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21. *Dislocation Flow Through a Layered Earth Structure**.

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Abstract

In the paper are discussed the boundary conditions for dislocation density current in the layered media. The assumption of dislocation flow brings, to some extent, an explanation of earthquake distribution along a hypocentral surface. The angle of the inclination which in Benioff's graphs suffers rapid change at about 300 km is in our case changing gradually according to respective changes of rigidity μ. The relative rate of stress accumulation could be calculated basing on shape of hypocentral plane.

1. Introduction

Recent developments of dislocation theory (Bilby,[1),2)] Kondo,[3-6] Hollander,[7-9] Kosevich[10,11)]) and its applications to earthquake problems (Teisseyre[12)]) imply that the distribution of dislocations and their mutual interactions in the earth interior could be treated in an uniform way by the method of continuous dislocation field.

In an earlier paper[13)] we discussed the problem of fold deformation; here we intend to approach the dynamic problems of dislocation flow.

We assume here that a deformation of an element of medium is

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*) Communicated by K. KASAHARA.

1) B. A. BILBY, R. BOULOUGH, E. SMITH, *Proc. Roy. Soc.*, **231** (1955), 236.
2) B. A. BILBY, E. SMITH, *Proc. Roy. Soc.* A, **232** (1956), 481.
3) K. KONDO, *RAAG Memoirs*, **1**, Div. C (1955), 361.
4) K. KONDO, *RAAG Memoirs*, **1**, Div. D (1955), 458.
5) K. KONDO, *RAAG Memoirs*, **1**, Div. D (1955), 484.
6) K. KONDO, *RAAG Memoirs*, **2**, Div. D (1958), 470.
7) E. F. HOLLÄNDER, *Czech. J. Phys.* B, **10** (1960), 409.
8) E. F. HOLLÄNDER, *Czech. J. Phys.* B, **10** (1960), 479.
9) E. F. HOLLÄNDER, *Czech. J. Phys.* B, **10** (1960), 551.
10) A. M. KOSEVICH, *Jour. Eks. i Teoret.*, **45** (1962), 637.
11) A. M. KOSEVICH, *Usp. Fiz. Nauk*, **84** (1964), 579.
12) R. TEISSEYRE, *Bull. Earthq. Res. Inst.*, **44** (1966), 153.
13) R. TEISSEYRE, *loc. cit.*, 12).

given by deformation vector u_i and by distortion tensor u_{is}:

$$dX_i = dx_i + du_i + u_{is}dx_s \tag{1}$$

The respective coordinate transformation presents anholomic transformation. The dislocation density tensor is related to distortion tensor u_{mn} by the following relation (we restrics our consideration to first order terms only)

$$\left.\begin{aligned} \oint u_{mn}dx_n &= \iint \alpha_{mn}d\sigma_n \\ \alpha_{mn} &= \varepsilon_{kln}u_{mk,l} \end{aligned}\right\} \tag{2}$$

where index m refers to Burgers vector, while n to tangent vector to dislocation lines.

From the last equation the continuity of dislocation density follows:

$$\alpha_{mn,n} = 0 \tag{3}$$

We assume further, that in our medium stress momentum vanishes; hence we have[14)]

$$u_{[ik],ss} = 0 \tag{4}$$

where

$$u_{[ik]} = \frac{1}{2}[u_{ik} - u_{ki}]$$

In several cases we can assume that equation (4) is fulfilled by symmetric field

$$u_{[ik]} = 0 \tag{5}$$

The equilibrium equations $\tilde{p}_{ik,k} = 0$, could be expressed in terms of u_{is} using stress-strain relation[15)]

$$\left.\begin{aligned} p_{ik} &= \lambda\delta_{ik}u_{s,s} + 2\mu u_{(i,k)} \\ \bar{p}_{ik} &= \lambda\delta_{ik}u_{ss} + 2\mu u_{(ik)} \end{aligned}\right\} \tag{6}$$

$$\tilde{p}_{ik,k} = p_{ik,k} + \bar{p}_{ik,k} \tag{7}$$

where: stress tensor $\tilde{p}_{ik}$ splits in parts corresponding to elastic deformation p_{ik} and plastic deformation $\bar{p}_{ik}$ connected with distortion u_{is}; symbol $u_{(is)}$ denotes $1/2(u_{is} + u_{si})$.

14) *loc. cit.*, 12).
15) *loc. cit.*, 12).

Mainly we will deal here with plastic deformation only and in several applications we will also assume $u_{ss}=0$, that is volume element remaining constant under plastic deformation.

Considering dynamic problems we will describe the dislocation movements by means of tensor of dislocation density current. The definition of dislocation current density tensor could be given through integral relation representing time change of dislocation density inside a volume V and on the other side by the dislocation flow through surrounding surface:

$$c\iiint\dot{\alpha}_{ar}d\tau=-\iint I_{ars}d\sigma_s \tag{8}$$

or in form of continuity equation

$$c\dot{\alpha}_{ar}+I_{ars,s}=0 \tag{9}$$

where:

I_{ars}—dislocation current density; index a refers to Burgers vector; indexes r, s to dislocation line tangent and to direction of dislocation flow respectively;

c—shear wave velocity.

