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5. An Investigation into the Nature of Microtremors.

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1. Introduction

Studies of destructive earthquakes occurring throughout the world revealed that the design of earthquake resistant structures depends mainly on the physical properties of subsoil layers on which the structure rests. In general, damage is greater on soft and weak ground than it is on rigid ground. Through the extensive studies of microtremors, Kanai and others proposed that such properties of the ground could be deduced from the analysis of microtremors.

At present, however, there are two schools of thought concerning the nature of microtremors. The first school says that microtremors are mainly steady state body waves[1]. The second says they are mainly surface waves[2]. Therefore, a better understanding of the nature of microtremors is essential, before they can be applied to a problem. This paper describes an approach to find out which one of the two schools of thought really explains the nature of microtremors.

Several steps were carried out in order to approach the present

1) M. ISHIMOTO, *Bull. Earthq. Res. Inst.*, **15** (1937), 697-704.
K. KANAI, T. TANAKA and K. OSADA, *Bull. Earthq. Res. Inst.*, **32** (1954), 199-209.
K. KANAI and T. TANAKA, *Bull. Earthq. Res. Inst.*, **39** (1961), 97-114.
K. KANAI, T. TANAKA and S. YOSHIZAWA, *Bull. Earthq. Res. Inst.*, **43** (1965), 577-588.
E. SHIMA, *Bull. Earthq. Res. Inst.*, **40** (1962), 187-259.

2) P. BYERLY, *Bull. Seism. Soc. Amer.*, **37** (1947), 291-297.
Y. TOMODA and K. AKI, *Zisin*, [ii], **5** (1957), 17-22, (in Japanese).
K. AKI, *Zisin*, [ii], **8** (1955), 99-107, (in Japanese).
K. AKAMATSU, *Zisin*, [ii], **9** (1956), 21-39 (in Japanese); *Bull. Earthq. Res. Inst.*, **39** (1961), 23-75.
K. AKI, *Bull. Earthq. Res. Inst.*, **35** (1957), 415-456.
K. TAZIME, *Rep. Seism. Explor. Group of Japan*, **37** (1965), 27-32, (in Japanese).

problem. The first step was to determine the underground structures and the elastic constants of the subsoil layers. This was done by obtaining P- and S-wave information at various sites in the Tokyo Metropolitan Area and has been reported already[3]. The second step was to obtain microtremor recordings at those places in order to determine the predominant frequencies of microtremors. The third step was to obtain the spectral responses of the ground, which were derived through the application of the multiple reflection theory. This step was needed to qualify the previously mentioned first school of thought concerning the nature of microtremors. The fourth step was to derive the dispersion curves of Rayleigh and Love waves and to calculate the frequencies of the minimum group velocities at each place. This step was needed to qualify the second school of thought concerning the nature of microtremors.

After proceeding through these steps, the predominant frequencies of the power spectral density functions of the microtremors were compared with both of the frequencies of the minimum group velocities of Rayleigh and Love waves, and with the predominant peaks of the spectral responses of the ground.

2. Experiments and Results

The experiments were carried out at Otaku in the Tokyo Metropolitan Area and can be seen in Fig. 1, as follows:

1. Yukigaya Junior High School
2. Ikegami Primary School
3. Ikegami Second Primary School
4. Omori Senior High School
5. Onnazuka Primary School
6. Omori Third Primary School

Seismic exploration was done at the aforementioned places to obtain the ground structures and the elastic constants of the different layers of each location.

The summary of the derived underground structures for P- and S-profiles and the geological formations are shown in Fig. 2.

Microtremor recordings were made at the same places with two

3) H. KAWASUMI, E. SHIMA, Y. OHTA, M. YANAGISAWA, A. ALLAM and K. MIYAKAWA, *Bull. Earthq. Res. Inst.*, **44** (1966), 731-747, (in Japanese).

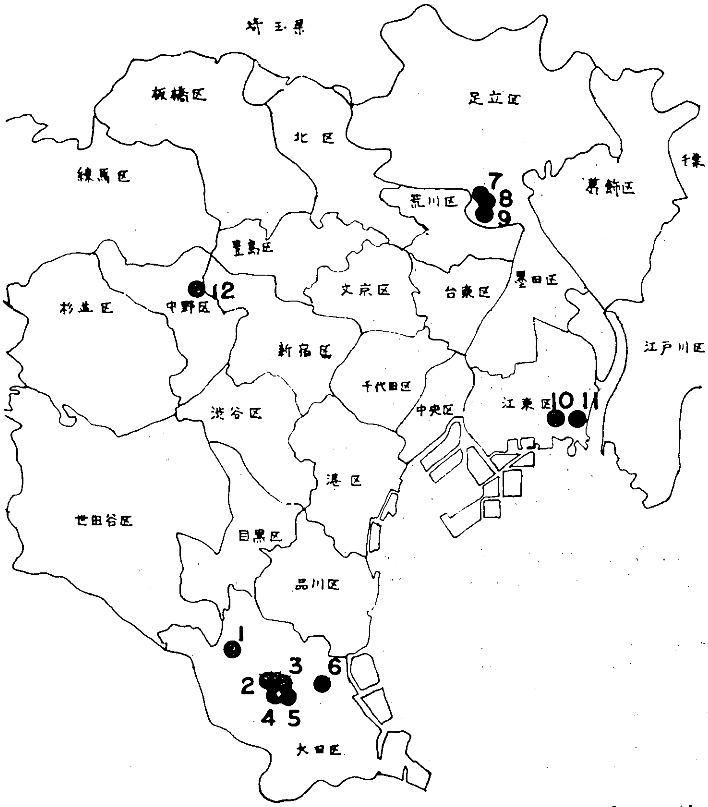

Fig. 1. Map of the Tokyo Metropolitan Area showing the observation sites.

horizontal components to determine the predominant frequency of microtremors at each site. We used two components and arranged them in two different directions so that we could see the effect of outside disturbances on the recordings.

