	1
49	20	11	6	1	19	5	3	1	39	16	9	2
50	27	9	0	0	25	7	1	0	52	16	1	0
51	23	8	4	1	26	9	3	0	49	17	7	1
52	103	42	18	11	13	2	0	0	116	44	18	11
53	40	19	4	3	15	4	0	0	55	23	4	3
54	32	17	6	2	19	4	3	1	51	21	9	3
55	27	15	12	1	20	6	2	0	47	21	14	1
56	28	10	5	1	19	3	1	0	47	13	6	1
57	27	7	2	0	26	4	0	0	53	11	2	0
58	32	17	12	7	18	6	0	0	50	23	12	7
59	23	5	0	0	24	9	4	2	47	14	4	2
60	86	32	15	7	20	4	1	0	106	36	16	7
61	23	7	2	2	22	13	6	0	45	20	8	2
62	23	12	7	3	16	4	1	0	39	16	8	3
63	15	4	1	1	16	2	2	2	31	6	3	3
64	14	7	2	1	23	4	3	0	37	11	5	1
Total	1179				894				2073			

Table 2a. The values of α and β in the expression $\log N = \alpha - \beta \log \varDelta$ for N of every five years.

	Region A		Region B		Region (A+B)	
	α	β	α	β	α	β
1924-1928	8.70±0.18	2.87±0.07	11.50±0.09	4.09±0.03	9.85±0.18	3.22±0.07
25- 29	8.89 0.20	2.90 0.08	12.33 0.50	4.44 0.20	10.08 0.16	3.32 0.06
26- 30	9.00 0.16	2.96 0.06	12.59 0.57	4.57 0.23	10.24 0.24	3.39 0.09
27- 31	10.15 0.20	3.45 0.08	12.47 0.44	4.51 0.17	11.17 0.26	3.79 0.10
28- 32	9.83 0.25	3.31 0.10	13.62 0.79	5.03 0.31	10.94 0.34	3.70 0.14
29- 33	7.67 0.16	2.33 0.07	14.02 0.91	5.21 0.36	8.79 0.16	2.75 0.06
30- 34	7.40 0.17	2.22 0.07	10.90 0.79	3.91 0.31	8.32 0.05	2.56 0.02
31- 35	7.68 0.32	2.33 0.13	11.48 0.32	4.14 0.13	8.55 0.25	2.64 0.10
32- 36	7.52 0.33	2.27 0.13	11.03 0.25	3.99 0.10	8.30 0.21	2.55 0.08
33- 37	7.39 0.31	2.22 0.12	11.06 0.16	3.98 0.06	8.24 0.22	2.53 0.09
34- 38	8.88 0.84	2.95 0.33	9.98 0.60	3.35 0.24	9.81 0.70	3.18 0.28
35- 39	9.28 1.15	3.12 0.46	10.84 1.01	3.70 0.40	10.47 1.06	3.45 0.42
36- 40	8.84 1.02	2.96 0.40	10.89 0.90	3.73 0.36	10.35 0.92	3.41 0.37
37- 41	9.17 1.22	3.11 0.48	10.91 0.98	3.73 0.39	10.55 1.06	3.50 0.42
38- 42	7.81 0.72	2.55 0.29	9.85 0.71	3.28 0.28	9.38 0.68	3.00 0.27
39- 43	7.51 0.30	2.42 0.12	8.57 0.33	2.85 0.13	8.34 0.24	2.63 0.09
40- 44	6.83 0.08	2.12 0.03	7.99 0.36	2.65 0.14	7.62 0.11	2.34 0.05
41- 45	7.00 0.13	2.18 0.05	7.74 0.37	2.53 0.15	7.62 0.23	2.33 0.09
42- 46	6.93 0.24	2.15 0.10	7.41 0.56	2.38 0.22	7.45 0.35	2.26 0.14
43- 47	7.43 0.14	2.35 0.05	8.44 0.51	2.83 0.20	8.14 0.24	2.55 0.10
44- 48	8.12 0.03	2.66 0.01	9.84 0.10	3.43 0.04	9.08 0.06	2.96 0.03
45- 49	9.27 0.55	3.14 0.22	9.48 0.08	3.25 0.03	9.67 0.32	3.19 0.13
46- 50	10.12 0.51	3.50 0.20	10.23 0.17	3.56 0.07	10.48 0.34	3.53 0.14
47- 51	9.76 0.48	3.34 0.19	11.88 1.12	4.23 0.44	10.91 0.68	3.69 0.27
48- 52	9.02 0.33	2.92 0.13	11.82 1.09	4.20 0.43	9.96 0.42	3.23 0.17
49- 53	8.95 0.39	2.86 0.16	13.22 1.37	4.82 0.54	9.87 0.43	3.19 0.17
50- 54	9.01 0.42	2.88 0.17	13.20 1.34	4.82 0.53	9.88 0.42	3.18 0.17
51- 55	8.64 0.52	2.71 0.21	12.91 1.48	4.70 0.58	9.49 0.56	3.01 0.22
52- 56	8.70 0.54	2,73 0.21	12.77 1.05	4.67 0.41	9.47 0.52	3.01 0.21
53- 57	9.69 1.06	3.22 0.42	13.23 1.02	4.84 0.40	10.81 0.98	3.61 0.39
54- 58	8.35 0.84	2.66 0.33	13.37 1.06	4.90 0.42	9.71 0.78	3.15 0.31
55- 59	8.63 0.81	2.80 0.32	12.03 0.64	4.32 0.25	9.91 0.70	3.25 0.28
56- 60	8.65 0.28	2.75 0.11	12.13 0.50	4.36 0.20	9.66 0.25	3.11 0.10
57- 61	8.50 0.11	2.70 0.04	11.86 1.21	4.22 0.48	9.52 0.30	3.05 0.12
58- 62	7.99 0.13	2.48 0.05	11.51 1.35	4.08 0.53	8.97 0.33	2.82 0.13
59- 63	8.72 0.14	2.81 0.05	9.78 0.69	3.36 0.28	9.31 0.27	2.98 0.11
60- 64	8.36 0.18	2.67 0.07	11.17 1.30	3.95 0.52	9.28 0.36	2.97 0.14

