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7. *Tsunami in an L-shaped Canal [IV]* —*the fourth Approximation*—.

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Abstract

In this work, numerical solutions of the waves in the vicinity of the crooked part of an L-shaped canal including the buffer domain were obtained under the fourth approximation. Unlike the preceding works, the expressions of the *buffer domain* are incorporated into the simultaneous equations as members of the unknowns in order to examine numerically the behavior of the waves in this domain. Then the variations of the amplitudes and phases are illustrated figuratively for the range $0<kd<1.0$ (k: a wave number of the incident wave, d: a width of the canal).

I. Introduction

In a series of papers[1),2),3),4)] entitled "Tsunami in an L-shaped Canal", we have treated long waves in a canal in which a train of periodic waves is invading from a branch of the channels.

In the present work, the study of long waves in a canal of uniform width is made under the approximations

$$\left.\begin{aligned} \sin kd &\simeq \sum_{n=0}^{3} (-1)^n \frac{(kd)^{2n+1}}{(2n+1)!} \\ \cos kd &\simeq \sum_{n=0}^{3} (-1)^n \frac{(kd)^{2n}}{(2n)!} \end{aligned}\right\} \tag{1}$$

(k: a wave number of the incident waves, d: a width of the canal) for the expressions of the buffer domain.

1) T. Momoi, *Bull. Earthq. Res. Inst.*, **40** (1962), 719.
2) T. Momoi, *Bull. Earthq. Res. Inst.*, **41** (1963), 581.
3) T. Momoi, *Bull. Earthq. Res. Inst.*, **42** (1964), 449.
4) T. Momoi, *Bull. Earthq. Res. Inst.*, **43** (1965), 749.

The last two papers 3) and 4) will be referred to as papers A and B respectively in later discussions.

In the works treated so far, only solutions of the domains other than the buffer domain have been obtained, while in this work the numerical analysis of all the domains, including the buffer domain, is carried out with the aid of an electronic computer.

2. Theory

The fundamental equation used in this study is an equation of a long wave, i.e.,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\zeta=0 \tag{2}$$

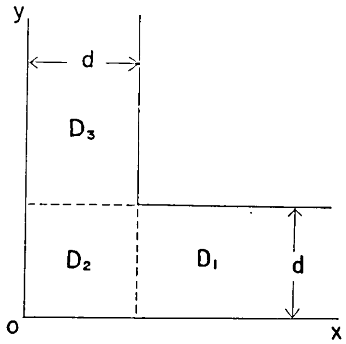

Fig. 1. Geometry of a canal.

for periodic waves, where $k=\omega/c$ (c: a velocity of a long wave, ω: an angular frequency of surging waves).

Then, according to paper A, the formal expressions under the condition such that

$$\frac{\partial\zeta}{\partial n}=0$$

at the rigid boundaries ($\partial/\partial n$: normal derivative to the rigid boundaries) are (refer to Fig. 1):—

in the domain D_1,

$$\zeta_1=\zeta_0 e^{-ikx}+\sum_{m=0}^{\infty}\zeta_1^{(m)}\cos\frac{m\pi}{d}y\cdot e^{+ik(m)x}\ ; \tag{3}$$

in the domain D_2,

$$\zeta_2=\sum_{f_2}A_2(f_2)\cdot\cos k_2^{(x)}x\cdot\cos k_2^{(y)}y\ ; \tag{4}$$

in the domain D_3,

$$\zeta_3=\sum_{m=0}^{\infty}\zeta_3^{(m)}\cos\frac{m\pi}{d}x\cdot e^{+ik(m)y}\ ; \tag{5}$$

where the used notations are exactly the same as those in paper A, and where, since our analysis is limited to the case of a canal of uniform width, the substitution $d_1=d_3=d$ is made in the above formal expressions. Communicating the above expressions (3) to (5) by use of the conditions

$$\left.\begin{aligned} \zeta_i &= \zeta_j \\ \frac{\partial \zeta_i}{\partial s} &= \frac{\partial \zeta_j}{\partial s} \end{aligned}\right\}$$

(ζ_i and ζ_j denote the wave heights in the neighbouring domains and $\partial/\partial s$ a derivative normal to the boundary of the adjacent domains) and applying the operators

$$\int_0^d \cos\frac{m\pi}{d}\left\{\begin{matrix} xdx \\ ydy \end{matrix}\right\} \quad (m\text{: non-negative integers}),$$

we have the following relations (refer to (19)-(26) of paper A):—

$$(\zeta_0 e^{-ikd} + \zeta_1^{(0)} e^{+ikd})d = \sum_{f_2} A_2(f_2) \cos k_2^{(x)} d \cdot \frac{1}{k_2^{(y)}} \sin k_2^{(y)} d\ , \tag{6}$$

$$\zeta_3^{(0)} e^{+ikd} d = \sum_{f_2} A_2(f_2) \frac{1}{k_2^{(x)}} \sin k_2^{(x)} d \cos k_2^{(y)} d\ , \tag{7}$$

$$-ikd\zeta_0 e^{-ikd} + ikd\zeta_1^{(0)} e^{+ikd} = -\sum_{f_2} A_2(f_2) k_2^{(x)} \sin k_2^{(x)} d \cdot \frac{1}{k_2^{(y)}} \sin k_2^{(y)} d\ , \tag{8}$$

$$ikd\zeta_3^{(0)} e^{+ikd} = -\sum_{f_2} A_2(f_2) \frac{1}{k_2^{(x)}} \sin k_2^{(x)} d \cdot k_2^{(y)} \sin k_2^{(y)} d\ , \tag{9}$$

$$\begin{aligned} \frac{1}{2}\zeta_1^{(m)} e^{+ik(m)d} &= (-1)^m \sum_{f_2} A_2(f_2) \cos k_2^{(x)} d \sin k_2^{(y)} d \\ &\quad \times \frac{k_2^{(y)} d}{(k_2^{(y)} d)^2 - (m\pi)^2} \quad (m=1, 2, 3, \cdots)\ , \end{aligned} \tag{10}$$

$$\begin{aligned} \frac{1}{2}\zeta_3^{(m)} e^{+ik(m)d} &= (-1)^m \sum_{f_2} A_2(f_2) \cos k_2^{(y)} d \sin k_2^{(x)} d \\ &\quad \times \frac{k_2^{(x)} d}{(k_2^{(x)} d)^2 - (m\pi)^2} \quad (m=1, 2, 3, \cdots)\ , \end{aligned} \tag{11}$$

$$\begin{aligned} i \cdot \frac{1}{2} k^{(m)} d\zeta_1^{(m)} e^{+ik(m)d} &= (-1)^{m+1} \sum_{f_2} A_2(f_2) k_2^{(x)} d \sin k_2^{(x)} d \sin k_2^{(y)} d \\ &\quad \times \frac{k_2^{(y)} d}{(k_2^{(y)} d)^2 - (m\pi)^2} \quad (m=1, 2, 3, \cdots)\ , \end{aligned} \tag{12}$$