From eq. (8) and (2) follows

$$I_{ars}=c\varepsilon_{krs}\dot{u}_{ak} \tag{10}$$

which is equivalent to

$$\dot{u}_{am}=\frac{1}{2c}\varepsilon_{mrs}I_{ars} \tag{11}$$

It is also worth noting here that in equation of motion

$$\tilde{p}_{am,m}=\rho\dot{v}_a \tag{12}$$

the acceleration of volume element $\rho\dot{v}_a$ is given by time derivations of vector u_a only.

2. Formulation of boundary problem

We would like to consider in this paper a medium having layered structure, boundary surfaces forming parallel planes. Let us choose the coordinate system as on Fig. 1, and consider a boundary plane e.q. $x_3=0$, separating two neighbouring layers with constents λ, μ for $x_3>0$

and λ_0, μ_0 for $x_3<0$. The continuity of stress components $\tilde{p}_{i3}$ is now expressed by

$$\tilde{p}_{i3}=\tilde{p}^0_{i3}\ ; \qquad x_3=0 \tag{13}$$

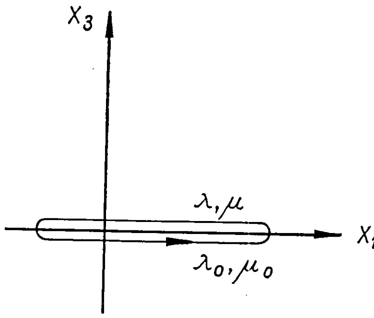

Fig. 1.

However, according to (6) stress tensor splits into elastic and plastic (distortional) parts, hence from the condition (13) we get for plastic stresses

$$\bar{p}_{i3}=\bar{p}^0_{i3}-\varDelta p_{i3}\ ; \qquad x_3=0 \tag{14}$$

where: $\varDelta p_{i3}$ denotes the differences in elastic stresses.

The aim of this paper is to investigate behavior of dislocation density and dislocation current in the horizontally layered earth structure. Especially we will deal with characteristics and changes of these fields as resulting from boundary conditions.

From eq. (14) we have further (for $u_{ss}=0$)

$$\mu u_{(3k)}=\mu_0 u^0_{(3k)}-\frac{1}{2}\varDelta p_{3k} \tag{15}$$

We can easily prove that any differentiating along a boundary does not change this condition, so we get also

$$\mu u_{(3k),s}=\mu_0 u^0_{(3k),s}-\frac{1}{2}\varDelta p_{3k,s}\ ; \qquad s=1,2 \tag{16}$$

Let us now consider the contour integral crossing boundary (Fig. 1), and let its surface tend to zero by assumption that lengths of elements in the x_3 direction are reduced to zero. Thus the contour integral becomes reduced to line integral along boundary, passing over it and below it. On the other hand, however, we can transform this contour integral into surface integral with dislocation density as integrand. If the later is continuous this surface integral will tend to zero as its surface does:

$$\oint v_{ms}dx_s=\int(u_{ms}-u^0_{ms})dx_s=\iint\alpha_{mn}d\sigma_n=0$$

However, in a general case we could expect some distribution of dislocation density along boundary plane, if so the density α_{mn} will be no longer continuous at this plane. Instead of the above written equation we will get the equation defining linear density of dislocation distribution on the boundary plane $x_3=0$:

$$\int(u_{ms}-u_{ms}^0)dx_s=\int\beta_{ms}dx_s\ ; \qquad s=1, 2 \tag{17}$$

Hence in general we get further conditions for $x_3=0$:

$$u_{ms}-u_{ms}^0=\beta_{ms}\ ; \qquad s=1, 2 \tag{18}$$

From eq. (18) follows also

$$u_{ms,t}-u_{ms,t}^0=\beta_{ms,t}\ ; \qquad s=1, 2;\ t=1, 2 \tag{19}$$

In the case of dynamic problem we obtain that boundary conditions remain the same for time derivatives of the above quantities. The respective boundary equations for time derivatives we will denote by ($\dot{15}$), ($\dot{16}$), ($\dot{18}$), ($\dot{19}$).

3. Static problem

Some of the above given conditions could be expressed in terms of dislocation density. From eq. (2) and (19) we get

$$\alpha_{m3}-\alpha_{m3}^0=\beta_{m1,2}-\beta_{m2,1} \tag{20}$$

This states that possible discontinuities in dislocation density α_{i3} would be compensated by derivatives of linear density of dislocations along boundary.

Adopting symmetric distortion (5) we get further from (16)

$$\mu\alpha_{33}-\mu_0\alpha_{33}^0=\frac{1}{2}\Delta p_{32,1}-\frac{1}{2}\Delta p_{31,2} \tag{21}$$

where the right side is given by derivatives of elastic stress differences at boundary $x_3=0(\Delta p_{3i}=p_{3i}-p_{3i}^0)$. Comparing (20) with eq. (21) we get (putting $\mu=\mu_0+\Delta\mu$) the condition of consistence:

$$\frac{1}{\mu}\left(\frac{1}{2}\Delta p_{32,1}-\frac{1}{2}\Delta p_{31,2}-\Delta\mu\alpha_{33}^0\right)=\beta_{31,2}-\beta_{32,1} \tag{22}$$

which reduced to

$$\frac{1}{2}\Delta p_{32,1}-\frac{1}{2}\Delta p_{31,2}=\Delta\mu\alpha^0_{33} \tag{23}$$

if linear density vanishes at boundary. Further if one postulates the continuity of elastic stresses, then either $\alpha^0_{33}=0$, or distortion elements are not symmetric: $u_{31}\neq u_{13}$, $u_{32}\neq u_{23}$.

