BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 44 (1966), pp. 793-810

43. *A Dislocation Model of Earthquake Swarms.*

By Keichi KASAHARA,
Earthquake Research Institute
and
Roman TEISSEYRE*,
International Institute of Seismology and Earthquake Engineering, Tokyo.

(Read May 24, 1966.-Received June 29, 1966.)

1. Introduction

The activity of an earthquake swarm, represented by frequency of occurrence, depends obviously on stress concentration and on the properties of the medium.

The importance of medium structure in this problem was pointed out clearly in papers by K. Mogi[1),2)]. Mogi distinguished three types of shock pattern according to material characteristics: 1) in a homogeneous medium the main shock occurs without preceding foreshocks, but after it many aftershocks could follow, 2) in a fairly heterogeneous medium there could be expected a small number of foreshocks, but 3) in extremely heterogeneous medium the activity is revealed in the swarm type shocks.

Statistical properties of earthquake swarms were investigated very carefully, but rarely could one find some ideas on mechanism governing earthquake sequences. We should, however, refer here to the paper of T. Terada[3)], who proposed a comparative model of swarm mechanism based on observation of the Ito as well as of the Kwanto District earthquakes. This mechanism supposes a group consisting of a large number of latent origins of earthquake which are gradually ripening or approaching a critical state. The time of attaining such a state is distributed about the mean value according to some statistical law.

---

*) On leave during 1965-1966 academic year from Institute of Geophysics, Polish Academy of Sciences, Warszawa.

1) K. MOGI, *Bull. Earthq. Res. Inst.*, **41** (1963), 595-614.

2) K. MOGI, *Bull. Earthq. Res. Inst.*, **41** (1963), 615-658.

3) T. TERADA, *Bull. Earthq. Res. Inst.*, **10** (1932), 29-35.

It seems, however, that the single events in swarm series are not independent between themselves, but that there should exist specific interaction and dynamical connection between developments of separate shocks. The dislocation theory could provide here a suitable tool to describe the swarm process, when taking a group of dislocations into account. The model based on the dislocation theory is, of course, a great simplification of the real process, but its relatively simple form presents many advantages in comparison with the theory of cracks.

The numerous achievements in crystalography and, generally, in solid state physics suggest that the application of the dislocation theory to the continuous media could also bring some explanation of main developments in earthquake processes. Medium structure presents undoubtedly a great role in the process of earthquake swarm; the complicated characteristics of medium, including all its internal failures, would be simplified, in the present attempt, to the homogeneous medium with a number of internal sources representing origins of development of dislocation nuclei.

The classification by Mogi could be also related to our concept of dislocation sources nuclei. Mogi's heterogeneous medium could thus correspond to a medium which would have a great number of sources, while a homogeneous one would be characterized by a small number of sources.

The dislocation theory is applied here to describe the whole process of shock swarm. Previously applications of the dislocation theory were made either to a single earthquake theory[4),5)] or to the sequence of earthquakes in the problem of aftershock series[6)].

2. Basic structure of the model

Let us take an elastic medium of parallelpiped shape ($V=L_0H_0D_0$), which undergoes dislocation processes to cause earthquake swarms in it (Fig. 1a). We assume that a certain number of dislocation sources are distributed in our active volume, that is to say, dislocations are being produced at definite places in the medium under the action of stress field. Roughly speaking, a dislocation is repeated by a pair of dislocation lines, around which the bounding energy of dislocation is accumulated. After physical considerations on their properties, we know that these lines are

4) S. DROSTE and R. TEISSEYRE, *Sci. Rep. Tohoku Univ.*, Ser. 5, Geophys., **11** (1959), 1.
5) R. TEISSEYRE, *Acta Geoph. Pol.*, **9** (1961), 3.
6) R. TEISSEYRE, *Acta Geoph. Pol.*, **12** (1964), 23.