$$\begin{aligned} i \cdot \frac{1}{2} k^{(m)} d\zeta_3^{(m)} e^{+ik(m)d} &= (-1)^{m+1} \sum_{f_2} A_2(f_2) k_2^{(y)} d \sin k_2^{(y)} d \sin k_2^{(x)} d \\ &\quad \times \frac{k_2^{(x)} d}{(k_2^{(x)} d)^2 - (m\pi)^2} \quad (m=1, 2, 3, \cdots)\ , \end{aligned} \tag{13}$$

where

$$k^{(m)}d=\sqrt{(kd)^2-(m\pi)^2}\ .$$

Applying the approximation (1) to the right-hand sides of (6)-(9) and retaining the terms up to the sixth order of $k_2^{(z)}d(z=x\ or\ y)$, we have:—

$$\zeta_0 e^{-ikd}+\zeta_1^{(0)}e^{+ikd}=\sum_{m,n=0}^{m+n\leqq 3}(-1)^{m+n}\frac{1}{(2m)!}\cdot\frac{1}{(2n+1)!}\times\sum_{f_2}A_2(f_2)(k_2^{(x)}d)^{2m}(k_2^{(y)}d)^{2n}\ , \tag{14}$$

$$\zeta_3^{(0)}e^{+ikd}=\sum_{m,n=0}^{m+n\leqq 3}(-1)^{m+n}\frac{1}{(2m+1)!}\cdot\frac{1}{(2n)!}\times\sum_{f_2}A_2(f_2)(k_2^{(x)}d)^{2m}(k_2^{(y)}d)^{2n}\ , \tag{15}$$

$$-ikd\zeta_0 e^{-ikd}+ikd\zeta_1^{(0)}e^{+ikd}=\sum_{m,n=0}^{m+n\leqq 2}(-1)^{m+n+1}\frac{1}{(2m+1)!}\cdot\frac{1}{(2n+1)!}\times\sum_{f_2}A_2(f_2)(k_2^{(x)}d)^{2m+2}(k_2^{(y)}d)^{2n}\ , \tag{16}$$

$$ikd\zeta_3^{(0)}e^{+ikd}=\sum_{m,n=0}^{m+n\leqq 2}(-1)^{m+n+1}\frac{1}{(2m+1)!}\cdot\frac{1}{(2n+1)!}\times\sum_{f_2}A_2(f_2)(k_2^{(x)}d)^{2m}(k_2^{(y)}d)^{2n+2}\ . \tag{17}$$

Provided that the application of the theory is limited to the range

$$kd<\pi\ , \tag{18}$$

the following expansion might be possible:

$$\frac{1}{(k_2^{(z)}d)^2-(m\pi)^2}=\frac{-1}{(m\pi)^2}\cdot\sum_{n=0}^{\infty}\left(\frac{k_2^{(z)}d}{m\pi}\right)^{2n}\qquad (z=x\ \text{or}\ y;\quad m=1,2,3,\cdots)\ . \tag{19}$$

Applying the approximation (1) to the right-hand sides of (10)-(13) and substituting (19) into these equations, we have the following (retaining the terms up to the sixth order of $k_2^{(z)}d$ ($z=x$ or y)):—

$$\frac{(-1)^{m+1}}{2}\zeta_1^{(m)}e^{+ik^{(m)}d}=\sum_{p,q,n=0}^{p+q+n\leqq 2}\frac{(-1)^{p+n}}{(m\pi)^{2q+2}}\cdot\frac{1}{(2p)!}\cdot\frac{1}{(2n+1)!}\times\sum_{f_2}A_2(f)(k_2^{(x)}d)^{2p}(k_2^{(y)}d)^{2q+2n+2}\ , \tag{20}$$

$$\frac{(-1)^{m+1}}{2}\zeta_3^{(m)}e^{+ik^{(m)}d}=\sum_{p,q,n=0}^{p+q+n\leq 2}\frac{(-1)^{p+n}}{(m\pi)^{2q+2}}\cdot\frac{1}{(2p)!}\cdot\frac{1}{(2n+1)!}$$
$$\times\sum_{f_2}A_2(f_2)(k_2^{(y)}d)^{2p}(k_2^{(x)}d)^{2q+2n+2}\ , \tag{21}$$

$$i\cdot\frac{(-1)^{m+1}}{2}k^{(m)}d\zeta_1^{(m)}e^{+ik^{(m)}d}=\sum_{p,q,n=0}^{p+q+n\leq 1}\frac{(-1)^{p+n+1}}{(m\pi)^{2q+2}}\cdot\frac{1}{(2p+1)!}\cdot\frac{1}{(2n+1)!}$$
$$\times\sum_{f_2}A_2(f_2)(k_2^{(x)}d)^{2p+2}(k_2^{(y)}d)^{2q+2n+2}\ , \tag{22}$$

$$i\cdot\frac{(-1)^{m+1}}{2}k^{(m)}d\zeta_3^{(m)}e^{+ik^{(m)}d}=\sum_{p,q,n=0}^{p+q+n\leq 1}\frac{(-1)^{p+n+1}}{(m\pi)^{2q+2}}\cdot\frac{1}{(2p+1)!}\cdot\frac{1}{(2n+1)!}$$
$$\times\sum_{f_2}A_2(f_2)(k_2^{(y)}d)^{2p+2}(k_2^{(x)}d)^{2q+2n+2}\ , \tag{23}$$

where m is a positive integer.

In the equation (2) we have, after a separation of the variables, the relation of the wave numbers

$$(k_2^{(x)})^2+(k_2^{(y)})^2=k^2 \tag{24}$$

Eliminating the terms with respect to $k_2^{(y)}$ from the equations (14) to (17) and (20) to (23) by use of (24), the following are obtained:—

$$a_{1,1}X_1+a_{1,5}X_5+a_{1,6}X_6+a_{1,7}X_7+a_{1,8}X_8=a_{1,9} \tag{25}$$
(from (14)) ,

$$a_{2,2}X_2+a_{2,5}X_5+a_{2,6}X_6+a_{2,7}X_7+a_{2,8}X_8=0 \tag{26}$$
(from (15)) ,

$$a_{3,1}X_1+a_{3,6}X_6+a_{3,7}X_7+a_{3,8}X_8=a_{3,9} \tag{27}$$
(from (16)) ,

$$a_{4,2}X_2+a_{4,5}X_5+a_{4,6}X_6+a_{4,7}X_7+a_{4,8}X_8=0 \tag{28}$$
(from (17)) ,

$$a_{5,3}X_3+a_{5,5}X_5+a_{5,6}X_6+a_{5,7}X_7+a_{5,8}X_8=0 \tag{29}$$
(from the equation (20) for $m=1$) ,

$$a_{6,4}X_4+a_{6,6}X_6+a_{6,7}X_7+a_{6,8}X_8=0 \tag{30}$$
(from the equation (21) for $m=1$) ,