Table 2b. The values of α and β for N of every ten years and forty years. The values of α are reduced to those for N of five years.

	Region A		Region B		Region (A+B)	
	α	β	α	β	α	β
1924-1933	8.07±0.05	2.53±0.02	12.32±0.34	4.46±0.13	9.26±0.11	2.96±0.05
29- 38	7.96 0.32	2.50 0.13	10.80 0.47	3.76 0.19	9.21 0·31	2.93 0.12
34- 43	8.16 0.54	2.67 0.21	9.36 0.44	3.13 0.17	9.11 0.47	2.92 0.19
39- 48	7.83 0.15	2.54 0.06	9.08 0.18	3.08 0.07	8.69 0.10	2.78 0.04
44- 53	8.65 0.27	2.79 0.11	11.07 0.42	3.93 0.16	9.55 0.26	3.09 0.10
49- 58	8.67 0.53	2.76 0.21	13.30 1.22	4.86 0.48	9.78 0.54	3.16 0.22
54- 63	8.52 0.48	2.73 0.19	11.04 0.74	3.90 0.27	9.48 0.50	3.05 0.20
1924-1963	8.34 0.27	2.67 0.10	10.59 0.38	3.70 0.15	9.35 0.27	3.01 0.11

the number N of earthquakes during the period of every five years as shown in equation (6), the coefficients α and β in each region were determined by the method of least squares;

$$N = N_{i-4} + N_{i-3} + N_{i-2} + N_{i-1} + N_i \,, \tag{6}$$

where $i = 1928 - 1964$.

The obtained values of α and β are shown in Table 2a, together with the probable errors.

Then, for the sake of comparison, values of α and β were determined for N of every ten years, i.e., 1924-1933, 1929-1938, 1934-1943, 1939-1948, 1944-1953, 1949-1958, 1954-1963, and for N of forty years from 1924 to 1963. The values are shown in Table 2b.

3. Obtained result and consideration

With i (the year in equation (6)) on the abscissa and corresponding value of β on the ordinate, Figs. 2a, 2b and 2c were prepared presenting the Regions A, B and (A+B), respectively. In these figures black dots and thin vertical lines indicate the values of β and the probable error in Table 2a, and open circles are the values of moving average of every three years.