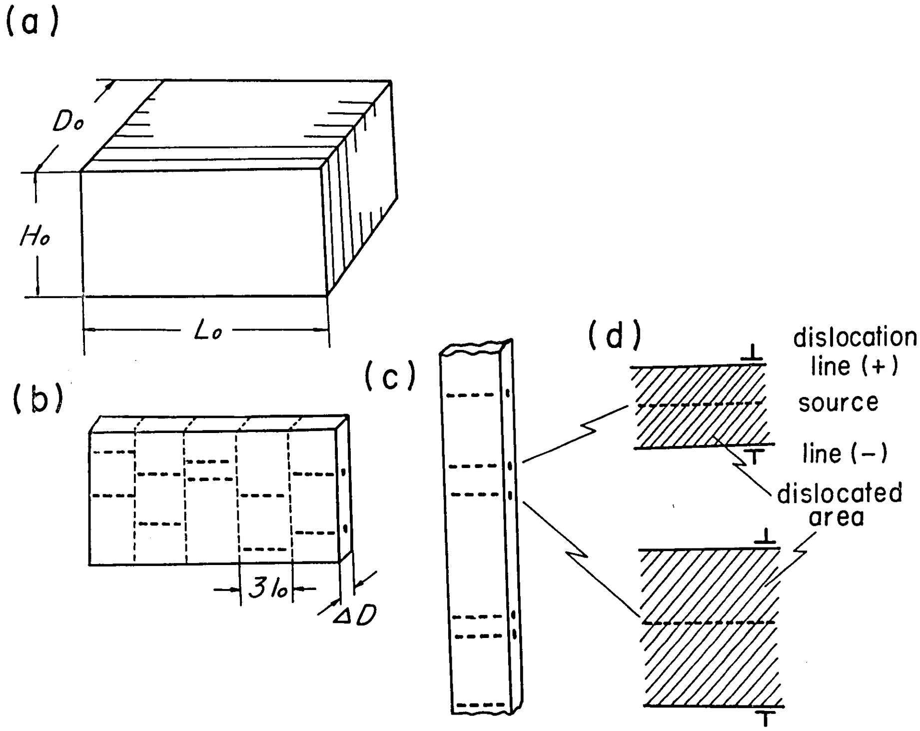


Fig. 1. Basic structure of the model.

of opposite sense to one another. Hence a dislocated area inside the medium is bounded by a pair of dislocation lines of opposite signs, or shortly by a pair of dislocations (Fig. 1d). Development of the dislocated area is therefore replaced by an outward movement of dislocations (lines).

Dislocation processes are probably controlled by various conditions of external stresses and the internal constitution of the medium. We will not discuss here the generating mechanism in detail, but we will only assume that an external stress (*f.i.* shear) generates, at numbers of fixed places in the medium (called sources), small dislocation pairs of edge- or screw-types (called elementary dislocations).

A dislocation line is conventionally represented by a cylinder with its radius proportional to the square root of energy concentration, which is given as the square of a displacement vector b_0—the Burgers vector of dislocation. The radius (ρ_0) and the associated energy (e) are given respectively as follows:

$$\rho_0 = \frac{\mu b_0}{2\pi s}, \tag{1}$$

$$e = \frac{\mu b_0^2 l_0}{2\pi} \ln \frac{2\rho}{\rho_0}, \tag{2}$$

where, ρ_0 radius of dislocation, l_0 length of dislocation,

b_0 Burgers vector, ρ separation of dislocation lines,

s shearing strength, μ rigidity modulus.

In the following we also assume that the minimum separation is equal to the sum of the dislocation radii $2\rho_0$. The energy of an elementary dislocation is thus given by $e_0 = \frac{\mu b_0^2 l_0}{2\pi} \ln 2$, when it is of the screw type.

Returning to properties of the active volume, let us take further assumptions for simplicity's sake in mathematical treatment. As shown in Fig. 1a-c, we slice the volume into α sheets of plates so that they may be several times as thick as the supposed radius of dislocation cylinders, ρ_0. Then we cut each slice into β pieces of narrow strips, where their width is taken as $3l_0$ (l_0 is average length of dislocations). Hence, the total length of the strips becomes $L = \alpha\beta L_0$.

We now assume that the above-stated treatments (slicing and cutting) of the volume do not seriously affect dislocation processes, that is to say, all of the dislocation sources are involved in either of the strips evenly and all the dislocation lines will move only in parallel to the strips. Thus our model has been reduced to a linear system with randomly distributed n sources in it. The external stress, which is tentatively considered as shearing, is governing the time rate of dislocation-pair production; practical considerations would be later limited to the case of unchanged external shearing stress in time and also to constant dislocation-pair production.

3. Dynamic development of activity

We will consider dynamic development in the successive time intervals τ_i. Starting at the moment $t=0$, we assume that at all n sources the dislocation pairs with minimum spacing $2\rho_0$ are generated, one pair in each dislocation source. Average spacing between dislocations (of different pairs) is thus given by $2d_0 = \frac{L - 2\rho_0 n}{n}$. This situation refers to the beginning of the first interval τ_0. We assume that the dislocation can move under the influence of the external shearing stress p_0. The velocity of dislocation movement describes the rate at which the dislocation surfaces spread outwards in regard to position of their sources. According to the equation of motion for dislocation lines[7], we have

$$v_0 = p_0 - s \qquad \text{or} \qquad v_0 = \frac{p_0 - s}{\nu}, \tag{3}$$

where, v_0 average dislocation velocity,
ν viscosity coefficient for slipping processes along layerlets.

The average velocity v_0 refers thus to stress level p_0, and one could omit any assumption regarding s and ν, when assuming directly some value of v_0 instead of p_0.

