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51. *Response Analysis of Tall Buildings to Strong Earthquake Motions.*
Part 1
Linear Response of Core-wall Buildings.

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Abstract

The elastic response of a core-wall building to earthquake ground motions is analyzed. The core-wall building is assumed to be represented by a vibratory system having a uniform shear beam and a uniform cantilever with elastic restraint to rotation. The equation of vibration for this system is derived and the solution is given by modal expression. Using this modal solution the maximum response values of six core-wall buildings to two recorded earthquakes are obtained and the results are compared with those computed by modal analysis of "root mean square" type.

1. Introduction

Recent investigations on the response analysis in the field of earthquake engineering have contributed to clarify the dynamic behavior of building structures during an earthquake. However, in order to establish the dynamic design method of tall buildings for strong earthquake motions many problems are still left unsolved. It is intended in this series of investigation to solve some problems related to earthquake response of tall buildings. The purpose of this first paper is to clarify the general feature of the elastic response of a core-wall building to earthquake ground motions.

When a building has a box-shape wall extending over the height of the building as shown in Fig. 1, it is stated to be the core-wall building. Because of the existense of the box-shape wall it will not be adequate for this type of structure to be treated as a shear type multi-mass vibratory system as in the case of moment resisting frames, for

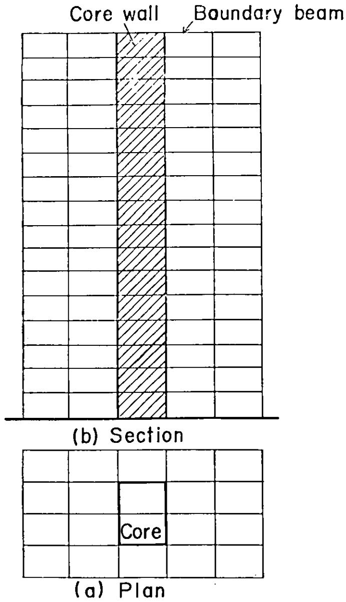

Fig. 1. Core-wall Building.

the bending deformation of the wall must be considered. In the present paper the following reduced vibratory system is considered to make the response analysis of the core-wall building. The system employed is a combination of a unifrom shear beam type structure and a uniform cantilever type structure having elastic restraint to rotation, these structures representing the open frames and the core-wall, respectively.

2. Assumptions

To investigate the general effect of earthquake ground motions on the stress distributions of the structure, it is assumed that:

(1) The masses and the stiffnesses are uniformly distributed over the height of the building.

(2) All the members of the structure are distorted within an elastic range.

(3) The open frames vibrate as a shear beam type structure, and the core walls behave as a cantilever standing on the founda-

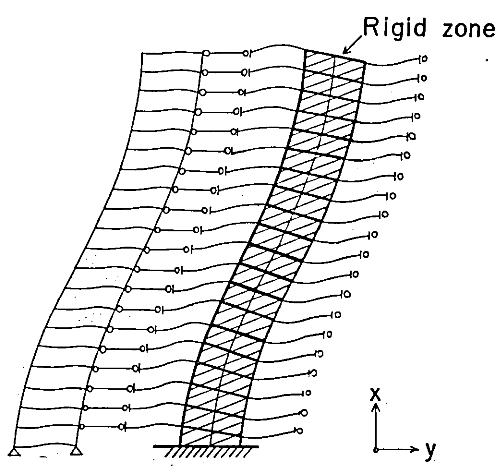

Fig. 2. Reduced vibratory system.

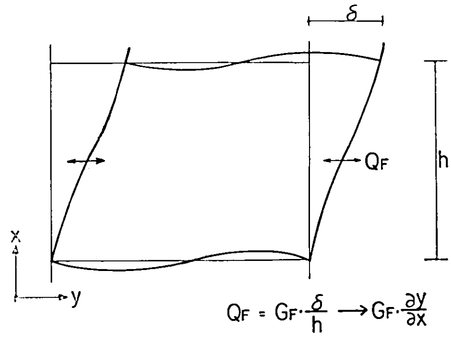

Fig. 3. Rigidity of open frames.

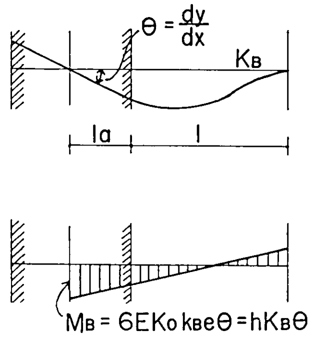

Fig. 4. Rigidity of boundary beams.

tion and are subjected to restaint action from the adjacent beams.

(4) There is no foundation rotation.

With these assumptions the system can be analyzed as a combination of shear beam type structure and a flexural cantilever type structure having restraint proportional to its rotation.

3. Derivation of the equation of motion

Open frames:

As the open frames are assumed to be the shear beam, the shear at a section located a distance x from the base is expressed as follows:

$$Q_F(x) = G_F \frac{\partial y}{\partial x}. \tag{1}$$

Where G_F is the distributed rigidity of the frame (see Fig. 3).

Core walls:

The core wall is represented by the cantilever and the boundary effect from the adjacent beams is taken into account as the flexural resistance proportional to the slope $\partial y/\partial x$. Then the bending moment of the wall is expressed as

$$M_W(x) = -EI\frac{\partial^2 y}{\partial x^2} = -\int_x^H Q_W(x)dx' + \int_x^H K_B \frac{\partial y}{\partial x} dx'. \tag{2}$$

Where EI is the flexural rigidity of the wall and K_B is a sum of the effective stiffness of adjacent beams (see Fig. 4). Differentiating equation (2), the expression for the shear of the wall is obtained.

$$Q_W(x) = -EI\frac{\partial^3 y}{\partial x^3} + K_B\frac{\partial y}{\partial x}. \tag{3}$$

Equation of motion:

Substituting the above fundamental relationship (1) and (3) into the D'Alembert's principle, the following differential equation of motion is derived.

$$EI\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} - (G_F + K_B)\frac{\partial^2 y}{\partial x^2} = 0 \tag{4}$$

where $\rho A = M/H$ = mass of the building per unit length.