$$a_{7,3}X_3+a_{7,6}X_6+a_{7,7}X_7+a_{7,8}X_8=0 \tag{31}$$
(from the equation (22) for $m=1$) ,

$$a_{8,4}X_4+a_{8,6}X_6+a_{8,7}X_7+a_{8,8}X_8=0 \tag{32}$$
(from the equation (23) for $m=1$) ,

where

$$\left.\begin{aligned}&X_1=\zeta_1^{(0)},\ X_2=\zeta_3^{(0)},\ X_3=\zeta_1^{(1)},\\&X_4=\zeta_3^{(1)},\ X_5=\sum_{f_2}A_2(f_2)\ ,\\&X_6=\sum_{f_2}A_2(f_2)(k_2^{(x)}d)^2,\ X_7=\sum_{f_2}A_2(f_2)(k_2^{(x)}d)^4\ ,\\&X_8=\sum_{f_2}A_2(f_2)(k_2^{(x)}d)^6\ ,\end{aligned}\right\} \tag{33}$$

and where

$$\left.\begin{aligned}&a_{1,1}=-e^{+ikd}\ ,\\&a_{1,5}=1-\frac{1}{3!}(kd)^2+\frac{1}{5!}(kd)^4-\frac{1}{7!}(kd)^6\ ,\\&a_{1,6}=\left(-\frac{1}{2!}+\frac{1}{3!}\right)+\left(\frac{1}{2!}\cdot\frac{1}{3!}-\frac{2}{5!}\right)(kd)^2\\&\qquad+\left(-\frac{1}{2!}\cdot\frac{1}{5!}+\frac{3}{7!}\right)(kd)^4\ ,\\&a_{1,7}=\left(\frac{1}{4!}-\frac{1}{2!}\cdot\frac{1}{3!}+\frac{1}{5!}\right)\\&\qquad+\left(-\frac{1}{4!}\cdot\frac{1}{3!}+\frac{1}{2!}\cdot\frac{2}{5!}-\frac{3}{7!}\right)(kd)^2\\&a_{1,8}=-\frac{1}{6!}+\frac{1}{4!}\cdot\frac{1}{3!}-\frac{1}{2!}\cdot\frac{1}{5!}+\frac{1}{7!}\ ,\\&a_{1,9}=\zeta_0e^{-ikd}\ ,\end{aligned}\right\} \tag{34}$$

$$\left.\begin{aligned}&a_{2,2}=-e^{+ikd}\ ,\\&a_{2,5}=1-\frac{1}{2!}(kd)^2+\frac{1}{4!}(kd)^4-\frac{1}{6!}(kd)^6\ ,\\&a_{2,6}=\left(-\frac{1}{3!}+\frac{1}{2!}\right)+\left(\frac{1}{3!}\cdot\frac{1}{2!}-\frac{2}{4!}\right)(kd)^2\\&\qquad+\left(-\frac{1}{3!}\cdot\frac{1}{4!}+\frac{3}{6!}\right)(kd)^4\ ,\\&a_{2,7}=\left(\frac{1}{5!}-\frac{1}{3!}\cdot\frac{1}{2!}+\frac{1}{4!}\right)\\&\qquad+\left(-\frac{1}{5!}\cdot\frac{1}{2!}+\frac{1}{3!}\cdot\frac{2}{4!}-\frac{3}{6!}\right)(kd)^2\ ,\end{aligned}\right\} \tag{35}$$

$$a_{2,8}=-\frac{1}{7!}+\frac{1}{5!}\cdot\frac{1}{2!}-\frac{1}{3!}\cdot\frac{1}{4!}+\frac{1}{6!}\,,$$

$$\left.\begin{aligned}
a_{3,1}&=-ikde^{+ikd}\,,\\
a_{3,6}&=-1+\frac{1}{3!}(kd)^2-\frac{1}{5!}(kd)^4\,,\\
a_{3,7}&=\left\{-\frac{1}{(3!)^2}+\frac{2}{5!}\right\}(kd)^2\,,\\
a_{3,8}&=-\frac{2}{5!}+\frac{1}{(3!)^2}\,,\\
a_{3,9}&=-ikd\zeta_0 e^{-ikd}\,,
\end{aligned}\right\} \tag{36}$$

$$\left.\begin{aligned}
a_{4,2}&=-ikde^{+ikd}\,,\\
a_{4,5}&=-(kd)^2+\frac{1}{3!}(kd)^4-\frac{1}{5!}(kd)^6\,,\\
a_{4,6}&=1-\frac{1}{3!}(kd)^2+\left\{-\frac{1}{(3!)^2}+\frac{3}{5!}\right\}(kd)^4\,,\\
a_{4,7}&=\left\{\frac{2}{(3!)^2}-\frac{4}{5!}\right\}(kd)^2\,,\\
a_{4,8}&=\frac{2}{5!}-\frac{1}{(3!)^2}\,,
\end{aligned}\right\} \tag{37}$$

$$\left.\begin{aligned}
a_{5,3}&=-\frac{1}{2}e^{+ik^{(1)}d}\,,\\
a_{5,5}&=\frac{1}{\pi^2}\left\{(kd)^2-\frac{1}{3!}(kd)^4+\frac{1}{5!}(kd)^6\right\}\\
&\quad+\frac{1}{\pi^4}\left\{(kd)^4-\frac{1}{3!}(kd)^6\right\}+\frac{1}{\pi^6}(kd)^6\,,\\
a_{5,6}&=\frac{1}{\pi^2}\left\{-1+\left(-\frac{1}{2!}+\frac{2}{3!}\right)(kd)^2\right.\\
&\quad\left.+\left(\frac{1}{2!}\cdot\frac{1}{3!}-\frac{3}{5!}\right)(kd)^4\right\}\\
&\quad-\frac{1}{\pi^4}\cdot 2(kd)^2-\frac{1}{\pi^6}\cdot 3(kd)^4\,,
\end{aligned}\right\} \tag{38}$$

$$\left.\begin{aligned}
a_{5,7}&=\frac{1}{\pi^2}\left\{\left(\frac{1}{2!}-\frac{1}{3!}\right)\right.\\
&\quad\left.+\left(\frac{1}{4!}-\frac{1}{2!}\cdot\frac{2}{3!}+\frac{3}{5!}\right)(kd)^2\right\}\\
&\quad+\frac{1}{\pi^4}\left\{1+\left(1-\frac{1}{2!}\right)(kd)^2\right\}\\
&\quad+\frac{1}{\pi^6}\cdot 3(kd)^2\ ,\\
a_{5,8}&=\frac{1}{\pi^2}\left(-\frac{1}{4!}+\frac{1}{2!}\cdot\frac{1}{3!}-\frac{1}{5!}\right)\\
&\quad+\frac{1}{\pi^4}\left(-\frac{1}{2!}+\frac{1}{3!}\right)-\frac{1}{\pi^6}\ ,
\end{aligned}\right\}$$

