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20. *Long Waves around a Circular Island* [I].

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Abstract

In this work, the study of long waves around a circular island is made numerically. The most important feature among the exposed facts is such that scattered waves are emitted from two parts, that is to say, one is from the windward part of the coast of a circular island and another from the leeward part.

The diffraction of waves is one of the most important problems related to the propagation of waves. In various branches of physics, especially in optics, the diffraction phenomena have been well investigated theoretically and experimentally. From the seismological point of view, the diffraction of long waves of a tsunami also deserves close investigation. A number of authors have been engaged in the investigation of this phenomena. Many problems in this field, however, remain unsolved.

In this paper, the diffraction of long waves around a circular island is studied. The study is carried out by calculating a serial solution with the aid of an electronic computer.

2. Theory

The used model is a cylindrical island situated in the waters extending horizontally to infinity with uniform depth.

Referring to Fig. 1, the polar coordinates (r, θ) are centered at the midpoint of a circular island, the basal line being taken in the counter-direction of the incident wave travel. Let ζ and k be the wave height and the wave number of the incident waves, the latter of which is related to the angular frequency ω and the velocity of a long wave c in such a way that

$$\omega = kc \ .$$

Provided that the sea depth is uniform, the equation of a long

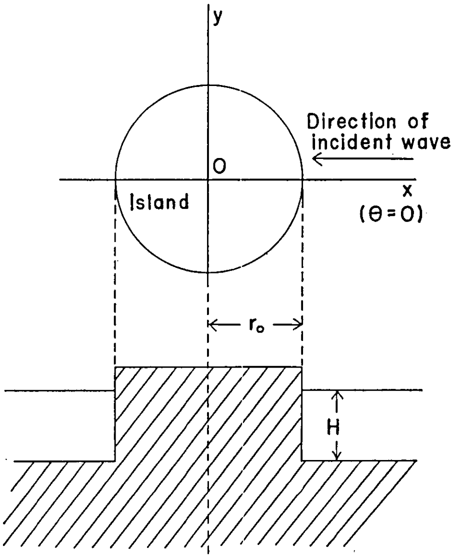

Fig. 1. A geometry of used model.

wave for periodic waves is then

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+k^2\right)\zeta=0 \quad (1)$$

The boundary condition at the rigid wall of the circular island is

$$\frac{\partial \zeta}{\partial r}=0 \quad (2)$$

Let the incident wave ζ_{in} be

$$\zeta_{in}=\zeta_0 e^{+ikx}\ , \quad (3)$$

where the time factor $\exp(+i\omega t)$ is omitted as usual (this convention is followed in the following development of the theory), the only real part has physical meaning, ζ_0 is the amplitude of the incident waves, and x is a component of the cartesian coordinate (refer to Fig. 1).

Let the radius of the given island be r_0. Then the solution of the equation (1) becomes as follows.

$$\left.\begin{aligned}
&\zeta=\zeta_{in}+\zeta_{sc}\ ,\\
&\zeta_{sc}=\sum_{m=0}^{\infty}A_m \cos m\theta\, H_m^{(2)}(kr)\\
&A_m=-\frac{J_0'(kr_0)}{H_0^{(2)\prime}(kr_0)}\zeta_0 \qquad (\text{for } m=0)\\
&\quad=2(-1)^{n+1}\frac{J_{2n}'(kr_0)}{H_{2n}^{(2)\prime}(kr_0)}\zeta_0\\
&\qquad\qquad (\text{for } m=2n,\ n=1,2,3,\cdots)\\
&\quad=i\cdot 2(-1)^{n+1}\frac{J_{2n+1}'(kr_0)}{H_{2n+1}^{(2)\prime}(kr_0)}\zeta_0\\
&\qquad\qquad (\text{for } m=2n+1,\ n=0,1,2,\cdots)
\end{aligned}\right\} \quad (4)$$

and ζ_{in} is described in (3).

In the above reductions, the only cosine terms are retained owing to the symmetry of the phenomenon with respect to the x-axis (refer to Fig. 1), $A_m(m=0, 1, 2, \cdots)$ are determined so as to satisfy the condition (2), and the relation

$$e^{+ikx} = J_0(kr) + 2\sum_{n=1}^{\infty}(-1)^n \cos 2n\theta J_{2n}(kr)$$
$$+ i\cdot 2\sum_{n=0}^{\infty}(-1)^n \cos(2n+1)\theta J_{2n+1}(kr)$$

is employed. In (4), ζ_{sc} denotes the scattered waves which must be of the outgoing ones at a point distant from the island. Using the formal expression (4), numerical calculations are carried out for the waves around an island with the aid of an electronic computer.

