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26. *A Further Study of Diffraction of Tsunami Invading a Semi-circular Peninsula.*

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Abstract

In this paper, further calculations of the waves around a peninsula were made. Then it is found that: (1) large wave height takes place near the root of a peninsula (which has already been ascertained in the previous paper), (2) rotating waves appear in a nearby part of the peninsula, (3) the incident waves are reflected in two parts, *i.e.*, near the root of a peninsula and in a part of the tip of a headland and, between the above two zones of reflection, a discriminative line of discontinuity of phases is found.

1. Introduction

In the previous work[1)] entitled "Diffraction of Tsunami Invading a Semi-circular Peninsula" (this paper will be referred to as paper I in the following), it has been verified that a huge wave height occurs in a nearby part of the root of a peninsula. In this report, further computations are made to inquire into the behaviors of the waves diffracted around a peninsula.

2. Theory

According to paper I, when the surging periodic waves are directed perpendicular to the straight coast (refer to Fig. 1), the expression of the waves arround a semi-circular peninsula is given as follows:—

$$\zeta = 2\zeta_0 \cos kx + \sum_{m=0}^{\infty} A_{2m}^{(2)} \cos 2m\theta \cdot H_{2m}^{(2)}(kr), \quad (1)$$

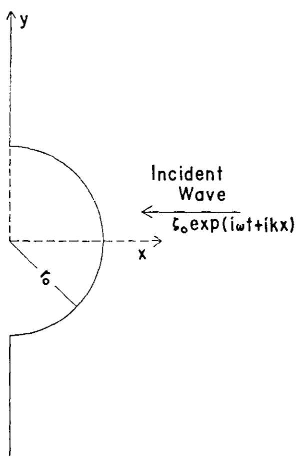

Fig. 1

1) T. Momoi, *Bull. Earthq. Res. Inst.*, **41** (1963), 589.

where

$$A_{2m}^{(2)}=(-1)^{m+1}2\varepsilon_m\cdot\frac{J'_{2m}(kr_0)}{H_{2m}^{(2)\prime}(kr_0)}\cdot\zeta_0 \tag{2}$$

$$(\varepsilon_m=1 \text{ for } m=0 \text{ and } 2 \text{ for } m\geqq 1)\,,$$

and where the used definitions and notations are exactly the same as those in paper I (refer to (7) and (9) of paper I).

3. Numerical Calculations and Discussions

In the previous paper I, the calculations along a semi-circular peninsula and a straight coast were made for parameter $kr_0=4$ alone. Then, we came to the conclusion that a large wave height is expected at a part near the root of a semi-circular peninsula as a result of the coupling of the directly incident waves and the diffracted ones (around a peninsula).

In this article, overall pictures of the behaviors of the waves in the surrounding part of the peninsula are presented for the cases of $kr_0=1$, 2 and 3.

To begin with, the equi-amplitude lines of the resultant waves, as shown in (1), are visualized respectively in Figs. 2, 4 and 6 for $kr_0=1$, 2 and 3, while the equi-phase lines are given in Figs. 3, 5 and 7 for $kr_0=1$, 2 and 3. According to these figures, the following facts are known. Maximum wave heights are seen in the vicinity of the root of a peninsula in Figs. 2, 4 and 6 (The occurrence of a maximum, however, is not so trivial for the case of $kr_0=1$ in Fig. 2. For this purpose, a further calculation along a straight coast is necessitated.) These maximums are explained from the figures of the phases (Figs. 3, 5 and 7) in such a way that the waves are, as a result of the coupling of incident, reflected and diffracted waves, propagated along the coast of a peninsula toward the straight coast and collide with it producing a maximum.

When one sees figures relevant to phases (Figs. 3, 5 and 7), these figures are marked by an existence of rotating waves. Such a rotation is interpreted to be caused by an inflow of the waves along the circular coast of the peninsula. Comparing the figures of phases (Figs. 5 and 7) and those of amplitudes (Figs. 4 and 6), it is found that the centers of the rotation have zero amplitudes and supposedly zero-gradient amplitudes, so that the vorticity component

