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9. *Density Field of Dislocations and Fold Deformation Problem.**

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1. Introduction

In different branches of physical field theories there is a connection between field quantities and geometry of system. Formally such a relation can be brought in to evidence by presenting the field equations in an arbitrary curvilinear coordinate system; field quantities and components of metric tensor form an interdependent arrangement.

Two basic quantities of deformation theory, deformation tensor and tensor of dislocation density, possess their own geometrical equivalents. Often to present a geometrical equivalent one associates coordinate system with real points of continuous medium. In such a way with medium deformation occurs simultaneously the deformation of coordinate system. Many interesting relations may then be brought from differential geometry theory to the problems of continuous medium deformation.

This idea is closely related with problems of plasticity. Present theories on plastic deformation and plastic flow are based on the conception of dislocation displacements, considering either a system of single dislocations in a system of slip planes or—in a more general way—by introducing a continuous field of dislocations.[1)-9)]

The discontinuity of the displacements Δu_i of a single Volterra dislocation is described by the formula

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*) Communicated by S. Omote.

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1) B. A. BILBY, R. BOULLOUGH, and E. SMITH, *Proc. Roy. Soc., A*, **231** (1955), 236.

2) B. A. BILBY and E. SMITH, *Proc. Roy. Soc., A*, **232** (1956), 481.

3) K. KONDO, *RAAG Memoirs*, **1**, Div. C (1955), 361.

4) K. KONDO, *RAAG Memoirs*, **1**, Div. D (1955), 458.

5) K. KONDO, *RAAG Memoirs*, **1**, Div. D (1955), 484.

6) K. KONDO, *RAAG Memoirs*, **2**, Div. D (1958), 470.

7) E. KRÖNER, *Arch. Rational Mech. & Anal.*, **4** (1960), 273.

8) J. F. NYE, *Acta Metallurg.*, **1** (1953), 158.

9) R. TEISSEYRE, *Bull. Seism. Soc. Amer.*, **54** (1964), 1059-1072.

$$\Delta u_i = b_i + \varepsilon_{iks}\omega_k x_s \qquad (1)$$

where: b_i-Burgers vector, ω_k-rotation vector.

In the case of continuous distribution of the dislocations we can determine the discontinuity of displacement along an arbitrary contour with the formula[10)]

$$\overrightarrow{\Delta\xi} = \overrightarrow{i_\mu}\alpha^{\mu\nu}n_\nu d\sigma \qquad (2)$$

where: $\alpha^{\mu\nu}$-dislocation tensor given by the expression

$$\alpha^{\mu\nu} = \varepsilon^{\nu\alpha\beta}(S^{\mu}_{\cdot\alpha\beta} + R^{\mu}_{\cdot\alpha\eta\beta}\xi^{\eta}) \qquad (3)$$

The torsion tensor $S^{\mu}_{\cdot\alpha\beta}$ corresponds in physical respect to the Burgers vector b_i for single dislocation.

Similarly as the Rieman-Christoffel tensor $R^{\mu}_{\cdot\alpha\eta\beta}$ the torsion tensor refers to an auxiliary space which has to be defined generally as a non-Riemanian space with an affine connexion object and torsion. The geometric concepts and an auxiliary space can by introduced after K. Kondo[11),12)] by means of the following procedure. Since the real medium has in general a complex pattern of internal stresses and inhomogeneities, we can brings the surroundings of every point of the medium to an ideal state with no internal stresses. The elements of the medium successively obtained in this manner will however in general not form a continuous and connected whole. Excesses and lacks will occur in a medium which elements are brought to a stressless state. In order to obtain a connected description of the ideal medium (i.e. with neither lack nor excess) we must therefore introduce special space geometry. Thus the deformation state of the real medium can be described by means of the geometrical properties of a space, into which the ideal state could be inserted. Formula (2) tells us in particular to what extent the ends of a closed circuit are shifted after having brought its elements successively to a stressless state.

In an earlier paper[13)] we used the inverse procedure, placing the ideal medium in the Euclidean space. Introduction of a deformation with continuous distribution of the dislocations leads to description of the real medium in a non-Riemanian space with affine connection and torsion.

10) K. KONDO, *loc. cit.*, 3)-6).
11) K. KONDO, *loc. cit.*, 3).
12) K. KONDO, *loc. cit.*, 4).
13) R. TEISSEYRE, *loc. cit.*, 9).