The microtremors were recorded by a velocity type seismometer having a natural period of one second. The signals from the seismometers were integrated and amplified. Hence, the output signals are proportional to the ground displacement.

The only theory which is applicable to analyze the microtremors is the generalized harmonic analysis[4]. Thus, the autocorrelation functions

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4) For instance, see,
W. B. DAVENPORT and W. L. ROOT, *An Introduction to the Theory of Random Signals and Noise*(McGraw-Hill, 1958).
N. WIENER, *Extrapolation, Interpolation and Smoothing of Stationary Time Series with Engineering Application* (John Wiley & Sons 1950).

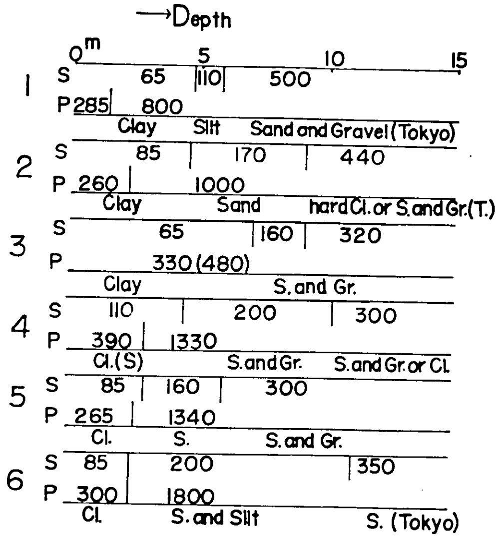

Fig. 2. Summary of the derived underground structures for P- and S-profiles and the corresponding geological formations.

were calculated by means of the automatic correlator, through which the power spectral density functions were derived.

The dispersion curves of Rayleigh and Love waves were derived from the P- and S-wave information to ascertain the frequency of the minimum group velocity at each location. As we know nothing about the source spectrum, we assumed that the spectrum of the noise source is white, which implies that the amplitude is maximum when the group velocity is minimum.

The spectral responses were derived to study the vibrational characteristics of the ground at these sites. These could be computed from the known elastic constants and densities of different layers of the ground at the site by the application of the theory of multiple reflections of seismic waves in the soil layers[5].

The ground structure, which was derived from S-wave information, is not the same as that derived from P-wave information. Due to this fact, we derived the dispersion curves of Rayleigh waves for two cases, A and B; (Fig. 3). We assumed two layers in case A and three layers in case B. The thickness of the first layer in case A is equal

5) R. TAKAHASI, *Bull. Earthq. Res. Inst.*, 33 (1955), 259-264.
K. KANAI, *Bull. Earthq. Res. Inst.*, 30 (1952), 31-37; *Bull. Earthq. Res. Inst.*, **31** (1953), 219-226.

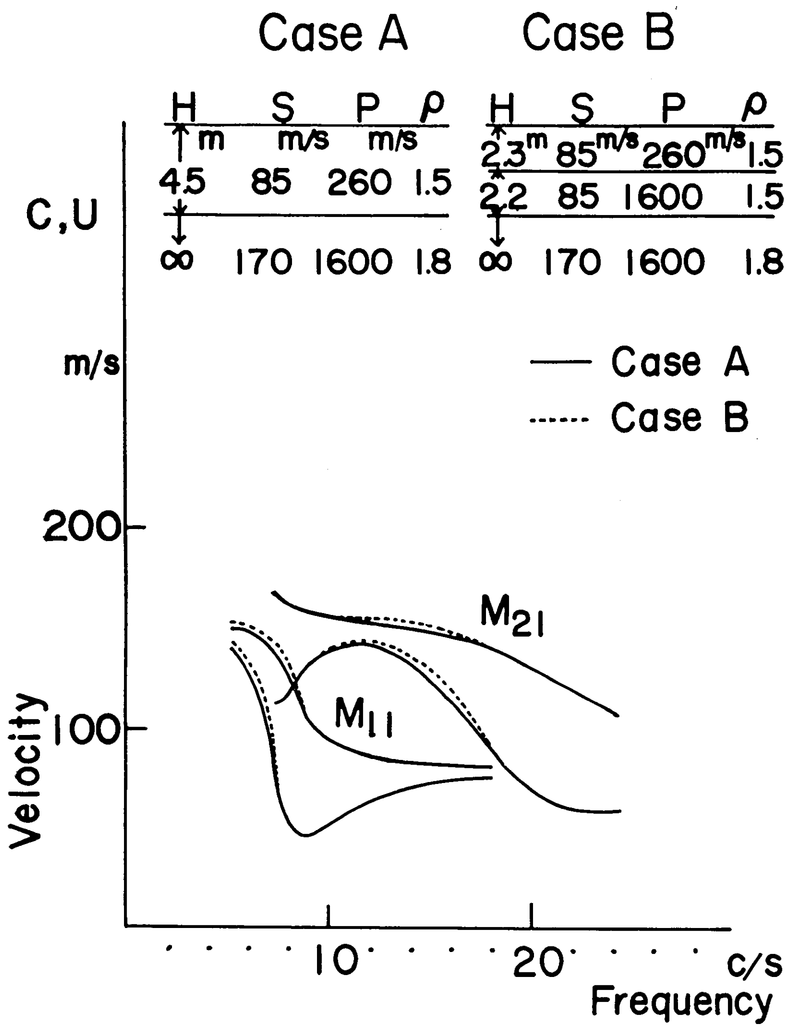

Fig. 3. Dispersion curves of Rayleigh waves for two cases, A and B.

to the sum of the thicknesses of the first and second layers in case B. All the other parameters are the same in both cases except for the P-wave velocity in the second layer of case B. As can be seen from Fig. 3, there is no significant difference between the dispersion curves of M_{11} and M_{21} in both cases. This proves that the frequency of the minimum group velocity in Rayleigh waves will not change much even if the structure derived from P- waves is different from that derived from S-waves. Hence, in our cases, the differences in S- and P-profiles will not make a significant change in the derivation of the dispersion curves for Rayleigh waves.