$$\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

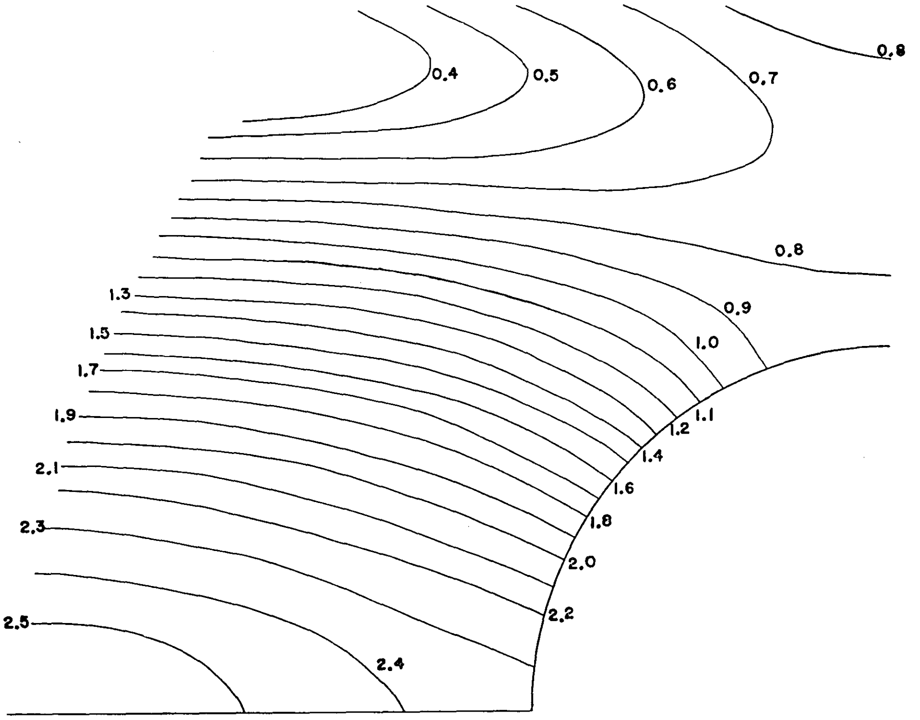

Fig. 2. Variation of amplitude for $kr_0=1.0$.

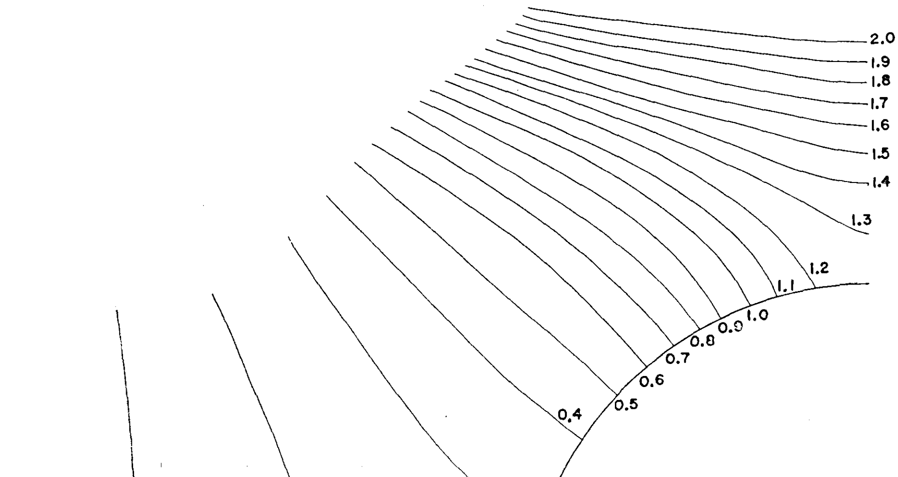

Fig. 3. Variation of phase for $kr_0=1.0$.