Formula (2) describes the displacement (after deformation) of the ends of a closed contour, defined in the ideal state of the medium. Both descriptions are obviously equivalent; the first one is consistent with physical meaning, while the other is more convenient for theoretical computation of deformation models[14].

It was mentioned in the foregoing that the density field of the dislocation is related to plastic deformation. In seismological and geological application we are interested in two types of plastic deformation. In the first, primary sense of the word, plastic deformation takes place after transgression of the elastic limit and is related to processes occurring in the vicinity of earthquake foci.

The theory of plastic deformations, based on a model of dislocation field density, had been applied in our previously quoted paper[15] to the case of a geological fold under the specific conditions of thin stratification. In this second case we propose to use the term "plastic" deformation in quotation marks where such a distinction appears desirable. In this manner will be emphasized the great physical difference which exists between these two deformation types in spite of a similar mathematical construction of the respective theories. The first type occurs namely on transgression of the limit of elasticity, while the second type, related to the properties of a medium, allows simultaneously elastic deformation and "plastic" one (displacements along the layers).

The thus defined "plastic" deformation in the limit for the case of very thin layerlets may be described by a continuous field of dislocations, the medium corresponding to a compound anisotropic system.

2. Dislocation density tensor

Following several authors[16)–22)] we will introduce the distortion tensor $u^{i}_{\cdot s}$ (or u_{is}, when lower indexes are preferable) which describes deformation state in the case of continuous dislocation distribution.

Simple elastic deformation of a medium is defined by the vector field

14) *Ibid.*
15) R. Teisseyre, *loc. cit.*, 9).
16) B. A. Bilby, R. Boullough, and E. Smith, *loc. cit.*, 1).
17) E. Kroner, *loc. cit.*, 7),
18) E. F. Holländer, *Czech. J. Phys.*, B **10** (1960), 409.
19) E. F. Holländer, *Czech. J. Phys.*, B **10** (1960), 479.
20) E. F. Holländer, *Czech. J. Phys.*, B **10** (1960), 551.
21) T. Mura, *Phil. Mag.*, **8** (1963), 843.
22) A. M. Kosevich, *Usp. Fiz. Nauk*, **84** (1964), 579.

u^i. This means that a point with the coordinates x^i in a rectangular system was shifted to a point with the coordinates X^i in that system

$$X^i = x^i + u^i \tag{4}$$

The corresponding relations for increments are obviously

$$dX^i = dx^i + du^i \tag{5}$$

In the case of continuous dislocation field on the above deformation is superimposed the displacement field $u^i_{\cdot k}$, which has a nonholonomic character. In this way we obtain instead of formula (5) the expression

$$dX^i = dx^i + du^i + u^i_{\cdot k} dx^k \tag{6}$$

The nine independent quantities $u^i_{\cdot k}$ form a distortion tensor, and they cannot be derived from a set of three components of displacement vector. This mean that the expression (6) is not integrable. Often we approach a deformation problem by maintaining the numerical invariance of the coordinates of the material points. This means that a point having the coordinate values x^i prior to deformation will keep them also after deformation. This postulate produces a change in the character of the system of coordinates; in lieu of the rectilinear system x^i we obtain a curvilinear system ξ^i, but for the same material points the numerical invariance condition $\xi^i \underset{\text{Num.}}{=} x^i$ still holds. If the deformation has furthermore the character of a nonholonomic transformation expressed by $u^i_{\cdot k}$, the set ξ^i will in general not form an Euclidean space, as was noted in the introductory part. For tensors and any other objects in non-Euclidean space we shall use for indexes the Greek letter. For general deformations including the dislocation fieds it was proved[23),24)] that a set of the ξ^μ-quantities is related to a non-Riemanian space.

From the expression (6) we obtain correspondingly

$$\left.\begin{aligned} \frac{\partial X^i}{\partial \xi^\mu} &= \bar{C}^i_\mu = \delta^i_\mu + u^i_{,\mu} + u^i_{\cdot\mu} \\ \frac{\partial \xi^\mu}{\partial X^i} &= C^\mu_i = \delta^\mu_i - u^\mu_{,i} - u^\mu_{\cdot i} \end{aligned}\right\} \tag{7}$$

The torsion tensor to which we are restricting our consideration is:[25)]

23) B. A. BILBY, and E. SMITH, *loc. cit.*, 2).

