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77. *Deformation of an Anisotropic Visco-elastic Medium due to Internal Force under Initial Stress and its Significance in the Activities of Earthquake Swarms.*

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1. Introduction

The earthquake swarms at Matsushiro, during their daily activities, have been raising up many questions among earth scientists about the process and nature of earthquake genesis. The precise observations, which have been currently undertaken, are successfully revealing many interesting phenomena of such earthquake activities. At the present stage of such investigation, it will not be too early to make any mathematical approach for finding the possible kinematical process involved in the activities of earthquake swarms. Such mathematical analysis would be useful for advancing the understanding of mutual relations among various observational quantities and also for clarifying the possible mechanism of earthquake genesis.

As for the kinematical process of earthquake swarms, there are several existing theories. T. TERADA[1)] regarded the train of earthquake swarm as a certain statistical distribution of which example is seen in the fall of camellia flowers. M. ISHIMOTO[2,3)] and F. KISHINOUYE[4)] independently presented the theories that the earthquake swarms are accompanied by the crustal deformations. T. MATUZAWA[5,6)] has remarked that the spatial distribution of aftershocks is restricted within the uplifted

1) T. TERADA, *Bull. Earthq. Res. Inst.*, **10** (1932), 29-35.
2) M. ISHIMOTO and R. TAKAHASI, *Bull. Earthq. Res. Inst.*, **8** (1930), 438 and 456.
3) M. ISHIMOTO, *Zisin*, [i], **9** (1937), 108-117.
4) F. KISHINOUYE, *Zisin*, [i], **8** (1936), 590-591; *Bull. Earthq. Res. Inst.*, **15** (1937), 785-827.
5) T. MATUZAWA, *Bull. Earthq. Res. Inst.*, **14** (1936), 38-67.
6) T. MATUZAWA, *Study of Earthquakes* (Uno-shoten, Tokyo, 1964), III.

area. S. KUSAKABE[7] and H. BENIOFF[8] have regarded the sequence of aftershocks as the phenomena of the elastic aftereffect. K. MOGI[9,10] has presented the view that the earthquake swarms are the phenomena of elastic shock in the heterogeneous media. These theories have made valuable contributions for advancing the understanding of the genesis mechanism of earthquake swarms. However, since these theories are not formulated as problems in the field of kinematics, they are not always clear regarding what boundary conditions are postulated in the process of earthquake swarms. Therefore, when one tries to formulate kinematical processes involved in these theories, one often comes across several ambiguities.

Thus, in this paper, an attempt is made to formulate a kinematical process for the activity of earthquake swarms as a boundary value problem. The purpose of such an attempt is to derive various mutual relations which are thought to exist among observational quantities about the earthquake swarms.

The problem is settled as follows. Firstly, a visco-elastic medium is postulated as the medium within which earthquake swarms take place. Secondly, the medium is supposed to be under initial stress. Thirdly, certain internal forces are supposed to act as the source of deformation. It is well known that the activity of an earthquake swarm is accompanied by the crustal deformation, which remains almost permanently even after the cessation of its activity. In order to account for such a semi-permanent deformation and also for the time dependency of the activity, it will be natural to postulate that the medium is visco-elastic. The viscous property of the medium is thought to be due to the weathered substances, liquid or liquified materials which exist along discontinuities of various kind within the hard rock mass. The elastic property is supposed to be due to the hard part of the rock mass of the earth's crust. The medium is supposed to be composed of the laminations of the elastic elements and viscous elements. Anisotropy, which appears from the lamination model, has been taken into consideration throughout the analysis as a generality of the treatment.

It will be a widely accepted view that earthquake swarms are the phenomena which take place under initial stress. In spite of this general

7) S. KUSAKABE, *Publ. Earthq. Invest. Comm.*, **17** (1904), 33.
8) H. BENIOFF, *Bull. Seis. Soc. Amer.*, **41** (1951), 31-62.
9) K. MOGI, *Bull. Earthq. Res. Inst.*, **40** (1962), 107-124.
10) K. MOGI, *Bull. Earthq. Res. Inst.*, **41** (1963), 615-658.

view, it seems that the study on the effect of the initial stress is not always satisfactory. Recently, however, M. A. BIOT,[11] has developed a method of the systematic treatment for the mechanics of deformation under initial stress. This enables us to deal with the above problem.

In order to formulate the kinematical process of earthquake swarms as a boundary value problem, the internal forces acting at an interface are introduced as the source of the deformation. As regards the existence of such internal force for the development of earthquake swarms, there are several different views. The case of the non-existence of the internal force, however, will be included in this case as the limiting one of very small value of internal force.

Thus, in this paper, we will treat of the deformation of an anisotropic visco-elastic medium, composed of elastic and plastic elements, due to internal force under initial stress. For simplifying the mathematical analysis, we will assume that the medium is incompressible, the deformation restricted in the plane strain and the speed of deformation sufficiently slow as to allow the inertia term to be neglected. The effect of gravity is not taken into consideration in this paper. The internal force is supposed to act at a plane lying in the infinite medium. The deformation is also assumed to be local and vanishes at the large distance. As for the initial stress, the principal one is supposed to act along the direction of the coordinate. The internal force is supposed to act vertically to the interface like liquid pressure. Under these restrictions, the relation between the deformation and the internal force is derived by the method developed by BIOT.

In order to complete the problem we have to know the relation between the process of deformation and the activities of the earthquake swarm. The activities of earthquake swarm will be considered with respect to several parameters such as daily number of small shocks, magnitude of shocks, radiated energy, etc. In this paper, the frequency of small shock occurrence will be taken into consideration as the parameter of the activity. Then, the frequency of small shock occurrence will be related to the process of deformation by the ideas which have been developed in the theory of dislocation.

The occurrence of small shocks is considered to be accompanied by the sudden formation of small slips. These small slips are, according to the theory of dislocations,[12] believed to occur as the result of accumu-

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11) M. A. BIOT, *Mechanics of Incremental Deformations* (John Wiley, 1965).
12) A. H. COTTRELL, *Dislocations and Plastic Flow in Crystals* (Oxford, 1953).

lation and migration of numerous dislocations. Moreover, these numerous dislocations are thought to be produced by the plastic deformation of the medium. In consequence of all this, the occurrence of small shocks will be considered to be related to the plastic deformation of the medium. As regards the quantitative relation between them, several formulae are given in the theory of dislocations.[13,14] In this paper, we will use the relation that the density of edge dislocations is proportional to the curvature of the plastic deformation. We also assume that the number of small shocks is proportional to the number of small slips, and that the latter is proportional to the density of dislocations. Thus, the number of small shocks is proportional to the curvature of the plastic deformation. If we represent this relation in terms of the time rate, it becomes that the frequency of small shock occurrence, for example, the daily number of small shocks, is proportional to the speed of the plastic deformation in the progress of the curvature. Since the curvature of the deformation is geometrically expressed by the spatial second derivative of the displacement, the above relation will be represented, as the first approximation, as being that the frequency of the small shock occurrence is proportional to the speed of plastic deformation as far as the time sequence is concerned, and to the curvature of the deformation as far as the spatial distribution is concerned. The total number of small shocks is proportional to the curvature of the plastic deformation. In this way, the activity of the earthquake swarm is related to the process of deformation of the medium.

In 2, the above problem is formulated as the boundary value problem by Biot's method, and the solution is given in the general form. In 3, the relation of the internal force and the deformation is obtained for the liquid-like internal force at the interface. In 4, the deformation responses are computed for several types of internal force with respect to a single harmonic component of the deformation. In 5, are examined the characteristic features of mutual relations among the number of small shocks, deformation of the medium, and the internal force. In 6, a consideration is given for the kinematical process of earthquake swarms.

2. Boundary value problem

In this section, we will show that the problem of deformation due

13) A. H. COTTRELL, *ibid.*, 12), p. 29.

14) J. J. GILMAN, "Microdynamics of Plastic Flow at Constant Stress," *Jour. Appl. Phys.*, **36** (1965), 2772-2777.

to internal force under initial stress is treated as the boundary value problem. The methods of analysis of a problem of this kind have been developed by BIOT.[15] We will follow his method.

For simplicity, we will assume that the media are anisotropic, visco-elastic, incompressible and that the deformation is restricted in plane strain. We will consider principal initial stress s_{11} and s_{22}, and assume $s_{12}=0$. The effect of gravity will not be taken into consideration in this paper. The internal forces are supposed to act along the interface $y=0$, and are functions of time and a co-ordinate x. The speed of development of the deformation is supposed to be so slow that the inertia effect can be neglected. Under the above restrictions we will formulate the problem following BIOT's method.

The mechanical system of the deformation of continuous media is, in general, completely described by the systems of those basic equations which are equilibrium equations, stress$\sim$strain relations, and strain displacement relations. First, the stress$\sim$strain relations will be derived for the media which are composed of elastic elements and plastic elements (2·1). As is shown by BIOT[16] the media are represented by an equivalent anisotropic, visco-elastic medium. From the system of basic equations, the field equation will be derived for displacement potential (2·2). The field equation is, then, solved under the boundary condition given at the interface (2·3 and 2·4).

The formulations will be represented in Laplace transform with respect to time t with parameter p. This procedure bases upon the corresponding principle[17] between visco-elastic media and purely elastic media, which states that Laplace transformed equations in the visco-elastic media have the same forms of equations in the elastic media, and that the deformation coefficients in the transformed space are expressed by functions of p. Therefore, the solutions will be first obtained in Laplace transformed space, the solutions as functions of t will then be obtained by applying the inverse transformation.

In order to treat the internal force which is a function of x, we will use the Fourier transformation with respect to x with parameter ξ. We will also consider the deformation which vanishes at large distances. That is, we will consider localized deformation within an infinite medium.

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15) M. A. BIOT, *ibid.*, 11).
16) M. A. BIOT, *ibid.*, 11), chap. 4, § 2.
17) M. A. BIOT, *ibid.*, 11), chap. 6, § 3, p. 359.

2.1. Media and their stress～strain relations

Let us suppose that the earth's crust is made up of laminated media of elastic elements and plastic elements. The elastic elements are thought to correspond to hard rock mass, and plastic elements to correspond to soft thin layers which develop along discontinuities within rock mass. These may consist of fractured zone, fissures, joints, unconformities or interface of plane of stratification. The property of plastic elements will be represented, for the first approximation, by viscous liquid. It is shown by BIOT[18] that the visco-elastic property of such laminated media is represented by an equivalent continuous visco-elastic media.

For simplicity let us assume that the materials are incompressible and the deformations restricted in plane strain. Then, according to BIOT[18], the stress～strain relations of such equivalent continuous media are given by

$$\begin{cases} s_{11}-s=2\hat{N}e_{xx} \\ s_{22}-s=2\hat{N}e_{yy} \\ s_{12}=2\hat{Q}e_{xy}\ , \end{cases} \tag{2.1}$$

where s_{11}, s_{22}, and s_{12} are the incremental stress components referred to the co-ordinate axis after deformation. s is the two-dimensional mean stress defined by

$$s=\frac{1}{2}(s_{11}+s_{22})\ . \tag{2.2}$$

Incremental strain components are defined by

$$e_{xx}=\frac{\partial u}{\partial x}\ ,\quad e_{yy}=\frac{\partial v}{\partial y}\ ,\quad e_{xy}=\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right), \tag{2.3}$$

where u and v denote the displacement component in x and y directions respectively. Incompressibility is expressed by the condition

$$e_{xx}+e_{yy}=0\ . \tag{2.4}$$

$\hat{N}$ and $\hat{Q}$ are deformation coefficients in operational form, and are expressed by those composing elements as

$$\begin{cases} \hat{N}=\alpha_1\hat{N}_1+\alpha_2\hat{N}_2 \\ 1/\hat{Q}=\alpha_1/\hat{Q}_1+\alpha_2/\hat{Q}_2\ . \end{cases} \tag{2.5}$$

18) M. A. BIOT, *ibid.*, 11), p. 360.