3. Numerical Calculations and their Discussions

3,1. *Height and Phase of Resultant Waves*

In this section, the wave height and phase of the resultant waves (being composed of the incident and scatterred waves) are calculated by the formal solution (4) with the help of an electronic computer for specified values $kr_0 = 0.5$ and 1.0. The reduction of these values to the relation between a wave-lengh (λ) and a diameter ($D = 2r_0$) of the circular island yields

$$\lambda = 2\pi D \qquad (\text{for } kr_0 = 0.5)$$
$$\sim 6D$$

and

$$\lambda = \pi D \qquad (\text{for } kr_0 = 1.0)$$
$$\sim 3D\,.$$

The above values of wave-length are considered as being of the order of or rather larger than those of tsunamis for actual islands in the ocean.

The variations of the resultant waves are presented in Figs. 2, 3, 6 and 7, of which the first two are relevant to a parameter $kr_0 = 0.5$ and the last two to $kr_0 = 1.0$.

According to Figs. 2 and 6 (showing the variations of amplitude), it is found that a portion of large amplitude extends parabolically from windward part of the island to leeward. Large amplitude in the windward part of the island is caused by reflection of the incident waves in this part. Further, a parabolic extension of a portion of high waves is attributed to the waves diverted to both sides of the island

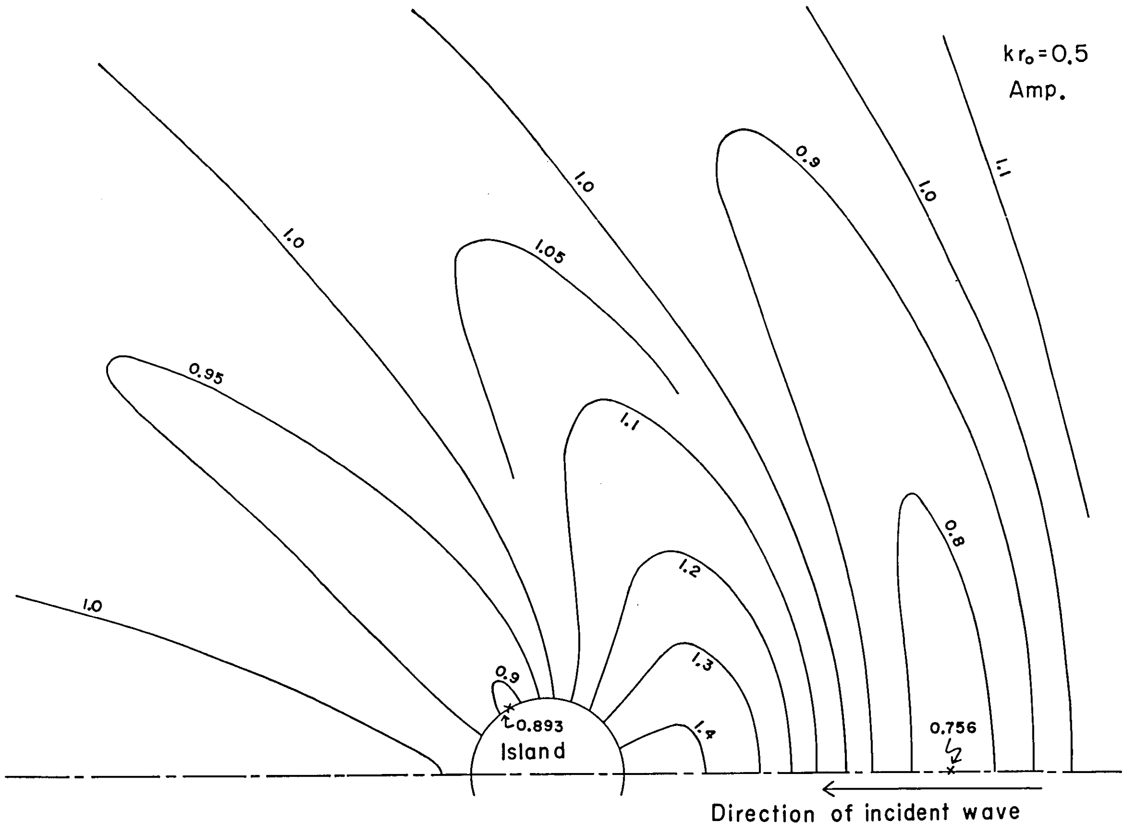

Fig. 2. A variation of amplitude of a wave around an island for a specified parameter $kr_0=0.5$ (the stated values in the contours denote the magnitudes of amplitude, i.e. $|\zeta|$).