If any physical phenomenon is considered to be a stationary random process, then the properties of such a phenomenon can hypothetically be considered at any instant of time by computing average values over the collection of sample functions which describe the process[6]. As microtremors are good examples of such stationary random processes, in our problem we used recordings covering about five minute intervals. We took four separate intervals of time from the recordings, each interval lasted about fifty seconds. The power spectral density function for each was calculated.

Fig. 4 shows a comparison of the microtremor derivations at three places for both of the two horizontal components, H_1 and H_2. The power spectral densities for four sample functions were calculated from each component as represented by A, B, C, and D. The dark line represents the sample mean of the sum of the four individual sample functions. At Omori, the predominant peak is at frequency 2 cps in H_1 and H_2. At Omori Third the predominant frequency for H_1 is at frequency 3.7 cps, while in that of H_2 there are two peaks, one at 3.7 cps and the other at 4.5 cps. At Onnazuka there are two peaks in H_1, one at 2 cps and the other at 3.7 cps, while in H_2 one is at 2 cps and the other is at 2.7 cps.

Comparing these results, we found that in Omori the predominant frequency in H_1 coincides with that of H_2. In Omori Third, the coincidence is not as good as that in Omori. In Onnazuka there is even less of a coincidence between H_1 and H_2. This may be due to the fact that long sample records should be required for the analysis of microtremors. In Omori the sample time was sufficient to give a good sample mean. However, in Omori Third and Onnazuka the sampling time should have been much longer to obtain good sample means. Also the number of sample functions should be increased. Hence, the sufficient sampling time and the number of sample functions must be selected for each individual case.

We feel our assumption is justifiable in calculating the sample means of the power spectral density functions in our cases. Also, to record

6) S. GOLDMAN, *Information Theory* (Prentice-Hall, 1953).
loc. cit., 4).
J. S. BENDAT and A. G. PIERSOL, *Measurement and Analysis of Random Data* (John Wiley & Sons, 1966).

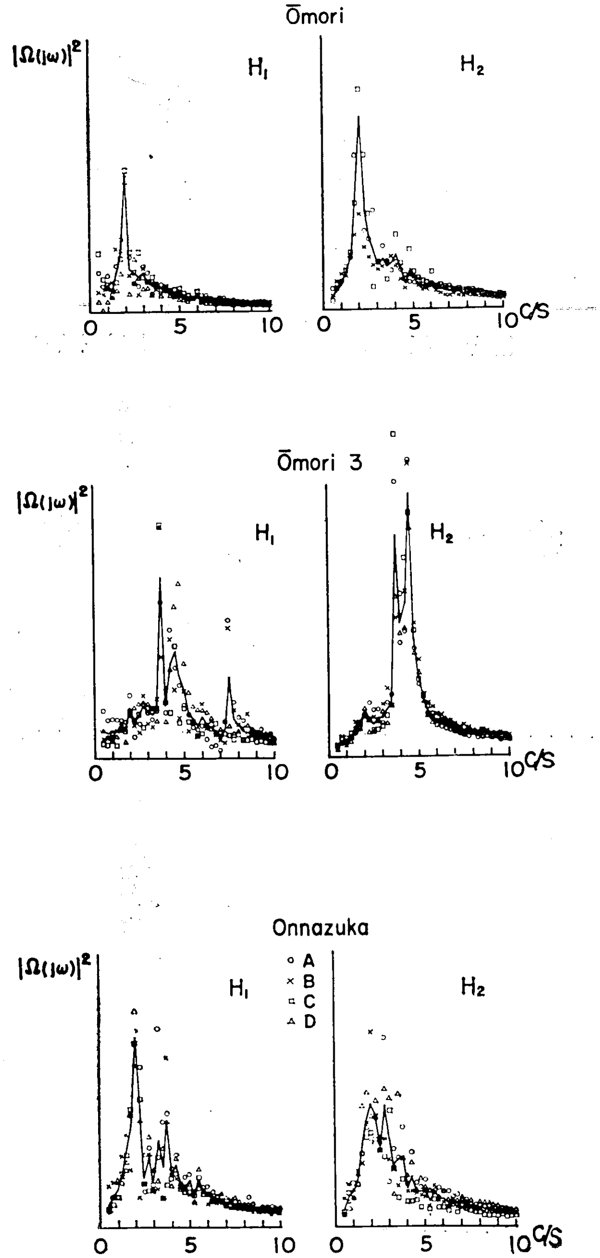

Fig. 4. The microtremor derivations at Omori, Omori 3rd and Onnazuka for both H_1 and H_2. The dark line represents the sample mean of the sum of the four individual sample functions A, B, C and D.