4. Dynamic problem

For time dependent problem we will try to express boundary conditions in terms of dislocation current tensor components. Using (11) we get from $(\dot{15})$ $(I_{abc}=-I_{acb})$:

$$\frac{\mu}{c}\varepsilon_{(k|ps|}I_{3)ps}-\frac{\mu_0}{c_0}\varepsilon_{(k|ps|}I^0_{3)ps}=-\Delta\dot{p}_{3k} \tag{24}$$

If distortion u_{mn} is symmetric we can omit symmetry operator ( ):

$$\left.\begin{aligned}
&\frac{\mu}{c}I_{323}-\frac{\mu_0}{c_0}I^0_{323}=-\frac{1}{2}\Delta\dot{p}_{31}\\
&\frac{\mu}{c}I_{331}-\frac{\mu_0}{c_0}I^0_{331}=-\frac{1}{2}\Delta\dot{p}_{32}\\
&\frac{\mu}{c}I_{312}-\frac{\mu_0}{c_0}I^0_{312}=-\frac{1}{2}\Delta\dot{p}_{33}
\end{aligned}\right\} \tag{25}$$

From the condition $(\dot{18})$ we get

$$\left.\begin{aligned}
&\frac{1}{c}I_{m23}-\frac{1}{c_0}I^0_{m23}=\dot{\beta}_{m1}\\
&\frac{1}{c}I_{m31}-\frac{1}{c_0}I^0_{m31}=\dot{\beta}_{m2}
\end{aligned}\right\} \tag{26}$$

Comparing eq. (25) and (26) for $m=3$ we come to following conditions of consistency:

$$\left.\begin{aligned}
&\frac{1}{\mu}\left(\frac{1}{2}\Delta\dot{p}_{31}+\frac{\Delta\mu}{c_0}I^0_{323}\right)=-\dot{\beta}_{31}\\
&\frac{1}{\mu}\left(\frac{1}{2}\Delta\dot{p}_{32}+\frac{\Delta\mu}{c_0}I^0_{331}\right)=-\dot{\beta}_{32}
\end{aligned}\right\} \tag{27}$$

This condition corresponds directly to condition (22) for static fields;

returning to original equation (15), (18) we notice that (22) could be split in two separate equations. Here again from eq. (27) it follows that if there is no generation of linear dislocation density $\dot{\beta}_{ms}=0$ at boundary, then either elastic stress is not continuous or current components $I_{323}=I^0_{323}=0$, $I_{331}=I^0_{331}=0$, or at least distortion is not symmetric $u_{31}\neq u_{13}$, $u_{32}\neq u_{23}$.

In the last case of non symmetric distortion u_{is} we should return to equation (24) and write following condition instead of (25):

$$\left.\begin{aligned}
&\frac{\mu}{2c}(I_{323}+I_{112})-\frac{\mu_0}{2c_0}(I^0_{323}+I^0_{112})=-\frac{1}{2}\Delta\dot{p}_{31}\\
&\frac{\mu}{2c}(I_{331}+I_{212})-\frac{\mu_0}{2c_0}(I^0_{331}+I^0_{212})=-\frac{1}{2}\Delta\dot{p}_{32}\\
&\frac{\mu}{c}I_{312}-\frac{\mu_0}{c_0}I^0_{312}=-\frac{1}{2}\Delta\dot{p}_{33}
\end{aligned}\right\} \tag{29}$$

Condition (26) remains without change.

5. Dislocation flow through a layered structure

Let us now consider in detail changes at boundary of dislocation current components I_{abc} for the most important cases.

We assume that some external field of tectonic forces, represented here by stress tensor p_{ik}, causes dislocation flow. However, it is not necessery to specify the nature of these forces, but some conclusions regarding intensity of stress accumulation at different depths will be deduced further on from the consistency equations (27). Moreover, it is neither necessery to specify whether we have upward or downward direction of flow, nor its velocity. In the discussion on dislocation flow in a layered medium, we will refer to these stress components involved in the considered dynamic process.

Case A: Burgers vector of dislocations parallel to x_2, dislocation line parallel to x_2, velocity of dislocation propagation in the plane x_1x_3.

This case describing screw dislocation field represented by components $I_{22\alpha}$, $\alpha=1,3$. Transformation properties in plane x_1x_3 of $I_{22\alpha}$ are the same as for vector.

This process is connected with the p_{32} and p_{12} components of stresses. The p_{32} field is involved in boundary conditions.