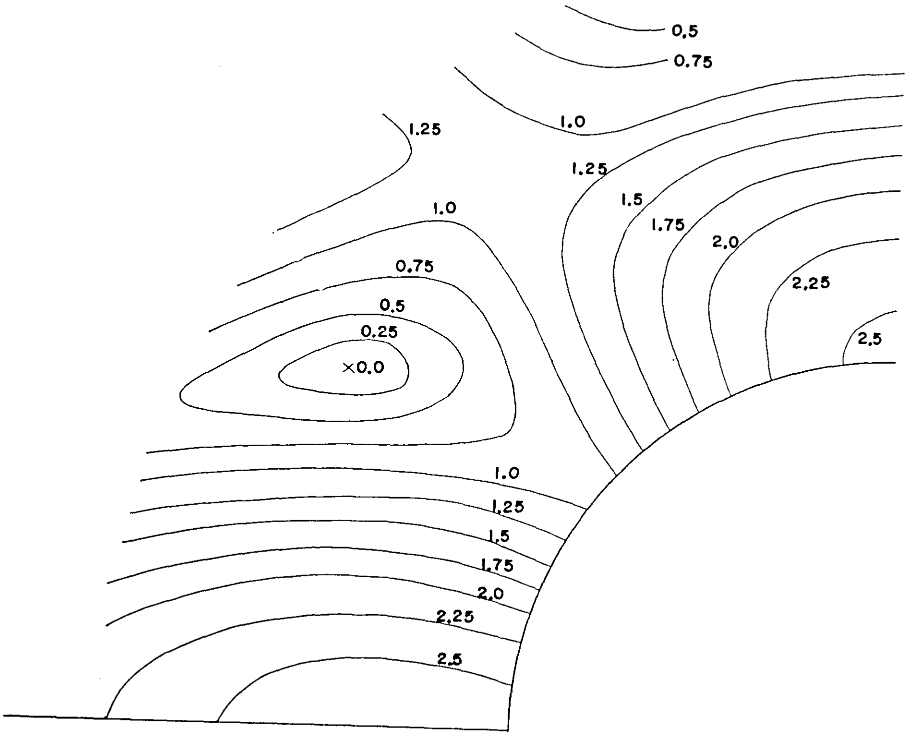

Fig. 4. Variation of amplitude for $kr_0=2.0$.

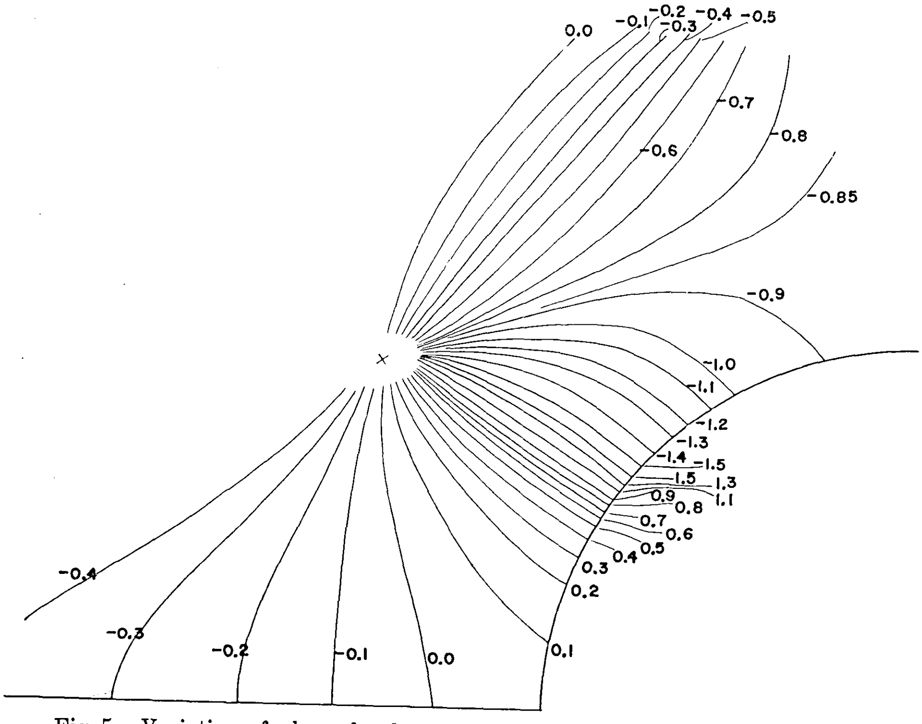

Fig. 5. Variation of phase for $kr_0=2.0$ (Stated values of phases are considered in the range of principal value of tangent).