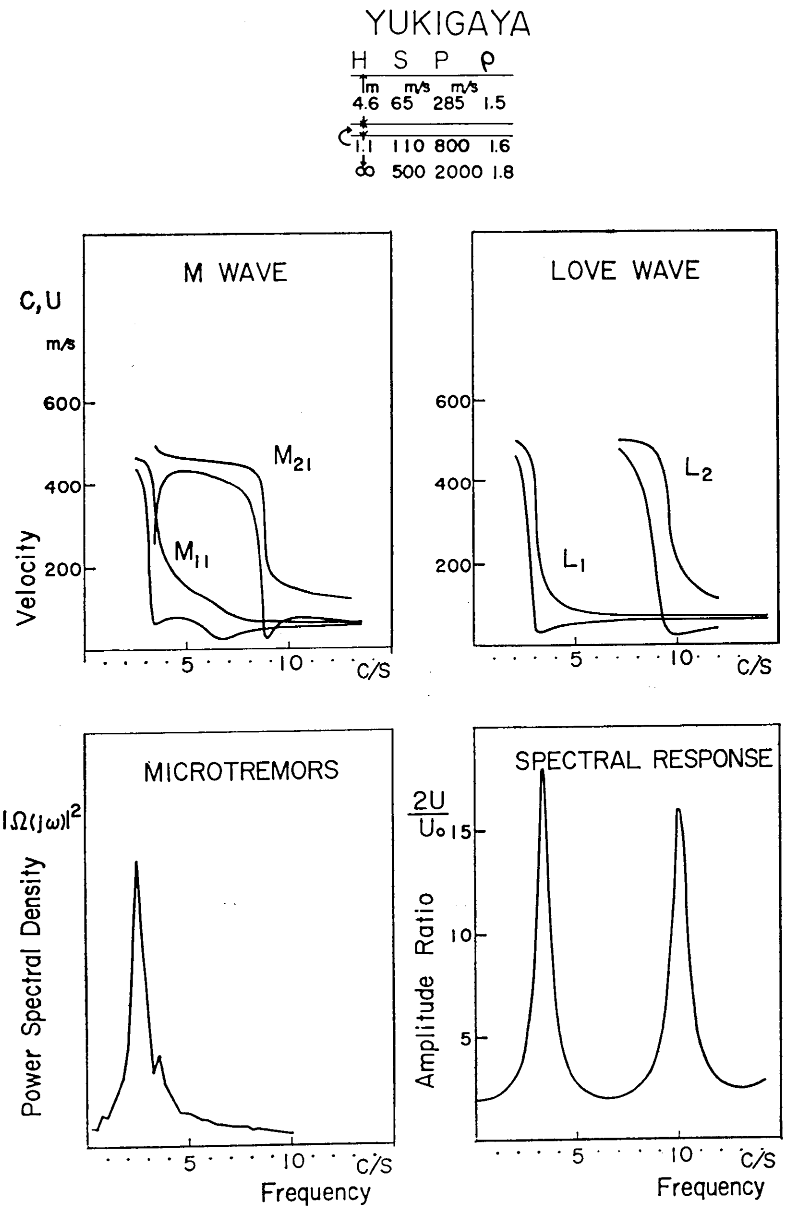

Fig. 5. The derivations at Yukigaya Junior High School. The underground structure at Yukigaya is shown at the top of the page.

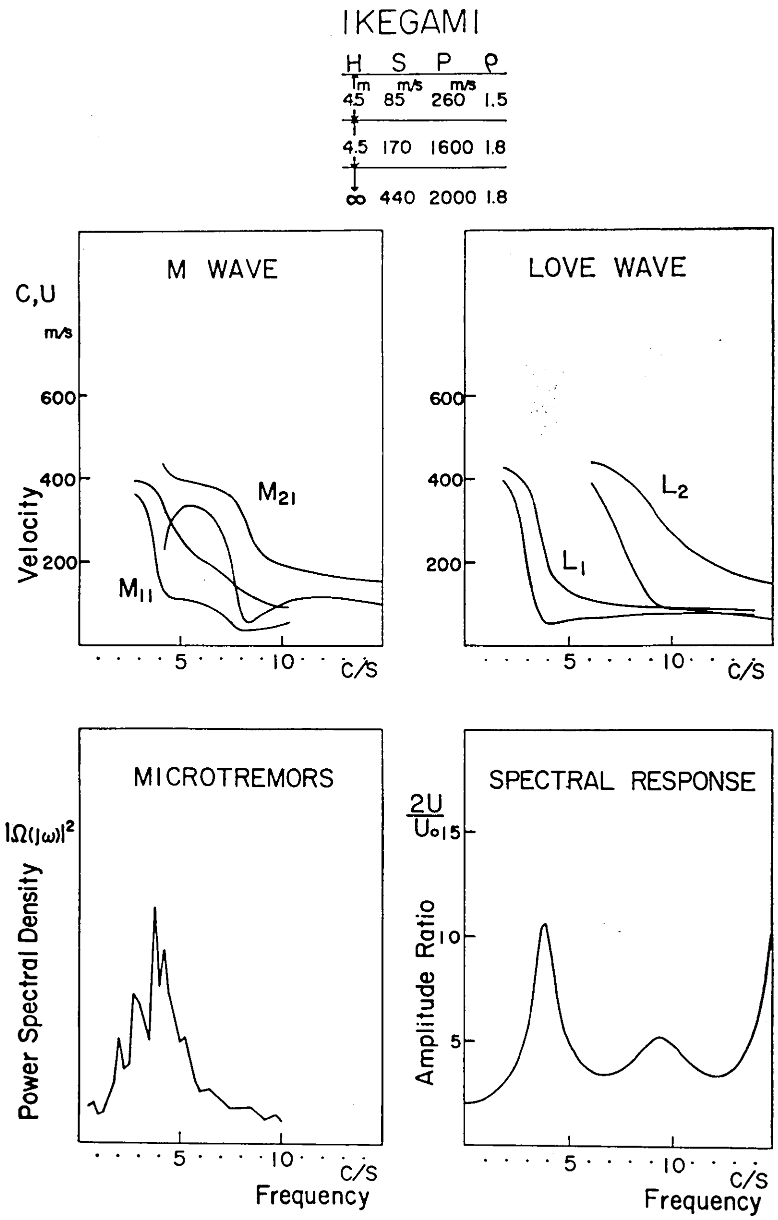

Fig. 6. The derivations at Ikegami Primary School. The underground structure at Ikegami is shown at the top of the page.