Referring to Fig. 2 we have for the direction of velocity

$$\mathrm{tg}\alpha = \frac{v_3}{v_1} = \frac{I_{223}}{I_{221}} \tag{30}$$

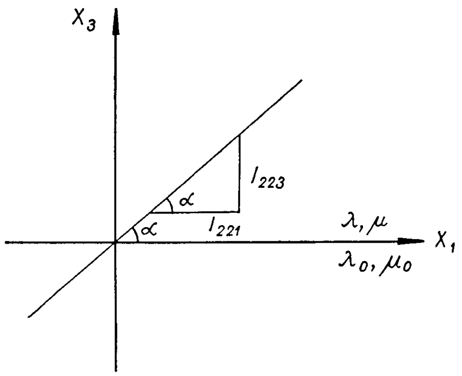

Fig. 2.

Assuming symmetric distortion we get from (11)

$$\left.\begin{aligned} I_{112} &= -I_{121} = I_{323} = -I_{332} \\ I_{113} &= -I_{131} = I_{232} = -I_{223} \\ I_{221} &= -I_{212} = I_{313} = -I_{331} \end{aligned}\right\} \tag{31}$$

Hence (30) could be expressed as follows

$$\mathrm{tg}\alpha = -\frac{I_{223}}{I_{331}} \tag{32}$$

Using boundary conditions (25), (26) we can express tg in two ways

$$\left.\begin{aligned} \mathrm{tg}\alpha &= -\frac{I^0_{223} + c_0\dot{\beta}_{21}}{I^0_{331} + c_0\dot{\beta}_{32}} \\ \mathrm{tg}\alpha &= -\frac{I^0_{223} + c_0\dot{\beta}_{21}}{\dfrac{\mu_0}{\mu} I^0_{331} - \dfrac{c_0}{2\mu}\Delta\dot{p}_{32}} \end{aligned}\right\} \tag{33}$$

where I^0_{223}, I^0_{331}, c_0 refer to first layer.

Here and further on we assume that any components of linear density β_{ms} not related to index of boundary surface $x_3=0$ vanish, that is

$$\beta_{mn} = 0 \; ; \qquad m, n \neq 3 \tag{34}$$

Thus only β_{3i} could be different from zero. Then we get from equations (33):

$$\mathrm{tg}\alpha = \mathrm{tg}\alpha_0 \left(1 + \frac{c_0\dot{\beta}_{32}}{I^0_{331}}\right)^{-1}$$

$$\frac{1}{\mu}\mathrm{tg}\alpha = \frac{1}{\mu_0}\mathrm{tg}\alpha_0 \left(1 - \frac{c_0\Delta\dot{p}_{32}}{2\mu I^0_{331}}\right)^{-1}$$

For a system of n layers this result can be easily generalized to

$$\left.\begin{aligned} \frac{\mathrm{tg}\alpha_0}{\mathrm{tg}\alpha_n} &= 1 + \frac{c_0 \sum_1^n \dot{\beta}_{32}}{I^0_{331}} \\ \frac{\frac{1}{\mu_0}\mathrm{tg}\alpha_0}{\frac{1}{\mu_n}\mathrm{tg}\alpha_n} &= 1 - \frac{c_0(\dot{p}^n_{32} - \dot{p}^0_{32})}{2\mu_0 I^0_{331}} \end{aligned}\right\} \tag{35}$$

From the observational data on earthquake distribution along some surfaces (e.q. hypocentral plane) we can estimate angle $\alpha(x_3)$, then from eq. (35) a relative course of $\dot{p}_{32}(x_3)$ and $\sum_1^n \dot{\beta}_{32}$ could be determined. Namely from eq. (27) follows that

$$-\frac{c_0(\dot{p}^n_{32} - \dot{p}^0_{32})}{2\mu_0 I^0_{331}} = \frac{c_0}{I^0_{331}} \sum_1^n \dot{\beta}_{32} + \frac{\mu_n - \mu_0}{\mu_0}\left(1 + \frac{c_0}{I^0_{331}} \sum_1^n \dot{\beta}_{32}\right)$$

and combining this result with difference of equations (35) we get $(I^0_{331} = -I^0_{221})$:

$$\mathrm{tg}\alpha_n = \mathrm{tg}\alpha_0 \left(1 - \frac{c_0}{I^0_{221}} \sum_1^n \dot{\beta}_{32}\right)^{-1} \tag{36}$$

Assuming here $\sum_1^n \dot{\beta} = 0$ we get simple relation

$$\alpha_n = \alpha_0 \tag{37}$$

Case B: dislocation lines parallel to x_2, Burgers vector parallel to direction of propagation, both in the $x_1 x_3$ plane.

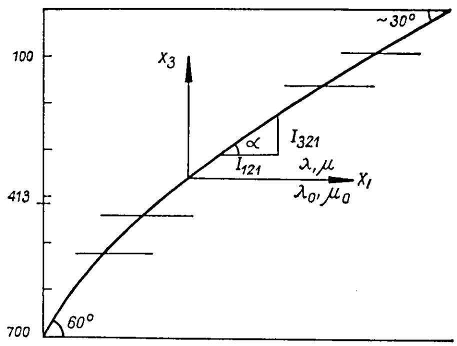

Fig. 3.

This case is represented by edge dislocation flow and is given by components $I^{\alpha 2 \beta}$; $\alpha, \beta = 1, 3$ (Fig. 3), $\alpha || \beta$ (symbolic notation).

This process is connected with the p_{31}, p_{33}, p_{11} components of stresses. The p_{31} and p_{33} fields are involved in boundary conditions.