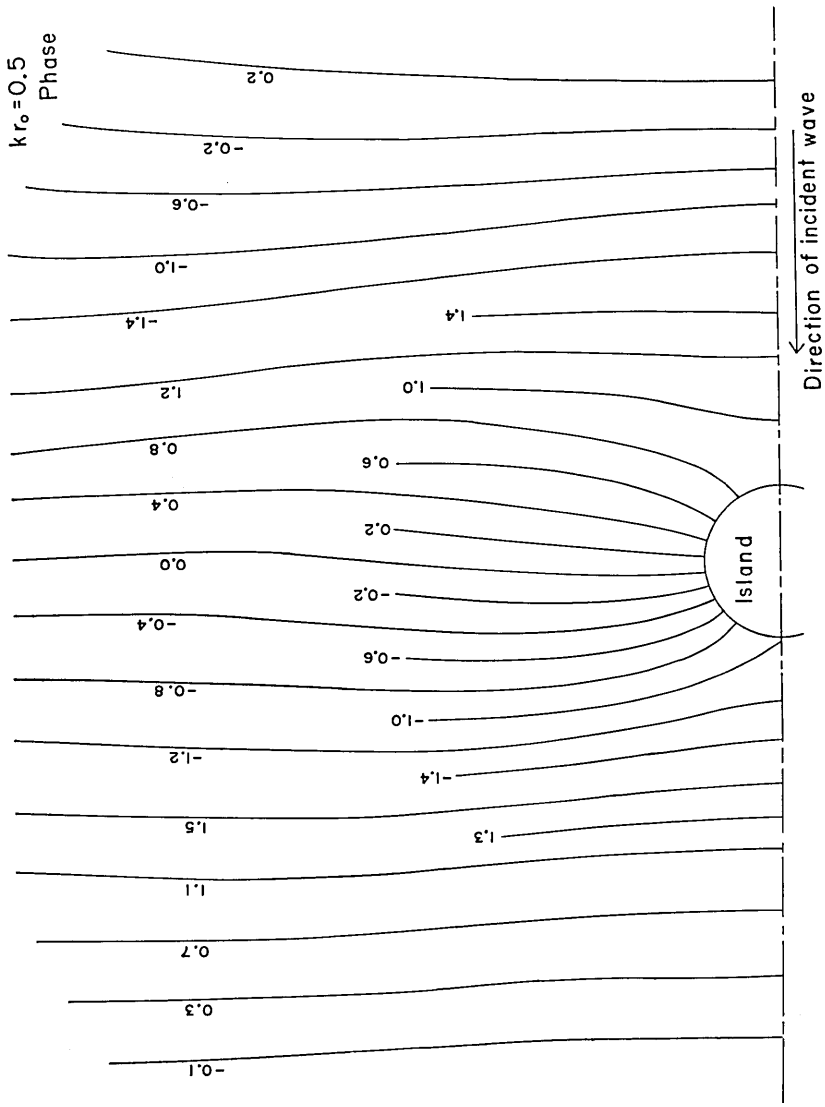

Fig. 3. A variation of phase of a wave around an island for a specified parameter $kr_0=0.5$ (the stated values in the contours denote $\arg\zeta$, which were calculated in the range of the principal value of *tangent*).