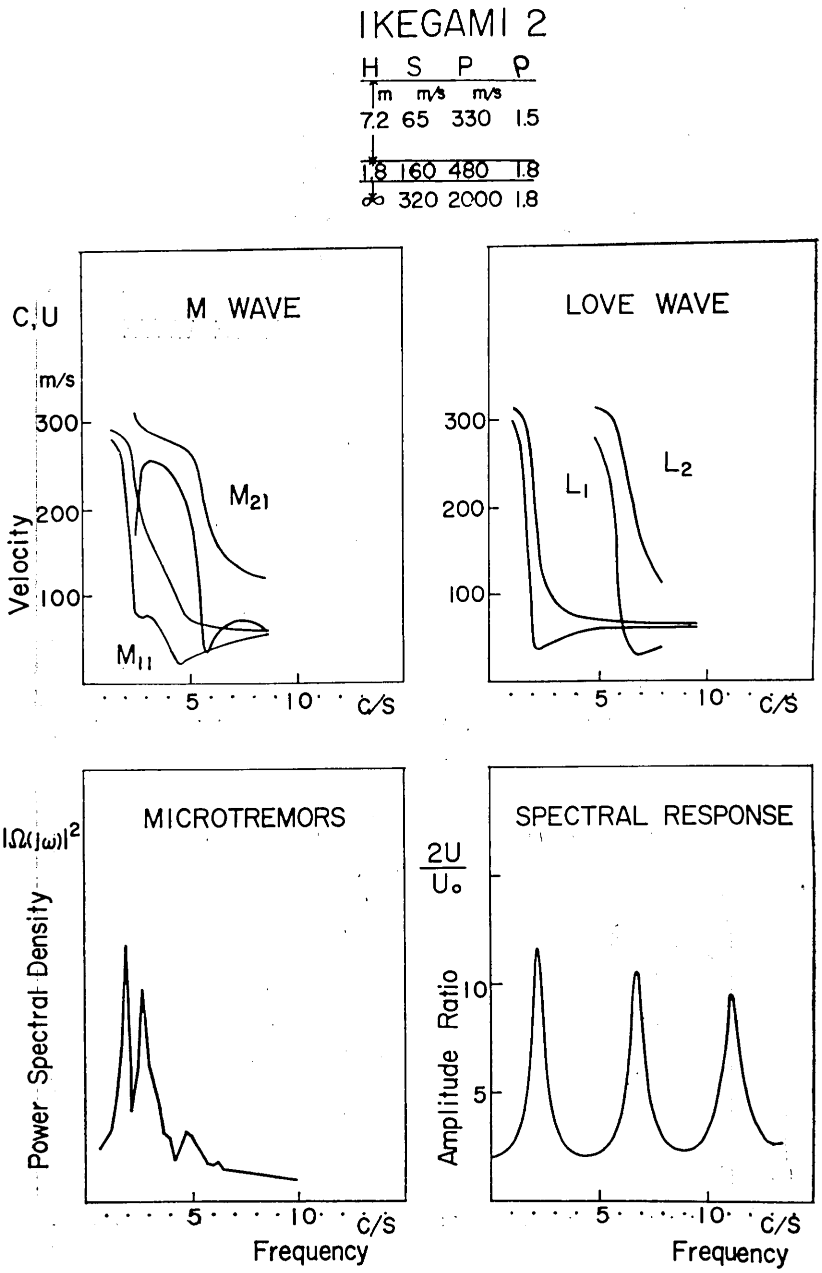

Fig. 7. The derivations at Ikegami 2nd Primary School. The underground structure at Ikegami 2nd is shown at the top of the page.