From these diagrams it is certain that the value of β in each region shows secular variation. Also, it is interesting to see that the mode of variation differs largely between Region A and Region B, which can be summarized as follows: The period of secular variation is about 10 years in Region A, whereas in Region B about 20 years, nearly twice the former. The range of variation of β in Region A is about 2.1–3.4 (when β is converted into b by equation (5), using Ichikawa's value, $\gamma=2.7$, we get $b \fallingdotseq 0.77-1.25$). In Region B, on the other hand, the range of β is about $2.5-4.9$ ($b \fallingdotseq 0.93-1.81$). The value of β obtained for N of a duration of 40 years is 2.67 ($b \fallingdotseq 0.99$) in Region A and is 3.70 ($b \fallingdotseq 1.37$) in Region B (these values being represented by a thin horizontal line in Figs. 2a and 2b). In Region (A+B), the mode of the secular variation of β is similar to that of Region A, but the average value of β is 3.01 ($b \fallingdotseq 1.11$) (the value being represented in Fig. 2c) and is larger than that of Region A in consequence of the large average value of Region B.

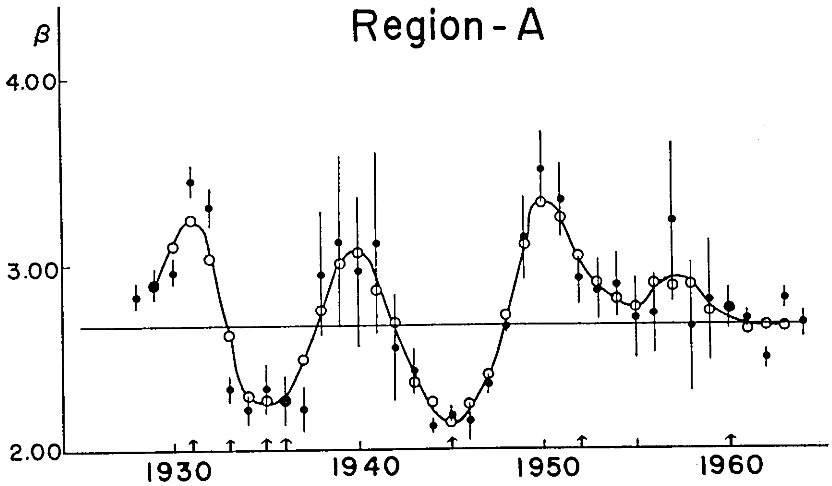

Fig. 2a. Secular variation of β in Region A. Black dots and thin vertical lines indicate the values of β and the probable errors, the open circles being the values of moving average of every three years. The arrows show the years for which the sum of energy released exceeded about 10^{23} ergs.

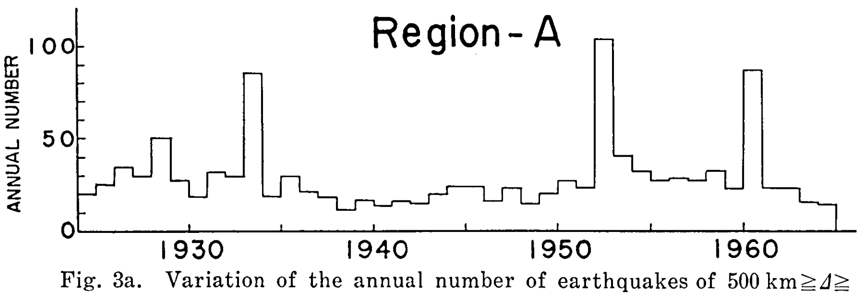

Fig. 3a. Variation of the annual number of earthquakes of 500 km$\geqq \Delta \geqq$ 200 km in Region A.

There is another interesting fact. That is, in Region A the amplitude of variation of β becomes conspicuously smaller from about 1955, the period also becoming shorter, and the value of β approaches the average ($\beta=2.67$) of 40 years, while no similar tendency is noticed in Region B.