Assuming the triangular distribution of distances, symmetric about do, between dislocations in the interval: $0 \leq d \leq 2d_0$ (the mean value d_0 has thus here an arithmetic sense), we can define an average time $t_0 = \frac{d_0}{v_0}$ required to finish this stage of process by mutual annihilation of all dislocations considered here (this being equivalent to the whole area being uniformly slipped along parallel dislocation planes). The average interval of single occurrence of each "join and release" process of dislocation energy discharge is then expressed by time $T_0 = 2\frac{t_0}{n}$, giving the frequency of occurrence during first interval of activity $(\tau_0 < t_0)$: $\eta_0 = \frac{\tau_0}{T_0}$. Each occurrence is represented by energy release[8] corresponding to the energy of the elementary dislocations:

$$E = e_0 = \frac{\mu b_0 l_0}{2\pi} \ln 2. \tag{4}$$

We would like now to define the time-interval τ_0 as connected with a rate of stress field expenses on dislocation pair production by the following condition: $\tau_0 v_0 = 2\rho_0$, where $2\rho_0$ is minimum spacing between dislocations of a pair. Thus the above conditions express that after time τ_0 a new set of pairs is generated in each source. All relations related to the present time interval are here summarized,

$$2d_0 = \frac{L - 2\rho_0 n}{n}, \tag{5}$$

7) *loc. cit.*, 5).
8) R. TEISSEYRE, *Acta Geoph. Pol.*, **12** (1964), No. 2.

$$t_0=\frac{d_0}{v_0}, \quad T_0=2\frac{t_0}{n}, \quad \frac{\tau_0}{T_0}=\eta_0, \quad v_0\tau_0=2\rho_0. \tag{6}$$

From relations (6) also follows:

$$\eta_0 d_0=n\rho_0. \tag{7}$$

After time interval τ_0 we next consider the second time interval τ_1; in our active volume a new set of dislocations should be considered as related to the same group of sources. Among the newly created n elementary dislocation pairs, $N_1=n-\eta_0$ now form double pairs, while η_0 form single ones (because there have remained at the $n-\eta_0$ sources the pairs belonging to the previous τ_0 interval). A double pair can be temporarily treated as a pair with the Burgers vector twice as great as a single pair: $b_1=2b_0$. Later we will return to this question. Simultaneously we would assume that dislocation lines limiting these dislocation surfaces are also separated by double distance $4\rho_0$. Hence the average dislocation spacing in pairs $2\rho_1$ will be expressed as follows,

$$2\rho_1=\frac{\eta_0 2\rho_0+(n-\eta_0)4\rho_0}{n}, \quad \text{or} \quad \rho_1=2\rho_0-\frac{\eta_0}{n}\rho_0. \tag{8}$$

From this immediately follows average value of distances between neighboring dislocations (belonging to adjacent pairs):

$$2d_1=\frac{L-2\rho_1 n}{n}. \tag{9}$$

Taking analogy to the previous interval (τ_0), we obtain,

$$t_1=\frac{d_1}{v_1}, \quad T_1=2\frac{t_1}{n}, \quad \frac{\tau_1}{T_1}=\eta_1, \quad v_1\tau_1=2\rho_0, \tag{10}$$

$$\eta_1 d_1=\eta_0 d_0=n\rho_0=\text{const.}. \tag{11}$$

Among the η_1 occurrences in the τ_1 interval, however, some could be related to dislocations of double and others to dislocations of single pairs. Number of single pairs equals η_0, double $n-\eta_0$; respective probabilities are given by:

$$\alpha_0=\frac{\eta_0^2}{n^2}, \quad \alpha_{10}=\frac{2\eta_0(n-\eta_0)}{n^2}, \quad \alpha_1=\frac{(n-\eta_0)^2}{n^2}=\frac{N_1^2}{n^2}, \tag{12}$$

where, α_0 probability of join-process of two dislocations belonging to elementary pairs,

α_{10} probability of dislocations from double and single pair,

α_1 probability related to join-process of two dislocations belonging to double pairs. Hence respective number of occurrences representing the greatest energy release in this interval ($E_1=4e_0$) is given by $\alpha_1\eta_1=\frac{N_1^2}{n^2}\eta_1$.

In the third interval we will have, in analogy, the following relations. The average spacing between dislocations of a given source: $2\rho_2$, where

$$\rho_2=3\rho_0-\frac{\eta_0}{n}\rho_0-\frac{\eta_1}{n}\rho_0 . \tag{13}$$

This relation expresses that dislocations spread out at a rate $\frac{\rho_0}{\tau_i}=v_i$, the first two terms on the right side being equal to $\rho_1+\rho_0$, whereas the third term bringing a decrease corresponding to the relative amount of dislocation discharges $\frac{\eta_1}{n}$ multiplied by ρ_0.

The number of sources unaffected by the previous energy release now equals

$$N_2=(n-\eta_0)\left(\frac{n-\eta_1}{n}\right). \tag{14}$$

This represents the decrease of the former number of unaffected pairs $N_1=(n-\eta_0)$ by the factor equal to ratio of unaffected sources in the τ_1 interval, $n-\eta_1$, to the total number of possible occurrences, n. The probability of the strongest shocks in the τ_2 interval is given by

$$\alpha_2=\frac{N_2^2}{n^2} , \tag{15}$$

and corresponding number of the strongest occurrences is given by $\alpha_2\eta_2$. We are now able to express these relations for an arbitrary time interval (the $(k+1)$-th interval), as given in Table 1.

The average velocity of dislocation movement is related not only to the external stresses but also results from local stress concentration in a considered region. An increase of local stress concentration is caused by dislocation production at individual sources.

In the $(k+1)$-th time interval the unaffected source (by previous

Table 1. Relation describing the $(k+1)$-th interval of activity.