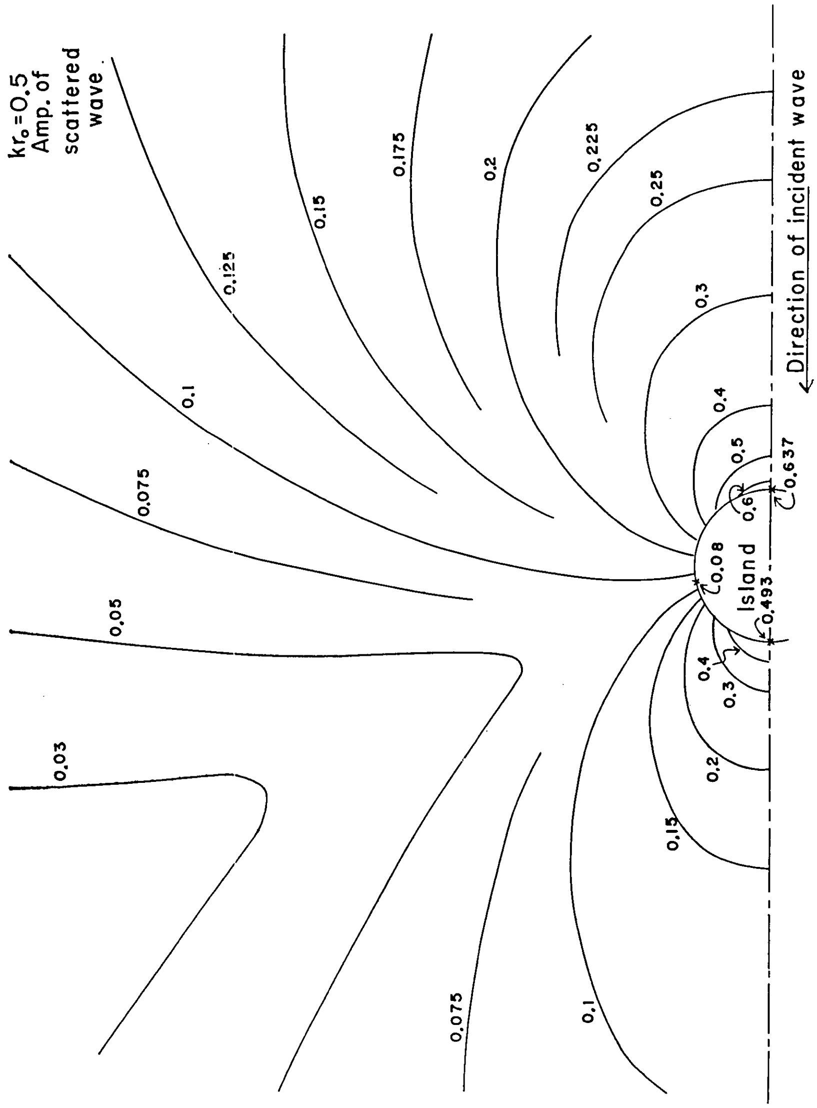

Fig. 4. A variation of amplitude of a scattered wave around an island for a specified parameter $kr_0=0.5$ (the stated values in the contours denote the magnitude of amplitude of a scattered wave, i.e. $|\zeta_{sc}|$).