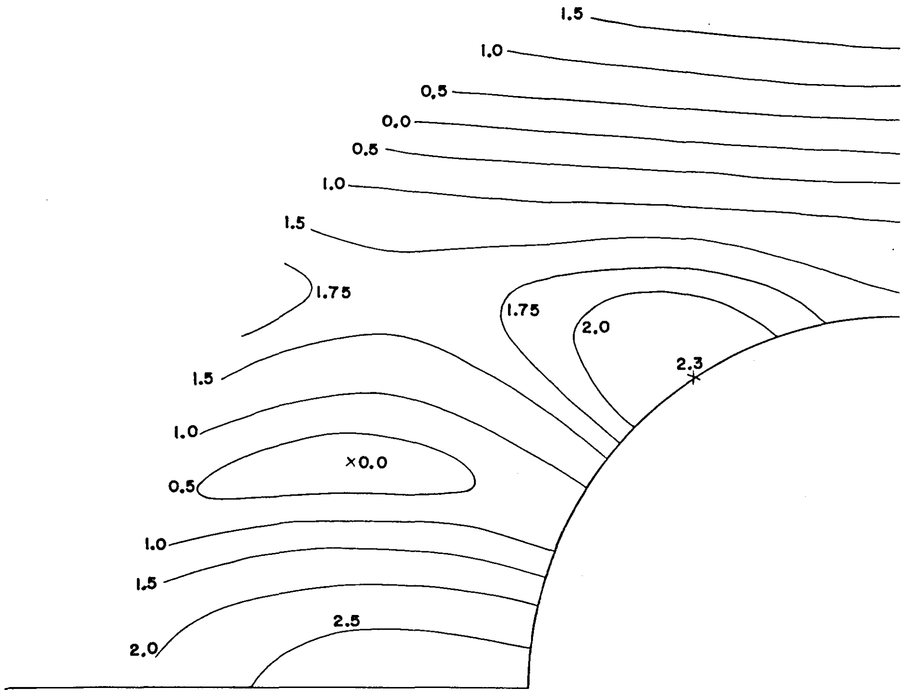

Fig. 6. Variation of amplitude for $kr_0=3.0$.

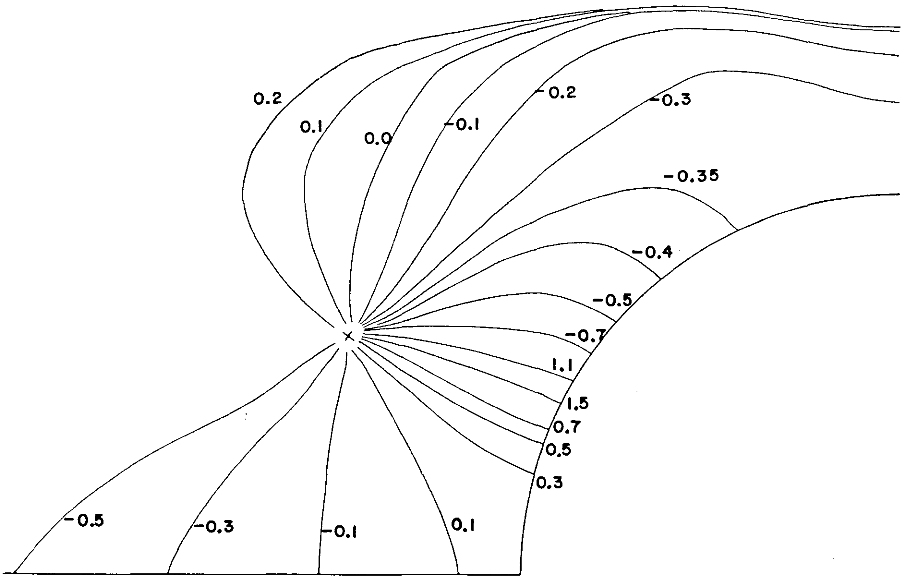

Fig. 7. Variation of phase for $kr_0=3.0$ (Stated values are considered in the range of principal value of tangent).

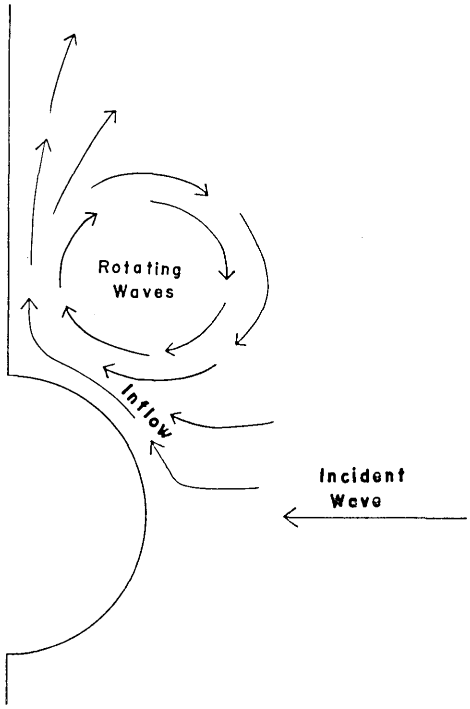

Fig. 8.

(u and v are the x- and y-components of velocities) disappears at these points.

The above mentioned behaviors of the waves are explained schematically in Fig. 8.