Direction of propagation can now be determined by following ratios

$$\frac{I_{321}}{I_{121}} = \frac{I_{323}}{I_{123}} = \frac{I_{123}}{I_{121}} = \frac{I_{323}}{I_{321}} = \mathrm{tg}\alpha\ ; \quad \frac{I_{323}}{I_{121}} = \mathrm{tg}^2\alpha \tag{38}$$

because first and third index refers to Burgers vector and velocity vector respectively. This can of course be easily verified by transformation properties. From the above equations immediately follows:

$$I_{123}=I_{321}\rightarrow\dot{u}_{11}+\dot{u}_{33}=0$$

and also:

$$I_{121}I_{323}=I_{123}I_{321}>0$$

Last equation states that $I_{121}=-c\dot{u}_{13}$ and $I_{323}=c\dot{u}_{31}$ shall have the same sign, hence distortion u_{13} cannot be symmetric here.

The boundary conditions (26) and (29) bring together with (38) the following combinations:
dividing the first equation of (29) by the first one of (26):

$$\mu(1-\mathrm{ctg}^2\alpha)-\mu_0(1-\mathrm{ctg}^2\alpha_0)\left(1+\frac{c_0\dot{\beta}_{31}}{I^0_{323}}\right)^{-1}=-\frac{c_0\Delta\dot{p}_{31}}{I^0_{323}}\left(1+\frac{c_0\dot{\beta}_{31}}{I^0_{323}}\right)^{-1} \tag{39}$$

dividing the first equation of (29) by the third one of (29):

$$\mathrm{ctg}2\alpha-\mathrm{ctg}2\alpha_0\left(1+\frac{c_0\Delta\dot{p}_{33}}{2\mu_0I^0_{321}}\right)^{-1}=\frac{c_0\Delta\dot{p}_{31}}{2\mu_0I^0_{321}}\left(1+\frac{c_0\Delta\dot{p}_{33}}{2\mu_0I^0_{321}}\right)^{-1} \tag{40}$$

dividing the first equation of (26) by the third one of (29):

$$\frac{1}{\mu}\mathrm{tg}\alpha-\frac{1}{\mu_0}\mathrm{tg}\alpha_0\left(1+\frac{c_0\Delta\dot{p}_{33}}{2\mu_0I^0_{321}}\right)^{-1}=\frac{c_0\dot{\beta}_{31}}{\mu_0I^0_{321}}\left(1+\frac{c_0\Delta\dot{p}_{33}}{2\mu_0I^0_{321}}\right)^{-1} \tag{41}$$

From these three equations we have to determine the course of α, $\dot{\beta}_{31}$, $\Delta\dot{p}_{31}$, $\Delta\dot{p}_{33}$ for given distribution of μ and c. To this aim let us investigate the effect of summation of boundary conditions (39), (40), (41) over several layers. As one can see from eq. (26) (29) we will get for n layers instead of qualities $\dot{\beta}_{31}$, $\Delta\dot{p}_{31}$, $\Delta\dot{p}_{33}$, the sum of these quantities related to each boundary $\sum_1^n\dot{\beta}_{31}$; $(\dot{p}^n_{31}-\dot{p}^0_{31})$; $(\dot{p}^n_{33}-\dot{p}^0_{33})$. The system (39), (40), (41) could be written taking also (38) into account, as follows:

$$\left.\begin{aligned}
&\left(1+\frac{c_0}{I^0_{323}}\sum_1^n\dot{\beta}_{31}\right)\mu_n(1-\mathrm{ctg}^2\alpha_n)=\mu_0(1-\mathrm{ctg}^2\alpha_0)-\frac{c_0}{I^0_{323}}(\dot{p}^n_{31}-\dot{p}^0_{31})\\
&\mathrm{ctg}\alpha_n\left[1+\frac{c_0}{2\mu_0I^0_{323}}(\dot{p}^n_{33}-\dot{p}^0_{33})\mathrm{tg}\alpha_0\right]=\mathrm{ctg}\alpha_0+\frac{c_0}{2\mu_0I^0_{323}}(\dot{p}^n_{33}-\dot{p}^0_{33})\mathrm{tg}\alpha_0\\
&\frac{1}{\mu_n}\mathrm{tg}\alpha_n\left[1+\frac{c_0}{2\mu_0I^0_{323}}(\dot{p}^n_{33}-\dot{p}^0_{33})\mathrm{tg}\alpha_0\right]=\frac{1}{\mu_0}\mathrm{tg}\alpha_0\left(1+\frac{c_0}{I^0_{323}}\sum_1^n\dot{\beta}_{31}\right)
\end{aligned}\right\} \tag{42}$$