$$\left.\begin{aligned}
a_{6,4}&=-\frac{1}{2}e^{+ik^{(1)}d}\ ,\\
a_{6,6}&=\frac{1}{\pi^2}\left\{1-\frac{1}{2!}(kd)^2+\frac{1}{4!}(kd)^4\right\}\ ,\\
a_{6,7}&=\frac{1}{\pi^2}\left\{\left(\frac{1}{2!}-\frac{1}{3!}\right)\right.\\
&\quad\left.+\left(\frac{1}{2!}\cdot\frac{1}{3!}-\frac{2}{4!}\right)(kd)^2\right\}\\
&\quad+\frac{1}{\pi^4}\left\{1-\frac{1}{2!}(kd)^2\right\}\ ,\\
a_{6,8}&=\frac{1}{\pi^2}\left\{\frac{1}{4!}-\frac{1}{2!}\cdot\frac{1}{3!}+\frac{1}{5!}\right\}\\
&\quad+\frac{1}{\pi^4}\left(\frac{1}{2!}-\frac{1}{3!}\right)+\frac{1}{\pi^6}\ ,
\end{aligned}\right\}\tag{39}$$

$$\left.\begin{aligned}
a_{7,3}&=-\frac{1}{2}\cdot ik^{(1)}de^{+ik^{(1)}d}\ ,\\
a_{7,6}&=-\frac{1}{\pi^2}\left\{(kd)^2-\frac{1}{3!}(kd)^4\right\}-\frac{1}{\pi^4}(kd)^4\ ,\\
a_{7,7}&=-\frac{1}{\pi^2}\left\{-1+\frac{1}{3!}(kd)^2\right\}+\frac{1}{\pi^4}\cdot 2(kd)^2\ ,\\
a_{7,8}&=-\frac{1}{\pi^4}\ ,
\end{aligned}\right\}\tag{40}$$

$$\left.\begin{aligned}
a_{8,4}&=-\frac{1}{2}\cdot ik^{(1)}de^{+ik^{(1)}d}\ ,\\
a_{8,6}&=-\frac{1}{\pi^2}\left\{(kd)^2-\frac{1}{3!}(kd)^4\right\}\ ,\\
a_{8,7}&=-\frac{1}{\pi^2}\left\{-1+\frac{1}{3!}(kd)^2\right\}-\frac{1}{\pi^4}(kd)^2\ ,\\
a_{8,8}&=\frac{1}{\pi^4}\ .
\end{aligned}\right\} \tag{41}$$

It must be noted here that, since our treatment is confined to the case $kd<\pi$, the factor $i\cdot k^{(1)}d$ involved in the first expressions of (38)-(41) is real.

Now the relations (25) to (32) denote simultaneous equations with eight unknowns X_j ($j=1, 2, 3, \cdots, 8$) given by (33), of which the coefficients are shown in (34) to (41). Hence the physical quantities shown in the right-hand sides of the expressions (33) readily begin to be known, if one uses an electronic computer.

Eliminating the terms with respect to $k_2^{(y)}d$ from (20) and (21) by use of the relation of the wave number (24), the higher modes of the waves $\zeta_j^{(m)}$ ($j=1, 3$; $m=2, 3, 4, \cdots$) in the domain D_j ($j=1, 3$) become as follows:—

$$\zeta_1^{(m)}=(-1)^{m+1}2e^{-ik^{(m)}d}(B_1^{(m)}X_5+C_1^{(m)}X_6+D_1^{(m)}X_7+E_1^{(m)}X_8) \tag{42}$$

and

$$\zeta_3^{(m)}=(-1)^{m+1}2e^{-ik^{(m)}d}(C_3^{(m)}X_6+D_3^{(m)}X_7+E_3^{(m)}X_8)\ , \tag{43}$$

where

$$\left.\begin{aligned}
B_1^{(m)}&=\frac{1}{(m\pi)^2}\left\{(kd)^2-\frac{1}{3!}(kd)^4+\frac{1}{5!}(kd)^6\right\}\\
&\quad+\frac{1}{(m\pi)^4}\left\{(kd)^4-\frac{1}{3!}(kd)^6\right\}\\
&\quad+\frac{1}{(m\pi)^6}(kd)^6\ ,\\
C_1^{(m)}&=\frac{1}{(m\pi)^2}\left\{-1+\left(-\frac{1}{2!}+\frac{2}{3!}\right)(kd)^2\right.\\
&\quad\left.+\left(\frac{1}{2!}\cdot\frac{1}{3!}-\frac{3}{5!}\right)(kd)^4\right\}\\
&\quad-\frac{1}{(m\pi)^4}\cdot 2(kd)^2-\frac{1}{(m\pi)^6}\cdot 3(kd)^4\ ,
\end{aligned}\right\} \tag{44}$$

$$\left.\begin{aligned}
D_1^{(m)}&=\frac{1}{(m\pi)^2}\left\{\left(\frac{1}{2!}-\frac{1}{3!}\right)\right.\\
&\quad\left.+\left(\frac{1}{4!}-\frac{1}{2!}\cdot\frac{2}{3!}+\frac{3}{5!}\right)(kd)^2\right\}\\
&\quad+\frac{1}{(m\pi)^4}\left\{1+\left(1-\frac{1}{2!}\right)(kd)^2\right\}\\
&\quad+\frac{1}{(m\pi)^6}\cdot 3(kd)^2\,,\\
E_1^{(m)}&=\frac{1}{(m\pi)^2}\left(-\frac{1}{4!}+\frac{1}{2!}\cdot\frac{1}{3!}-\frac{1}{5!}\right)\\
&\quad+\frac{1}{(m\pi)^4}\left(-\frac{1}{2!}+\frac{1}{3!}\right)-\frac{1}{(m\pi)^6}\,,
\end{aligned}\right|$$

$$\left.\begin{aligned}
C_3^{(m)}&=\frac{1}{(m\pi)^2}\left\{1-\frac{1}{2!}(kd)^2+\frac{1}{4!}(kd)^4\right\},\\
D_3^{(m)}&=\frac{1}{(m\pi)^2}\left\{\left(\frac{1}{2!}-\frac{1}{3!}\right)\right.\\
&\quad\left.+\left(\frac{1}{2!}\cdot\frac{1}{3!}-\frac{2}{4!}\right)(kd)^2\right\}\\
&\quad+\frac{1}{(m\pi)^4}\left\{1-\frac{1}{2!}(kd)^2\right\},\\
E_3^{(m)}&=\frac{1}{(m\pi)^2}\left(\frac{1}{4!}-\frac{1}{2!}\cdot\frac{1}{3!}+\frac{1}{5!}\right)\\
&\quad+\frac{1}{(m\pi)^4}\left(\frac{1}{2!}-\frac{1}{3!}\right)+\frac{1}{(m\pi)^6}\,,
\end{aligned}\right\}\qquad(45)$$

and where X_j ($j=5, 6, 7, 8$) are as expressed by (33).