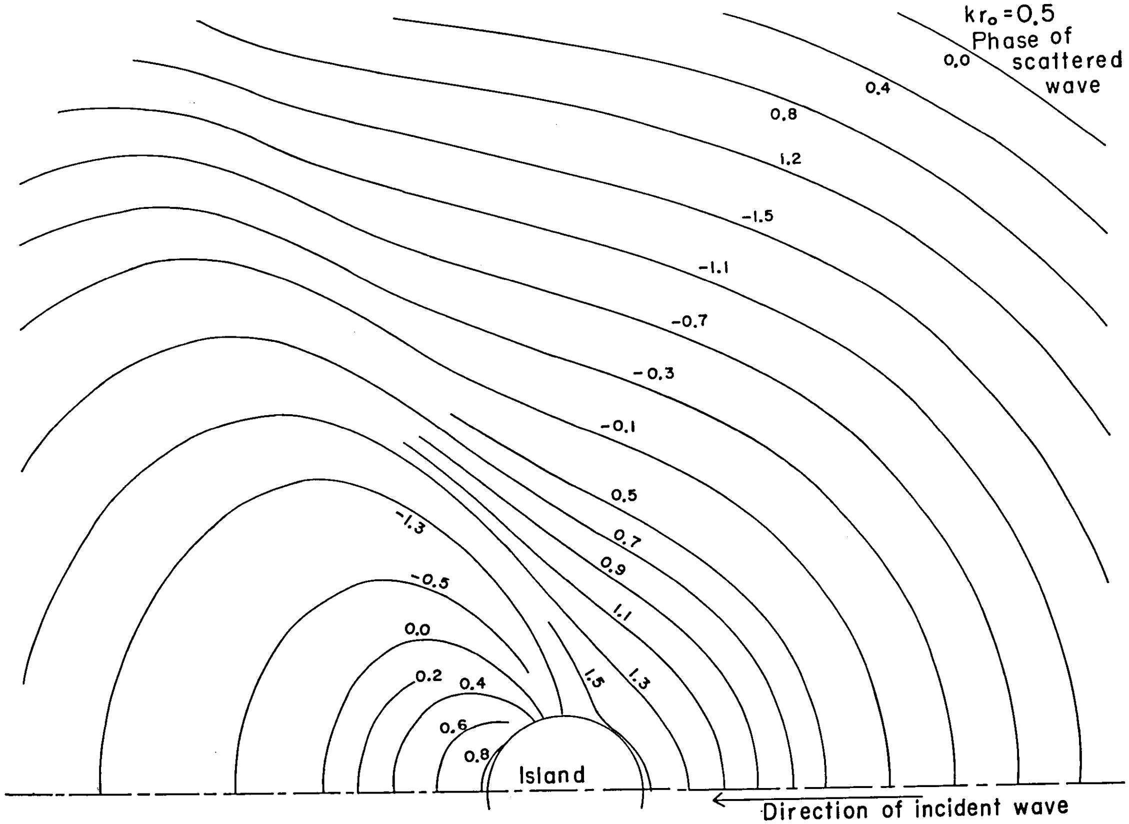

Fig. 5. A variation of phase of a scattered wave around an island for a specified parameter $kr_0=0.5$ (the stated values in the contours denote $\arg\zeta_{sc}$, which were calculated in the range of the principal value of *tangent*).

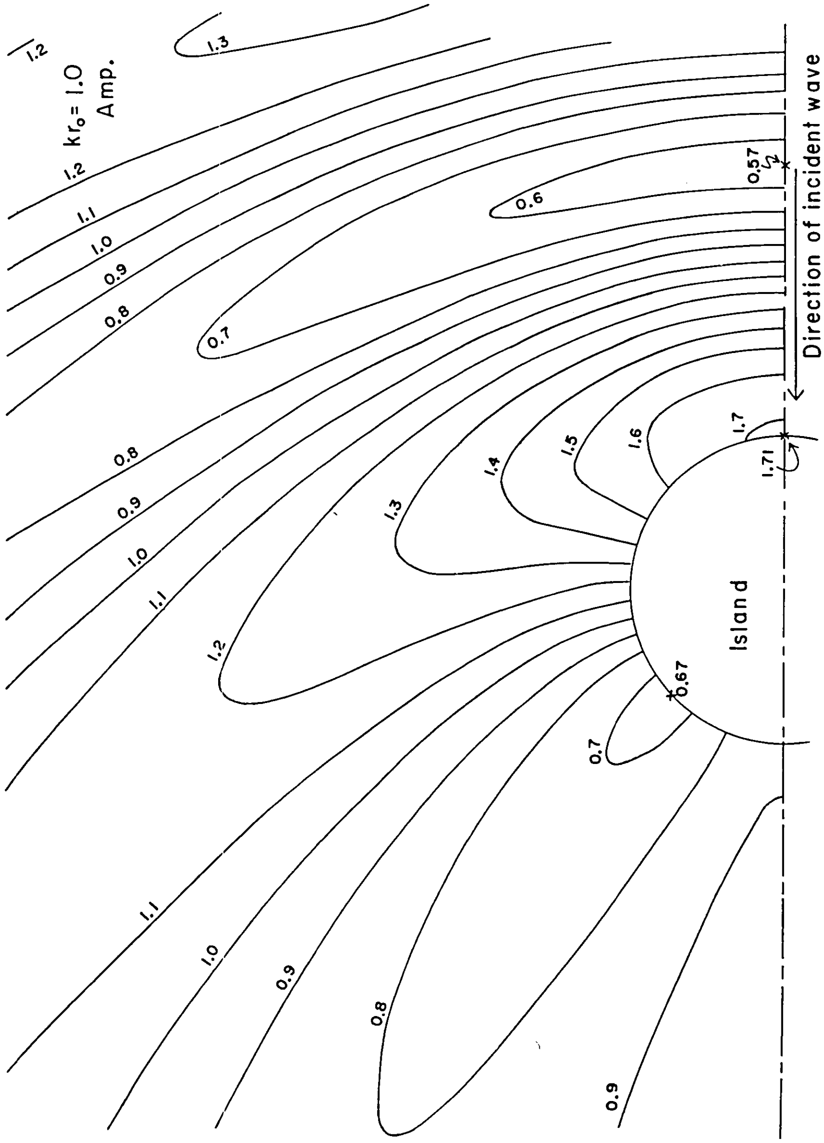

Fig. 6. A variation of amplitude of a wave around an island for a parameter $kr_0=1.0$ (the stated values in the contours denote the magnitude of amplitude, i.e. $|\zeta|$).