Relation	Relation at $\tau_i=\tau_0,\ v_i=v_0$	Meaning
$\rho_k=(k+1)\rho_0-\frac{\rho_0}{n}\sum_{i=0}^{k-1}\eta_i$		half of dislocation spacing around sources
$2d_k=\frac{L-2\rho_k n}{n}$		average distance between dislocations of different pairs (sources)
v_k	v_0	average dislocation velocity (related to total stresses in this interval)
$t_k=\frac{d_k}{v_k}$	$t_k=\frac{d_k}{v_0}$	half-time (i.e. mean time) required for energy release of all dislocations
$T_k=2\frac{t_k}{n}$		frequency of occurrences expressed by their average time
$\eta_k=\frac{\tau_k}{T_k}$	$\eta_k=\frac{\tau_0}{T_k}$	number of occurrences in the $(k+1)$-th interval
$\frac{\eta_k}{\tau_k}=\frac{1}{T_k}$	$\frac{\eta_k}{\tau_0}=\frac{1}{T_k}$	frequency of skocks (e.g. daily)
$\tau_k v_k=2\rho_0$	$\tau_0 v_0=2\rho_0$	condition for length of time interval required for generation of a new set of pairs in dislocation sources
$\eta_k d_k=n\rho_0=\text{const}$		auxiliary relation
$N_k=n\prod_{i=0}^{k-1}\left(1-\frac{\eta_i}{n}\right)$		number of unaffected sources from beginning until this interval
$\alpha_k=\frac{N_k^2}{n^2},\ \alpha_k\eta_k$		probability and number of the strongest shocks in the $(k+1)$-th interval
$E_{(k+1)}=m^2e_0;\ m\leq k+1$	$m\leq\frac{t}{\tau_0}$	energy of the strongest shocks

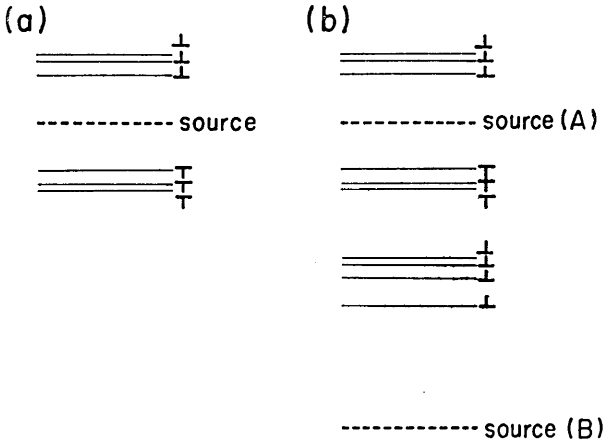


Fig. 2. Structure of multiple-dislocation source.

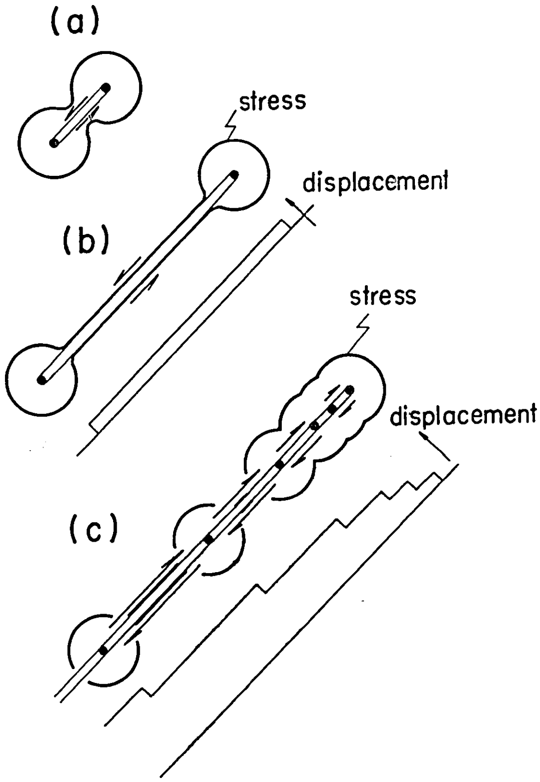


Fig. 3. Stress concentration in a multiple-dislocation source.

energy release occurrences) represents a series of positive and negative dislocations situated with a source between them (Fig. 2a). The dislocations in a series are not independent; in this case a mutual interaction should be taken into account. In the result the dislocations are pushed near the first outward (leading) dislocation, where the greatest stress concentration will occur[9].

We should, however, return to the problem of stress distribution around dislocations. Small dislocation area bounded by two dislocation lines has stress distribution schematically indicated at Fig. 3a. For greater distance between the edges of a dislocated area the stresses are concentrated almost only around the border of dislocation, that is, along the dislocation lines (Fig. 3b-dislocation lines are here perpendicular to the drawing plane). For a dislocation series the schemative stress distribution is shown in Fig. 3c. We can see that at least some dislocation near the first leading dislocation can be treated as one dislocation with appropriately greater Burgers vector (at Fig. 3c: first three dislocations). This can be formally brought into evidence by calculation of distances between dislocations in a series and then comparing those distances with dislocation radii. Let us assume that dislocations had originally equal Burgers vector b_0. Then, for $(k+1)$ dislocations of the same sign we have the following situations[10]:

1) As a result of dislocation interactions the concentrated stress is acting on the first leading dislocation, being totally $(k+1)$ times greater than the average (external) stress field: $(k+1)p$, where p is external field.