If one substitutes the solutions obtained from the simultaneous equations (25)–(32) into the higher modes of the waves $\zeta_j^{(m)}$ ($j=1, 3$; $m=2, 3, 4, \cdots$), i.e., (42) and (43), the behavior of the waves in the domains D_1 and D_3 are to be inquired into through the formal expressions of the wave heights (3) and (5), of which the actual calculations are carried out with the aid of an electronic computer.

In the foregoing works[1)–4)], the solution of the waves in the domain D_2 (the buffer domain) has not been obtained and, in this study, the numerical analysis of the waves in this domain is made using an electronic computer, the procedure of which is described hereunder:—

Since

$$d \geqq x(\text{or } y) \geqq 0$$

in the buffer domain D_2, the following approximation is permissible from the approximations (1), i.e.,

$$\cos k_2^{(z)} z \simeq \sum_{n=0}^{3} (-1)^n \frac{(k_2^{(z)} z)^{2n}}{(2n)!} \quad (z=x \text{ or } y) .$$

Substituting the above expression into (4) and retaining the terms up to the sixth order of $k_2^{(z)} z$ ($z=x$ or y), (4) becomes

$$\zeta_2 = \sum_{m,n=0}^{m+n\leqq 3} (-1)^{m+n} \frac{1}{(2m)!} \cdot \frac{1}{(2n)!} \cdot \sum_{f_2} A_2(f_2)(k_2^{(x)} x)^{2m} (k_2^{(y)} y)^{2n} .$$

Using the relation of the wave number (24), the above expression is reduced further to the following (eliminating the terms with respect to $k_2^{(y)}$):—

$$\zeta_2 = K_5 \cdot X_5 + K_6 \cdot X_6 + K_7 \cdot X_7 + K_8 \cdot X_8 , \tag{46}$$

where

$$\left.\begin{aligned}
K_5 &= 1 - \frac{1}{2!}(ky)^2 + \frac{1}{4!}(ky)^4 - \frac{1}{6!}(ky)^6 , \\
K_6 &= y^2 \left\{ \frac{1}{2!} - \frac{2}{4!}(ky)^2 + \frac{3}{6!}(ky)^4 \right\} \\
&\quad - \frac{1}{2!} x^2 \left\{ 1 - \frac{1}{2!}(ky)^2 + \frac{1}{4!}(ky)^4 \right\} , \\
K_7 &= y^4 \left\{ \frac{1}{4!} - \frac{3}{6!}(ky)^2 \right\} \\
&\quad - \frac{1}{2!} x^2 y^2 \left\{ \frac{1}{2!} - \frac{2}{4!}(ky)^2 \right\} \\
&\quad + \frac{1}{4!} x^4 \left\{ 1 - \frac{1}{2!}(ky)^2 \right\} , \\
K_8 &= \frac{1}{6!} y^6 - \frac{1}{2!} \cdot \frac{1}{4!} x^2 y^4 \\
&\quad + \frac{1}{4!} \cdot \frac{1}{2!} x^4 y^2 - \frac{1}{6!} x^6 ,
\end{aligned}\right\} \tag{47}$$

and where X_j ($j=5, 6, 7, 8$) are described in (33).

Since the factors X_j ($j=5, 6, 7, 8$) have already been solved by the simultaneous equations (25)-(32), the behavior of the waves in the buffer domain D_2 are examined numerically through the expression (46), of which the actual calculations are also made by use of an electronic computer.

3. Numerical Analysis and Discussion

To begin with, the calculations of the heights of the transmitted and reflected waves are made, of which the variations are depicted in Fig. 2. According to this figure, the result of the theory under the third approximation is found to be in use for the range $0 \leqq kd \leqq 1.0$. As far as the application range of the theory under the fourth approximation is concerned, a further development of the generalized theory is required.

The variations of the phases of the transmitted and reflected waves derived under the fourth approximation are presented in Fig. 3, according to which a good agreement of the third and fourth approximations is seen up to $kd=1.5$.

Therefore, the discussions made in paper B for the wave heights and phases are along the same lines.

We consider next the overall variations of the wave heights and phases in the neighbouring part of the corner of the canal. The actual computations are carried out by use of an electronic computer, following the procedures described in the foregoing section. The behaviors of the amplitudes are depicted in Figs. 4a, 5a, 6a, 7a and 8a for the parameters $kd=0.1, 0.3, 0.5, 0.7$ and 0.9 respectively, while those of the phases are drawn in Figs. 4p, 5p, 6p, 7p, and 8p.

Passing through all the figures of the amplitudes (Figs. 4a, 5a, 6a, 7a, and 8a), these variations are marked with the contours of an elliptic form which have their center in the very point of the outer corner of the canal. Such variations are explained as follows. The incident waves advance toward the wall AB (see Fig. 9) and collide therewith, to be diffracted primarily towards the leading canal along the wall AB, being reflected partially to the canal through which the waves invaded. It must be noted here that when the collision of the incident waves with the wall occurred the waves diffracted along the wall AB did not stagnate. That is to say, no appreciable retardation of the phases of the advancing waves is seen in the figures of the phases (Figs. 4p, 5p, 6p, 7p, and 8p). Through the five figures relevant to the amplitudes, the horizontal patterns of the contours in the crooked part of the canal resemble one another, but the vertical shapes differ from one another with a tendency such

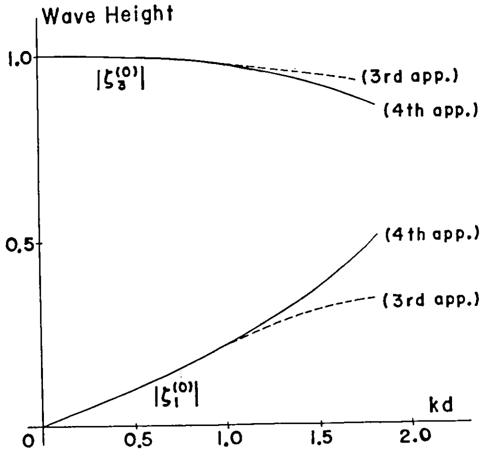

Fig. 2. Variations of amplitudes of reflected and advancing waves.

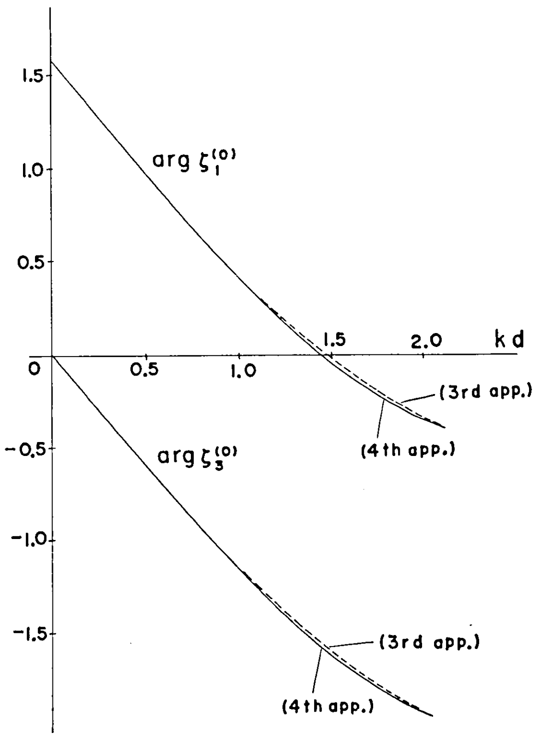

Fig. 3. Variations of phases of reflected and advancing waves.