24) K. KONDO, *loc. cit.*, 4).

25) *Ibid*·

$$S^{\mu}_{\cdot\alpha\beta} = -C^{\mu}_{[i,k]}\bar{C}^{k}_{\beta}\bar{C}^{i}_{\alpha} \tag{8}$$

and differs from zero provided the alteration

$$C^{\mu}_{[i,k]} = \frac{1}{2}(u^{\mu}_{\cdot i,k} - u^{\mu}_{\cdot k,i}) \tag{9}$$

does not vanish. Expression (9) differs from zero for nonholonomic transformation $u^{m}_{\cdot i}$.

Confining our attention to linear expression, we obtain from (8) the expression for the torsion tensor

$$S^{\mu}_{\cdot\alpha\beta} = u^{\mu}_{\cdot[\beta,\alpha]} \tag{10}$$

Here, and further on we shall not distinguish between upper and lower indexes. Also ordinary differentiation will be used instead of covariant differentiation needed in a nonlinear theory. From the Eqs. (3) and (10) we get (in the linear approximation we come back to Latin indexes):

$$\alpha_{mn} = \varepsilon_{kln} u_{m[k,l]} \tag{11}$$

This basic equation was derived by Kondo[26] Bilby, Boullough, Smith[27),28] and Kröner.[29] Defining the dislocation density tensor by integral relation:

$$\iint \alpha_{mn} d\sigma_n = -\oint u_{ms} dx_s \tag{12}$$

we will immediately obtain the same result given by (11). Indeed, transforming the contour integral on the left side of Eq. (12) to the surface integral we get

$$\alpha_{mn} = \varepsilon_{nsk} u_{ms,k} = \varepsilon_{nsk} u_{m[s,k]}$$

From Eq. (11) it follows immediately the so-called continuity equation for dislocation tensor

$$\alpha_{ms,s} = 0 \tag{13}$$

Eqs. (12) and (13) have been generalized by Holländer[30] to the time dependence cases. Thus, for example, the time derivation of Eq. (12) defines the shrface integral of the dislocation density current.

26) K. KONDO, *loc. cit.*, 3)-6).
27) B. A. BILBY, R. BOULLOUGH and E. SMITH, *loc. cit.*, 1).
28) B. A. BILBY and E. SMITH, *loc. cit.*, 2).
29) E. KRÖNER, *loc. cit.*, 7).
30) E. F. HOLLÄNDER, *loc. cit.*, 18)-20).

The above introduced distortion tensor u_{ik} permits us to define the deformation tensor according the general rule

$$e_{ik}=u_{(ik)} \tag{14}$$

where the brackets $u_{(ik)}$ denote $1/2(u_{ik}+u_{ki})$. The antisymmetrical part of distortion tensor forms the rotation tensor

$$\omega_{ik}=u_{[ik]} \tag{15}$$

Reciprocal formula to the Eqs. (14) and (15) states

$$u_{ik}=e_{ik}+\omega_{ik} \ ,$$

generally it is assumed that the rotation tensor vanishes[31),32)]. However, in the case of thin stratification of the medium (as we will discuss in the next part) we believe that there may arise a field of couple stresses associated with rotation tensor. The presence of the couple stresses (stress moments) is usually not taken into account. The hypothesis of occurrence of couple stresses seems particularly plausible in the case of "plastic" deformation in a thin-layered medium. The field of couple stresses, should satisfy Cauchy's equation II which can be presented in the form[33)]

$$m_{kij,k}+\rho l_{ij}+P_{[ij]}=0$$

where: m_{kij} -couple stress (antisymmetric in the indexes i, j)
$P_{[ij]}$ -antisymmetric part of the stress tensor (in our case the tensor P_{ij} is symmetric)
l_{ij} -volume moments of forces (in our case we shall assume $l_{ij}=0$)

Our equation is now simplified to:

$$m_{kij,k}=0 \tag{16}$$

Density of the volume energy related with the occurrence of the field of couple stresses amounts to:[34)]

$$\overline{W}=m_{kij}\omega_{ij,k}$$

31) E. KRÖNER, *loc. cit.*, 7).
32) E. F. HOLLÄNDER, *loc. cit.*, 18).
33) A. C. ERINGEN, *Nonlinear Theory of Continuous Media* (McGraw-Hill, 1962).
34) A. C. ERINGEN, *loc. cit.*, 33).