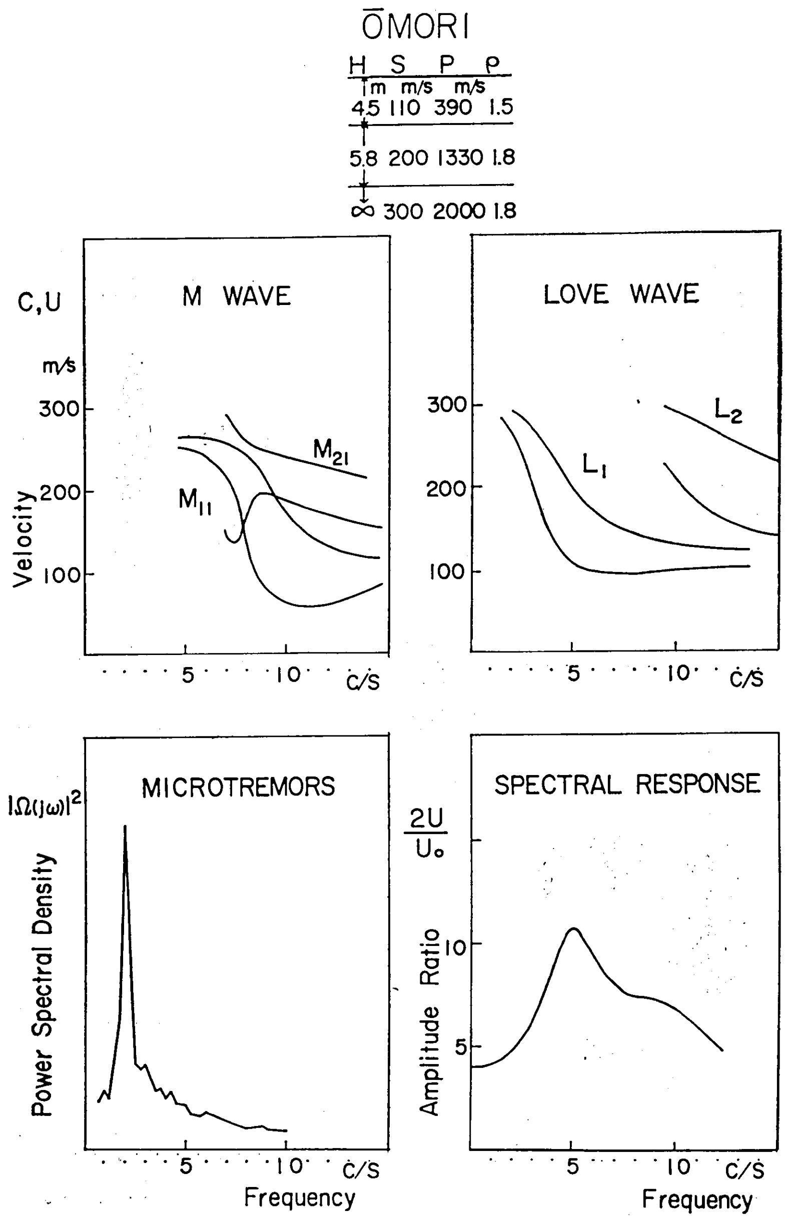

Fig. 8. The derivations at Omori Senior High School. The underground structure at Omori is shown at the top of the page.

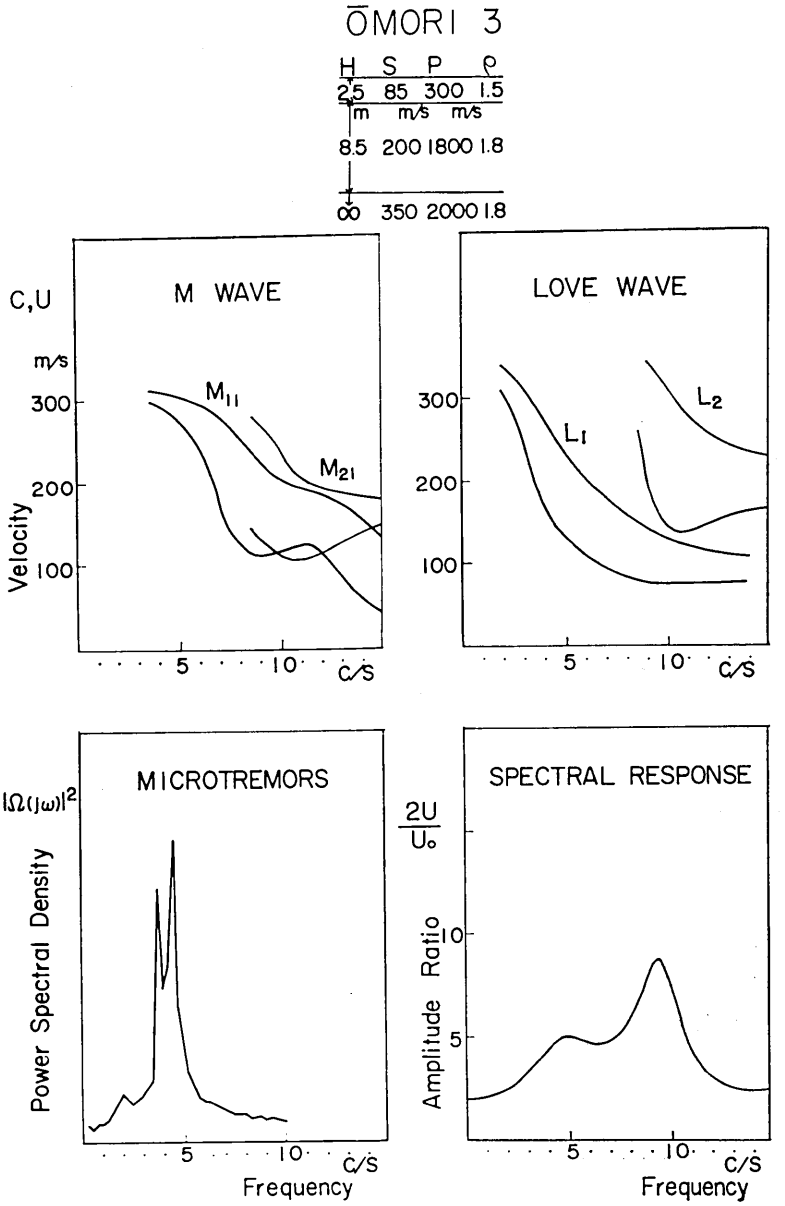

Fig. 9. The derivations at Omori 3rd Primary School. The underground structure at Omori 3rd is shown at the top of the page.