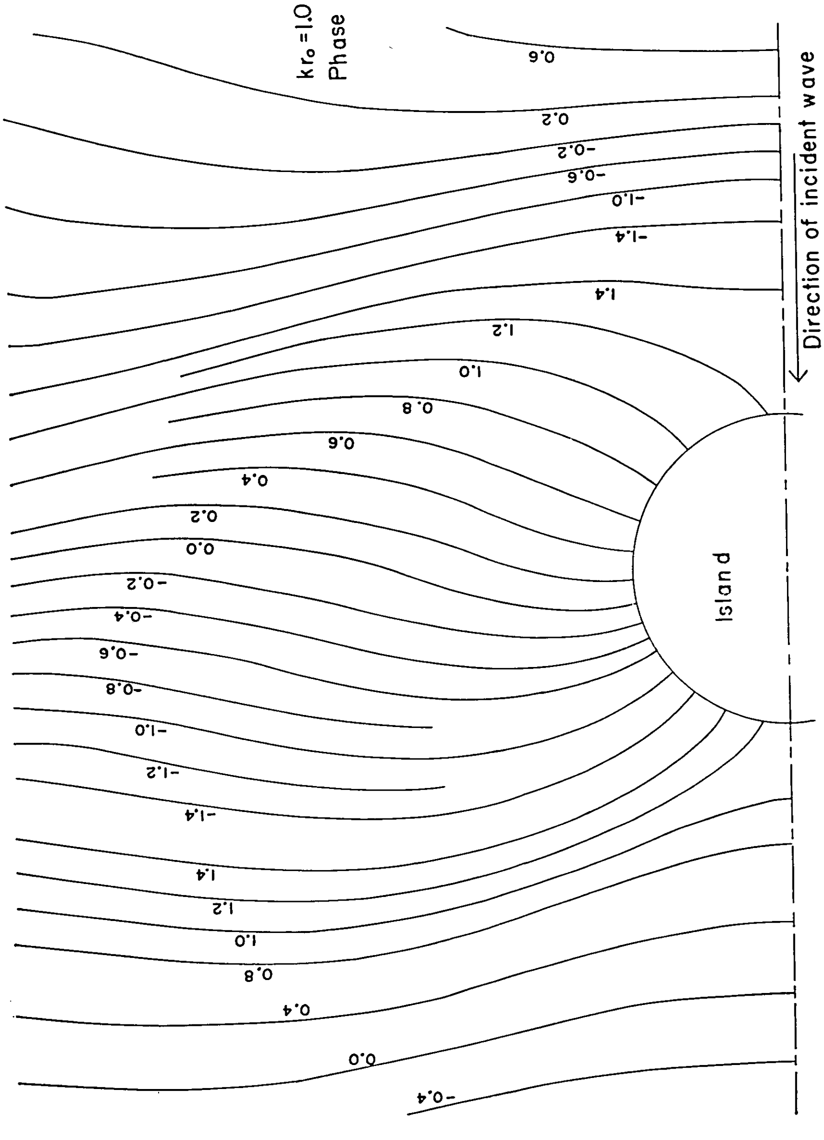

Fig. 7. A variation of phase of a wave around an island for a parameter $kr_0=1.0$ (the stated values in the contours denote arg ζ, which were calculated in the range of the principal value of *tangent*).