Similarly as through stress-strain relations we connect the stress tensor with the elastic deformation tensor, we shall now assume the linear relation between stress moments and rotation tensor derivations. We introduce now a modulus $\bar{\mu}$ which in the case of stratified medium characterises the physical properties on contacts of layerlets, and thus we get:

$$m_{kij}=2\bar{\mu}\omega_{ij,k}$$

Using this relation to the Eq. (16) we immediately obtain

$$\omega_{ij,kk}=u_{[ij],kk}=0 \tag{17}$$

Thus, the Laplace equation should be satisfied by the rotation tensor components. The equations for symmetric part of the distortion tensor follow from the stress-strain relation. For ideal elastic medium we have

$$P_{ik}=\delta_{ik}\lambda e_{ss}+2\mu e_{ik} \tag{18}$$

Hence, according to the Eq. $P_{ik,k}=0$ we obtain

$$\lambda u_{(ss),i}+2\mu u_{(ik),k}=0 \tag{19}$$

3. The problem of a fold

We follow here the general ideas contained in the previous paper,[35] but the new treatment is now required according to the above given considerations on dislocation densities and distortion tensors. Let's consider the folding problem for a homogeneous, elastic medium; the elastic deformation may be then presented by a deformation field u_i. For a two dimensional case we write for the coordinate changes acc. to Eq. (5):

$$dX=dx+du\ , \qquad dY=dy+dv\ . \tag{20}$$

Let's take as the example the symmetric anticlinal fold[36]

$$\left.\begin{aligned} &u=\beta xy \exp(-hr^2)\ , \\ &v=\beta(\varepsilon+|y|)\exp(-hr^2),\ \varepsilon>0,\ \beta>0,\ h>0\ , \end{aligned}\right\} \tag{21}$$

and the trough and ridge fold

$$\left.\begin{aligned} &u=\beta xy \exp(-hr^2)\ , \\ &v=-\beta|y|x\exp(-hr^2),\ \beta>0,\ h>0\ . \end{aligned}\right\} \tag{22}$$

35) R. TEISSEYRE, *loc. cit.*, 9).
36) R. TEISSEYRE, *loc. cit.*, 9).

The stress field associated with the above deformations u_i should satisfy the equilibrium equation

$$P_{ik,k} = -\rho f_i \tag{23}$$

where the forces f_i are regarded as responsible for maintenance of deformation.

In the case of a thin stratification of the medium we recognize the difference in the state of deformation due to displacements along the medium's layerlets. In the limit case of infinitesimally thin layerlets we come to the case of a continuous distribution of dislocations. The deformation should now be described not only by the u_i vector field but also by the u_{ik} distortion field. In our case we have

$$\left.\begin{aligned} dX &= dx + du + \varphi dx + \bar{\varphi} dy \\ dY &= dy + dv + \psi dx + \bar{\psi} dy \end{aligned}\right\} \tag{24}$$

where: $\varphi = u_{11}$, $\bar{\varphi} = u_{12}$, $\psi = u_{21}$, $\bar{\psi} = u_{22}$. The stresses $\tilde{P}_{ik}$ associated with u_i and u_{ik} should as a whole satisfy the equation identical to (23). Hence we deduce that the stresses $\bar{P}_{ik} = \tilde{P}_{ik} - P_{ik}$ associated with distortion u_{ik} ought to satisfy the homogeneous equation

$$\bar{P}_{ik,k} = 0 \tag{25}$$

This being the equation for symmetric distortion $u_{(ik)}$, when for rotation $u_{[ik]}$ we have the Laplace equation (17).

Additional condition for u_{ik} is imposed by requiring slipping deformation to be parallel to the fold slope, that means the dislocational displacements should occur along the particular layerlets. As the layerlets are defined by $d\xi_2 = 0$ this condition leads to the relation:[37)]

$$\frac{dY}{dX} = \frac{\psi}{\varphi} = \frac{dv}{du + dx} = \gamma\,, \quad \text{at} \quad dy \underset{\text{Num.}}{=} d\xi_2 = 0 \tag{26}$$

Further on we will resticrt our considerations to the case of a sharp fold, putting $h \gg 1$ in the formulas (21), (22). The slope condition (26) takes then the form

$$\gamma = -2\beta h(\varepsilon + |y|)\, x \exp(-hr^2) \tag{27}$$

for the symmetric anticlinal fold, and

37) R. TEISSEYRE, *loc. cit.*, 9).

$$\gamma = 2\beta h \mid y \mid x^2 \exp(-hr^2) \tag{28}$$

for the through and ridge fold respectively.