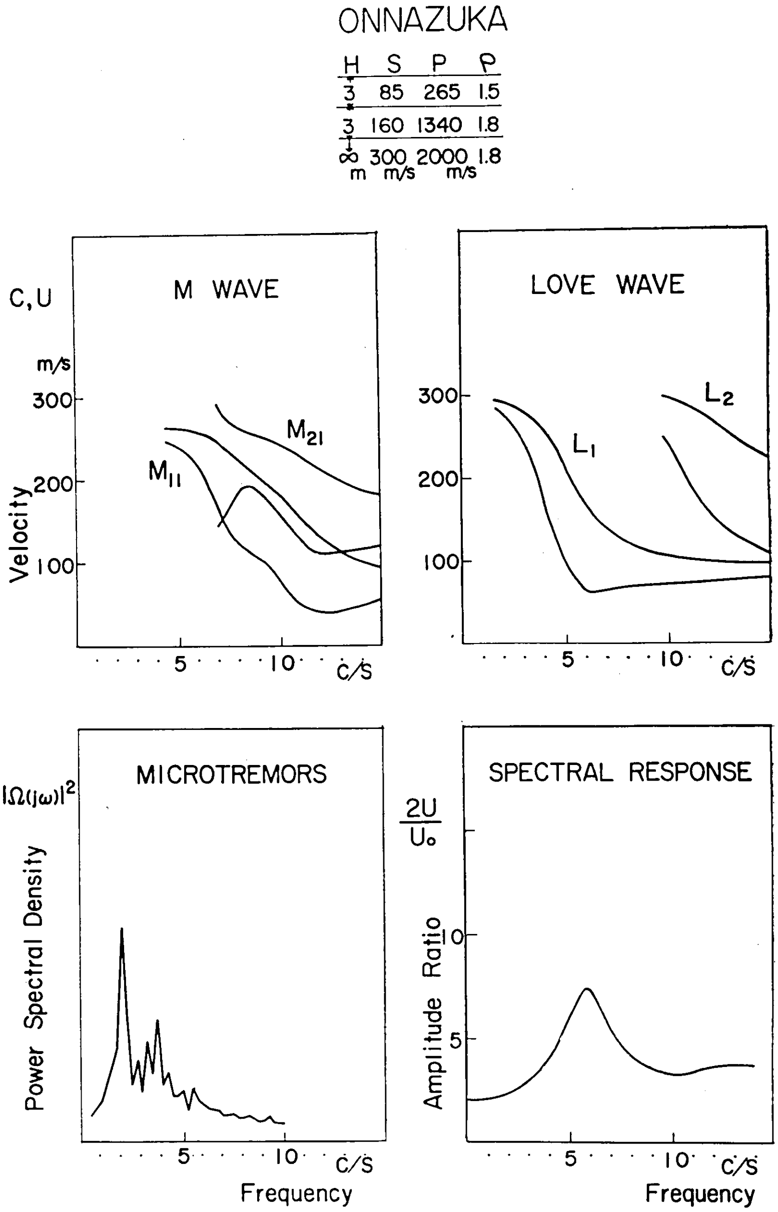

Fig. 10. The derivations at Onnazuka Primary School. The underground structure at Onnazuka is shown at the top of the page.