From the second and third we get here

$$\mu_n \operatorname{ctg}\alpha_n \operatorname{tg}\alpha_0\left(1+\frac{c_0}{I^0_{323}}\sum_1^n \dot{\beta}_{31}\right)=\mu_0 \operatorname{ctg}\alpha_0 \operatorname{tg}\alpha_n+\operatorname{tg}\alpha_n \operatorname{tg}\alpha_0 \frac{c_0}{2I^0_{323}}(\dot{p}^n_{31}-\dot{p}^0_{31})$$

and then comparing with first equation we obtain

$$\left(1+\frac{c_0}{I^0_{323}}\sum_1^n \dot{\beta}_{31}\right)\mu_n \operatorname{tg}\alpha_0(\operatorname{ctg}\alpha_n+\operatorname{tg}\alpha_n)=\mu_0 \operatorname{tg}\alpha_n(\operatorname{ctg}\alpha_0+\operatorname{tg}\alpha_0)$$

hence

$$\left(1+\frac{c_0}{I^0_{323}}\sum_1^n \dot{\beta}_{31}\right)=\frac{\mu_0(1+\operatorname{ctg}^2\alpha_0)}{\mu_n(1+\operatorname{ctg}^2\alpha_n)} \tag{43}$$

Assuming here $\sum_1^n \beta_{31}=0$ (no production of linear dislocation density at boundaries) we get

$$\mu_0(1+\operatorname{ctg}^2\alpha_0)=\mu_n(1+\operatorname{ctg}^2\alpha_n) \tag{44}$$

there we can easily find that for $\mu_n>\mu_0$ is $\alpha_n>\alpha_0$.

Case C: dislocation lines in the x_1x_3 plane, Burgers vector also in this plane, but perpendicular to dislocation line element $\overrightarrow{d\xi}$, velocity of propagation and Burgers vector parallel.

This process is connected with the p_{32} and p_{12} components of stresses. The p_{32} components is involved in boundary conditions.

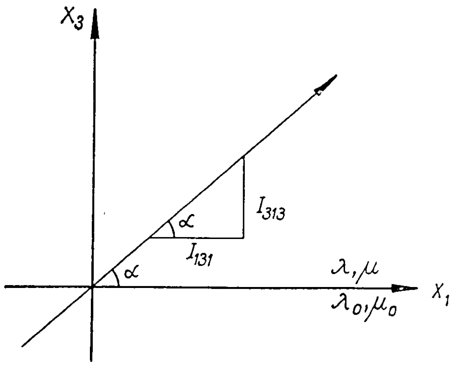

Fig. 4.

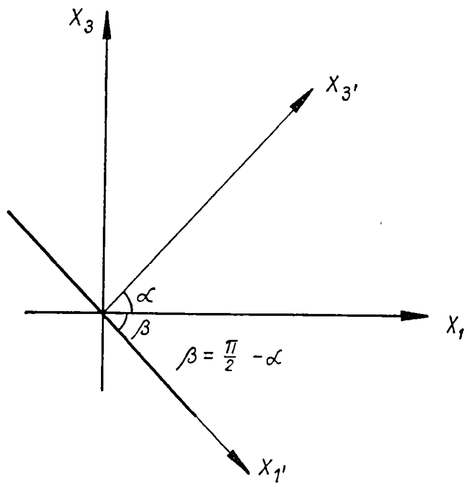

Fig. 5.

This case corresponds to edge dislocation field and is represented by components $I^{\alpha\beta\gamma}$ with $\alpha \| \gamma; \alpha \perp \beta$ (symbolic notation), Fig. 4. Only different from zero components are here I_{131}, I_{313}.

To determine the value of ratio $(I_{313})/(I_{131})$ a simple geometric reasoning could be erroneous, as we don't know how to deal with dislocation line components. So, here we shall for rigorous proof return to transformation properties. We take transformation in x_1x_3 plane by simple rotation between the system $x_{1'}x_{3'}$ and x_1x_3. In the system $x_{1'}x_{3'}$ we assume that $I_{1'3'1'}=0$ and dislocation field is represented only by a component $I_{3'1'3'}$ (Fig. 5). Then we get for the components in the x_1x_3 system $I_{313}=\sin\alpha I_{3'1'3'}$; $I_{131}=-\cos\alpha I_{3'1'3'}$
Hence

$$\frac{I_{313}}{I_{131}}=-\mathrm{tg}\alpha \tag{45}$$

Using now boundary conditions (25) and second equation of (26) we get $(\dot{\beta}_{12}=0)$:

$$\mathrm{tg}\alpha_n=\mathrm{tg}\alpha_0+\frac{c_0}{I^0_{131}}\sum_1^n\dot{\beta}_{32} \tag{46}$$

Similarly combining first and second equation of (26) we get

$$\mu_n\mathrm{tg}\alpha_n=\mu_0\mathrm{tg}\alpha_0-\frac{c_0}{2I^0_{131}}(\dot{p}^n_{32}-\dot{p}^0_{32})$$

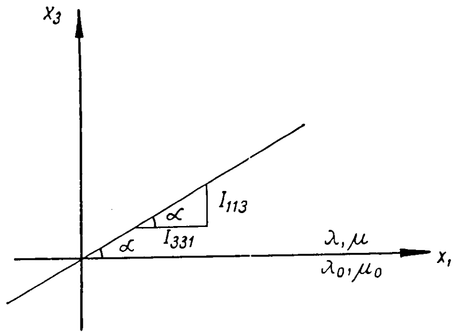

Fig. 6.

but together with consistency condition (27) we will return again to (46).

Assuming that $\sum_1^n\dot{\beta}_{32}=0$ we get here

$$\alpha_n=\alpha_0 \tag{47}$$

as the condition for propagation direction.