$\hat{N}_1$, $\hat{N}_2$, $\hat{Q}_1$, and $\hat{Q}_2$ are deformation coefficients of composing elements. α_1 and α_2 represent fractions of the unit thickness of the composite medium, and are

$$\alpha_1 + \alpha_2 = 1 \ . \tag{2.6}$$

The deformation is also expressed by alternative deformation coefficients $\hat{M}$ and $\hat{L}$, which are defined by

$$\begin{cases} \hat{M} = \hat{N} + \dfrac{1}{4} P \\ \hat{L} = \hat{Q} + \dfrac{1}{2} P \ , \end{cases} \tag{2.7}$$

$$P = S_{22} - S_{11} \ , \tag{2.8}$$

where S_{11} and S_{22} are initial stresses. These coefficients are used to represent the stress$\sim$strain relation referred to the co-ordinate axis before deformation. These coefficients are also expressed by those of composing elements as

$$\begin{cases} \hat{M} = \alpha_1 \hat{M}_1 + \alpha_2 \hat{M}_2 \\ 1/\hat{L} = \alpha_1/\hat{L}_1 + \alpha_2/\hat{L}_2 \ . \end{cases} \tag{2.9}$$

As a special case of the medium, if we take the purely elastic medium for the elastic elements and the purely viscous liquid for the plastic elements, the deformation coefficients of each material are, as given by BIOT,[19]

$$\begin{cases} \hat{L}_1 = L_1 \ , \quad \hat{M}_1 = M_1 \ , \\ \hat{L}_2 = \hat{M}_2 = \eta p \ , \end{cases} \tag{2.10}$$

and

$$\begin{cases} \hat{L} = \dfrac{p}{p+r} L_r \\ \hat{M} = M + pM' \ . \end{cases} \tag{2.11}$$

where

$$\begin{cases} L_r = L_1/\alpha_1 \ , \quad r = (\alpha_2 L_1)/(\alpha_1 \eta) \ , \\ M = \alpha_1 M_1 \ , \quad M' = \alpha_2 \eta \ . \end{cases} \tag{2.12}$$

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19) M. A. BIOT, *ibid.*, 11), p. 360.

η is the coefficient of viscosity being defined by

$$\begin{cases} t'_{12}=\eta\dfrac{\partial e_{xy}}{\partial t} \\ s_{11}-s=\eta\dfrac{\partial e_{xx}}{\partial t} , \end{cases} \tag{2.13}$$

where t'_{12} is the incremental stress component referred to the co-ordinate before deformation.

If the deformation coefficients of viscous elements are sufficiently small compared with the elastic element, namely, if $\hat{L}_1 \gg \hat{L}_2$ and $\hat{M}_1 \gg \hat{M}_2$, the equations (2.11) become approximately as

$$\begin{cases} \hat{L} \fallingdotseq \eta p/\alpha_2 \\ \hat{M} \fallingdotseq \alpha_1 M_1 . \end{cases} \tag{2.14}$$

This approximation will be used in 4 for the analytical evaluation of the force$\sim$displacement relation at the interface.

2.2. Field equations

Let us consider the case where the initial stress field is given by S_{11} and S_{22} referred to x and y axes (cf. Fig. 2.1). S_{11} and S_{22} are taken positive when they are extensive. Then the equilibrium equations[20] are given by

$$\begin{cases} \dfrac{\partial s_{11}}{\partial x}+\dfrac{\partial s_{12}}{\partial y}-P\dfrac{\partial \omega}{\partial y}=0 \\ \dfrac{\partial s_{12}}{\partial x}+\dfrac{\partial s_{22}}{\partial y}-P\dfrac{\partial \omega}{\partial x}=0 , \end{cases} \tag{2.15}$$

where P is the difference of initial stress as given in the equation (2.8), and ω is a local rotation defined by

$$\omega=\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right) . \tag{2.16}$$

Introducing the displacement potential ϕ, which is defined by

$$\begin{cases} u=-\partial\phi/\partial y \\ v=\partial\phi/\partial x , \end{cases} \tag{2.17}$$

we get the field equation of ϕ as

20) M. A. BIOT, *ibid.*, 11), p. 38, equ. (6.17).

$$\frac{\partial^4\phi}{\partial y^4}+2m\frac{\partial^4\phi}{\partial y^2\partial x^2}+k^2\frac{\partial^4\phi}{\partial x^4}=0\ , \tag{2.18}$$

where

$$\begin{cases} m=(2\hat{N}-\hat{Q})/(\hat{Q}+P/2)=(2\hat{M}-\hat{L})/\hat{L} \\ k^2=(\hat{Q}-P/2)/(\hat{Q}+P/2)=(\hat{L}-P)/\hat{L}\ . \end{cases} \tag{2.19}$$

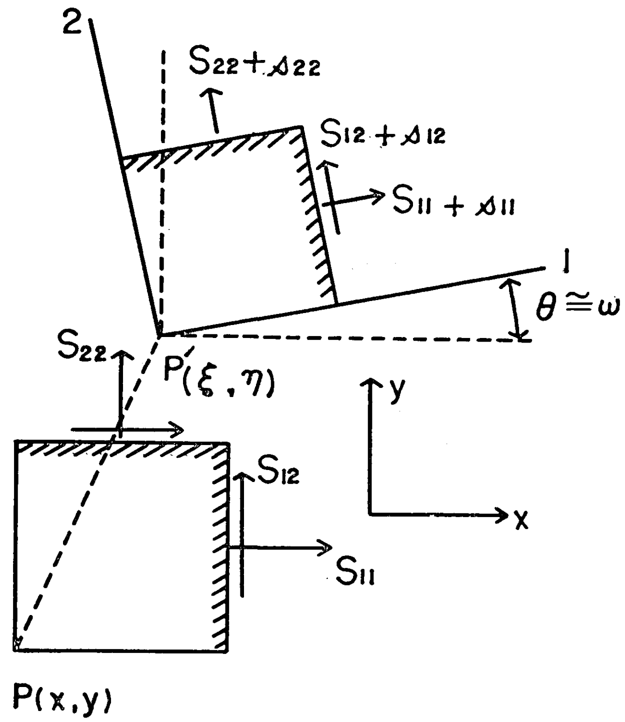

Fig. 2.1. Representation of the initial stresses S_{11}, S_{22}, S_{12} and the incremental stresses s_{11}, s_{22}, s_{12}. (After BIOT)

2.3. Boundary conditions

Let us now consider the deformation due to certain internal forces which are acting along a plane within an infinite medium. In order to formulate this as a boundary value problem, let us take this plane as $y=0$, and separate the medium into region (I) and (II) according to $y\gtrless 0$ (cf. Fig. 2.2). For each region, boundary forces are given at this interface $y=0$. The components of incremental forces acting along the interface are denoted by $\Delta f_x^{(i)}$ and $\Delta f_y^{(i)}(i=\text{I, II})$. Then, these incremental boundary forces are obtained from the general expressions of boundary forces,[21] and are

$$\begin{cases} -\Delta f^{\mathrm{I}}_x = \Delta f^{\mathrm{II}}_x = s_{12} + Pe_{xy} - S_{22}\dfrac{\partial v}{\partial x} \\ -\Delta f^{\mathrm{I}}_y = \Delta f^{\mathrm{II}}_y = s_{22} \qquad\qquad + S_{22} e_{xx} \,. \end{cases} \tag{2.20}$$

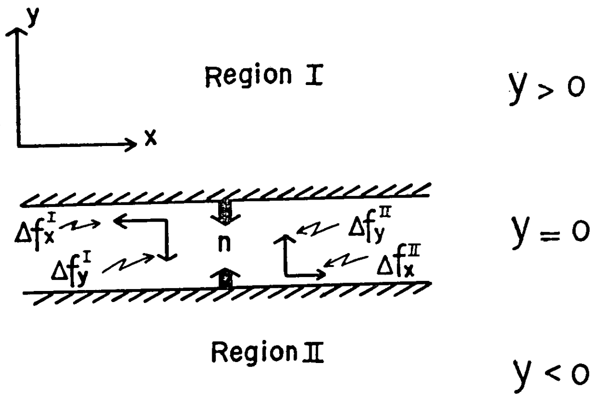

Fig. 2.2. Boundary forces at the interface $y=0$. (After BIOT)

As is seen in this equation, the condition of stress continuity holds at the interface $y=0$.

Since the field of deformation is described by the displacement potential ϕ the solution is first obtained for the certain displacements which are given at the interface. Then the solutions are related to the internal forces by using the boundary force$\sim$displacement relation of the equation (2.20).

2.4. Solutions

We apply Fourier transformation for x to the field equation (2.18). Then, it becomes

$$\frac{d^4\bar{\phi}}{dy^4} - 2m\xi^2\frac{d^2\bar{\phi}}{dy^2} + k^2\xi^4\bar{\phi} = 0 \,, \tag{2.21}$$

where $\bar{\phi}$ denotes the Fourier transform of displacement function ϕ, being defined by

21) M. A. BIOT, *ibid.*, p. 41, equ. (6.27). In this equation, by putting $S_{12}=0$, $(n, x)_I = -\pi/2$, $(n, y)_I = \pi$, for region I, and $(n, y)_{II} = \pi/2$, $(n, x)_{II} = 0$ for region II, the equ. (2.20) is derived.

$$\bar{\phi}(\xi, y) \equiv F\{\phi(x, y; \xi)\} \equiv \int_{-\infty}^{\infty} \phi(x, y) e^{-i\xi x} dx \ . \tag{2.22}$$

General solutions of the equation (2.21), which decay at infinitely large distances, are given in the form of

$$\begin{cases} \bar{\phi}_{\mathrm{I}}(\xi, y) = Ae^{-\beta_1 \xi y} + Be^{-\beta_2 \xi y} \\ \bar{\phi}_{\mathrm{II}}(\xi, y) = Ce^{+\beta_1 \xi y} + De^{+\beta_2 \xi y} \quad (Re\ \ \beta_i > 0,\ \ i=1, 2) \ . \end{cases} \tag{2.23}$$

$A, B, C,$ and D are arbitrary constants which will be determinated by boundary conditions. β_1 and β_2, whose real parts are chosen as positive, are the roots of the equation

$$\beta^4 - 2m\beta^2 + k^2 = 0 \ , \tag{2.24}$$

and are

$$\begin{cases} \beta_1 = \sqrt{m + \sqrt{m^2 - k^2}} \quad (Re\ \ \beta_1 > 0) \\ \beta_2 = \sqrt{m - \sqrt{m^2 - k^2}} \quad (Re\ \ \beta_2 > 0) \ . \end{cases} \tag{2.25}$$

We denote the displacement at the interface $y=0$ by u_{10}, v_{10}, u_{20}, and v_{20} for region I and II respectively. Then, the boundary conditions are, at $y=0$,

$$\begin{cases} u_1(x, 0) = u_{10}(x) \ , \quad v_1(x, 0) = v_{10}(x) \ , \quad \text{for region I} \\ u_2(x, 0) = u_{20}(x) \ , \quad v_2(x, 0) = v_{20}(x) \ , \quad \text{for region II.} \end{cases} \tag{2.26}$$

The Fourier transformed boundary conditions are, then, at $y=0$,

$$\begin{cases} \bar{u}_1(\xi, 0) = \bar{u}_{10}(\xi) \ , \quad \bar{v}_1(\xi, 0) = \bar{v}_{10}(\xi) \ , \quad \text{for region I} \\ \bar{u}_2(\xi, 0) = \bar{u}_{20}(\xi) \ , \quad \bar{v}_2(\xi, 0) = \bar{v}_{20}(\xi) \ , \quad \text{for region II,} \end{cases} \tag{2.27}$$

where notation bar denotes the Fourier transform of a function with respect to x, as shown in the equation (2.22).

By substituting general solutions of the equation (2.23) into the equation (2.26) with the equation (2.17), A, B, C and D are determined. Thus, we have the solution of displacement potential as

$$\bar{\phi}_{\mathrm{I}}(\xi, y) = \frac{1}{\xi(\beta_1 - \beta_2)} \{(\bar{u}_{10} - i\beta_2 \bar{v}_{10}) e^{-\beta_1 \xi y} - (\bar{u}_{10} - i\beta_1 \bar{v}_{10}) e^{-\beta_2 \xi y}\} \quad (y > 0) \ ,$$

$$\bar{\phi}_{\mathrm{II}}(\xi, y) = \frac{1}{\xi(\beta_1 - \beta_2)} \{(\bar{u}_{20} - i\beta_2 \bar{v}_{20}) e^{+\beta_1 \xi y} + (\bar{u}_{20} + i\beta_1 \bar{v}_{10}) e^{+\beta_2 \xi y}\} \quad (y < 0) \ . \tag{2.28}$$

The solutions of displacements are, from the equation (2.28) and the equation (2.17),

$$\begin{cases} \bar{u}_1(\xi, y) = -\xi\bar{\phi}_{\mathrm{I}}' \\ \bar{v}_1(\xi, y) = -i\xi\bar{\phi}_{\mathrm{I}} , \end{cases} \tag{2.29}$$

where the notation prime denotes the differentiation with respect to ξy,

$$\phi' \equiv \partial\phi/\partial(\xi y) . \tag{2.30}$$

The solutions of strain components are, from the equation (2.29) and the equation (2.3),

$$\begin{cases} \bar{e}_{xx}(\xi, y) = -\bar{e}_{yy}(\xi, y) = i\xi^2\bar{\phi}' \\ \bar{e}_{xy}(\xi, y) = -\dfrac{1}{2}\xi^2(\bar{\phi}+\bar{\phi}'') . \end{cases} \tag{2.31}$$

The solutions of the stress components are, from the stress$\sim$strain relations (2.1), (2.2), and equilibrium equations (2.15),

$$\begin{cases} \bar{s}(\xi, y) = -i\xi^2\left[\left(2\hat{N}-\hat{Q}+\dfrac{P}{2}\right)\phi'-\left(\hat{Q}+\dfrac{P}{2}\right)\phi'''\right] , \\ \bar{s}_{11}(\xi, y) = -i\xi^2\left[\left(-\hat{Q}+\dfrac{P}{2}\right)\phi'-\left(\hat{Q}+\dfrac{P}{2}\right)\phi'''\right] , \\ \bar{s}_{22}(\xi, y) = -i\xi^2\hat{L}[(2m+1)\phi'-\phi'''] , \\ \bar{s}_{12}(\xi, y) = -\xi^2\hat{Q}[\phi+\phi''] , \end{cases} \tag{2.32}$$

where m is given in the equation (2.19). The explicit expressions of the solutions in terms of displacement, strain components and stress components are obtained by substituting the solution of ϕ, the equation (2.28), into the equations (2.29), (2.31) and (2.32). The expressions of the solutions in the original x, y space will be obtained by applying inverse Fourier transformation to them.

