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29. *The Crust and Upper Mantle Structure in Japan.*

Part 3. An anisotropic model of the structure in Japan.

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1. Introduction

Aki and Kaminuma (1963) found that the model 6EJ which explains the observed phase velocity of Rayleigh waves in Japan, does not explain the observed phase velocity of Love waves. They suggested that the discrepancy might be due to either an anisotropy of the crust-mantle or errors in Love wave velocities. Since the same results are obtained in Part 1 of the present paper (Kaminuma; 1966), we may conclude that the discrepancy cannot be considered as that due to errors in observation. Brune and several other authors have studied phase velocity dispersions of both Love and Rayleigh waves and shown that no anisotropy is required for constructing structural models (for example Brune and Dorman; 1963). But it is impossible to explain phase velocity data in Japan with such a model considering the influence of the low velocity layer in the upper mantle and the sphericity of the earth.

McEvilly (1964) found similar results to those obtained by us from the surface wave analysis. He assumed different velocities of SH and SV waves at depths from 20 to 262 km to explain the observed discrepancy.

In this paper, we explain the discrepancy by assuming anisotropy having a transversely isotropic symmetry for the crust-mantle.

2. A transversely isotropic symmetry

The matrix of the elastic constants for a medium with a transversely isotropic symmetry (Love; 1944) is

$$\begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{C_{11}-C_{12}}{2} \end{pmatrix}$$

Accordingly, the stresses are expressed, taking z-axis as the vertical one, as follows;

$$\begin{aligned} Xx &= C_{11}e_{xx}+C_{12}e_{yy}+C_{13}e_{zz} , \\ Yy &= C_{12}e_{xx}+C_{11}e_{yy}+C_{13}e_{zz} , \\ Zz &= C_{13}e_{xx}+C_{13}e_{yy}+C_{33}e_{zz} , \\ Xy &= \frac{C_{11}-C_{12}}{2}e_{xy} , \\ Yz &= C_{44}e_{yz} , \\ Zx &= C_{44}e_{zx} . \end{aligned}$$

For an isotropic body,

$$C_{11}=C_{33}=\lambda+2\mu ; \qquad C_{13}=\lambda ;$$
$$\frac{C_{11}-C_{12}}{2}=C_{44}=\mu .$$

In the transversely isotropic medium, these five elastic constants are independent.

Matuzawa (1943) obtained the velocities of elastic waves propagated in the above anisotropic medium. For waves propagated to a direction specified by direction cosine $(0, m, n)$, in the y-z plane, he found,

$$\left.\begin{aligned} \rho V_p^2 &= \frac{B+\sqrt{B^2-4A}}{2} \\ \rho V_{SV}^2 &= \frac{B-\sqrt{B^2-4A}}{2} \\ \rho V_{SH}^2 &= \frac{C_{11}-C_{12}}{2}m^2+C_{44}n^2 \end{aligned}\right\} \qquad (1)$$

$$A=-(C_{13}+C_{44})m^2n^2+(C_{11}m^2+C_{44}n^2)(C_{44}m^2+C_{33}n^2) ,$$

$$B=C_{11}m^2+C_{33}n^2+C_{44}\ ,$$

where V_p, V_{SV} and V_{SH} are the velocities of P, SV and SH waves, respectively, and ρ is the density in the anisotropic medium. For transmission along the y-direction perpendicular to the z-axis, $n=0$, the velocities are

$$V_p^2=\frac{C_{11}}{\rho}\ , \qquad V_{SV}^2=\frac{C_{44}}{\rho}\ , \qquad V_{SH}^2=\frac{C_{11}-C_{12}}{2}\ .$$

3. A model of layered anisotropic medium

In equation 1, we assume the relation of elastic constants as follows;

$$\frac{C_{33}}{C_{11}}=1\ ; \qquad \frac{C_{13}+2C_{44}}{C_{11}}=1\ ,$$

$$\frac{C_{11}-C_{13}}{2}>C_{44} \text{ or } \frac{C_{11}-C_{12}}{2}=C_{44}(1+\delta)\ ,$$

which implies that the medium is isotropic for P and SV waves whose velocities in this medium are constants for any directions. The velocities of P and SV are $V_p=\sqrt{\frac{C_{11}}{\rho}}$ and $V_{SV}=\sqrt{\frac{C_{44}}{\rho}}$ respectively. Only the velocity of SH wave depends on the direction.

$\frac{C_{11}-C_{12}}{2}>C_{44}$ means that the modulus of rigidity for Yz and Zx are smaller than that for Xy. In such a medium, velocities whose particle displacement is Love type are expected to be higher than those of waves of Rayleigh type.

In this layered medium with transversely isotropic symmetry Rayleigh waves exist independently of Love waves; there is no coupling between motions of Rayleigh and Love types. As the velocity of Rayleigh waves is not affected by the velocity of SH waves, there is no infulence of $\frac{C_{11}-C_{12}}{2}$ on the phase velocity of Rayleigh waves (Anderson; 1961). As Stonely (1949) has pointed out, the modulus C_{44} corresponds to the isotropic rigidity for Rayleigh wave motion, whereas the corresponding modulus for Love wave motion is $\frac{C_{11}-C_{12}}{2}$.