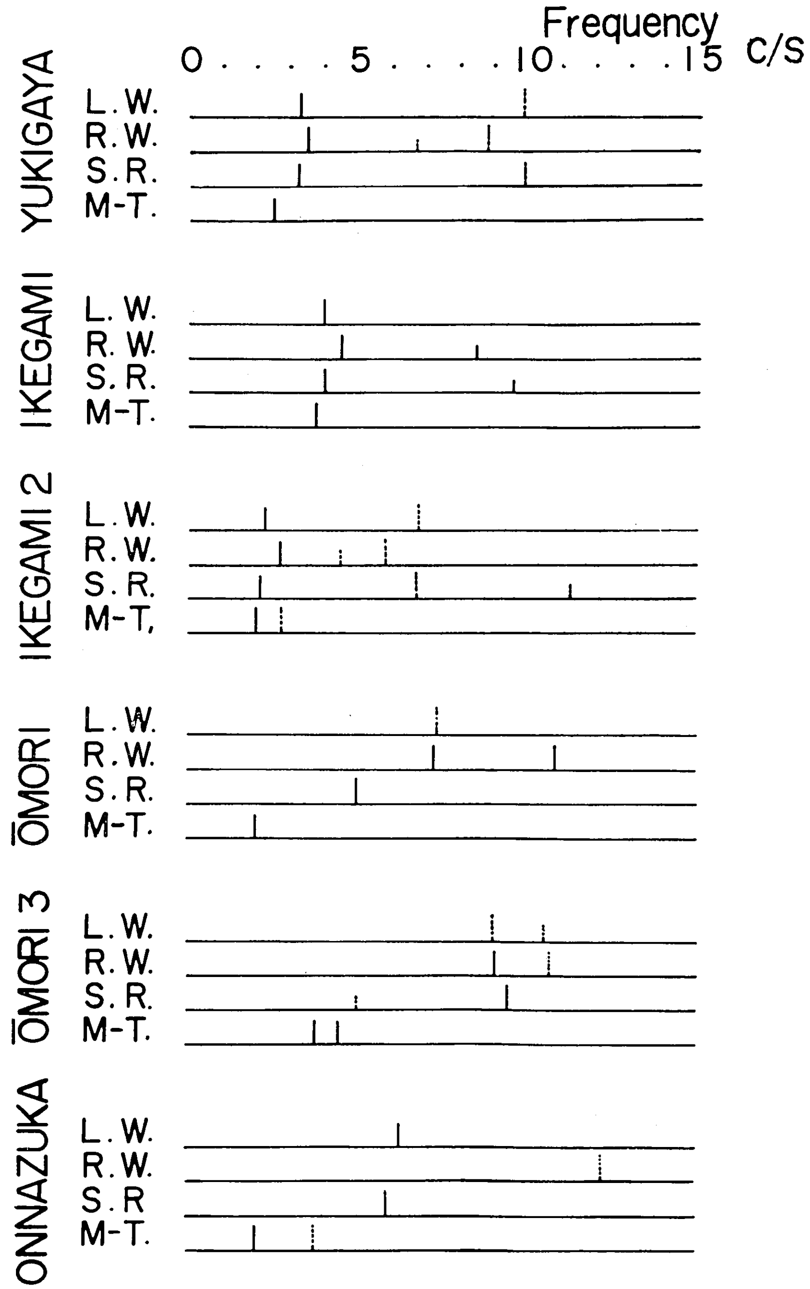

Fig. 11. Summary of all the results. The frequencies of the minimum group velocities of Love waves and those of Rayleigh waves, the predominant frequencies of the spectral responses and those of microtremors are represented by the long dark strips. The other strips show the ones which are not so clear.

microtremors at any site, we can use only one of the two components.

Our first place to make a comparison between the results is Yukigaya Junior High School. The underground structure at Yukigaya is shown in Fig. 5. The frequency of the minimum group velocity of M_{11} is 3.3 cps, of L_1 3.2 cps, the predominant frequency of the spectral response which was calculated from the assumed structural profiles through the application of the multiple reflection phenomenon is 3.2 cps, and of the microtremors which was calculated from the experimental results of the sample functions is 2.5 cps.

In the same way, derivations of the other places are shown in Fig. 6 to Fig. 10.

Fig. 11 shows the summary of all the derivations. We can divide our results into two groups; the first group is Yukigaya, Ikegami and Ikegami Second. In this group the frequencies of the minimum group velocities of Love waves, Rayleigh waves and the peak frequencies of the spectral responses coincide with those of the microtremors. The second group is Omori, Omori Third and Onnazuka. In this group no significant correlation between them was found. It is noted that the peak frequencies of the spectral responses agree well with the frequencies of the minimum group velocities of Love waves, even in the case of the latter group. However, no attempt was made to explain the above mentioned result in the present paper, and this has been left for future study.

3. Discussion

As was shown in the first group, there is a strong correlation between the frequencies of the minimum group velocities of Rayleigh waves, Love waves and the peak frequency of the spectral response and of the microtremors. According to the boring data near the sites, the third layers of these sites are believed to be hard, there being no indication of the existence of a softer layer beneath the third layer. In other words, the boundaries of the subsoil layers and the basement are distinct in these sites. And it may be that the increment of the wave velocities beneath the third layer is small enough so as not to effect much the shape of the dispersion curves.

So, at least in these sites, we can say that the microtremors can be treated either as steady state body waves and/or as surface waves with the same result.

Regarding the second group there is no significant correlation between the various frequencies. We believe this is due, at least, to two causes; first, the microtremors have lower energies at higher frequencies, due to absorption, etc. Second, the wave lengths of the surface waves are very long at the frequency of microtremors, so that the dispersion curves will be affected by the wave velocity of the deeper portion. Actually, the boring data showed the possibility of the existence of low velocity layers at depth, however, we could not get the insitu S- and P-wave information at such depths. For further study, the necessity of this type of information is certainly indicated.

4. Summary

Many seismologists obtained different results concerning the nature of microtremors. Some proposed that they are mainly steady state body waves, and others suggested that they are mainly surface waves. To qualify each hypothesis the following steps were followed:

1. P- and S-wave information was obtained at Otaku in the Tokyo Metropolitan Area, from which the elastic constants, densities and underground structures were derived.
2. Microtremors were recorded at the same locations. The power spectral density functions were computed to determine the predominant frequency of the microtremors of each place.
3. The dispersion curves for Rayleigh and Love waves were derived to determine the frequencies of the minimum group velocities.
4. The spectral response was computed through the application of the multiple reflection phenomena to determine the predominant peak of the ground response.

After proceeding through all of these steps, the predominant frequencies of the power spectral density functions of the microtremors were compared with the frequencies of the minimum group velocities of Rayleigh and Love waves. Also, they were compared with the predominant frequency of the spectral response of the ground. From these comparisons, we may conclude that the microtremors can be treated as steady state body waves and/or as surface waves with the same results at the places where the boundary of the subsoil and the basement is distinct. However, for a definite conclusion more detailed study along this line is inevitable.

5. Acknowledgements

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5. 常時微動について

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常時微動は，交通機関などの人工的震動源又は風などによる自然震動源により誘起された震動が四方八方から集って来たものと考えられている．したがって，常時微動は，これ等の波が伝播してくる途中の地盤の物理的性質に関する情報をたくさんふくんでいるに違いない．このような観点から，工学的な目的に積極的に常時微動を利用しようという研究が年々盛んになりつつある．

常時微動をこのような目的に利用しようとすれば，先ずその本性を知っておかねばならないことは当然である．しかるに，これに関しては，現在，主として実体波からなるとする立場と，主として表面波からなるとする全く相反した2つの意見が対立している．本論文では，このような問題を解決する一つのこころみとして，先年都内でS波による地震探査を実施した調査地のうち大田区呑川流域の6ヶ所において常時微動の観測を行ない，その卓越周波数と，S波の地下構造から予想されるRayleigh波，Love波の極小群速度を与える周波数，又S波が真下から入射するとして重複反射をする場合に期待される卓越周波数を比較することにより，どれと一番あうかを調べた．その結果基盤と地表層がはっきりわかれているようなところでは，上記の周波数は皆よい一致を示すことが解った．つまりこのような場所では，常時微動が，実体波からなるとしても，表面波，或いは両方からなるとしてとりあつかっても卓越周期に関する限り結果は違はないことになる．