Now, the relations of these solutions to the internal forces are obtained by the internal force$\sim$displacement relation at the interface of the equation (2.20). We denote the normal and tangential component of the boundary forces by $\bar{q}_1(\xi)$, $\bar{\tau}_1(\xi)$, $\bar{q}_2(\xi)$ and $\bar{\tau}_2(\xi)$ for region I and II respectively,

$$\Delta\bar{f}_y^i = \bar{q}_i(\xi) , \quad \Delta\bar{f}_x^i = \bar{\tau}_i(\xi) , \quad (i=\mathrm{I}, \mathrm{II}) . \tag{2.33}$$

By substituting the equations (2.32), (2.31) and (2.29) into the equation

(2.20), we obtain

$$\begin{cases} \bar{q}_1 = i\xi\hat{L}\left\{\left(a_{12}+\dfrac{S_{22}}{\hat{L}}\right)\bar{u}_{10} - a_{22}i\bar{v}_{10}\right\} \\ \bar{\tau}_1 = \xi\hat{L}\left\{a_{11}\bar{u}_{10} - \left(a_{12}+\dfrac{S_{22}}{\hat{L}}\right)i\bar{v}_{10}\right\}, \end{cases} \tag{2.34}$$

and

$$\begin{cases} \bar{q}_2 = i\xi\hat{L}\left\{-\left(a_{12}+\dfrac{S_{22}}{\hat{L}}\right)\bar{u}_{20} - a_{22}i\bar{v}_{20}\right\} \\ \bar{\tau}_2 = \xi\hat{L}\left\{a_{11}\bar{u}_{20} + \left(a_{12}+\dfrac{S_{22}}{\hat{L}}\right)i\bar{v}_{20}\right\}, \end{cases} \tag{2.35}$$

where

$$\begin{cases} a_{11} = \beta_1 + \beta_2 \\ a_{22} = \beta_1\beta_2(\beta_1+\beta_2) \\ a_{12} = \beta_1\beta_2 - 1 . \end{cases} \tag{2.36}$$

These are the equations which give the relations between internal force and the displacements at the boundary. When the internal forces are given, the boundary displacements are computed from the equations (2.34) and (2.35). The displacement field, strain field, and stress field in terms of the internal force are, then, computed from the equations (2.28), (2.29), (2.31) and (2.32), by substituting the equations (2.34) and (2.35) into the equation (2.28). The expressions of the solution in the $x \sim y$ space will be obtained by applying inverse Fourier transformation to the above solutions. The expressions in the time domain will be obtained by applying inverse Laplace transformation. In the next section, we will show an example of deformations due to internal force of special forms which seems to be related closely to the kinematics of earthquake swarms.

3. Deformation due to internal force

Now we will consider such a special internal force that is, like liquid pressure, acting normally at the interface, $y=0$, towards both media, region I and II, (Fig. 2.2). We will assume that there is only the normal component of the internal force, q, and no tangential component, τ, at the interface. The internal force is supposed to be a function of co-ordinate x and time t, and is expressed by the product of the space function and the time function.

From the condition of stress continuity at the interface, $y=0$, we have the relations

$$\begin{cases} q_1(x,\ t) = -q_2(x,\ t) = q_0(x,\ t) & (3.1) \\ \tau_1(x,\ t) = -\tau_2(x,\ t) = 0\ . & (3.2) \end{cases}$$

Suffix 1 and 2 represents the boundary force to the region I and II respectively.

Since the deformations due to such internal force are supposed to be symmetric with respect to x axis, we have the relations from the equations (2.34) and (2.35),

$$\begin{cases} v_{10}(x,\ t) = -v_{20}(x,\ t) = v_0(x,\ t) & (3.3) \\ u_{10}(x,\ t) = u_{10}(x,\ t) = u_0(x,\ t)\ . & (3.4) \end{cases}$$

Therefore, we will hereafter calculate the solutions in the region I. The solutions in the region II are obtained by the above relations.

By substituting equation (3.2) into the equations (2.34), we obtain the relation,

$$\bar{u}_{10}(\xi,\ p) = \frac{a_{12} + (S_{22}/\hat{L})}{a_{11}}\, i\bar{v}_{10}(\xi,\ p)\ . \tag{3.5}$$

This means that, at the interface, tangential displacement, $\bar{u}_{10}$, is induced by the normal displacement, $\bar{v}_{10}$, and its amount is determined by the condition of vanishing of tangential force.

Then substituting the equation (3.5) into (2.34), we have

$$\bar{q}_{10}(\xi,\ p) = \xi\hat{L}\frac{a_{11}a_{22} - (a_{12} + S_{22}/L)^2}{a_{11}}\bar{v}_{10}(\xi,\ p)\ , \tag{3.6}$$

or

$$\bar{v}_{10}(\xi,\ p) = \frac{1}{\xi\hat{L}}\,\frac{a_{11}}{a_{11}a_{22} - (a_{12} + S_{22}/L)^2}\bar{q}_{10}(\xi,\ p)\ . \tag{3.7}$$

These represent the force$\sim$displacement relations at the boundary. There, a_{11}, a_{22}, and a_{12} are given in the equation (2.36) and are the functions of β. Since β is the root of the equation (2.24), we have the relations:

$$\begin{cases} \beta_1^2 + \beta_2^2 = 2m \\ \beta_1^2\beta_2^2 = k^2\ , \end{cases} \tag{3.8}$$

and

$$\beta_1+\beta_2=\sqrt{2(m+k)}\ , \tag{3.9}$$

where, m and k are, as given in the equation (2.19), functions of deformation coefficients and initial stress. Thus, from the equations (2.36), (3.8), and (3.9), we have

$$\begin{cases} a_{11}=\sqrt{2(m+k)}=\sqrt{2\{(2\hat{M}-\hat{L})/\hat{L}+\sqrt{(\hat{L}-P)/\hat{L}}\}} \\ a_{22}=k\sqrt{2(m+k)}=\sqrt{(\hat{L}-P)/\hat{L}}\sqrt{2\{(2\hat{M}-\hat{L})/\hat{L}+\sqrt{(\hat{L}-P)/\hat{L}}\}} \\ a_{12}=k-1=\sqrt{(\hat{L}-P)/\hat{L}}-1\ . \end{cases} \tag{3.10}$$

As a limiting case, when the initial stress is reduced to zero, these coefficients approach to $a_{12}\to 0$, $a_{11}=a_{22}\to 2\sqrt{\hat{N}/\hat{Q}}$. Therefore, the equation (3.6) is greatly simplified to

$$\bar{q}_{10}(\xi, p)=2\xi\sqrt{\hat{N}\hat{Q}}\,i\bar{v}_{10}(\xi, p)\ . \tag{3.11}$$

This is the force~displacement relation without the initial stress.

In order to make analytical examinations for the force~displacement relation, let us derive an approximate expression of the equations (3.6) and (3.7) for a moderate initial stress and for media which contain sufficiently weak plastic elements. If we assume $P/\hat{L}\ll 1$, that is, the amount of initial stress is sufficiently small compared with the deformation coefficient of the elastic elements, and also assume $\hat{M}\approx\hat{L}$, that is, the values of anisotropic elastic constants of the materials are of the same order, by neglecting the terms of the second order of $P/\hat{L}$, we have approximate expressions for the equations (3.6) and (3.10),

$$\bar{q}_{10}(\xi, p)\doteqdot\xi\gamma\left[\alpha\sqrt{\hat{L}}-\beta\frac{1}{\hat{L}}\right]\bar{v}_{10}(\xi, p)\ , \tag{3.12}$$

where

$$\begin{cases} \alpha=2\sqrt{\hat{M}}\{1-(P/4\hat{M})\} \\ \beta=\sqrt{\hat{M}}P \\ \gamma=\{1-(P/8\hat{M})\}\ . \end{cases} \tag{3.13}$$

As is seen in the above equations, the second term of the equation (3.12) is due to the effect of initial stress P.

By substituting the deformation coefficients of the equation (2.14) into the equation (3.12), we have finally

$$\bar{q}_{10}(\xi, p)=\xi\alpha'\left[\sqrt{p}-\delta\frac{1}{\sqrt{p}}\right]\bar{v}_{10}(\xi, p) , \tag{3.14}$$

or

$$\bar{v}_{10}(\xi, p)=\frac{1}{\xi\alpha'}\frac{\sqrt{p}}{(p-\delta)}\bar{q}_{10}(\xi, p) , \tag{3.15}$$

where

$$\begin{cases}\alpha'=2\sqrt{\alpha_1 M_1}\sqrt{\eta/\alpha_2} \\ \delta=P/(\eta/\alpha_2) .\end{cases} \tag{3.16}$$

These equations represent the force~displacement relation in the transformed space. The relations in the original space are obtained by applying the inverse Laplace transformation with respect to time t and the inverse Fourier transformation with respect to space x. By applying the inverse Fourier transformation to the equation (3.14), we have

$$\hat{q}_{10}(x, p)=F^{-1}\{\bar{q}_{10}(\xi, p)\} , \tag{3.17}$$

where

$$F^{-1}\equiv\frac{1}{2\pi}\int_{\infty}^{\infty}e^{+i\xi x}d\xi .$$

When the deformation and the internal force are given by a single harmonic element, being denoted by

$$\begin{cases}\hat{q}_{10}(x, p)=\hat{q}_0(p)\cos\dfrac{2\pi x}{\mathscr{L}} , & (3.18) \\ \hat{v}_{10}(x, p)=\hat{v}_0(p)\cos\dfrac{2\pi x}{\mathscr{L}} , & (3.19)\end{cases}$$

where $\mathscr{L}$ is the wavelength of the harmonic element, the equation (3.17) becomes

$$\hat{q}_0(p)=\frac{\alpha'}{\mathscr{L}}\left[\sqrt{p}-\delta\frac{1}{\sqrt{p}}\right]\hat{v}_0(p) . \tag{3.20}$$

By applying inverse Laplace transformation to the equations (3.17) and (3.20), we have

$$\begin{cases} q_{10}(x,\, t) = L^{-1} \cdot F^{-1}\{\bar{q}_{10}(\xi,\, p)\}\ , \\ q_{10}(\mathscr{L},\, t) = L^{-1}\left\{\hat{q}_0(p) \cos \dfrac{2\pi x}{\mathscr{L}}\right\}\ , \end{cases} \tag{3.21}$$

where

$$L^{-1} \equiv \frac{1}{2\pi i}\int_{c-j\infty}^{c+j\infty} e^{pt} dp\ . \tag{3.22}$$

In a similar way, we have

$$\begin{aligned} v_{10}(x,\, t) &= L^{-1} \cdot F^{-1}\{\bar{v}_{10}(\xi,\, p)\}\ , \\ v_{10}(\mathscr{L},\, t) &= L^{-1}\left\{\hat{v}_0(p) \cos \frac{2\pi x}{\mathscr{L}}\right\}\ . \end{aligned} \tag{3.23}$$

The equation (3.20) is approximate relation between the internal force and displacement at the interface of an anisotropic viscoelastic media under initial stress, and valid for moderate initial stress and for media of weak plastic elements.

In the following sections, we will examine the time function of the force$\sim$displacement relation in connection with the development of earthquake swarms. The evaluation of the spatial distribution of deformation will be left for further treatment.