2) The successive distances between dislocations are given as follows:

$$r_{i,i+1}=\frac{\mu b_0}{16\pi^2(k+1)\rho}(j_{i+1}^2-j_i^2)\,, \tag{16}$$

where j_i is the i-th root of the Bessel function J_1. These distances are comparatively small near leading dislocation and greater for internally lying dislocations, that is, for dislocations lying nearer their common source. All successive dislocations separated by the distance $r_{i,i+1}$, less than $2\rho_0$, shall be treated as one dislocation with Burgers vector resulting as their sum. To describe the situation properly one could use here the crack theory, but in a rough approximation, we can assume that the leading dislocation has now the appropriately greater Burgers vector mb_0, where m represents a number of dislocations gathered inside ranges of their

9) I. D. ESHELBY, F. C. FRANK and F.R.N. NABARRO, *Phil. Mag.*, **41** (1951), 351.
10) *loc. cit.*, 9).

dislocation radii $2m\rho_0$ (Fig. 3c). From the above result it follows that the number m in the $(k+1)$ interval is defined by the condition, $\frac{N_{i,i+1}}{2\rho_0} \leq 1$.

The joining process of dislocations proceeding from neighboring sources should be described by the joining of two series of dislocations and in this way we can explain the gradual increase of the stronger events with time.

The respective energy release corresponding to the strongest occurrence is given now by

$$E_{(k+1)} = m^2 \frac{\mu b_0^2 l_0}{2\pi} \ln 2 = m^2 e_0, \tag{17}$$

where m is smaller than the number of dislocations created around a given source, $m \leq k+1$. Or, for $\tau_i = \tau_0 = \text{const.}$, $m \leq \frac{t}{\tau_0}$, where t is the time from the beginning of swarm.

Next dislocations in this series could form immediate replicas according to successive contacts of dislocations from both series (Fig. 2b). The swarm process will reach its maximum level when the number of shocks in some τ_m interval becomes equal to the number of generated pairs, that is, equal to n. More exactly, one should here take into account different energy releases and compute an equivalent number of shocks as reduced to energy e_0 of the elementary dislocation.

The above presented considerations and their results will remain valid even if one assumes slow movements of randomly distributed sources.

4. Simplified theory of swarm development

According to former considerations the rate of generation of new dislocations in the assumed sources is equal to time τ_k. These dislocations either increase the total number of dislocations inside an active volume or increase Burgers vector of the previously existing dislocations related to the same source (as it was pointed out in regard to the first dislocation in series shown at Fig. 3c).

Let us now assume a simplified model of swarm development. First we assume that external stress remains constant $(p_i = p_0)$, then $v_i = v_0$. Also, the intervals of activity defined as time required for generating a new set of dislocation pairs in all n sources will remain constant, *i.e.* $\tau_i = \tau_0$. In the present model, we also consider that activity can be represented by the sum of activity related to the respective set of dislocations. Then the number of events can be synthesized by geometrical construction as explained in Fig. 4. If we denote here by η_i^* the partial activity function (number of shocks in time) related to each triangle separately, then the total daily frequency will be given by a sum, $\sum \eta_i^*$. Each triangle representing one set of dislocations will thus have a constant mean discharge time t_0. Let us assume that $r\tau_0 = t_0$, where, r is an integer.

Oscillation with the period τ_0 is expected due to this generation interval time and to those properties of dislocation interactions, which prevails time connected energy release in a number of annihilation processes of dislocations[11].

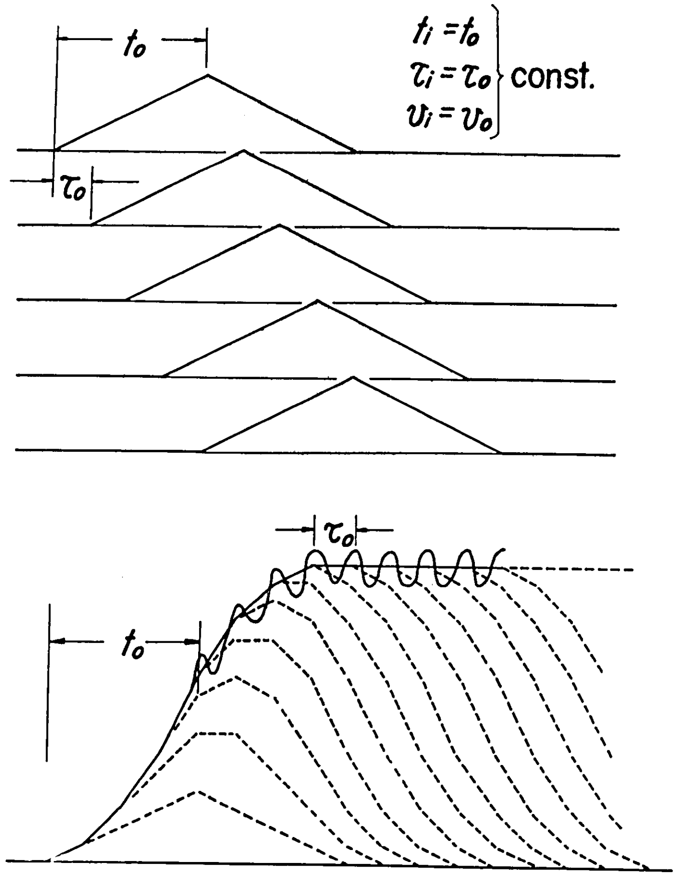


Fig. 4. Development of swarm activity.