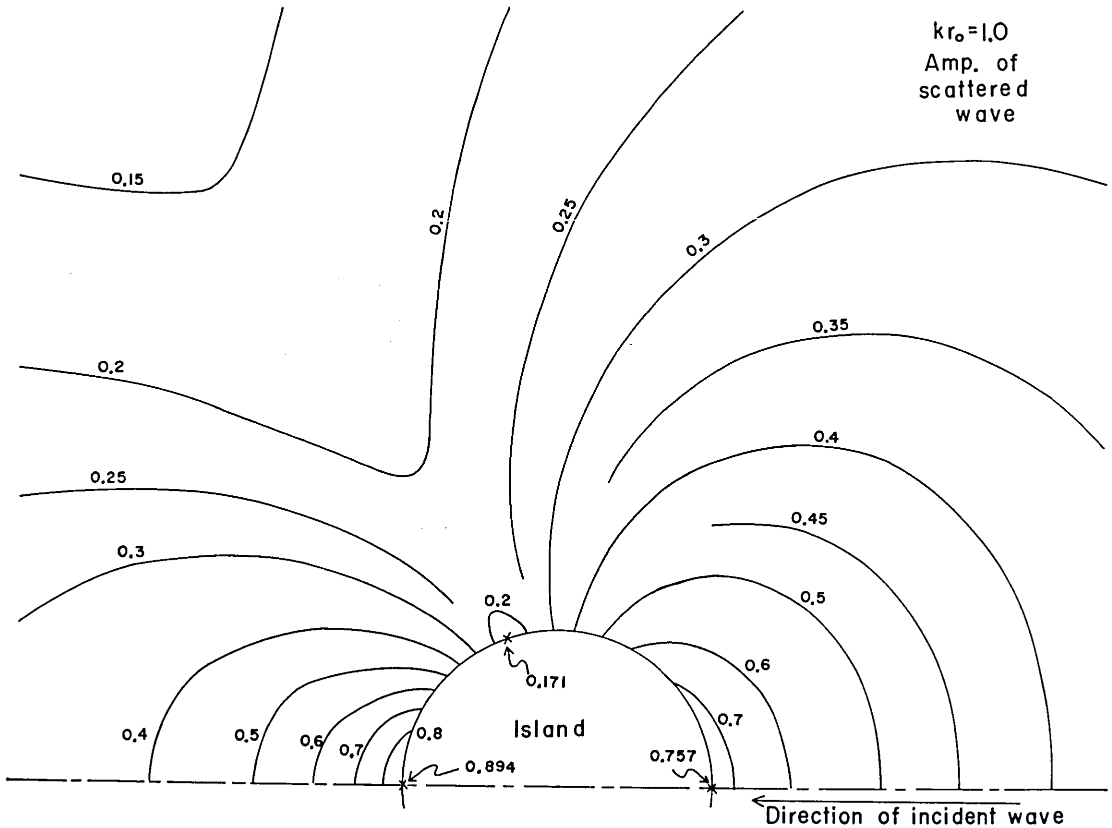

Fig. 8. A variation of amplitude of a scattered wave around an island for a parameter $kr_0=1.0$ (the stated values in the contours denote the magnitude of a scattered wave, i.e. $|\zeta_{sc}|$).

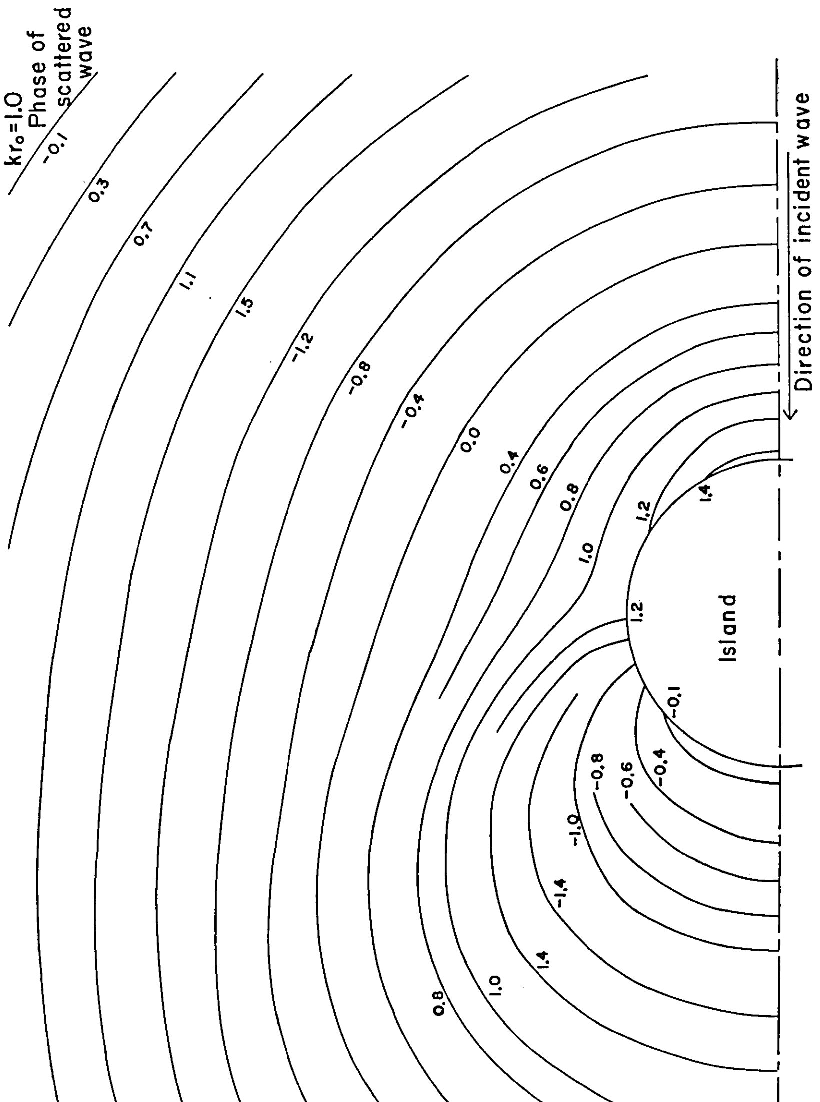

Fig. 9. A variation of phase of a scattered wave around an island for a parameter $kr_0=1.0$ (the stated values in the contours denote $\arg \zeta_{sc}$, which were calculated in the range of the principal value of *tangent*).

along the windward part of the coast. Such diversion of waves is seen in Figs. 3 and 7, which show the phase variations of waves for $kr_0=0.5$ and 1.0 respectively (the stated values in the figures are taken in the range of the principal value of *tangent*, i.e., $\pi/2>\arg\zeta>-\pi/2$). In these figures, some of the waves diverted along the coast, though that is not so clear for the case of $kr_0=0.5$ (see Fig. 3), advance straightly to leeward leaving from the island, which are considered to produce parabolic extension of large amplitude in the leeward waters of the island as the result of superposition of the incident waves. These behaviors are illustrated in Fig. 10.