Case D: dislocation lines in the x_1x_3 plane, Burgers vector parallel to dislocation line, direction of propagation perpendicular to dislocation lines but in the same plane x_1x_3 (Fig. 6).

This process is connected with the p_{32}, p_{12} components. The p_{32} component is involved in boundary conditions.

This case is represented by screw dislocation field, respective current components are $I_{\alpha\beta\gamma}$; $\alpha \parallel \beta$; $\alpha \perp \gamma (I_{113}, I_{331})$. Their ratio equals to

$$\frac{I_{113}}{I_{331}} = \mathrm{tg}\alpha \tag{48}$$

by the similar arguments as used for derivation (45). According to asymmetry of I_{abc} this ratio is just reverse of ratio given by (45).

Therefore we can easily get the corresponding equation by putting instead of tgα the value-ctgα:

$$\mathrm{ctg}\alpha_n = \mathrm{ctg}\alpha_0 - \frac{c_0}{I^0_{131}} \sum_1^n \dot{\beta}_{32} \tag{49}$$

and for $\sum_1^n \dot{\beta}_{32} = 0$ we get as before

$$\alpha_n = \alpha_0 \tag{50}$$

6. Application to earth structure model

Our starting point for discussion of dislocation flow are data concerning parameters μ, ρ, c in the earth mantle up to 700 km on one side

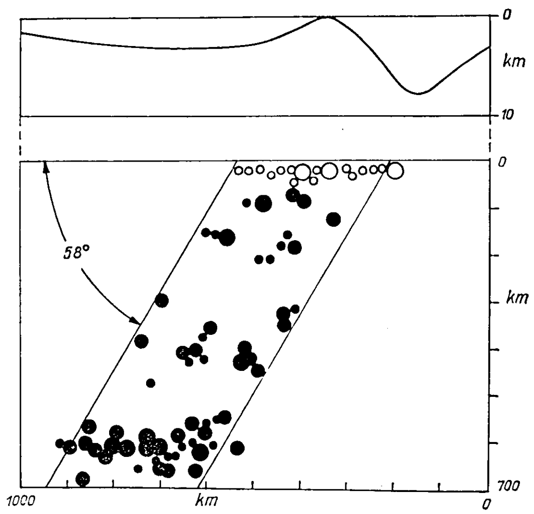

Fig. 7.

and the Benioff's graphs[16] representing typical tectonic models as given by hypocentral planes on the other side (Fig. 7 and Fig. 8). Our idea is that earthquakes which originate along a hypocentral plane are mutually connected. It means we assume that this plane represents a path of dislocation flow. However, we will not enter here into consideration where this flow originates. Before further discussion we should notice

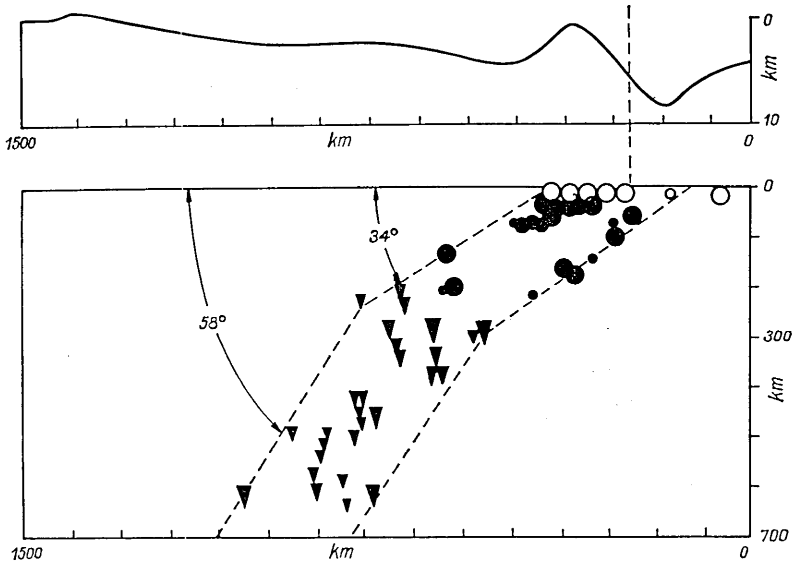

Fig. 8.

that estimation of hypocentral planes itself as well as angles of their inclinations were based on rather widely scattered points representing earthquake foci. For the case A of screw dislocation flow ($\vec{b} \parallel \vec{d\xi}$, $\vec{d\xi} \parallel \vec{i_2}$, $\vec{v} \perp \vec{i_2}$ $\vec{i_k}$-unit vectors of coordinate system) we get according to (37) the propagation path as straight line (Fig. 2).

For the case B of edge dislocation flow ($\vec{b} \perp \vec{d\xi}$, $\vec{b} \parallel \vec{v}$, $\vec{b} \perp \vec{i_2}$, $\vec{d\xi} \parallel \vec{i_2}$) according to equation (44) we get the path represented in Fig. 3, where angle α decreases with decreasing μ towards earth surface. Calculation was performed assuming for depth $z = 700$ km the value $\alpha = 60°$

For the case C of edge dislocation flow ($\vec{b} \perp \vec{d\xi}$, $\vec{b} \parallel \vec{v}$, $\vec{d\xi} \perp \vec{i_2}$, $\vec{v} \perp \vec{i_2}$) we get constant direction of flow.