11) *loc. cit.*, 6).

Repeated generation of dislocation pairs causes increasing activity as shown at Fig. 4. The mean half-time of discharge is assumed here constant and equal to t_0.

After time $2t_0=2r\tau_0$, the activity will reach its maximum level as we learn from Fig. 4; we notice that $2sn$ pairs of elementary dislocations have been created during this time.

Starting from this moment we have a stable activity with some oscillation with period τ_0. Between the time $2t_0=2r\tau_0$ and $4t_0=4r\tau_0$ the number of generated dislocation pairs will amount to $2rn$, as there are $2r$ time intervals. As the activity is at constant level, we are expecting also $2rn$ events corresponding to the elementary dislocations release, or a somewhat smaller number when taking compound release of dislocation series into account. From the above consideration we can deduce that mean activity in the period $(0, 2t_0)$ is half of the stationary one and the maximum number of dislocation pairs contained in the volume is equal to rn (Fig. 4). Thus the above simplified considerations lead us to some important relations as follows:

$$\text{time interval } (2t_0,\ 4t_0):\quad \left.\begin{array}{l}\text{number of pairs generated}=2rn\\ \text{number of pairs annihilated}=2rn\end{array}\right\} \tag{18a}$$

$$\text{time interval } (0,\ 2t_0):\quad \left.\begin{array}{l}\text{number of pairs generated}=2rn\\ \text{number of pairs annihilated}=rn\end{array}\right\} \tag{18b}$$

$$\text{maximum number of dislocation pairs at maximum level}=rn. \tag{18c}$$

Lastly we would like to present a simple geometric reasoning related to the danger of a very great earthquake due to *sui generis* spontaneous process of dislocation annihilations. This would depend upon how far the dislocation lines are from each other and this we can express simply by ratio of a minimum distance between dislocations at full activity $2d_m$ to the distance of elementary spacing $2\rho_0$, as follows:

$$H=\frac{2d_m}{2\rho_0}=\left(\frac{L-2\rho_0 rn}{rn}\right)/2\rho_0=\frac{L}{2\rho_0 rn}-1\,. \tag{19}$$

When $2d_m$ is small, that means when $2\rho_0 rn=\frac{\mu b_0 t_0 n}{\pi S\tau_0}$ approaches L, the possibility of simultaneous discharge becomes increasingly greater. The ratio H expresses to what extent our volume is fulfilled by the elementary dislocations.

5. The Matsushiro earthquake swarms explained by the present model

There have been a number of local shocks in the Matsushiro area since August 1965, which seem to offer an interesting example for the dislccational model. In the following, therefore, we shall apply a certain set of parameters to the model and examine to see if it explains the basic features of the sequence satisfactorily or not.

As reported by many seismologists, the swarm activity started in early August 1965. Then it developed to an activity level of 70−90 felt shocks and 700−800 unfelt shocks per day, so far as the JMA's reports are concerned (Fig. 5). The activity maintained this level for a considerably long time until March 1966 when it developed to a new level several times as high as the previous one. For simplicity's sake in treat-