The equations for distortion field (25) and (17) can now be put in the following form

$$\left.\begin{aligned} P_{1k,k} &= (\lambda+2\mu)\varphi_{,x} + \lambda\bar{\psi}_{,x} + \mu\bar{\varphi}_{,y} + \mu(\gamma\varphi)_{,y} = 0 \\ P_{2k,k} &= (\lambda+2\mu)\bar{\psi}_{,y} + \lambda\varphi_{,y} + \mu\bar{\varphi}_{,x} + \mu(\gamma\varphi)_{,x} = 0 \\ u_{[12],kk} &= (\bar{\varphi}-\gamma\varphi)_{,xx} + (\bar{\varphi}-\gamma\varphi)_{,yy} = 0 \end{aligned}\right\} \tag{29}$$

Among the possible solutions we choose one for the vanishing rotation tensor $u_{[12]}$. That means we put in Eqs. (29), $\bar{\varphi}=\gamma\varphi$, and then we obtain:

$$\left.\begin{aligned} (\lambda+2\mu)\varphi_{,x} + \lambda\bar{\psi}_{,x} + 2\mu(\gamma\varphi)_{,y} = 0 \\ (\lambda+2\mu)\bar{\psi}_{,y} + \lambda\varphi_{,y} + 2\mu(\gamma\varphi)_{,x} = 0 \end{aligned}\right\} \tag{30}$$

Eliminating from these equations the function $\bar{\psi}$ we get the second order differential equation for φ:

$$(\lambda+2\mu)\varphi_{,xy} + (\lambda+2\mu)\varphi_{,yy} - \lambda(\gamma\varphi)_{,xx} = 0 \tag{31}$$

We would like to find the solution of (31) under the assumption $h \gg 1$. Taking into account that the γ-slope function contains the factor $\exp(-hr^2)$, that means it has the form $f(x, y)\exp(-hr^2)$, we can confine any differential process to the exp-function:

$$\left.\begin{aligned} &\gamma_{,x} = -2hx\gamma\ , \qquad \gamma_{,y} = -2hy\gamma \\ &\gamma_{,xx} = 4h^2x^2\gamma\ , \qquad \gamma_{,xy} = 4h^2xy\gamma\ , \qquad \gamma_{,yy} = 4h^2y^2\gamma \end{aligned}\right\} \tag{32}$$

In Eq. (31) we have the terms:

$$\varphi_{,xy},\ \varphi\gamma_{,xx},\ \varphi_{,x}\gamma_{,x},\ \varphi_{,xx}\gamma,\ \varphi\gamma_{,yy},\ \varphi_{,y}\gamma_{,y},\ \varphi_{,yy}\gamma\ .$$

We assume that the derivations of φ are of the first order in $\exp -(hr^2)$, the function φ itself being of zero order, respectively. Then we will consider in Eq. (31) only the terms $\varphi_{,xy}$, $\varphi\gamma_{,xx}$, $\varphi\gamma_{,yy}$, meanwhile we will omit $\varphi_{,x}\gamma_{,x}$, $\varphi_{,xx}\gamma$, $\varphi_{,y}\gamma_{,y}$, $\varphi_{,yy}\gamma$ being proportional to $(\exp(-hr^2))^2$. These conditions allow to construct the function φ by means of the function $\exp[\exp(-hr^2)]$—for abbreviation we will write sometimes Exp—and by the error function

$$\Phi(\sqrt{h}\,r), \quad \text{where} \quad \Phi(z) = 2\int_0^z \exp(-t^2)dt\ .$$

This follows immediately when one takes the derivations of the Exp and Φ, which are proportional to $\exp(-hr^2)$. For example

$$\left.\begin{aligned}(\mathrm{Exp})_{,x} &= -2hx \exp(-hr^2)\,\mathrm{Exp}\\ \Phi_{,x} &= 2h^{1/2}r^{-1}x\exp(-hr^2)\end{aligned}\right\} \tag{33}$$

In this manner we can construct the following combination for φ:

$$\varphi = \exp[A(x, y)\exp(-hr^2)] - \Phi(\sqrt{h}\,r)\exp[B(x, y)\exp(-hr^2)] \tag{34}$$

where the function $A(x, y)$ and $B(x, y)$ are to be found from the Eq. (31). The form (34) assures also that φ tends to zero for $r\to\infty$, becruse Φ and Exp tend then to unity.