The reflected and diffracted waves except for the incident ones are considered here only for the waves of the parameter $kr_0=2$. Since the incident waves are, in our theory, expressed by $\zeta_0 e^{i\omega t+ikx}$, the reflected and diffracted waves ζ_{rd} are given as (from (1))

$$\zeta_{rd}=\zeta_0 e^{-ikx}+\sum_{m=0}^{\infty} A_{2m}^{(2)} \cos 2m\theta \cdot H_{2m}^{(2)}(kr)\ , \tag{3}$$

where $A_{2m}^{(2)}$ is presented in (2).

By use of the expression (3), the phase variation of the reflected and diffracted waves is drawn in Fig. 9 with the aid of an electronic computer. According to this figure, the reflections of the waves take place in two parts: one is in the root of a peninsula and the other near the tip of a headland. The zones of the reflections of the former and the latter are referred to as *A-zone* and *B-zone* in Fig. 10. At the boundary of the above two zones the waves advance very slowly from A-zone to B-zone, which is designated by *stagnation line* in Fig. 10.

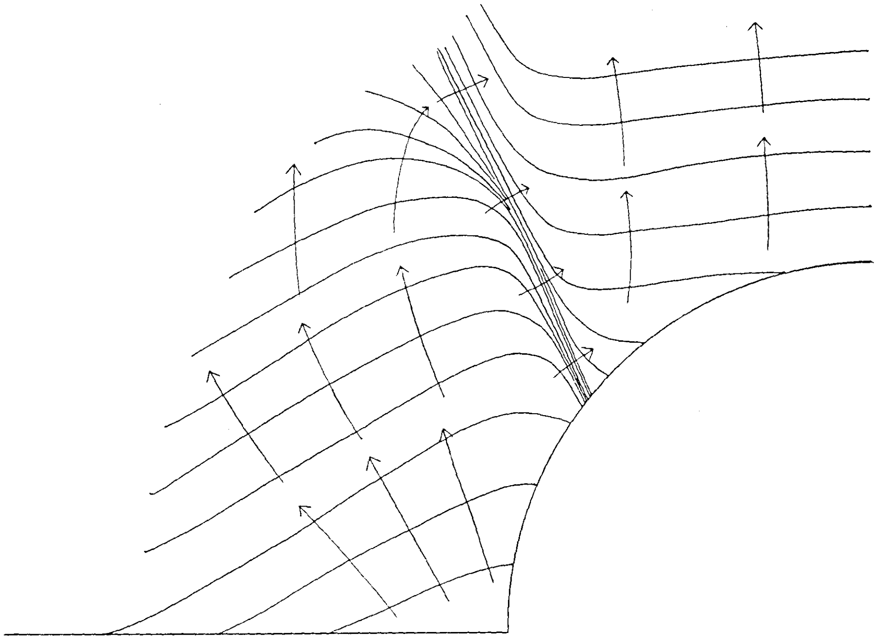

Fig. 9. Variation of phase of reflected and diffracted waves for $kr_0=2.0$ (arrows in the figure stand for direction of propagation of waves).

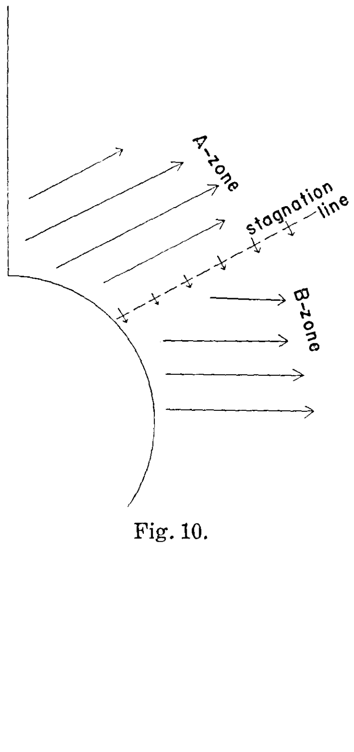

Fig. 10.

26. 半円形の半島をおそつた津波の回折（続報）

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前報告に続き本報告において，半島のまわりにおける津波の様子が論じられている．
数値計算の結果，次のことが知られる．
（1） 半島の根元近くに波高の大きい所が現われる（これはすでに前報告で知られている），
（2） 沖合に回転する波が現われる，
（3） 進入波は 2 つの部分で反射される，すなわち半島の根元の近くと半島の先端近くとである．そしてこの 2 つの反射波の間には一種の不連続線のようなものが存在する．