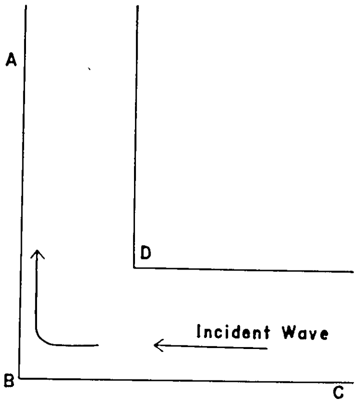

Fig. 9.

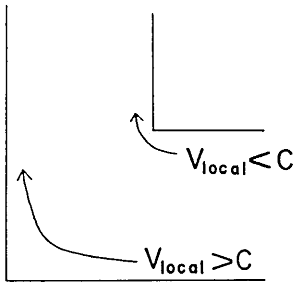

Fig. 10. V_{local}: local velocity of a wave, C: velocity of a long wave.

that their gradients begin to increase as kd increases. The contours of the amplitudes in the domain D_1, through which the incident waves came, run approximately perpendicular to the axis of the canal, reflecting the direction of the incidence of the waves. When kd increases, the contours of the amplitudes in the domain D_1 begin to incline (refer to Figs. 4a, 5a and 6a) until the small valleys appear in the upper side of the canal (refer to Figs. 7a and 8a). These inclinations and the appearances of the small valleys are likely to be caused by the increase of the amount of the reflected waves and their deviation from the axis of the canal. When kd increases, the wave height at the outer corner (B in Fig. 9) of the canal is gradually augmented, whilst that at the inner corner (D in Fig. 9) diminishes.

Another outstanding feature in the contours of the amplitude variations (Figs. 4a, 5a, 6a 7a and 8a) is such that though the shape of the contours is elliptic for the curves of $|\zeta| > 1.0$, when the $|\zeta|$-value decreases over about 1.0, the elliptical patterns are deformed (a kink appears on the circumference of the ellipse) to become diverging forms.

Next, our attention is directed to the behaviors of the phases in Figs. 4p, 5p, 6p, 7p and 8p.

Through all the figures of the phases, a general trend in the variations of these curves is very similar, though the contours of the amplitudes have not a little difference in the domain D_1.

In the nearby area of the inner corner of the canal, the neighbouring equi-phase lines run very closely as compared with those in other parts of the canal, which implies that the waves in the vicinity of the inner

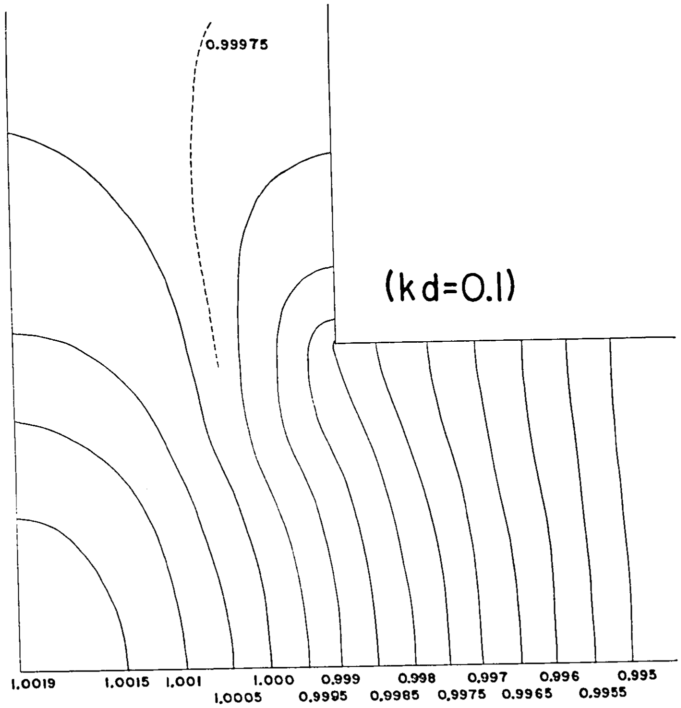

Fig. 4a. Variation of an amplitude for $kd=0.1$.

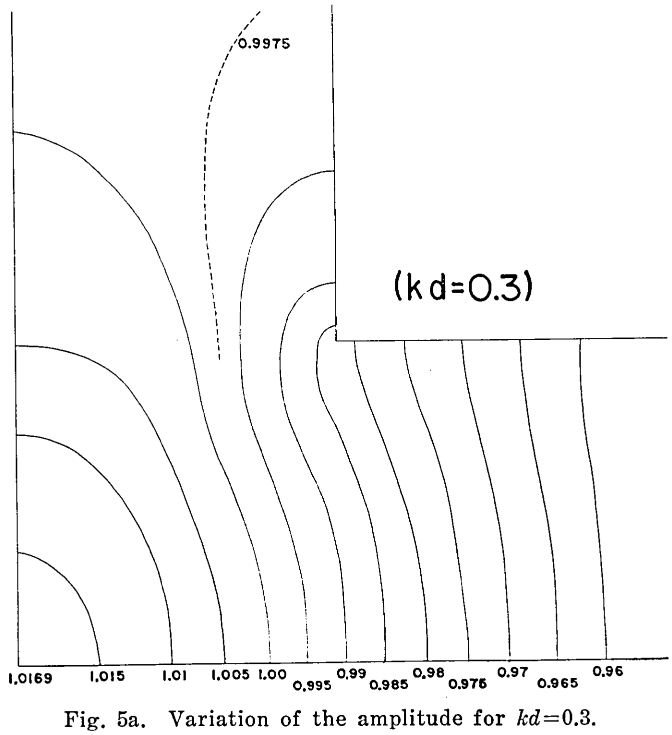

Fig. 5a. Variation of the amplitude for $kd=0.3$.