For the derivation of φ we get:

$$\left.\begin{aligned}\varphi_{,x} &= -2hxA\exp(-hr^2)\,\mathrm{Exp}^A - 2h^{1/2}r^{-1}x\exp(-hr^2)\,\mathrm{Exp}^B\\ &\quad + 2hxB\exp(-hr^2)\Phi\,\mathrm{Exp}^B\\ \varphi_{,xy} &= 4h^2xyA\exp(-hr^2)\,\mathrm{Exp}^A + 4h^{3/2}r^{-1}xy\exp(-hr^2)\,\mathrm{Exp}^B\\ &\quad - 4h^2xyB\exp(-hr^2)\Phi\,\mathrm{Exp}^B\end{aligned}\right\} \tag{35}$$

where we restrict considerations only to the terms containing first power of $\exp(-hr^2)$, following the differentiation rule analogous to (32). The expressions Exp^A, Exp^B denote $\exp[A\exp(-hr^2)]$ and $\exp[B\exp(-hr^2)]$, respectively. Inserting the expressions for φ and its derivations into Eq. (31) we get immediately for the symmetric anticlinal fold:

$$A = 2h\beta(\varepsilon + |y|)\left(y - \frac{\lambda}{\lambda+2\mu}\frac{x^2}{y}\right), \qquad B = A + C, \qquad C = \frac{1}{h^{1/2}r}\cdot\frac{1}{\Phi} \tag{36}$$

and for trough and ridge fold:

$$A = 2h\beta|y|x\left(-y + \frac{\lambda}{\lambda+2\mu}\frac{x^2}{y}\right), \qquad B = A + C, \qquad C = \frac{1}{h^{1/2}r}\cdot\frac{1}{\Phi} \tag{37}$$

The expression for φ then becomes

$$\varphi = \exp[A\exp(-hr^2)]\{1 - \Phi(\sqrt{h}\,r)\exp[C\exp(-hr^2)]\} \tag{38}$$

hence we can also see that φ is limited for $r=0$.

The full solution of the (30) system, including $\bar{\psi}$, obviously requires further calculations.

The non vanishing components of dislocation density tensor (11) can be expressed as follows

$$\alpha_{13}=u_{11,2}-u_{12,1}=\varphi,_y-(\gamma\varphi),_x$$
$$\alpha_{23}=u_{21,2}-u_{22,1}=(\gamma\varphi),_y-\bar{\psi},_x .$$

To calculate the conditional maxima of α_{13} along the lines $d\xi_2=0$, we should turn back to Eq. (2) and Eq. (12). Taking the contour integral alongated with the line $d\xi_2=0$ we can express $\Delta\xi_1$ as follows:

$$\Delta\xi_1=\iint\alpha_{13}dx\,dy=-\oint u_{1s}dx_s\approx-\int(u_{11,2}\Delta y)dx$$

where we reduced the contour integral to the two sides over dx extending along the line $d\xi_2=0$. Investigation of the α_{13} maxima is under this condition limited to the term $u_{11,2}=\varphi,_y$. Thus, we get the equation for maxima in respect to x:

$$\varphi,_{yx}=0 \tag{39}$$

Eq. (39) is equivalent to the following expression for the anticlinal fold:

$$x\left(y^2-\frac{\lambda}{\lambda+2\mu}x^2\right)(1-\Phi\mathrm{Exp}^C)=0 \tag{40}$$

and similarly in the case of the trough and ridge fold to:

$$x^2|y|\left(y^2-\frac{\lambda}{\lambda+2\mu}x^2\right)(1-\Phi\,\mathrm{Exp}^C)=0 \tag{41}$$

The factor $1-\Phi\,\mathrm{Exp}^C$ is always greater unity. From (40) follows immediately the equation for maxima

$$y=\pm\sqrt{\frac{\lambda}{\lambda+2\mu}}x$$

which are separated by minimum at $x=0$, referring to summit line of anticline.

The change of sign due to the factor x in (40) is related with the change of the Burgers vector of dislocations at opposite slopes of anticline. In the same way from (41) follows the solution for the lines of maxima of dislocation density:

$$y=\pm\sqrt{\frac{\lambda}{\lambda+2\mu}}x$$

The minima should be here also related to the summit and bottom of structure, that is to $x=\pm 1/\sqrt{2h}$, but according to our approximation $h \gg 1$, these lines are shifted to the $x=0$ line.