4. Numerical examples of internal force$\sim$displacement relation in the time domain

In this section we will obtain the displacement responses for several types of internal force in the time domain. And also we will obtain the patterns of internal force which are necessary to produce the given modes of deformation. The calculation will be made for the approximate force$\sim$displacement relation of the equations (3.20), (3.21), and (3.23). The responses, therefore, are limited for a single harmonic element of the deformation, and also for a special type of the medium which is composed of laminations of very hard elastic elements and very soft viscous elements as described in 2.

Case I: $q(t) = q_0 H(t)$

$$H(t) \equiv \begin{cases} 0 & (t<0) \\ 1 & (t>0) \end{cases} \tag{4.1}$$

This is the case where the internal force is suddenly applied in the form of step function. q_0 is the constant of its amount. The Laplace transform of the equation (4.1) is given by

$$\hat{q}(p) = q_0 \cdot \frac{1}{p} . \tag{4.2}$$

Substituting the equation (4.2) into the equation (3.20), we have

$$\hat{v}_0(p) = \frac{\mathscr{L} q_0}{\alpha'} \cdot \frac{1}{(p-\delta)\sqrt{p}} . \tag{4.3}$$

By applying inverse Laplace transformation[22] to the equation (4.3), we have

$$v_0(t) = \frac{\mathscr{L} q_0}{\alpha'} \delta^{-1/2} e^{\delta t} Erf(\sqrt{\delta t}) , \tag{4.4}$$

where Erf is an error function defined by

$$Erf(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-x^2} dx . \tag{4.5}$$

For the convenience of numerical representation, we will introduce parameter t_c, t', t_0' and β defined by

$$\begin{cases} t_c = 1/2\delta = (\eta/\alpha_2)/2P , \\ t' = t/t_0 , \\ \beta = t_0/t_c . \end{cases} \tag{4.6}$$

These parameters may be called as t_c: critical time, t': non-dimensional time, t_0: reference time, and β: coefficient of initial stress effect. Then, the equation (4.4) becomes

$$v_0(t) = \frac{\mathscr{L} q_0}{S_0} \cdot A(t') , \tag{4.7}$$

where

$$\begin{cases} A(t') = \sqrt{\frac{2}{\beta}} e^{(1/2)\beta t'} Erf\left(\sqrt{\frac{1}{2}\beta t'}\right) , \\ 1/S_0 = \sqrt{t_0}/2\sqrt{M_a}\sqrt{\eta_a} , \\ M_a = \alpha_1 M_1 , \quad \eta_a = \eta/\alpha_2 . \end{cases} \tag{4.8}$$

22) A. ERDÉLYI *et al.*, *Table of Integral Transforms*, Vol. I (McGraw-Hill, 1954), p. 233.

An approximation of this equation for small values of $\beta t'$ is obtained as

$$v_0(t) \doteqdot \frac{\mathscr{L} q_0}{S_0} \cdot \frac{2}{\sqrt{\pi}} \sqrt{t'} \left\{1 + \frac{1}{2}\beta t'\right\} . \tag{4.9}$$

The rate of displacement is obtained from the equation (4.3). By multiplying parameter p for both sides of the equation (4.3), we obtain

$$pv(p) = \frac{\mathscr{L} q_0}{\alpha'} \cdot \frac{\sqrt{p}}{(p-\delta)} . \tag{4.10}$$

Since the term $pv(p)$ is the Laplace transform of the rate of displacement, $dv/dt \equiv \dot{v}(t)$, the Laplace inversion[23] of the equation (4.10) results in

$$\dot{v}_0(t) = \frac{\mathscr{L} q_0}{\alpha'} \cdot \delta^{1/2} \left\{ \frac{1}{\sqrt{\pi}} \frac{1}{\sqrt{\delta t}} + e^{\delta t} Erf(\sqrt{\delta t}) \right\} . \tag{4.11}$$

By using the parameters of the equations (4.6) and (4.8), we have

$$\dot{v}_0(t) = \frac{\mathscr{L} q_0}{t_0 S_0} \cdot B(t') , \tag{4.12}$$

where

$$B(t') = \frac{1}{\sqrt{\pi}\sqrt{t'}} + \sqrt{\frac{\beta}{2}} e^{(1/2)\beta t'} Erf\left(\sqrt{\frac{1}{2}\beta t'}\right) . \tag{4.13}$$

An approximation of this equation for small values of $\beta t'$ is also obtained as

$$\dot{v}_0(t) \doteqdot \frac{\mathscr{L} q_0}{t_0 S_0} \cdot \frac{1}{\sqrt{\pi}\sqrt{t'}} \{1 + \beta t'\} . \tag{4.14}$$

In Fig. 4.1 is shown the responses of the displacement v and the rate of displacement $\dot{v}$ as functions of non-dimensional time $t' = t/t_0$, calculated from the exact equations of (4.7) and (4.12). The units of v and $\dot{v}$ are taken as $\mathscr{L} \cdot q_0/S_0$ and $(\mathscr{L}/t_0) \cdot (q_0/S_0)$ respectively. It is noticed in these figures that, corresponding to such a step type of internal force, the displacement starts to build up and continues to develop, the rate of deformation being at its maximum at the beginning and decreasing

23) A. ERDÉLYI *et al.*, *ibid.*, 22), p. 235.

gradually. The characteristic feature of the continuous progress of deformation with gradual fall in its rate is due to the effect of the viscoelastic property of the medium. It will also be noticed that the decrease of the deformation rate is approximately proportional to $1/\sqrt{t}$.

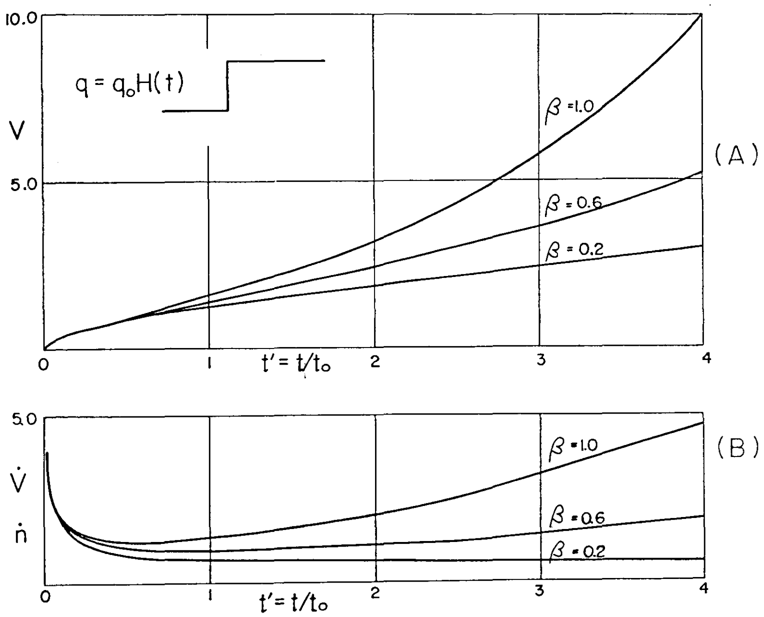

Fig. 4.1. Responses of the displacement v and the rate of displacement $\dot{v}$ at the boundary due to the step type internal force q. The units of $v, \dot{v}$ and t' are taken as non-dimensional values, being $\mathscr{L}q_0/S_0$, $\mathscr{L}q_0/(S_0t_0)$, and t/t_0 respectively. The parameter $\beta=t_0/t_c$ represents the amount of initial stress.

The effect of initial stress appears in the later stage of deformation in such a way that the deformation tends to develop with increasing time rate as is seen in Fig. 4.1 for the larger values of β. This characteristics may find some correspondence with the creep buckling of rock deformation under a constant loading.

Case II:

$$q(t)=q_0\{H(t)-H(t-\tau)\}$$

$$H(t-\tau)=\begin{cases}0 & t<\tau \\ 1 & t>\tau\end{cases} \tag{4.15}$$

This is the case where a constant internal force is applied for a finite duration τ. The responses of displacement v and its rate $\dot{v}$ are obtained from the responses of case I as having such differences as $v_{10}(t)-v(t-\tau)$, $\dot{v}_{10}(t)-\dot{v}(t-\tau)$.

In Fig. 4.2 is shown an example of this case, the period of duration τ being taken as $\tau=0.1$ and 0.5. As is seen in the figure, when the duration of the force is short compared to t_0, deformation decreases as the internal force disappears. This will represent the recovery of elastic deformation under the resistance due to viscous elements. However, the displacement does not show complete recovery, remains as a permanent deformation, which is due to the viscous property of the medium.

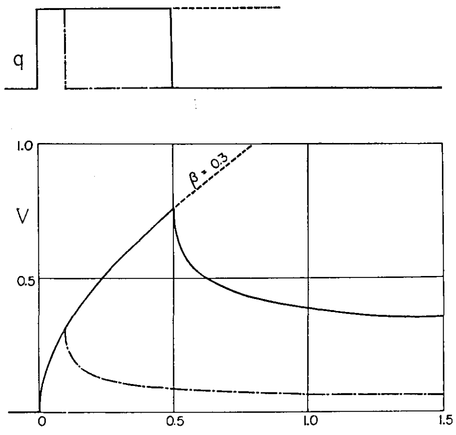

Fig. 4.2. Responses of the displacement due to a constant internal force of finite duration τ. Solid line is for $\tau=0.5$, and broken line is for $\tau=0.1$.

Case III:

$$q(t)=q_0(a+1)\frac{\sqrt{t'}}{t'+a}$$

$$=q_0(a+1)\sqrt{t_0}\frac{\sqrt{t}}{t+\alpha}$$

$$(a=\alpha/t,\ t'=t/t_0) \qquad (4.16)$$

This is the case where the internal force continuously develops and dies out. The function forms of the equation (4.16) are illustrated in Fig. 4.3-(A). These curves represent various patterns of time function according to the value of the papameter a. The coefficient $q_0(a+1)$ in the equation (4.16) is taken as such a normalization that the value $q(t)$ should have the value q_0 at $t'=1$. When $a=0.1$, the function represents a pulse-like force. When $a=10.0$, it represents an almost linearly increasing time function. When $a=1.0$ and $a=0.5$, it represents forces which suddenly build up and hold a nearly constant value. Therefore, we shall be able to examine the characteristics of deformations by this single function for several kinds of internal forces.

The Laplace transform of the equation (4.16) is given by[24)]

$$q(p)=q_0(a+1)\sqrt{t_0}\left[\sqrt{\frac{\pi}{p}}-\pi\sqrt{\alpha}\,e^{\alpha p}Erfc(\sqrt{\alpha p})\right] \qquad (4.17)$$

where $Erfc(x)$ is defined by

$$Erfc(x)=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-x^2}dx\ . \qquad (4.18)$$

24) A. ERDÉLYI *et al.*, *ibid.*, 22), p. 136.

Substituting the equation (4.17) into the equation (3.20), we have

$$\hat{v}_0(p)=\frac{\mathscr{L}q_0(a+1)\sqrt{t_0}\sqrt{\pi}}{\alpha'(p-\delta)}[1-\sqrt{\pi\alpha p}\, e^{\alpha p}Erf(\sqrt{\alpha p})] . \tag{4.19}$$

By using the convolution formula in the Laplace inversion[25] of the equation (4.19), we have

$$v_0(t)=\frac{\mathscr{L}q_0(a+1)\sqrt{t_0}}{\alpha'}\int_0^t H(t-\tau)e^{\delta(t-\tau)}(1/2)\sqrt{\alpha}(\tau+\alpha)^{-3/2}d\tau . \tag{4.20}$$

After a little calculation, with some approximations for small values of δt, we arrive at

$$v_0(t')=\frac{\mathscr{L}q_0}{S_0}(a+1)\sqrt{\pi}\left[1-\left(\frac{a}{t'+a}\right)^{1/2}\right]$$
$$\times\left[1+\frac{1}{2}\beta(t'+a)\left\{1-\left(\frac{a}{t'+a}\right)^{1/2}\right\}+O(\beta^2)\right] . \tag{4.21}$$

The response of the rate of displacement is also obtained for small values of δt,

$$\dot{v}_0(t')=\frac{\mathscr{L}q_0}{S_0 t_0}\cdot\frac{\sqrt{\pi}}{2}\left(\frac{a+1}{a}\right)\left[\left(\frac{a}{t'+a}\right)^{3/2}+\frac{\beta a}{\sqrt{2}}\left\{1-\sqrt{\frac{a}{t'+a}}\right\}\right] . \tag{4.22}$$

These responses are shown in Fig. 4.3-(B) and (C). It will be interesting to notice several special features in these curves.