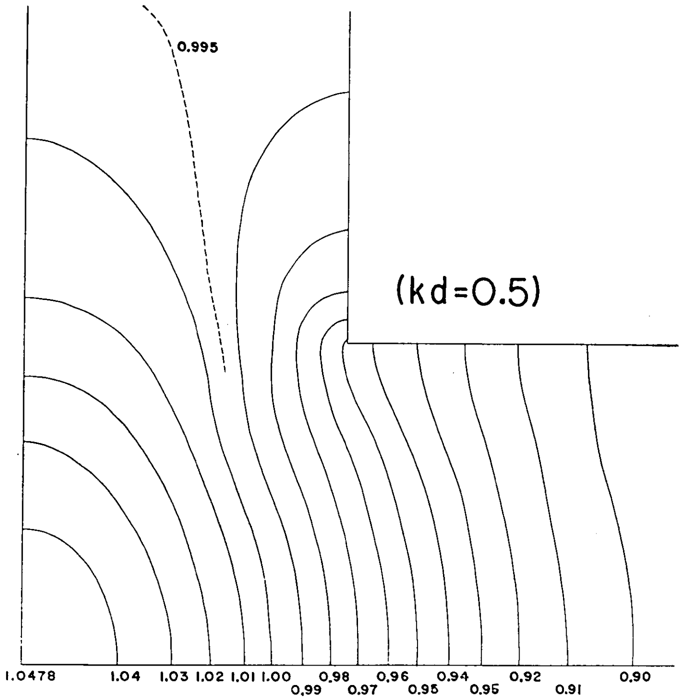

Fig. 6a. Variation of an amplitude for $kd=0.5$.

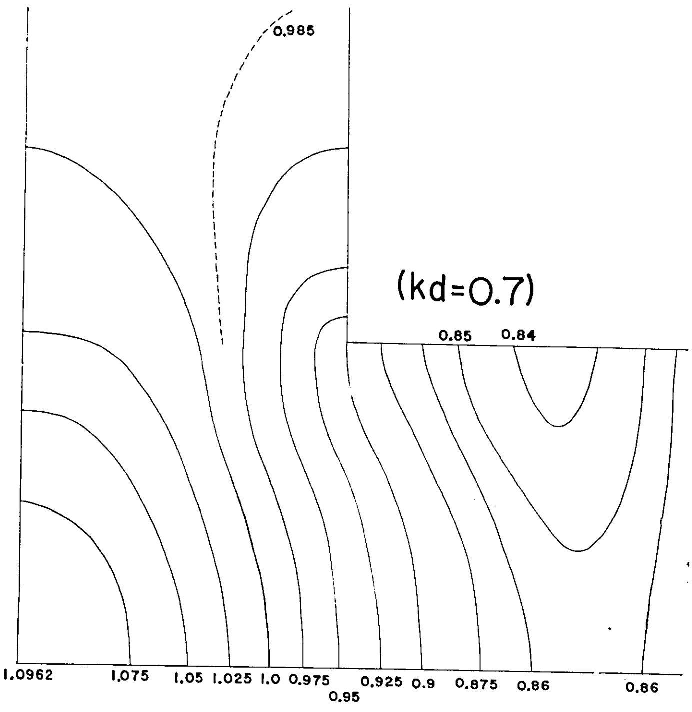

Fig. 7a. Variation of an amplitude for $kd=0.7$.

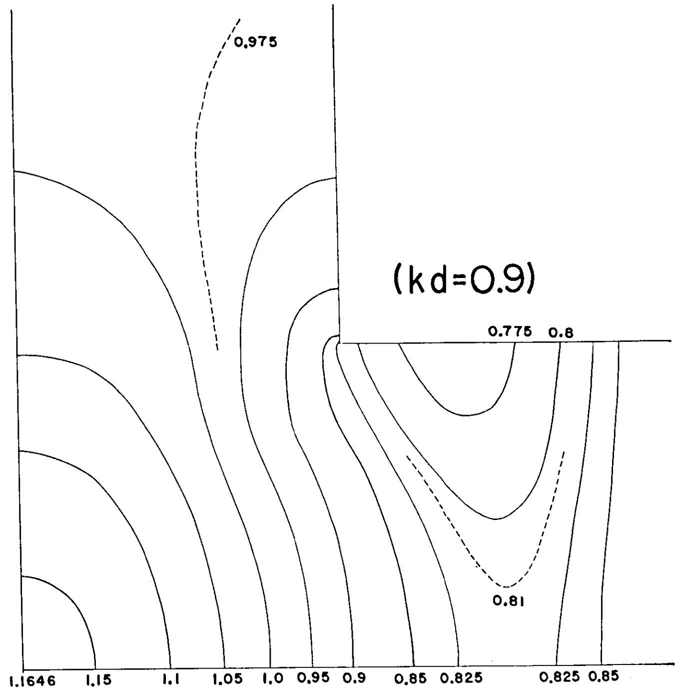

Fig. 8a. Variation of an amplitude for $kd=0.9$.

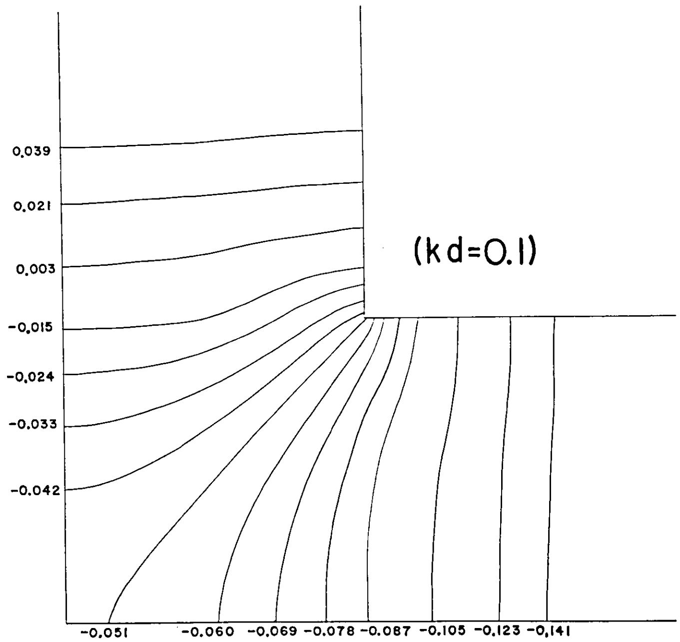

Fig. 4p. Variation of a phase for $kd=0.1$.

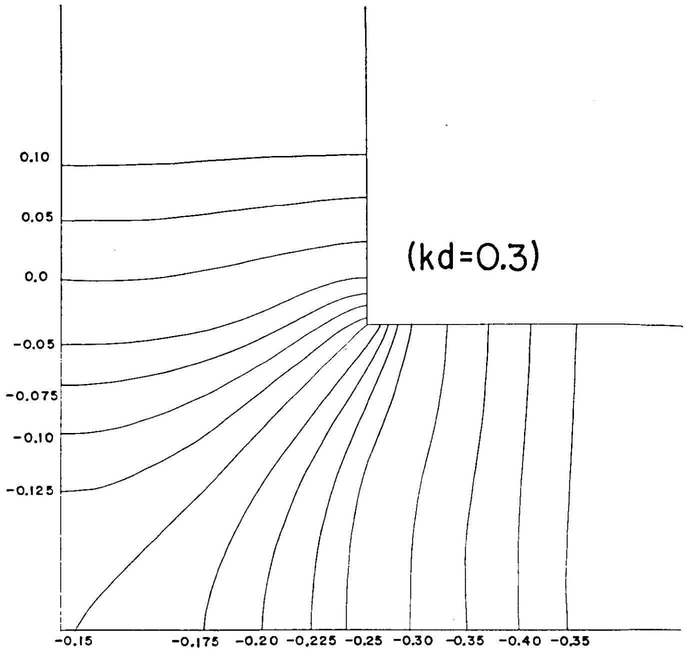

Fig. 5p. Variation of a phase for $kd=0.3$.