Anderson (1961) has obtained the periodic equation for Love waves in case of one layered anisotropic half space;

$$\tan 2kH\left(\frac{C^2}{\beta_1^2}-1\right)^{1/2}\left[\frac{N_1}{L_1}\right]^{1/2}=\left[\frac{L_2N_2}{L_1N_1}\right]^{1/2}\left[\frac{1-C^2/\beta_2^2}{C^2/\beta_1^2-1}\right]^{1/2}, \tag{2}$$

where C is the phase velocity of Love waves and $2H$ is the thickness of the anisotropic layer with constants N_1, L_1 and β_1 over an anisotropic half space with constants N_2, L_2, and β_2. N and L correspond with $\frac{C_{11}-C_{12}}{2}$ and C_{44}, respectively, and β is the velocity of SH wave; $\beta^2=N/\rho$.

For transmission along the y-direction perpendicular to the z-axis, $m=1$ and $n=0$ in equation 1, the velocities of shear waves are as follows;

$$V_{SH_H}\approx\sqrt{\frac{C_{44}}{\rho}}\left(1+\frac{\delta}{2}\right). \tag{3}$$

And for transmission along the z-direction, $m=0$ and $n=1$, that of shear waves is as follows;

$$V_{SH_V}=V_{SV_H}=\sqrt{\frac{C_{44}}{\rho}}, \tag{4}$$

where, the smaller suffixes H and V mean that the directions of wave propagation are horizontal and vertical respectively.

4. Anisotropic layered model in Japan

One layered model, J-S-01, of the crustal and upper mantle structure in Japan is given in Table 1. The physical constants of this model are

Table 1. Layer parameters for the model J-S-01.

H	Vp	Vs	ρ	σ
32 km	5.9 km/s	3.40 km/s	2.8 gr/cm^3	0.25
—	7.7	4.32	3.2	0.27

also determined in considersing the results of explosion seismic observation and surface wave studies, The dispersion curves of Rayleigh waves for this model show a satisfactory coincidence with observed data as shown in Fig. 1.

There is a significant discrepancy between the observed dispersion curve of Love waves and the theoretical curve based on this model. The observed phase velocities of Love waves are 0.1 to 0.2 km/s higher than those of the theoretical one. To explain this discrepancy, we introduce the type of anisotropy mentioned before in the above model of structure and calculated the dispersion curves of Love waves using equation 2. We set $N_1=L_1(1.0+\delta_1)$, and $N_2=L_2(1.0+\delta_2)$ in that equation and compute dispersion curves of Love waves. δ_1 and δ_2 are the degrees

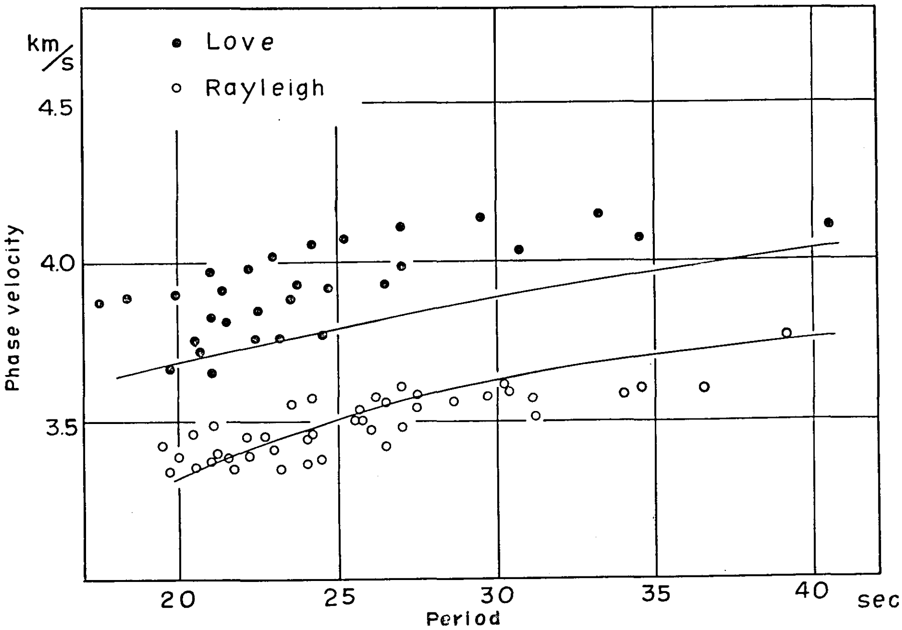

Fig. 1. The phase velocity curves for J-S-01. The data are the phase velocities in central Japan.

of anisotropy in the surface layer and the half space. After calculating the phase velocity with many combinations of δ_1 and δ_2, we have obtained one set of $\delta_1=0.04$ and $\delta_2=0.14$. From equation 3, the velocity of *SH* waves to the vertical direction is 2 per cent lower than that to the horizontal direction in the crust and 7 per cent in the upper mantle, which gives a satisfactory fit to the observed data. The dispersion curve is given in Fig. 2. If such anisotropy in the crust exists only in the lower part (McEvilly; 1964), the value may be taken a little larger than 2 per cent.