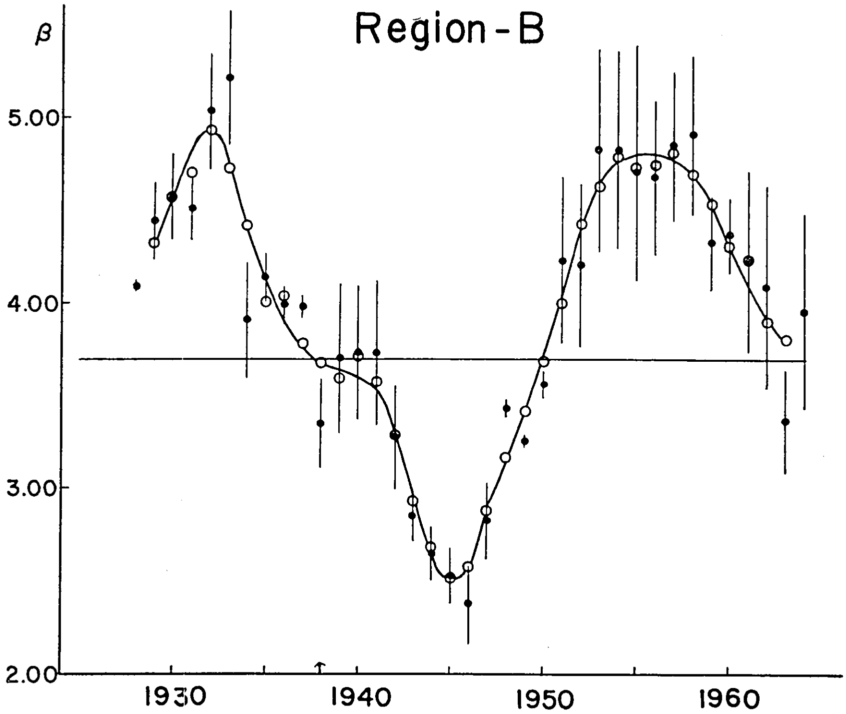

Fig. 2b. Secular variation of β in Region B.

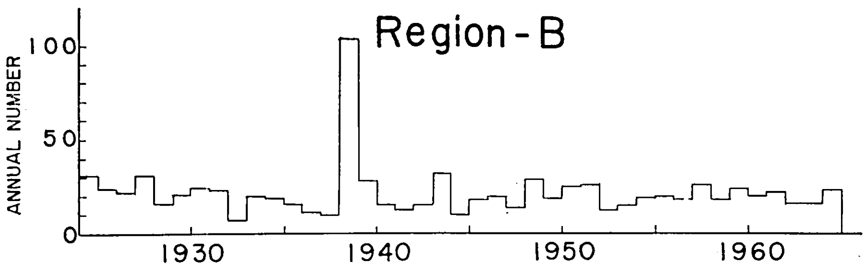

Fig. 3b. Variation of the annual number of earthquakes of $500\,\text{km} \geqq \Delta \geqq 200\,\text{km}$ in Region B.

In order to ascertain whether or not the variation of β has some connection with the earthquake activity in each region, the writer compared the variation of the annual number of earthquakes of $500\,\text{km} \geqq \Delta \geqq 200\,\text{km}$ (Figs. 3a and 3b) with the variation of β (Figs. 2a and 2b).

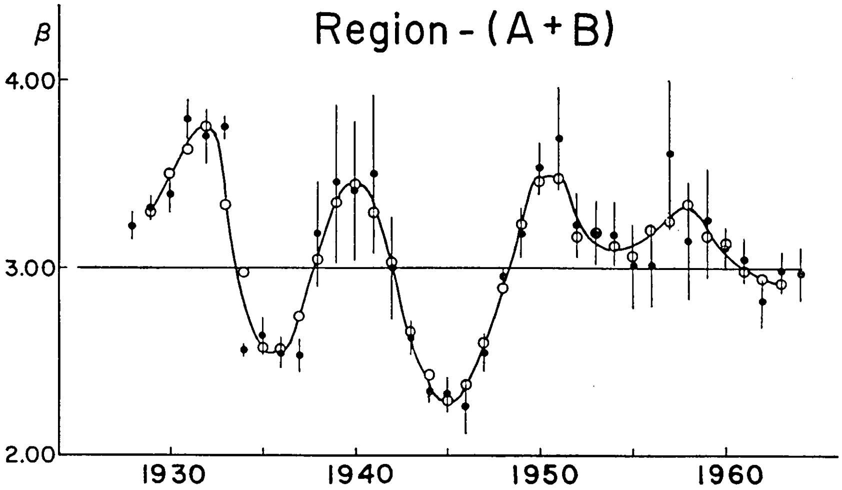

Fig. 2c. Secular variation of β in Region (A+B).