$a=0.1$: When the internal force suddenly builds up and then decreases with gradual fall, the displacement starts to increase approaching gradually to a certain value. On the other hand, the rate of displacement $\dot{v}$ decreases very rapidly. It is interesting to notice that there is such a state of deformation where the displacement continues to increase even though the internal force gradually turns to decrease.

$a=10.0$: When the internal force shows an almost linear increase, the displacement also shows a similar increase, the rate of displacement showing an almost constant value.

$a=1.0$ and 0.5: When the internal force starts to build-up to a certain value, the displacement shows a little slower build-up, the rate of displacement gradually decreasing.

As seen in the above cases, for such internal forces, the displacement

25) A. ERDÉLYI *et al.*, *ibid.*, 22), p. 131 and p. 267.

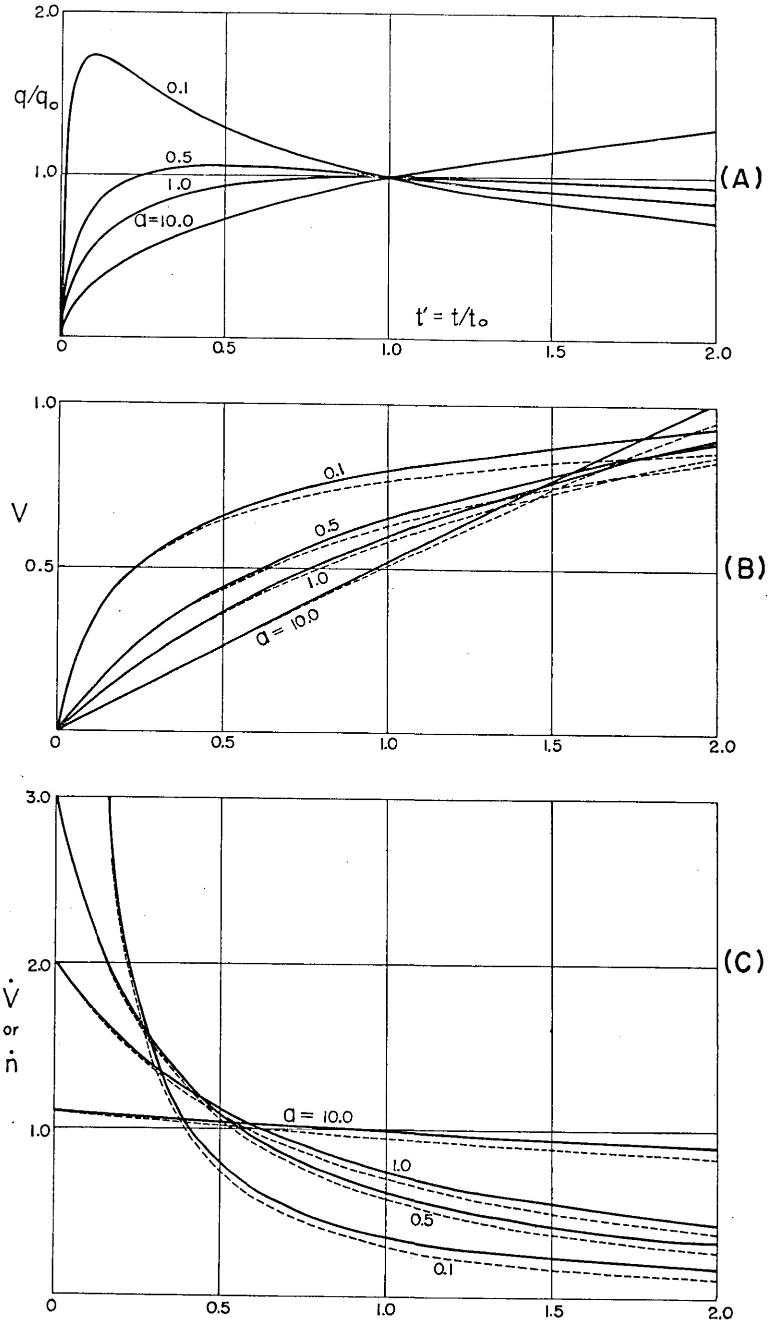

Fig. 4.3. (A) The patterns of internal forces which are represented by the equation $q=q_0(a+1)\sqrt{t'}/(t'+a)$, for $a=0.1, 0.5, 1.0$, and 10.0.

(B) Responses of the displacement for the internal force of (A). The unit of v is $\sqrt{\pi}\ \mathscr{L} q_0/S_0$.

(C) Responses of the rate of displacement for the internal force of (A). The unit of $\dot{v}$ is $(\sqrt{\pi}/2)\mathscr{L} q_0/(S_0 t_0)$. This curve is also read as the frequency $\dot{n}$ of the small shock occurrence in earthquake swarms as functions of non-dimensional time t'. Solid lines are for $\beta=0.1$, dotted lines are for $\beta=0$.

always increases, the rate of displacement decreasing. The gradient of the rate of displacement depends upon the mode of action of internal force. The rate of displacement easily decreases even though the internal force increases in the initial stage of deformation. Therefore, there is the necessity for some development of the internal force to increase the rate of deformation. Let us now examine the patterns of necessary internal forces for given modes of deformations.

Case IV: $$\dot{v}=\dot{v}_0 H(t) \tag{4.23}$$

This is the case where the deformation starts with a constant rate $\dot{v}_0$. The displacement associated with this case is of linear increase,

$$v(t)=\begin{cases}0 & t<0 \\ \dot{v}_0 t & t>0\,.\end{cases} \tag{4.24}$$

The Laplace transform of the equation (4.24) is

$$v(p)=\dot{v}_0\frac{1}{p^2}\,. \tag{4.25}$$

Substituting the equation (4.25) into the equation (3.20), we have

$$\hat{q}_0(p)=\frac{\alpha'}{\mathscr{L}}\left[\sqrt{p}-\delta\frac{1}{\sqrt{p}}\right]\dot{v}_0\frac{1}{p^2}\,. \tag{4.26}$$

Applying the inverse Laplace transformation[26] to the above equation, we have

$$q_0(t)=\frac{S_0 t_0}{\mathscr{L}}\dot{v}_0\frac{2}{\sqrt{\pi}}\left\{\sqrt{t'}\left(1-\frac{1}{3}\beta t'\right)\right\}\,. \tag{4.27}$$

In Fig. 4.4 is shown the curves of $\dot{v}_1$, v, and q as functions of $t'=t/t_0$, for the value $\beta=0.3$. As expected, the amount of the internal force, which is necessary to hold the constant rate of deformation, gradually increases.

Case V: $$\dot{v}=\dot{v}_0\{H(t)-H(t-\tau)\} \tag{4.28}$$

This is the case where deformation develops at a constant rate for a finite duration of period τ. The displacement associated with this case is

26) A. ERDÉLYI *et al.*, *ibid.*, 22), p. 235.

$$v(t)=v_0 t\{H(t)-H(t-\tau)\}\ . \tag{4.29}$$

The pattern of internal force which is necessary to produce such a deformation is given by the difference of the forces of the case IV which are separated by the period τ, being shown in Fig. 4.4 by solid and broken lines. As the rate of deformation vanishes, the internal force suddenly begins to decrease and continues to decrease with gradual fall in its rate. This pattern may find a correspondence to the stress relaxation in rock deformation under a constant displacement.

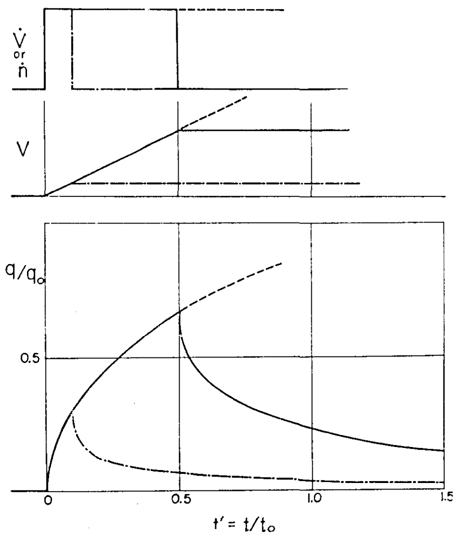

Fig. 4.4. The pattern of displacement v and internal force q which corresponds to the constant rate of deformation $\dot{v}$, which is computed by the equation (4.27). The unit of the internal force is $(2/\sqrt{n})S_0 t_0 \dot{v}_0/\mathscr{S}$. The parameter β is taken as $\beta=0.3$.

Case VI:

$$\dot{v}(t)=\dot{v}_0(a+1)\frac{\sqrt{t'}}{t'+a}=A\frac{\sqrt{t}}{t+\alpha}$$

$$(A=\dot{v}_0(a+1)\sqrt{t_0}\ ,\ a=\alpha/t_0,\ t'=t/t_0) \tag{4.30}$$

This is the case where the rate of displacement continuously increases and gradually begins to die out. $\dot{v}_0$ is a certain constant which represents the value of $\dot{v}$ at $t'=1.0$. These patterns are illustrated in Fig. 4.5-(A) for several values of parameter a. The displacement associated with this rate is obtained by integrating the equation (4.30), which becomes

$$v(t)=2A[\sqrt{t'}-\sqrt{a}\ \tan^{-1}\sqrt{t'/a}]\ . \tag{4.31}$$

The curves of displacements of the equation (4.31) are illustrated in Fig. 4.5-(B). There are not many differences in the forms of the displacement even though there are many differences in the forms of the rate of displacement. This feature will be easily understood by the relation that $\dot{v}$ is only the time gradient of the displacement v. Now, let us derive the differences of the internal forces which are necessary to bring about such differences in $\dot{v}$ as in the equation (4.30).

The Laplace transform of the equation (4.30) is given by

$$pv(p)=A\left[\sqrt{\frac{\pi}{p}}-\pi\sqrt{\alpha}\,e^{\alpha p}\,Erfc(\sqrt{\alpha p})\right]. \tag{4.32}$$

Substituting the equation (4.32) into the equation (3.20), we have

$$p\hat{q}_0(p)=\frac{\alpha' A}{\mathscr{L}}\sqrt{\pi}\left[\{1-\sqrt{\pi\alpha p}\,e^{\alpha p}\,Erfc(\sqrt{\alpha p})\}\right.$$
$$\left.-\delta\left\{\frac{1}{p}-\sqrt{\pi\alpha}\frac{e^{\alpha p}}{\sqrt{p}}Erfc(\sqrt{\alpha p})\right\}\right]. \tag{4.33}$$

By applying the inverse Laplace transformation[27] to the above equation (4.33), we have

$$\dot{q}_0(t)=\frac{\alpha' A\sqrt{\pi}}{\mathscr{L}}\left[\frac{1}{2}\sqrt{\alpha}\,(t+\alpha)^{-3/2}-\delta H(t)+\delta\sqrt{\alpha}\,(t+\alpha)^{-1/2}\right]. \tag{4.34}$$

Integrating the equation (4.34) with respect to time t, introducing non-dimensional time t', we have

$$q_0(t)=\frac{S_0 t_0 \dot{v}_0}{\mathscr{L}}(a+1)\left[\left(1-\sqrt{\frac{a}{t'+a}}\right)-\beta\left\{\frac{1}{2}t'-\sqrt{a}\sqrt{t'+a}\left(1-\sqrt{\frac{a}{t'+a}}\right)\right\}\right], \tag{4.35}$$

under the initial condition

$$q_0(0)=0 \qquad \text{at} \qquad t=0\,. \tag{4.36}$$

In Fig. 4.5-(C) are illustrated the patterns of q for several values of the parameter a, and β.

$a=10.0$: When the rate of displacement shows almost linear increase with time, the internal force should also increase almost linearly with time.

$a=0.1$: When the rate of displacement suddenly builds up to a certain value being followed by a gradual decrease, the internal force should also build up rapidly. It is noticed, however, that the internal force continues to increase even though the rate of deformation begins to decrease. In the later stage of deformation, after reaching some maximum value, the internal force gradually decreases.