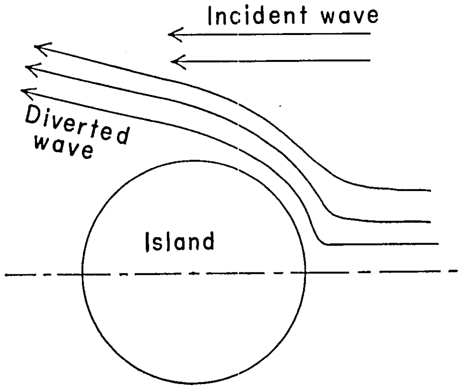

Fig. 10. A generation mechanism of parabolic extension of large amplitude.

In Figs. 2 and 6, another conspicuous feature is an appearance of a part of low amplitude at the root of a geometric shadow, which is sketched in Fig. 11. A mechanism of generation of the above low amplitude part might be proposed to be due to a departure of waves (including incident and diverted ones) from the coast of the island.

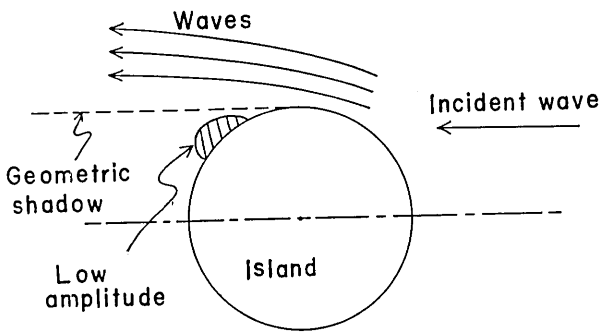

Fig. 11. A generation mechanism of low amplitude behind a circular island.

Comparing two figures relevant to the amplitude and phase of the resultant waves (Figs. 2, 3, 6 and 7), the variations for $kd=0.5$ are, as a whole, gentler than those for $kd=1.0$. This is reasonable from the fact that waves with longer wave-length are not so influenced by an obstacle as those with shorter wave-length.

3,2. *Height and Phase of Scattered Waves*

In this section, the wave height and phase of scattered waves are treated of. The numerical calculations for these waves are carried out by use of the expression ζ_{sc} in (4) for the same values of kd as those employed in Section 3,1, i.e., 0.5 and 1.0. The calculated results are shown in Figs. 4, 5, 8 and 9, of which the former two are relevant to $kd=0.5$ and the latter two to $kd=1.0$.

Inspection of Figs. 4 and 8 (the figures referring to the amplitudes of scattered waves) shows that regions of large amplitude extend to the axis of the direction of incident waves. Windward extension of the above region is due to the reflection of incident waves at the coast, while leeward extension being caused by the effect of geometric shadow of the island (in other words, the directivity of the incident waves).

According to Figs. 5 and 9 (the figures concerning the phase of scattered waves), the reflection of the incident waves at the windward part of the coast is seen definitely, of which the crest lines run in nearly circular form leaving from the island (in Figs. 5 and 9, the values denoting the phase variation are taken in the range of the principal value of *tangent*). In the above two figures, a conspicuous feature is an emission of scattered waves from the leeward coast of the island. Such leeward emission of scattered waves is considered as being due to a directivity of the incident waves, which makes, as a limiting case, a geometric shadow when the wavelength of the waves grows very small as compared with a dimension of the obstacle. Comparing Figs. 4 and 8, it is further found that the amplitude of scattered waves in the leeward part of the island is larger for $kr_0=1.0$ than that for $kr_0=0.5$ in accordance with the fact that the effect of the obstacle upon making a geometric shadow begins to be great with increasing kr_0.

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20. 円型の島のまわりの長波 [I]

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本報告においては円型の島のまわりの長波が数値解析されている.
得られた結果のうち，もつとも重要な事実は島による散乱波が 2 つの部分から放射されることである．すなわち，1 つは島の風上側の海岸から，もう 1 つは風下側の海岸からである.