The annual number of earthquakes in Region A is larger in 1933, 1952 and 1960, and in Region B it is larger in 1938. This is because the large earthquakes ($M>7.5$), accompanied by a large number of aftershocks, occurred in these years.

The comparison between the variation of β and that of annual number has revealed the following: The above-mentioned years, when the number of earthquakes was large, correspond to the years when the value of β was closest to minimum. One of the reasons for this will be given later. In Region A, however, β shows a conspicuous minimum around 1945 when the number of earthquakes was not comparatively large. On the other hand, neither Region A or Region B shows a tendency of a decrease in the number of earthquakes when the value of β attains its maximum.

These facts suggest that it would be difficult to discover an intimate correlation between the secular variation of β and the annual number of earthquakes. Therefore, the writer made an examination as stated below: The sum of energy released by the earthquake of $M \geqq 6.0$ in one year was calculated, and the years for which this sum exceeded about 10^{23} ergs were sorted out, as represented by the arrows in Figs. 2a and 2b. The energy of 10^{23} ergs corresponds to that released by one earthquake of $M=7.5$. As seen in the diagrams, release of great energy took place when the value of β was, or was becoming, minimum. This fact requires some explanation. The range of magnitude treated in this study was between $\Delta=200$ km ($M\doteqdot 5.2$) and $\Delta=500$ km ($M\doteqdot 6.5$), and is a relatively narrow range in terms of M. Within this range of magnitude, the fact is revealed that the rate of increase of occurrence of aftershoks is larger in large earthquakes ($\Delta=500$ km) than in small ones ($\Delta=200$ km). As a consequence, immediately after a large earthquake accompanied by a large number of aftershocks the value of β would become smaller. However, it must be particularly noted here that, in both Regions A and B, the minimum value of β corresponds to the increase in the amout of released energy. Especially around 1945, the minimum value of β is not due to the increase in the number of earthquakes but is ascribed to the increase in the amount of released energy. In 1945 any earthquakes accompanied by numerous aftershocks took place, but a few earthquakes of $M=6.5$–7.3 occurred so that the

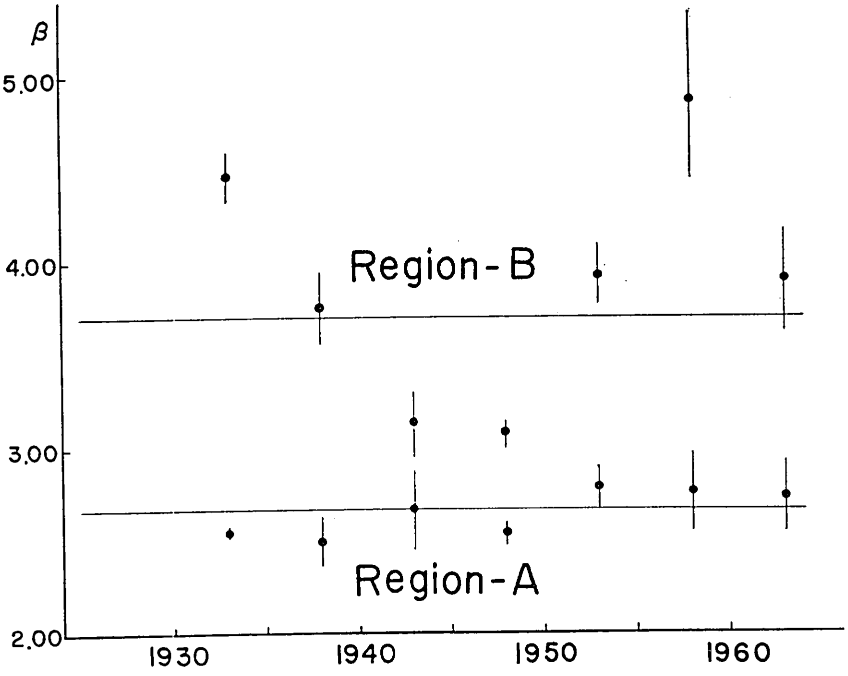

Fig. 4. Variation of β obtained for N of every ten years in Regions A and B.

released energy attained the amount of about 1×10^{23} ergs.