This is an effect of initial stress. When the value of the initial stress is large, the decrease of the internal force appears from the earlier stage of the deformation. This feature is shown in Fig. 4.6,

27) A. ERDÉLYI *et al.*, *ibid.*, 22), p. 267.

where the internal forces necessary to produce such deformation are shown with respect to several values of initial stress; ie. $\beta=t_0/(\eta_a/2P)=$

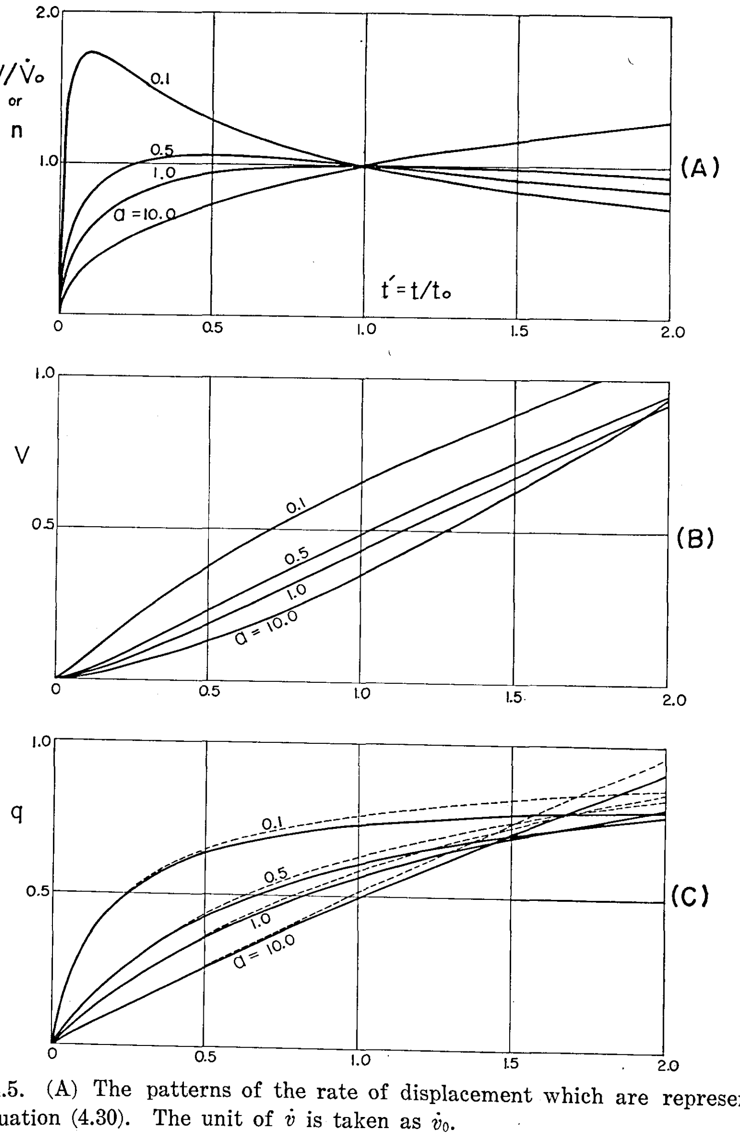

Fig. 4.5. (A) The patterns of the rate of displacement which are represented by the equation (4.30). The unit of $\dot{v}$ is taken as $\dot{v}_0$.

(B) The patterns of the displacement which correspond to the rate of deformation (A). The unit of v is $2\dot{v}_0 t_0$.

(C) The patterns of the internal forces which are necessary to produce the above patterns of displacement and the rate of the displacement. The unit of q is $S_0 t_0 \dot{v}_0/\mathscr{L}$. The solid lines are for $\beta=0.1$, the dotted lines are for $\beta=0$.

0.1, 0.5, and 1.0.

$a=1.0$ and 0.5: When the rate of displacement starts to build up approaching to a certain value, the internal force should also increase but at a slower speed than with the case of $a=0.1$.

Thus, from the above examples, it will be evident that the internal forces are required to increase as far as the rate of deformation increases. On the other hand, a decrease in the rate of deformation does not always mean a decrease of the internal force. Whether the internal force decreases or not is reflected upon the decreasing rate of the rate of deformation, and is also controlled by the amount of the initial stress. The effect of initial stress works so as to reduce the amount of the internal force which is necessary for producing the given mode of deformation.

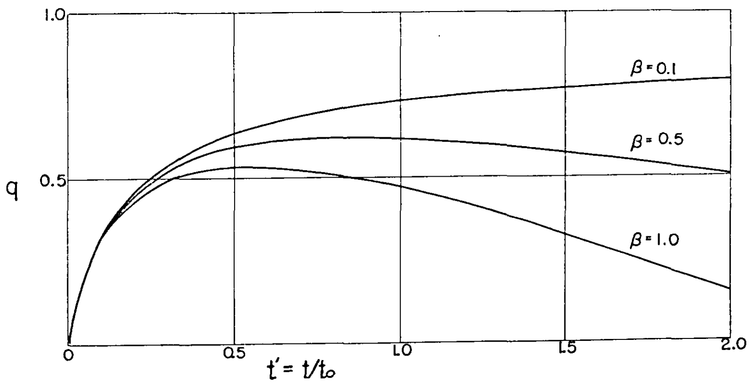

Fig. 4.6. The internal forces q which is necessary to produce the deformation with its rate $\dot{v}=\dot{v}_0(a+1)\sqrt{t'}/(t'+a)$, $(a=0.1)$. $\beta=t_0/t_c=t_0/(\eta/2P)$ represents the effect of initial stress. The unit of q is $S_0\dot{v}_0t_0/\mathscr{L}$.

5. Development of earthquake swarms

In the preceding sections, we have derived the deformations of an anisotropic visco-elastic medium, which is composed of purely elastic elements and weak plastic elements, due to internal force under initial stress. The next problem is to find the relations between these kinematics and the activities of earthquake swarms.

The terminology of "earthquake swarm" is used, according to MATUZAWA[28], to mean the sequence of earthquakes taking place in a certain limited range of time and space. As regards the range of time and

28) T. MATUZAWA, *Zisin-gaku* (Kadokawa-shoten, 1950), p. 248.

space, however, there are several different views. In the narrowest sense, as is seen in RICHTER's book[29], it is used to mean one of such classifications as foreshocks, aftershocks, and earthquake swarms. In this sense, the earthquake swarms are defined as a long series of large and small shocks with no one outstanding principal event.

On the other hand, ISHIMOTO[30] and KISHINOUYE[31] have presented further views that the concept of earthquake swarm could be used in another wider sense, including foreshocks and aftershocks. This is because of the idea that the genesis mechanisms of such are thought to be the same, being characterized by the crustal deformation.

In the broadest sense, the terminology is used by TERADA.[32] He has shown that the activities of earthquakes in the central part of Japan over several tens of years, having the great Kanto earthquake in 1923 at its centre, can be regarded as an earthquake swarm. In this paper, we will use the terminology of earthquake swarm in the sense of ISHIMOTO and KISHINOUYE.

The activities of earthquake swarms will be marked by the rise and fall in the frequency of small shock occurrence and also in the magnitude or energy. In this paper, we will consider the frequency of small shocks. As regards the development of larger shocks, it will be left for a further treatment.

Now let us consider the relation between frequency of small shocks of earthquake swarms and the deformations of the medium. It might be natural to suppose that occurrence of small shocks is related to that of dislocation within the earth's material as stated in 1. Any small shock will be considered to correspond to the sudden occurrence of a small slip which might have resulted from migration and accumulation of numerous dislocations which are produced by some other causes. Therefore, we shall be able to assume, as the first approximation, that the frequency of occurrence of small shocks is proportional to the time rate of dislocation genesis.

Let us denote the frequency of occurrence of small shocks by $\dot{n}$, which is defined by the number of small shocks per unit volume per a certain time interval during which the shocks are counted. The dimension of the frequency is number/cm^3/sec. Let us also denote the time

29) C. F. RICHTER, *Elementary Seismology* (W. H. Freeman and Company, San Francisco, 1958), Part I. Chap. 6.

30) M. ISHIMOTO, *loc. cit.*, 2) and 3).

31) F. KISHINOUYE, *loc. cit.*, 4).

32) T. TERADA, *loc. cit.*, 1).

rate of dislocation genesis per unit volume by the rate of dislocation density $\dot{\rho}$. The dimension of the dislocation density ρ is number of dislocation/cm^2. Then our assumption is written in the form:

$$\dot{n} = c\dot{\rho} \ , \tag{5.1}$$

where c is a proportional constant of which dimension is cm^{-1}.

Since it is shown in the theory of dislocation that there are some relations between dislocation density and deformations of the media, we shall be able to relate the frequency of earthquake occurrence $\dot{n}$ to the deformation of the medium. As a relation between dislocation density and deformations, we will use such a relation that the density of edge dislocation needed to produce a given bending deformation is inversely proportional to the radius of curvature r of the deformation,

$$\rho = \frac{1}{b} \cdot \frac{1}{r} \tag{5.2}$$

where b is Burgers vector, a unit of dislocation (cf. Fig. 5.1[33]). Since the radius of curvature is related to the displacement v by the geometrical relation,

$$\frac{1}{r} = \left| \frac{d^2 v}{dx^2} \right| , \tag{5.3}$$

we have the relation

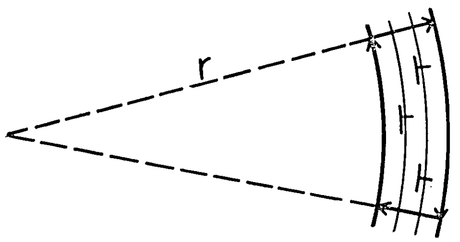

Fig. 5.1. The density ρ of edge dislocations in plastic bending is given by $\rho = 1/(r \cdot b)$. (After COTTRELL)

$$\dot{n} = \frac{c}{b} \left| \frac{d^2 \dot{v}}{dx^2} \right| , \tag{5.4}$$

where $\dot{v}$ denotes the rate of deformation, $\dot{v} \equiv dv/dt$.

When the deformation is given by a single harmonic component, $v = v_0 \cos (2\pi x)/\mathscr{L}$, the relation (5.4) becomes

$$\dot{n} = \frac{c}{b} \left(\frac{2\pi}{\mathscr{L}} \right)^2 \left| \dot{v}_0 \cos \frac{2\pi x}{\mathscr{L}} \right| \ , \tag{5.5}$$

where $\mathscr{L}$ is the wave length and v_0 is the amplitude. These are the relations which connect the frequency of small shocks with the deformations of the media.

33) A. H. COTTRELL, *Theory of Crystal Dislocations* (Gorden and Breach, 1962), p. 29-30.

From equations (5.4) and (5.5), it can be said that the frequency of small shocks is proportional to the rate of deformation as far as the time sequence is concerned, and it is also proportional to the curvature of the deformations as far as the spatial distribution is concerned.

Using the relation (5.5), the curves of $\dot{v}$ in Figs. 4.1~4.6 in 4 can be read as being those of $\dot{n}$. Thus, the schematic representation of the mutual relations among the three factors, frequency of small shock occurrence, deformation, and internal force, is obtained and presented in Fig. 5.2. From these figures, let us now examine the special features in the mode of development of $\dot{n}$ for the given types of internal force, and also in the mode of internal forces which are necessary to produce the given developments of earthquake swarms.

Case A: The internal force is suddenly applied and is held nearly constant. (Fig. 5.2. D)

This case corresponds to case I and the $a=0.5$ and 1.0 of case III in 4. The developments in the frequency of the earthquake swarm for this case are also presented in Fig. 4.1. (B) and Fig. 4.3. (C). The curves which are obtained for the rate of deformation are also the curves for the frequency of the small shocks of earthquake swarms because of the relation (5.5). As seen in these figures, the frequency of small shocks decreases with time. The form of its decay function is proportional to the inverse square root of time at the early stage. It is interesting to note that the frequency of small shocks decreases even though the internal force is held nearly constant.

The corresponding displacements are seen in Figs. 4.1. (A) and 4.3. (B). As seen in these figures, the displacement starts to build up rapidly and continues to develop with a gradual fall in its rate. The decrease of frequency of small shocks with time is, therefore, due to nothing but the decay of the speed of deformation.

The amount of deformation is computed from the equations (4.7), (4.6), and (3.16) by taking into account those numerical values which are the average elastic constant of the elastic elements $\alpha_1 M_1 = 10^{11}$ dynes/cm^2, the average viscous constant of plastic elements $\eta/\alpha_2 = 10^{22}$ dynes/cm^2/sec, the wave-length of the harmonic element $\mathscr{L} = 5\times 10^5$ cm, the reference time $t_0 = 100$ days $= 0.86\times 10^7$ sec, and the initial stress $p = 100$ bar $= 10^8$ dyne/cm^2. From equations (4.7), (4.6), and (3.16), we have

$$v(t') = \frac{\mathscr{L}\, q_0}{2\sqrt{M_a}\sqrt{\eta_a}} \cdot A(t') \tag{5.6}$$

Substituting the above numerical values into equation (5.6), the value of v is computed. In Table 1 are shown the values of v at $t'=1$ for several

Table 1. (A) The amount of the displacement v at the boundary one year after the application of a step type internal force q_0, being computed from the equation (5.6). (B) The critical time $t_c=\eta/P$ as functions of viscosity η and initial stress P.