The maxima lines of dislocation density are shown at fig. 1 and fig. 2. ($\beta=1, \varepsilon=0.3, h=5$). There are also shown the $\xi_2=0$ lines representing general feature of structures under consideration. It is perhaps for further investigations to decide whether the maxima lines of dislocation density may be referred to some hypocentral plane of earthquake occurrence.

The α_{23} component of the density tensor can be expressed by the

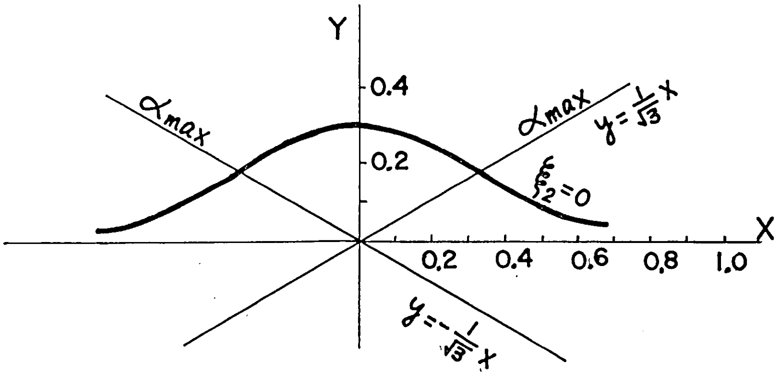

Fig. 1

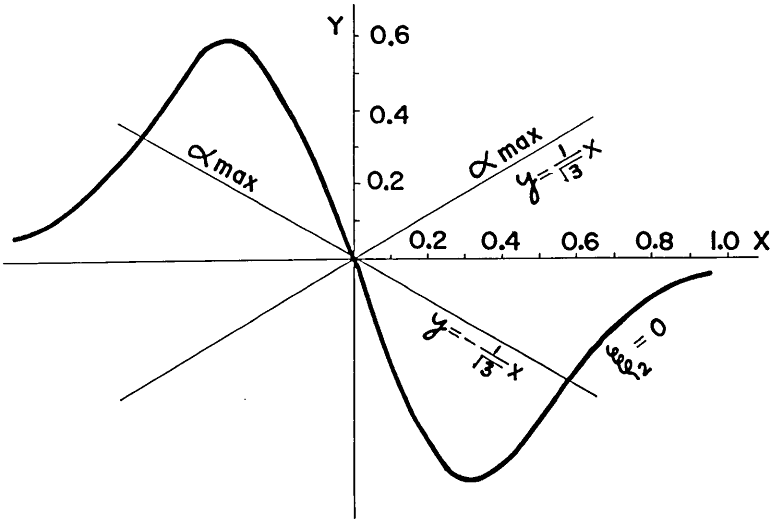

Fig. 2

help of the first of the (30) Eqs. We get:

$$\alpha_{23}=\frac{\lambda+2\mu}{\lambda}[(\gamma\varphi)_{,y}+\varphi_{,x}]$$

From this expression we could easily proceed to further calculations. It follows that the maxima of the α_{23} density are vertical lines, but according to our approximation $h\gg 1$ they are shifted to the $x=0$ line.

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9. 転位密度の場と褶曲変形の問題

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変形理論の 2 つの基本的量一変形テンソルと転位密度テンソルとは，それぞれ幾何学的相当量をもつている．しばしば幾何学的相当量を示すために，座標系を連続的媒質の実際の点と結びつける．このようにして，媒質の変形と同時に座標系の変形が起る．

一般に転位密度テンソルは，捩率テンソルと Riemann-Christoffel の曲率テンソルとによつて表現することが出来る．これらのテンソルは補助空間と関連していて，その補助空間の媒介によつて，実際に変形された媒質の特性が決定されるのである．一般的な考察および組み立てられた基礎方程式は，この論文においては，褶曲変形の問題に適用された．

もし媒質構造が，ごく薄い層のかさね合せによつて記述されるとするならば，褶曲作用を考える時には，薄い層にそつたすべりがおこることを考慮しなければならない．層の厚さを薄くした極限においては，その問題は連続的な転位分布の問題になる．この考えが 2, 3 の型の褶曲に適用された．