Summarizing the above-mentioned facts, it seems most likely that the variation of β has no direct relation with the number of earthquakes but is rather related to the amount of total energy released by the earthquakes. It is interesting to see that β begins to decrease several years before great energy was released, and that during the period of increasing values of β no release of great energy took place. This problem involves an important principle, so the writer will refrain from drawing any hasty conclusions here.

The values of β obtained for N of every ten years were plotted in Fig. 4. In Region B in the upper half of the diagram, variation of the 20 years period can still be observed, whereas in Region A the variation is obliterated since 10 years is almost the period of variation of β. Therefore, an examining the secular variability of β, the data to be dealt with should not be smoothed by the moving average of a very long window size, otherwise, existent variation is apt to be overlooked.

4. Acknowledgements

In conclusion the writer's deepest gratitude is expressed to Profs. Setumi Miyamura and Yasuo Satô of the Earthquake Research Institute, University of Tokyo, for valuable suggestions and the convenience of work. Thanks are also due to Miss Yoshiko Kotake who helped in the calculation work for the study.

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18. 地震の規模と回数の関係の永年変化について

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地震の規模 M とその発生回数 N との間の関係式 (1) または (2) における係数 b の値が，地域によつて異なることが最近のいくつかの研究によつて指摘された．今回の研究は，同一地域において，係数 b の値に永年変化があるかどうかを調べるのが目的であるが，$M<6.0$ の地震には資料に欠けるものがあるので，Mの代りに最大有感距離の対数値 $\log \Delta$ を用いて，これと M との関係式 (4) における係数 β の永年変化性を調べた．

1924 年から 1964 年までの 41 年間に，Fig. 1 に示した A, B および (A+B) 地域で起こつた深さ 60 km より浅い地震について，Δ=200, 300, 400, 500 km のものを選んで，それぞれの地域に対する β の値の年変化を求めた．結果は Table 2a, 2b, および Fig. 2a, 2b, 2c などに示した通りである．これらの結果から，β の値に明瞭な永年変化が存在することはほぼ間違いないもののよ

うである.

A 地域: $\beta \fallingdotseq 2.1 \sim 3.4$ ($b \fallingdotseq 0.77 \sim 1.25$) の範囲で約 10 年周期の変化が見られる. 1955 年頃より変化の振幅は小となり, 周期もやや短かくなりつつ, 40 年間の β の平均値 2.67 ($b \fallingdotseq 0.99$) に漸近しつつある傾向を示している.

B 地域: $\beta \fallingdotseq 2.5 \sim 4.9$ ($b \fallingdotseq 0.93 \sim 1.81$)の範囲で約 20 年周期の変化を示し, A 地域のそれとは著しく異つている. 40 年間の平均値は $\beta = 3.70$ ($b \fallingdotseq 1.37$) で A 地域より大である.

(A+B) 地域: A 地域での変化と似ているが, B 地域の影響で β の平均値は 3.01 ($b \fallingdotseq 1.11$) と A 地域のみの場合より大きくなつている.

β の変化と地震活動との関係を見るために, まず各年の地震数 (500 km$\geqq \Delta \geqq$200 km) の変化を図示すると Fig. 3a, 3b の如くなる. これらの図と β の変化 (Fig. 2a, 2b) とをそれぞれ比べると, 地震数の増減と β の値の増減とは必ずも一致しない. 特に A 地域で 1945 年頃の β の減少に対する地震数の変化には何ら関係を示さないし, β の増大に対して地震数の減少なども見られい.

次に各年の地震によるエネルギーが 10^{23} erg. 以上になつた年 (Fig. 2a, 2b の矢印) と β の変化とを比べると, 地震数との対応よりもより良好な一致が見られる. すなわち β の値が減少しつつある年またはそれが極小値に達した年に大きいエネルギーの放出がある. 一方 β の値が増大しつつある期間には決して大きいエネルギーの放出が起こつていない. これは興味ある事実であるが, しかしここでは軽々な結論は避けることとする.

β (あるいは b) の永年変化性を調べる場合, A 地域では 5 年間前後より長期間の地震数の総和から β の値を求めると, その永年変化性を見落す恐れがある (Fig. 4 参照).