η	(A) $v_{10}(t=1$ year)			(B) t_c	
	$q_0=100$ bar	$q_0=10$ bar	$q_0=1$ bar	$P=100$ bar	$P=1$ bar
10^{22}CGS	$2-3^{\text{mm}}$	$0.2-0.3^{\text{mm}}$	$2-3\times10^{-2\text{mm}}$	$3\times10^{6\text{year}}$	$3\times10^{4\text{year}}$
10^{20}	$2-3\times10^1$	$2-3$	$2-3\times10^{-1}$	3×10^4	3×10^2
10^{18}	$2-3\times10^2$	$2-3\times10^1$	$2-3$	3×10^2	3
10^{16}	$2-3\times10^3$	$2-3\times10^2$	$2-3\times10^1$	3	3×10^{-2}
10^{14}	$2-3\times10^4$	$2-3\times10^3$	$2-3\times10^2$	3×10^{-2}	3×10^{-4}

values of viscosity, which range from 10^{22} to 10^{16} (CGS), and for internal force of 100 bars~1 bar. As is seen in the equation (5.6) and in Table 1, the amount of v is proportional to q_0 and also to the inverse square root of the average viscosity of the plastic element. In Table 1, the value of the critical time $t_c=\eta/P$ is also shown as functions of the viscosity η and the initial stress P.

It is seen in the table that the deformation of the order of several hundreds of mm per year is produced by the internal force of 100 bars when the value of the average viscosity is of the order of 10^{22} CGS. When the internal force is as small as 1 bar, the viscosity should be as small as the order of 10^{18} CGS in order to produce a deformation of the same order.

Case B: The internal force gradually increases. (Fig. 5.2. A, B, and C.)

This case corresponds to the case $a=10.0$ of case III and case VI of 4. The development of the frequency of small shocks for this case is represented in Fig. 4.3 (C) for $a=10.0$ and in Fig. 4.5 (A) for $a=10.0$. The frequency of small shocks gradually increases as the internal force increases in the case of Fig. 4.5 (A), but it is not so in the case of Fig. 4.3 (C). Whether or not the frequency of the small shocks increases depends upon the rate of deformation.

Case C: The internal force rapidly builds up and is followed by a

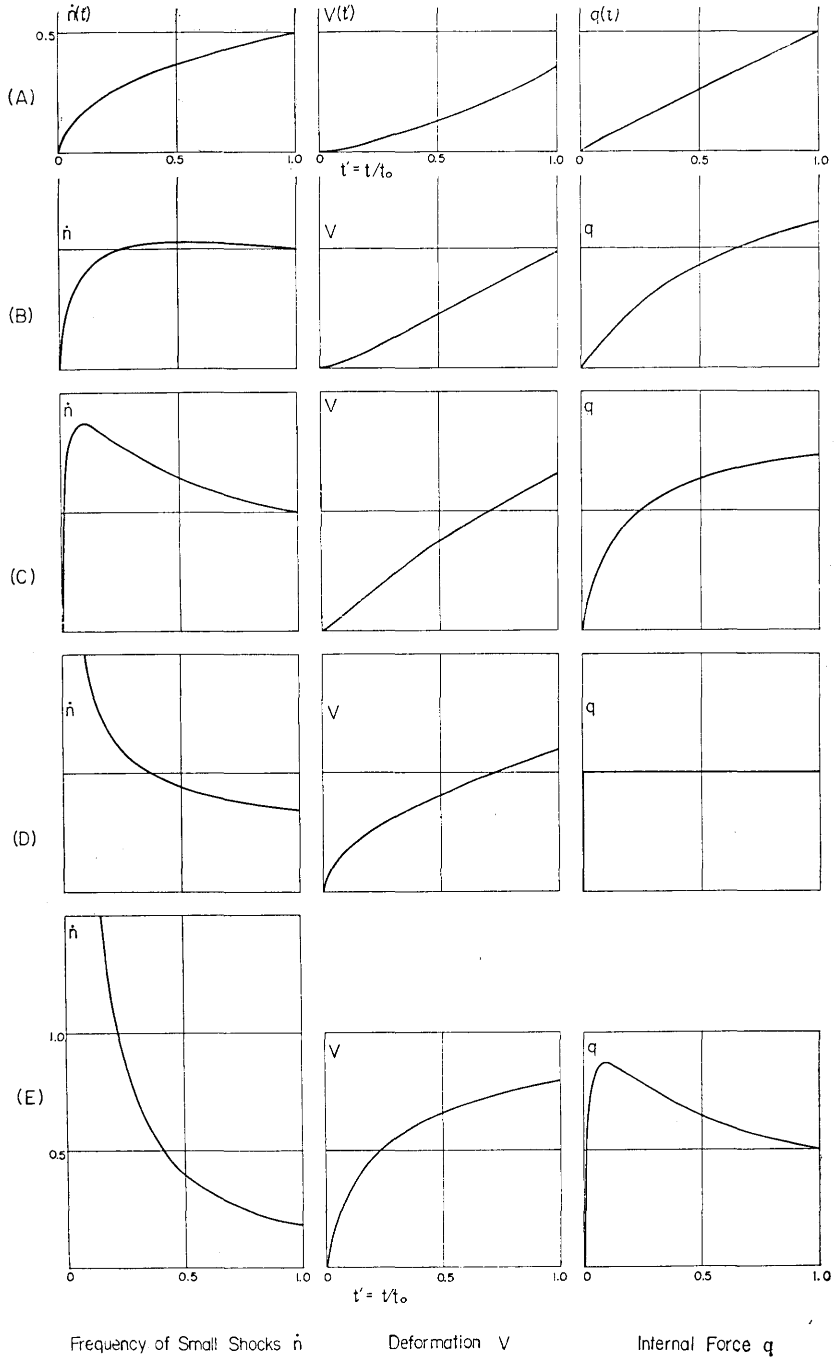

Fig. 5.2. The schematic representation of the mutual relations among the frequency of small shock occurrence $\dot{n}$, deformation v, and internal force q.

relatively rapid decrease. (Fig. 5.2. E)

This case corresponds to the case $a=0.1$ of case III in 4. As is seen in Fig. 4.3. (C), the frequency of the earthquake swarm is maximum at the beginning of its activity. The frequency then gradually decreases with time. The form of its decay function is proportional to $1/(t+a)^{3/2}$ in the ranges where the effect of internal stress is sufficiently small. Incidentally, the decay function of this case seems to be closely related to that of aftershocks. In order to examine the coefficient of the time exponent, the curves of frequency are plotted in log-log scale as shown in Fig. 5.3. Dotted lines show the case where the initial stress

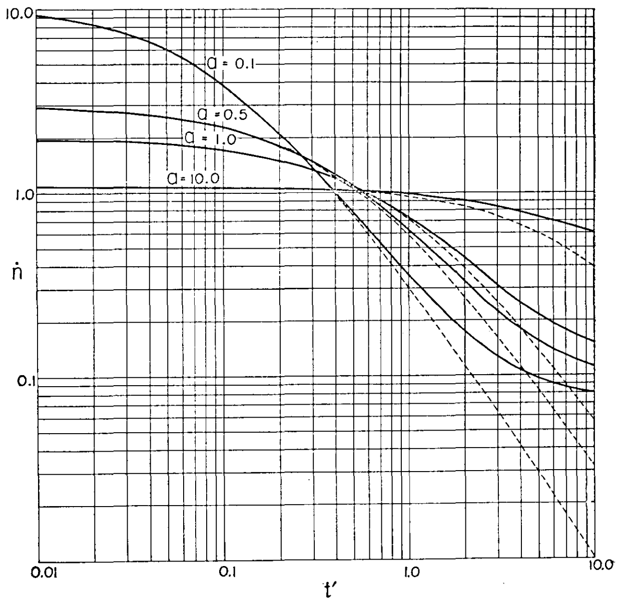

Fig. 5.3. The frequency of small shock occurrence as a function of non-dimensional time $t'=t/t_0$, which is computed from the equation (4.22). The dotted line shows the case of initial stress free. The solid line shows the case $\beta=0.3$.

is zero. It is seen in this figure that the exponent of the decay function depends upon the values of a and the values of initial stress. When $a=0.1$, $\dot{n}$ is approximately proportional to $t^{-3/2}$ for $t'>0.1$, and when $a=0.5$ and 1.0, $\dot{n}$ is approximately proportional to $t^{-1/2}$ for $t'>0.1$. Thus the exponent of the decay function depends upon the way the internal force is held. According to Utsu,[34] the time sequence of aftershock is

34) T. Utsu, *Geophys. Mag.*, **30** (1961), 546.

represented by the formula

$$\dot{n}=\frac{A}{(t+c)^p},$$

where A and c are constant and p ranges from 1.0 to 1.3 for destructive large shocks. Therefore, the time sequence of aftershock seems to correspond to the case when the internal force vanishes very rapidly and the effect of initial stress is very small.

Next, we will examine the forms of internal forces which are needed to produce given types in the frequency of small shocks in earthquake swarms.

Case D: The frequency of small shocks is nearly constant. (Fig. 5.2. B.)

This case corresponds to case IV and the case of $a=0.5$ and 1.0 of case VI of 4. As is seen in Fig. 4.3. (C) and Fig. 4.4, the internal force associated with such a pattern of frequency is an almost linearly increasing one. The deformations continue to grow-up as seen in Fig. 4.3. (B). Therefore, when the small shocks continue to occur with almost constant frequency, it does not mean that the activity of the earthquake is stationary, but that the internal force is increasing and the deformation is in progress at a constant time-rate.

Case E: The frequency of small shocks increases with time. (Fig. 5.2. A.)

This case corresponds to the initial stage of case VI in 4. In this case the internal force also increases with time. As is seen in Fig. 4.5, the rate of increase in frequency is controlled by the time function of internal force and the value of viscosity.

Case F: The frequency of small shocks decreases with time. (Fig. 5.2. C, D, and E.)

This case corresponds to the later stage of the case $a=0.1$ of case VI in 4. As is seen in Fig. 4.5 for $a=0.1$, the internal force does not necessarily decrease. On the contrary, the force continues to grow-up for a while with gradual fall in the rate of development. Therefore, there may be the case when the maximum internal force will appear after the maximum frequency of small shocks has passed. In such a case, the deformation may chance to meet the condition where the rate of internal force reaches zero under the continuous development of the deformation. In such a state of deformation, if its speed is kept constant, a part of the strain energy which is stored in the elastic elements of the medium may be radiated. This is one of the conditions of failure of elastic-plastic deformation as shown by JOHNSTON and GILMAN[35].

As regards the development of the deformation in this case, the decrease in frequency of small shock occurrence only means the slowing down of the speed of development of the deformation. Therefore, the cessation of the development of deformation will be indicated in the cessation of the small shock occurrence.

6. Considerations on a kinematical process of earthquake swarms

In the preceding sections we have derived the mutual relations which are expected among the frequency of small shock occurrence, deformation of the medium, and the internal forces acting at the interface in the process of the development of earthquake swarms. Since these relations are derived from a mathematical analysis of a hypothetical model under several restrictions, the confirmation of the existence of such relations in nature should be handed to the observations of the phenomena which are really taking place in nature. For the convenience of such comparisons, it would be worthwhile making a brief sketch, as a summary, of the kinematical process and its characteristic features of earthquake swarms.

Outline of the process:

Firstly, we have assumed that the media, within which the activities of an earthquake swarm are taking place, are visco-elastic ones, being composed of elastic elements and weak plastic elements. They are also assumed to be initially stressed. For simplicity, it is also assumed that the media are infinite, incompressible and the deformation restricted in plane strain, the principal initial stresses acting in the directions of the co-ordinates.

Under these conditions, if there is no additional internal force, nothing seems to happen except the buckling instability. This feature will be noticed in connection with the condition of the commencement of an earthquake swarm. If one does not need the presence of some internal force for such a commencement, one should take into consideration either buckling phenomena or some other alternative mechanics.

When a certain internal force begins to act at a plane interface in the medium of infinite extent, the deformation starts to develop. The

35) W. G. JOHNSTON and J. J. GILMAN, *Jour. Appl. Phys.*, **30** (1959), 141.

mode of such development is determined by the mode of action of internal force and the visco-elastic properties of the medium as solved in 3 and 4. As the plastic deformation develops, it produces numerous dislocations within the medium, and these are supposed to result in a finite number of small slips accompanied by small shocks (5). In such ways, the number of small shocks is thought to be related to the plastic deformation of the medium, and also to the activity of the internal force.