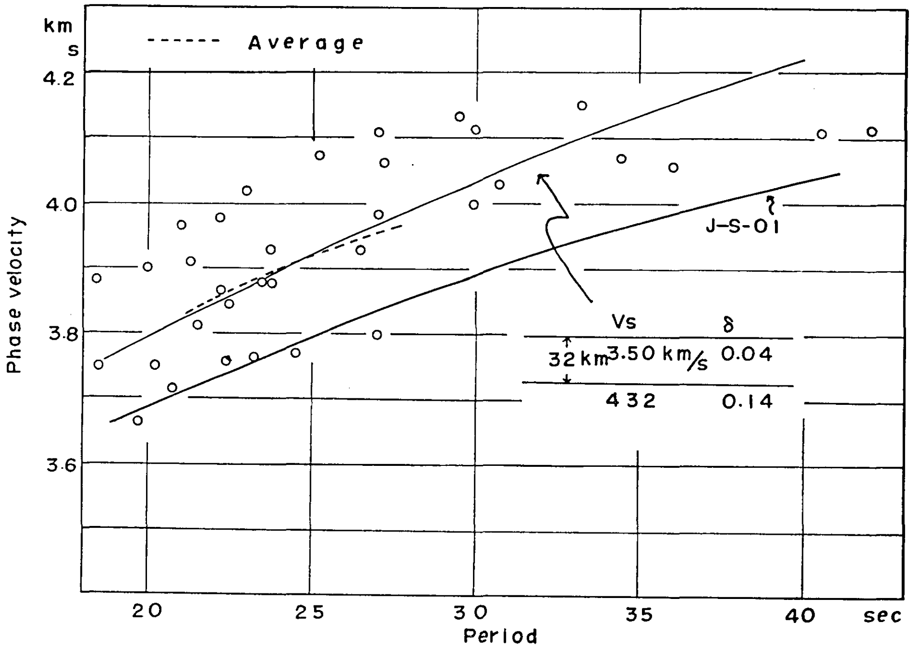

Fig. 2. The dispersion curves of Love waves for J-S-01 considering anisotropy.

5. Anisotropy and shear wave

If the anisotropy exists, the arrival times of bodily SV and SH waves will be different. Regarding the refracted shear waves, the arrival time of SH wave, recorded on the horizontal component, will arrive earlier than those of SV wave, recorded on the vertical component. When a shock occurs in the crust of J-S-01, the arrival time differences between SH and SV waves are 1.0 to 1.5 sec at the epicentral distance of 100 to 200 km.

It is in general difficult to identify the difference of the arrival of SV and SH waves. But it is found from the aftershock observations of the Niigata earthquake at the point where the epicentral distance was 170 km that the arrival times of SH waves, on the horizontal component, are 0.5 to 0.8 sec earlier than those of SV waves, on the vertical component (personal communication from Prof. Asada). If we consider that the differences are caused by the anisotropy, the observed values of difference are a little too small.

In the case of the direct S wave, the differences of arrival times between the SH and SV waves for J-S-01 are calculated as follows;

Epicentral distance	Depth of shock and time difference	
	50 km	80 km
50 km	0.4 sec	0.4 sec
100	1.1	1.0
150	1.6	1.7
175	2.0	2.0

6. Conclusion

The discrepancy between the observed phase velocities of Love and Rayleigh waves is explained by one model, taking transversal isotropy into consideration, in which the velocities of SH_H wave in the crust and upper mantle are 2 and 7 per cent respectively higher than those of SV_V wave. If this anisotropy in velocity exists in the crust and upper mantle, the arrival times of *SH* waves may be 1 to 2 sec earlier than those of *SV* wave at the epicentral distances 100 to 200 km. We have some evidence of the early arrival of *SH* from the aftershock observations of the Niigata earthquake.

It is possible, however, that the discrepancy can be explained through a combination of a lesser degree of anisotropy and a more detailed layer configuration in the crust-mantle than those given in the present paper.

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29. 日本の地殻及び上部マントルの構造

Part 3. 日本における異方性をもつ地下構造

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Part 1 で述べたように，中央日本では，Love 波および Rayleigh 波の分散が同じモデルで説明できないが，異方性を考えることにより，このくい違いを説明できる．

P 波と *SV* 波は，その伝播方向による速度の違いはないが，*SH* 波については，$V_{SH_H}=V_{SH_V}(1+\delta/2)$ というような速度を持つ媒質を考える．このような媒質では，Rayleigh 波の速度には変化はないが，Love 波の速度は等方性の場合に比べて速くなる．

等方性の一層モデル (J-S-01) の場合，Love 波の位相速度の理論値は，観測値よりも遅いが（第1図），地殻および上部マントルにそれぞれ 2%，7% の上述の異方性を考えると，同じモデルで Love 波の分散が説明できる（第 2 図）．