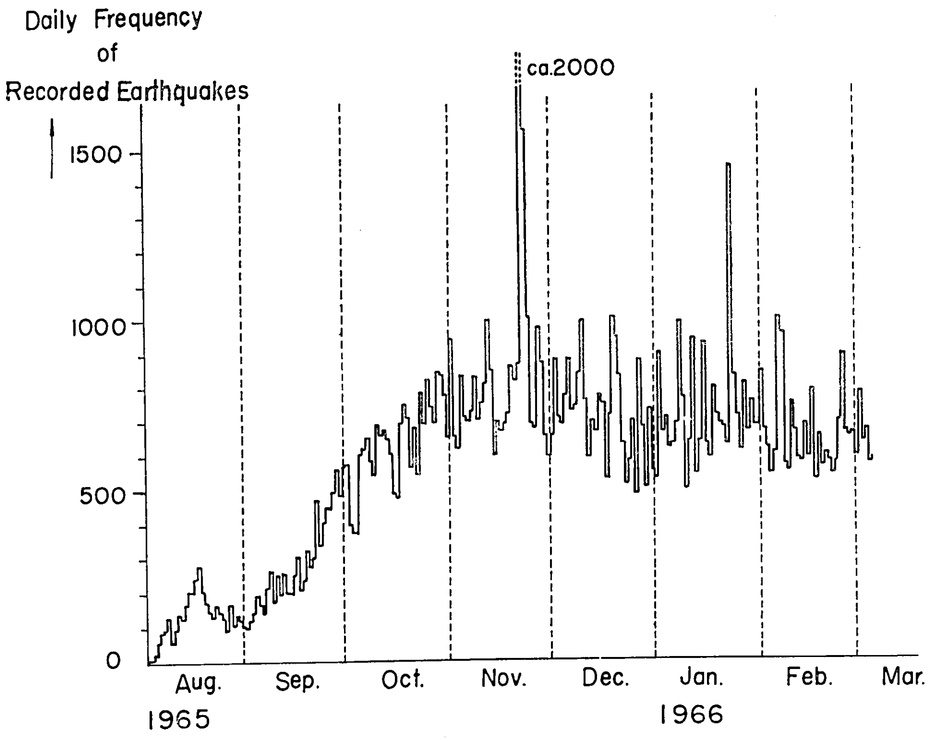


Fig. 5. Daily frequency of swarm earthquakes recorded at Matsushiro (after J.M.A.).

ment, however, let us limit our discussion to the sequences in the initial and the first stationary phases as illustrated in Fig. 5. We shall also assume that the general tendency of the present events is well represented by the JMA's records which are based on observations at the Matsushiro Seismological Station equipped with the WWSS system by USCGS. This assumption may be reasonable as the said station is located in the central part of the seismic area. We take the observational data by JMA and ERI into consideration and estimate $L_0D_0H_0$ at $10\times7\times4\,\mathrm{km}^3$ (Table 2). As to the parameters for the medium, we also assume μ and S at 10^{11} c.g.s. and 10^7 c.g.s., respectively. Low value of strength assumed here would be attributed to fracture mechanism along material failures.

Table 2. Parameters for the case of the Matsushiro earthquake swarms.

Parameter	Observed	Assumed	Computed	Remarks
μ	—	10^{11} c.g.s.	—	
S	—	$10^7\sim10^6$ c.g.s.	—	
$L_0D_0H_0$	$10\times7\times4$ km^3	—	—	
ΔD	—	0.5 km	—	
l_0	—	$10\rho_0$	30~130 m	
d_0	—	—	~1.5(~0.2km)	
$2\rho_0$	—	—	6~26 m	
b_0	—	—	0.18~0.032 cm	
e_0	—	10^{12} erg	—	$M=0\sim1$
E_{max}	See Fig. 7	—	See Fig. 7	
v_0	—	—	6~26 cm/hr	$\tau_0v_0=2\rho_0$
t_0	40 days	—	—	
τ_0	4~5 days	—	—	
r	—	—	~10	$t_0=r\tau_0$
$\sum\eta_i^*$	700~800	—	—	Daily freq. (stationary)
η_{max}	—	—	80	$\sum\eta_i^*=\eta_{max}\ (r-1)$
n	—	—	4×10^3	$n=\eta_{max}\ (r\tau_0-1)$
$2rn$	3×10^5	—	8×10^4	

Comparison of Fig. 5 with Fig. 4 leads us to estimate t_0 at about 40 days. For the purpose of deriving τ_0 from observations, we worked out a Fourier analysis on the daily frequency of shocks for the period of 120 days following October 25, 1965. So far as the present analysis

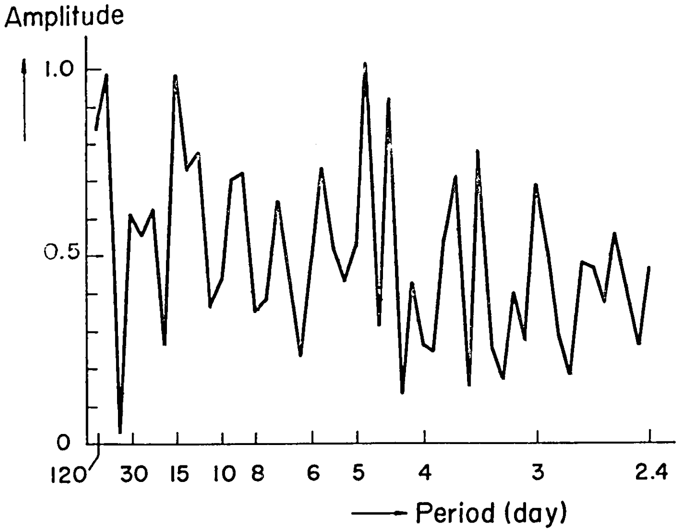


Fig. 6. Spectrum of daily change of earthquake numbers (Oct. 25, 1965 —— Feb. 22, 1966).

is concerned, there are two noticeable periods of harmonics ($14\sim15$ and $4\sim5$ days) besides a very low-frequency harmonics (Fig. 6). The first period is more likely to indicate the tides' influences on the sequence than to indicate the effect as illustrated in Fig. 6. So let us here assume that the second period is attributed to the present effect. We then obtain, $\tau_0=4\sim5$ days and $r=\frac{t_0}{\tau_0}\approx10$. We also obtain $\sum\eta_i^*=700\sim800$ as the average level of seismic activity in the stationary stage (Fig. 5). Simple consideration on the present model leads us to relations as follows,

$$\left.\begin{aligned}\sum\eta_i^*&=\eta_{\max}^*(r-1)\\ n&=\eta^*(r\tau_0-1)\end{aligned}\right\}. \tag{20}$$