Frequency of small shock occurrence versus deformation:

As stated in 5, the frequency of small shock occurrence in an earthquake swarm is considered to be proportional to the rate of deformation as far as the time sequence is concerned, and also proportional to the curvature of deformation as far as the spatial distribution is concerned. Therefore, the occurrence of small shocks in the earthquake swarm is thought to indicate that deformation is in progress, the larger values of frequency being considered to indicate a higher speed of deformation. The increase in the frequency indicates that the progress of deformation is accelerating, and the stationary values in the frequency indicate that the rate of deformation is almost constant. The decrease in the frequency indicates that the speed of deformation is slowing down.

The frequency of the small shock occurrence will be represented by the number of small shocks per day or per hour according to the time scale. However, since the occurrence of small shocks in the earthquake swarm is not uniform in the time sequence, there is some ambiguity about the mean value of the rate of occurrence when the frequency is represented by the daily number of occurrence. Therefore, the total number, i. e. the accumulated sum, of the small shocks will be also the other measure of the above relation, and is expected to be proportional to the total amount of the deformation.

ISHIMOTO[36] and KISHINOUYE[37] have already put forward the views that swarm of earthquake is the phenomena which are caused by a crustal deformation, in connection with the mechanism of the Itô earthquake swarm. The kinematical processes treated in this paper are not only in accordance with their views but also place an emphasis upon the dependence of frequency of small shock occurrence to the rate of deformation.

Frequency of small shock occurrence versus internal force:

Since the frequency of small shock occurrence is considered to be

36) M. ISHIMOTO, *loc. cit.*, 2) and 3).
37) F. KISHINOUYE, *loc. cit.*, 4).

caused by the deformation of the medium, and since the deformation is supposed to be caused by the internal force, the rise and fall in the frequency of small shock occurrence is supposed to be a reflection of the activity of the internal force. When the frequency increases or is stationary, it is thought to indicate that the internal force is increasing. Even though the frequency begins to decrease, it does not always indicate that the internal force also begins to decrease. Whether the internal force decreases or not is reflected on the rate of decay curve in frequency. When the internal force begins to decrease very rapidly, the exponent of decay function of frequency will be proportional approximately to $1/(t+a)^{3/2}$. When the internal force is held nearly constant, decay function of frequency will be proportional to $1/\sqrt{t}$.

Viscosity:

There is another interesting feature regarding the viscosity of the media in connection with the speed of deformation. As is seen in Table 1, when the value of the viscosity is $\eta=10^{22}$ CGS, the speed of the deformation is about 2~3 mm/year for an internal force of 100 bar. This value of the deformation is of the same order of the crustal deformation[38] due to its tectonic origin. The value of $\eta=10^{22}$, which is derived from the postglacial uplift of the Fennoscandia, will be considered as a normal value of the earth's crust.[39] Therefore, even though the internal force of about 100 bar is acting at a certain interface in the medium, the speed of the deformation is of the same order of that of tectonic movement as far as the viscosity of the medium is held as the value $\eta=10^{22}$ CGS. On the other hand, the speed of crustal deformation accompanied by earthquake swarm is reported as being as 0.75 mm/day by TSUBOI[40] in the Itô earthquake swarm of 1930. Therefore, in order to account for such a high speed of crustal deformation, the value of the viscosity should decrease approximately in the order of about $10^{-3}\sim 10^{-4}$ for an assumed internal force of about 100 bar.

It will be very interesting to notice that the reduction of the value of the viscosity is needed for the development of earthquake swarms. Such reduction of the viscosity will naturally be anticipated in the process of earthquake swarms if one accepts the viewpoint of the thermo-

38) A. OKADA, *Bull. Earthq. Res. Inst.*, **40** (1962), 431-493.
39) A. E. SCHEIDEGGER, *Principles of Geodynamics*, 2nd edition (Springer, 1963), p. 157.
40) C. TSUBOI, *Jap. Jour. Astr. Geophys.*, **10** (1932), 103.

dynamical activities for the process of earthquake genesis. For instance, the viscous properties of clayey materials, which develop along discontinuities in rock masses such as fractured zone, fissures, weathered unconformities, etc., may easily be changed by conditions of temperature, internal pressure, chemical reactions, etc. Such conditions may very likely be changed by the intrusion of magma, or the permeance of magmatic steam and gas into the earth's crust.

7. Summary and conclusions

As a possible kinematical process of earthquake swarms, the process of deformation of an anisotropic visco-elastic medium due to internal force under initial stress is considered.

The kinematical process is formulated and solved as a boundary value problem by the method developed by BIOT. Several relations are derived for the mutual relations among the deformation, rate of deformation, and the internal force.

Then the process of deformation is related to the activity of earthquake swarm by assuming that the small shocks of earthquake swarm are accompanied by the plastic deformation of the medium. The assumed relation is such that the frequency of small shock occurrence is proportional to the speed of the deformation as far as the time sequence is concerned, and to the curvature of the deformation as far as the spatial distribution is concerned.

Using these assumptions, the characteristic features of the development of earthquake swarms are examined for the mutual relations among the frequency of small shock occurrence, the process of deformation and the mode of internal force. The main results obtained are as follows:

(1) The presence of small shocks in the earthquake swarm is thought to indicate that the plastic deformation of the medium is in progress. Such small shocks continue to occur as far as the plastic deformation of the medium continues to progress.

(2) The increase in the frequency of small shock occurrence is thought as being an indication that the progress of the plastic deformation is accelerative.

(3) When the frequency of small shock occurrence is stationary, the plastic deformation is thought to increase almost linearly.

(4) When the frequency of small shock occurrence decreases, the speed of plastic deformation is thought to be slowing down.

(5) When the internal force is held stationary, the frequency of

small shock occurrence decreases, its decay function being proportional to $t^{-1/2}$.

(6) When the internal force begins to decrease very rapidly, the frequency of small shock decreases by the time function which is proportional to $(t+a)^{-3/2}$. This case may correspond to the earthquake swarms called after-shocks.

(7) As regards the effect of initial stress, unexpectedly, the large one is not found at the early stage of the activity of earthquake swarms. The major part of activities is controlled by the forced deformation due to internal force.

(8) The reduction of the viscosity of the medium is needed for the commencement of earthquake swarms.

Acknowledgement

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77. 初期応力下内部力源による異方粘弾性媒質の変形とその地震群の消長における意義

地震研究所　南　雲　昭　三　郎

群発地震の力学過程に対しては，従来寺田の理論，石本，岸上の地震群は地殻変形に伴うという理論，松沢の余震は隆起帯に起るという理論，塩谷，日下部，ベニオフ等の余震に対する弾性余効の理論，茂木の不均質媒質における微小破壊の偶発的発生によるという理論等がある．いずれも地震群活動の機構に対する理解を進める上に貴重な寄与が行なわれている．しかし，それらの所論は未だ力学場の問題として設定されていないため，境界条件をどのように考えているのか必ずしも明瞭でない場合が多い．したがつて，その力学過程を力学的に厳密に考えてゆこうとするとどうも曖昧な所が出て来てしまう．

この論文は地震群活動に対して，関係する諸量の相互関係を統一的に導くことの出来るような力学過程を formulate しようとする試みを行なつたものである．

問題の設定を次のように行なつてみた．まず地震群の生起する媒質としては粘弾性媒質を用うることにした．これは伊東群発地震の折にみられるように，地震群活動が地殻変動を伴い，その変形が半永久的変形として地震群活動終息後も残留することが知られており，その半永久的変形を説明するには粘弾性媒質が適切であると考えられるからである．粘弾性的性質については，弾性要素と

塑性要素の互層媒質から構成され，異方性があるものとした．次に，媒質は初期応力を受けている状態にある場合を考えることにした．それは，地震群は初期応力下において生起する現象であるということは多くの人々に受け入れられている考えであるにも拘わらず，初期応力による特徴が数理的に必ずしも十分に解析されているとは思えないからである．また最近 BIOT によつて，初期応力下の変形の問題の取扱い方が体系的にもまた具体的にも与えられて，この種の問題の取扱いが可能になつて来たからである．

次に力学過程を境界値問題として設定するために，変形活動の要因として内部力の作用を導入してみた．地震群活動における内部力の存否に関しては議論の分れるところであろうが，内部力が存在しない場合は，このような取扱いにおいて，内部力が極めて小さいという極限の場合に含まれると考えられるであろう．

このようにして，この論文では，弾性要素と粘性要素とから構成される異方粘弾性媒質の初期応力下における，内部力による変形の問題を取扱うことにした．数理解析を簡単にするために多くの仮定を行なつた．媒質は非圧縮，変形は平面歪に限られるとした．変形の進行は十分ゆるやかで慣性項が無視しうるものとみなした．また重力場の影響はこの論文においては簡単のために考慮しないことにした．内部力は無限媒質内の一平面上に作用するとし，それに伴う変形は局所的であり，十分遠方では消滅するものとした．初期応力は主応力が座標軸方向に作用するものとした．内部力としては，流体圧のように，境界面に垂直に作用する力を考えた．

次にわれわれはこのような変形過程と地震群活動との関係を知らねばならない．地震群の活動としては，小さい地震の頻度に関する消長，地震の規模に関する消長，放散エネルギーに関する消長等々，いろいろの観点から着目されるであろう．この論文ではまず小さい地震の頻度の消長を取扱うことにした．地震の規模，放散エネルギーなどの消長については別の取扱いにゆずることにする．

小さい地震の頻度と変形過程との関係については，転位論で展開されている考え方を借りることにする．まず小さい地震の発生は，小さな，しかし有限の大きさの slip が突然発生することに伴うと考える．その小さな slip は，転位論によれば，無数の転位の集積移動などによつて生起されると考えられ，その転位は塑性変形に伴つて発生すると考えられている．したがつて，結局，小さい地震の発生は媒質の塑性変形に依存すると考えることが出来る訳である．その間の定量的関係については，転位論において，いろいろの形式があることが知られている．この論文では，転位密度は塑性変形の曲率に比例するという刃状転位に関する関係式を用うることにし，また小さい地震の数と，slip の数と，転位の数との間に比例関係があるものと仮定することにする．すると小さい地震の数は塑性変形の曲率に比例するということになり，数を頻度でいい表わせば結局，小さい地震の頻度は塑性変形の曲率の進行速度に比例するという関係を得ることになる．曲率は幾何学的に変形の空間的2階微分で表わされる故，上記の関係は，第1近似として，小さい地震の頻度は，時間的には塑性変形速度に比例し，空間的には変形の曲率に比例するとも表現される．このようにして上記の力学過程と地震群の消長とを対応せしめることが出来る訳である．

まず§2 では上記の問題を BIOT の方法に従つて，境界値問題として formulate し，変位ポテンシャルを導入して解き，解の一般形式を求めた．次に §3 では内部力として，液体圧のように，境界面に垂直に作用する力を考え，内部力と変形との関係式を求め，また，純弾性要素と純粘性要素とから構成される媒質について近似式を求めた．次に §4 では空間的に単一の波長で表われる一つの調和成分要素について，内部力のいろいろのかかり方についての変形応答を求め，数値計算を行なつた．§5 において小さい地震の発生と媒質の変形とに関する関係を導入することによつて小さい地震の頻度と，媒質の変形と，内部力との相互関係の特徴を調べた．§6 において群発地震の動力学過程について若干の考察を行なつた．得られた主な結果は次の通りである．

(1) 小さい地震が群生していることは，媒質の塑性変形が進行していることを意味するものと考えられる．変形の進行が停止すれば，小さい地震の群生もまた止むものと考えられる．

(2) 小さい地震の頻度が次第に増加している場合は，塑性変形は加速的に進行していると考えられ，それに伴う内部力は同様に次第に増加しているものと考えられる．

(3) 小さい地震の頻度がほぼ一定値を保持している場合は，塑性変形はほぼ直線的に増加しているものと考えられ，それに伴う内部力は同様にほぼ直線的に増加しているものと考えられる．

(4) 小さい地震の頻度が減少する場合は，塑性変形の進行速度が遅くなることを意味する．そ

れに伴う内部力の変化はいろいろの場合がある.
(5) 内部力が保持されている場合は，小さい地震の頻度の減少はほぼ $t^{-1/2}$ に比例する．内部力が急速に消失する場合は小さい地震の頻度はほぼ $t^{-3/2}$ に比例する.
(6) いわゆる余震は内部力の急速に抜ける場合に相当すると考えられる.
(7) 初期応力の影響は，予想に反して，地震群活動の初期においてはあまり大きいものは認められなかつた．地震群活動の大部分は内部力による強制変形によつて支配されている.
(8) 地震群活動の開始に際しては，媒質の粘性係数の低下が必要であると考えられる.