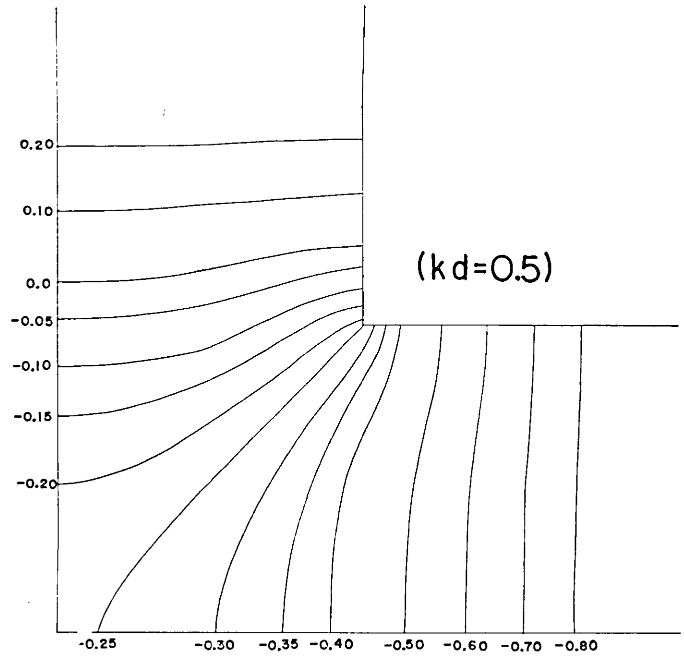

Fig. 6p. Variation of a phase for $kd=0.5$.

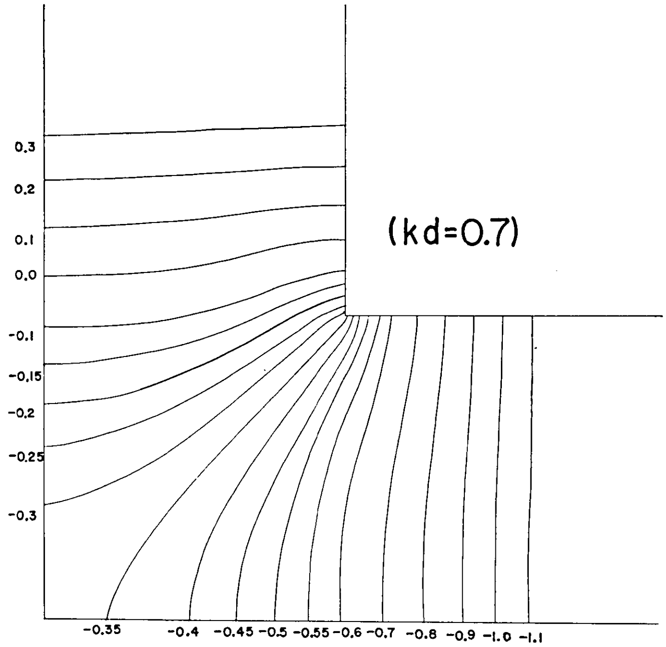

Fig. 7p. Variation of a phase for $kd=0.7$.

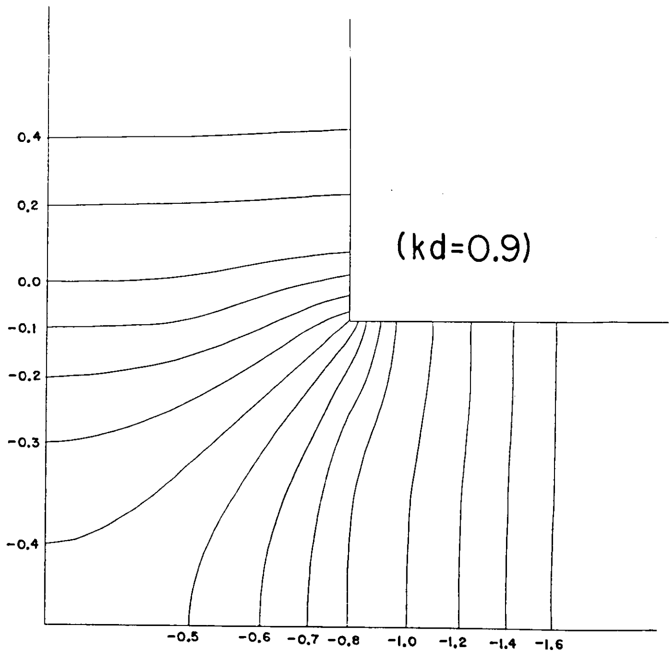

Fig. 8p. Variation of a phase for $kd=0.9$.

corner are propagated at a lower speed than in other parts. Contrarily in the nearby region of the outer corner of the canal, the equi-phase lines run more sparsely. This fact denotes that as the incident waves approach the crooked part of the canal the waves in the outer region are more rapidly accelerated reaching a maximum at the very point of the corner, the propagation velocity decreasing later on as the waves depart from the corner. At a point (A in Fig. 9) far distant from the crooked part of the canal, the transmitted waves advance with a speed of a long wave ($\sqrt{gH}$, where g is acceleration of gravity and H depth of sea). Comparing the distance ($\Delta\alpha_\infty$) of the neighbouring equi-phase lines at the above point with the local distance ($\Delta\alpha_{local}$) of the neighbouring phase lines in the vicinity of the crooked part of the canal, one finds that (1) the waves on the inner side of the crooked part (side D in Fig. 9) are propagated at a lower speed than that of a long wave ($\Delta\alpha_\infty > \Delta\alpha_{local}$) and (2) the waves making a detour along the wall (side B in Fig. 9) advance with a faster velocity than that of $\sqrt{gH}$ ($\Delta\alpha_\infty < \Delta\alpha_{local}$). This is explained figuratively in Fig. 10.

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7. L字水路における津波 [IV]

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本報告においては，L字水路の屈曲部近傍 (buffer domain を含んで) における波の数値解が第 4 近似の下に求められている．今迄の研究と異り，buffer domain の表現は一連の未知数として連立方程式の中に入られ，この領域の波の状態が数値的に調べられている．そして振幅と位相の変化が $kd=0\sim1.0$ (k: 進入波の波数，d: 水路の幅) の範囲について図的に説明されている．