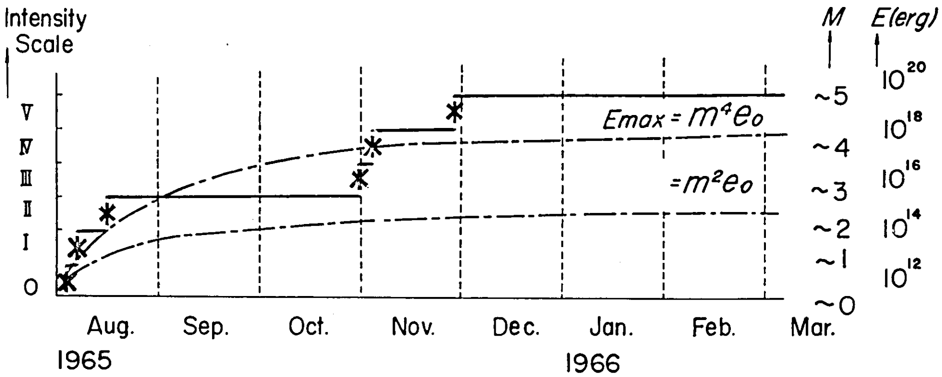
Hence, $\eta_{\max}^*$ and n are computed as 80 and 4×10^3, respectively.

Before entering the discussion on the size of an elementary dislocation, we have to take some assumption on the value of e_0; f.i. $e_0=10^{12}$ erg. This inevitably means that there are no earthquakes of energy less than the assumed level. It is obvious that there are numerous earthquakes of far less energy. However, we do take the above-stated assumption as our current interest lies in seeing the micro-seismicity with respect to a magnitude range of $M=0\sim 1$, which is the level detectable by the WWSS system at Matsushiro. By use of eqs. (1) and (4), we obtain $b_0=0.2$ cm, and accordingly, $2\rho_0=6$ m. On the basis of these results, the following parameters are estimated:

$$l_0=30\text{ m}, \quad \Delta D=0.5\text{ km}, \quad d_0=0.5\text{ km},$$

where we assumed that $\Delta D \approx d_0$ to represent uniform distribution of dislocation sources in the medium. All the parameters thus estimated are listed in Table 2, where we find several parameters computed in two different ways. For instance, the most probable separation of the dislocation sources, d_0, are given as 1.5 km by use of n, ΔD_0, and $L_0 D_0 H_0$. Another way to estimate it is to use v_0 and t_0, which gives $d_0=0.2$ km to show a reasonable agreement with the former, in the order of magnitude. The second parameter that works as the next check point is $2rn$, which is computed as 8×10^4 using r and n. The same parameter estimated alternatively (total number of shocks which occurred during t_0, in the stationary stage) is 3×10^5, which does not differ much from the previous value in the accuracy of order of magnitude estimation.

The most serious disagreement is noticed on the third check point, E_{max}. As given in eq. (17), energy of any shocks occurring in the $(k+1)$-th interval cannot be larger than $m^2 e_0$. In Fig. 7 is illustrated the E_{max}-



Feb. 7. Theoretical upper limit of earthquake magnitude (—— - ——) as compared with observations (*).

curve as a function of time which was computed on the basis of the above-stated parameters. Starmarks in the same figure show when the station received the first shocks of each of the increasing intensity scale.

The vertical scales on the right indicate energy and magnitude of shocks corresponding to the felt intensity. It is evident that the theoretical curve of E_{max} does not explain the observations.

The present disagreement seems to suggest our model's limit of application to earthquake phenomena. We should improve, more or less, the model so as it may promise better explanation of the nature. We tentatively consider that the following two modification might be useful for this purpose.

The first possibility is to consider that energy of the unit shock, e_0, increases to e_1, e_2, and so on as the activity develops. Another possible modification is to lessen the assumption that dislocation lines move only in one common direction. We do not know concretely what kind of modification should be applied to eq. (17) for E_{max}. But we may assume, as a test, that the index of m increases to 4 for the case of two-dimensional movement and interactions of dislocation lines. The chain line in Fig. 7 illustrates the said case, which fits the observations better.

Acknowledgement

The authors wish to acknowledge with thanks the valuable suggestions by Prof. C. Kisslinger of Saint Louis University and by Dr. S. Nagumo of this Institute. The present work was partly developed in the International Institute of Seismology and Earthquake Engineering, Tokyo, where one of the authors (R.T.) stayed in 1965–1966 as a UNESCO expert in seismology. He is very grateful to Dr. Omote, Acting Director of IISEE, for the facilities kindly provided.

---

43. 群発地震の転位理論模型

地震研究所 笠原 慶一

建築研究所 国際地震工学部 Roman TEISSEYRE

内部に多数の転位源（同型式）が存在する媒質が歪力の作用下に置かれた時，そこにどのような特性の転位発達型式が見られるかを簡単な模型について考察した．更に一例として松代における地震活動の時間的経過をとり，模型との比較を試みた．