For the case D of screw dislocation flow ($\vec{b} \parallel \vec{d\xi}$, $\vec{b} \perp \vec{v}$, $\vec{d\xi} \perp \vec{i_2}$, $\vec{v} \perp \vec{i_2}$) holds the same.

16) H. BENIOFF, "Seismic evidence for crustal structure and tectonic activity", Geol. Soc. Amer. Special Paper, **62** (1955), 61-73.

Cases A and B are here the most interesting as they represent the flow along hypocentral plane with dislocation line lying in this plane.

Relative value of the rate of stress accumulation can be now establish based on the given formulas.

From the second equation of (35) we get using (37), (31):

$$\frac{c_0}{2I^0_{221}}(\dot{p}^n_{32}-\dot{p}^0_{32})=\mu_n-\mu_0 \tag{51}$$

Similarly from (42) and (44) ($\dot{\beta}_{31}=0$) we can calculate relative course of $\dot{p}_{31}$ and $\dot{p}_{33}$:

$$\frac{c_0}{2I^0_{323}}(\dot{p}^n_{31}-\dot{p}^0_{31})=\mu_0-\mu_n \tag{52}$$

$$\frac{c_0}{2I^0_{323}}(\dot{p}^n_{33}-\dot{p}^0_{33})=\frac{1}{\mathrm{tg}\alpha_0}\left(\sqrt{\mu_0\mu_n\left(1+\frac{\mu_0-\mu_n}{\mu_0}tg^2\alpha_0\right)}-\mu_0\right) \tag{53}$$

Basing on the above given analysis it is possible to calculate the relative rate of stress accumulation for a given depth. (Fig. 9).

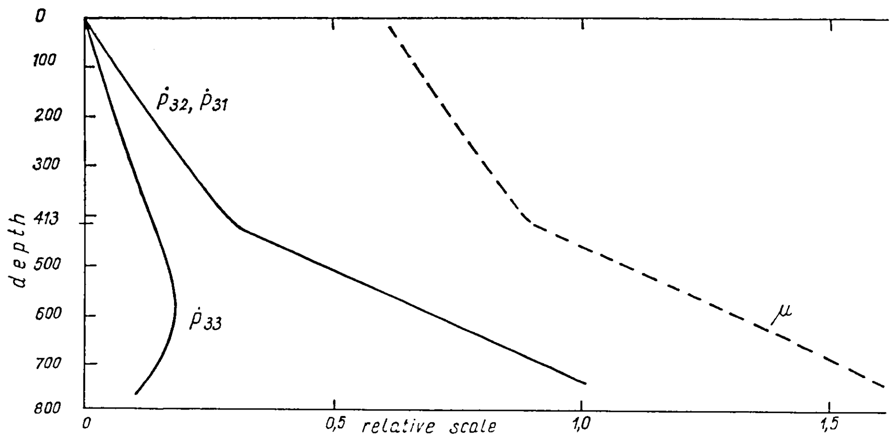

Fig. 9.

According to these preliminary studies we got thus the following results:

1) Assumption of dislocation flow explains roughly the earthquake distribution, the corresponding hypocentral surface forms curved surface for edge dislocation flow whose inclination angles fit to observational data (Figs. 3 and 8).

2) Changes of inclination of the above-mentioned surfaces are connected with changes of rigidity μ.

3) Relative value of stress rate $\dot{p}_{31}$, $\dot{p}_{32}$, $\dot{p}_{33}$ can be determined up to 700 km depth.

In conclusion we should add a few remarks. In the above given considerations on direction of dislocation movements we discussed the processes of dislocation flow, which represent a kind of "plastic" behaviour[17] of internal material in the earth.

It should be noted that these "plastic" properties of material could be to some extent modified just by existence of active slipping planes, as one can call planes of dislocation movements. Therefore such properties as angle of internal friction could be in reality influenced by structural changes due to dislocation processes.

In the given approach we have considered distortional deformation as a more general type of deformation in an elastic, horizontally stratified medium. Nevertheless, this deformation is characterized by some plastic features; therefore the possibility can be considered that stress-strain relation for distortional part of deformation (lower of equations (6)) should be governed by other (plastic) constants.

The above consideration were applied here to dislocation density field, as a kind of approximation of single dislocation distribution. Thus, it seems resonable to extrapolate those results also for the case of single dislocation. That case has some parallelism with the results obtained by Kasahara[18] on equilines of displacements due to a buried fault in a layered medium; however physical mechanism seems to be quite different.

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21. 成層構造における転位帯の発達

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前報 (震研彙報 **44** (1966), 153-165) で導入した連続的転位分布の理論を成層構造媒質に適用して，弾性定数の深さに対する変化が転位帯の発達の仕方にどのように反映するかを調らべた．基本型式が edge 型である転位帯が地下深所から地表に向つてななめに伸びている場合を想定すると，その傾きは剛性率の高い深所において急であり，地表に向い剛性率が低下するにつれて緩やかになることが期待される．これは島孤地震帯において見られる深発浅発地震発生面の様相 (第 8 図参照) と調和するように見える．

17) *loc. cit.* 12).

18) K. KASAHARA, *Bull. Earthq. Res. Inst.*, **42** (1964), 609.