If the system has the damping force proportional to the strain velocity, the equation of motion (4) is modified as

$$EI\left(1 + r_i\frac{\partial}{\partial t}\right)\frac{\partial^4 y}{\partial x^4} + \rho A\frac{\partial^2 y}{\partial t^2} - (G_F + K_B)\left(1 + r_i\frac{\partial}{\partial t}\right)\frac{\partial^2 y}{\partial x^2} = 0 \tag{5}$$

where r_i is the coefficient of internal friction.

Putting $\alpha^2 = \rho A H^4/EI = MH^3/EI$, $\gamma = (G_F + K_B)H^2/2EI$

and $y = Y + y_0$ (y_0 = ground displacement), equation (5) becomes

$$\left(1 + r_i\frac{\partial}{\partial t}\right)\frac{\partial^4 Y}{\partial x^4} + \frac{\alpha^2}{H^4}\frac{\partial^2 Y}{\partial t^2} - \frac{2\gamma}{H^2}\left(1 + r_i\frac{\partial}{\partial t}\right)\frac{\partial^2 Y}{\partial x^2} = -\frac{\alpha^2}{H^4}\frac{d^2 y_0}{dt^2}. \tag{6}$$

4. Solution by modal expression

The general solution of equation (6) can be obtained as the sum of the solution of free vibration and the particular solution, both of which are expressed by superposition of modal values.

$$Y = \sum_{s=1}^{\infty} {}_s\beta \cdot {}_su(x) \cdot {}_sq'(t) + \sum_{s=1}^{\infty} {}_s\beta \cdot {}_su(x) \cdot {}_s\bar{q}(t) = \sum_{s=1}^{\infty} {}_s\beta \cdot {}_su(x) \cdot {}_sq(t). \tag{7}$$

Where ${}_su(x)$, ${}_sq(t)$ and ${}_s\beta$ are the normal function, normal coordinate and the participation factor in s-th mode, respectively. The normal coordinate ${}_sq(t)$ is obtained as the solution of the following ordinary differential equation:—

$$\frac{d^2{}_sq}{dt^2} + 2{}_sh \cdot {}_sn \cdot \frac{d{}_sq}{dt} + {}_sn^2 \cdot {}_sq = -\frac{d^2 y_0}{dt^2}. \tag{8}$$

Where ${}_sh$ and ${}_sn$ are the fraction of critical damping and natural circular frequency without damping in s-th mode, respectively. The participation factor ${}_s\beta$ is determined in this case by

$$ {}_s\beta = \int_0^H {}_su(x)\,dx \Big/ \int_0^H {}_su^2(x)\,dx\,. \tag{9}$$

The expression of the normal function ${}_su(x)$ takes the form

$$u(x) = C_1 \sin(p_1x/H) + C_2 \cos(p_1x/H) + C_3 \sinh(p_2x/H) + C_4 \cosh(p_2x/H) \tag{10}$$

or

$$u(\xi) = C_1 \sin p_1\xi + C_2 \cos p_1\xi + C_3 \sinh p_2\xi + C_4 \cosh p_2\xi \tag{10$'$}$$

where

$$p_1 = \sqrt{\sqrt{\alpha^2n^2+\gamma^2}-\gamma}\,, \quad p_2 = \sqrt{\sqrt{\alpha^2n^2+\gamma^2}+\gamma} \tag{11}$$

$$\xi = x/H\,.$$

To compute above values natural frequency n must be determined first. n is calculated by the frequency equation which is derived from the boundary conditions. In the case where the core-wall is fixed at the base the following frequency equation is obtained:

$$2p_1^2p_2^2 - p_1p_2(p_1^2-p_2^2)\sin p_1 \sinh p_2 + (p_1^4+p_2^4)\cos p_1 \cosh p_2 = 0\,. \tag{12}$$

Accordingly, the following expressions for the displacement, shears and moments are derived as the modal values: —

Displacement: —

$$\begin{aligned} Y = [&(p_1^2\cos p_1 + p_2^2\cosh p_2)(p_2\sin p_1\xi - p_1\sinh p_2\xi) \\ &- p_1p_2(p_1\sin p_1 + p_2\sinh p_2)(\cos p_1\xi - \cosh p_2\xi)]Ce^{int}\,. \end{aligned} \tag{13}$$

Shear in the open frames: —

$$\begin{aligned} Q_F = [&p_1p_2(p_1^2\cos p_1 + p_2^2\cosh p_2)(\cos p_1\xi - \cosh p_2\xi) \\ &+ p_1p_2(p_1\sin p_1 + p_2\sinh p_2)(p_1\sin p_1\xi + p_2\sinh p_2\xi)]\left\{\frac{G_F}{H}\right\}Ce^{int}\,. \end{aligned} \tag{14}$$

Shear in the core wall: —

$$\begin{aligned} Q_W = \Bigg[&p_1p_2(p_1^2\cos p_1 + p_2^2\cosh p_2)\left\{\left(p_1^2 + \frac{K_BH^2}{EI}\right)\cos p_1\xi \right. \\ &\left. + \left(p_2^2 - \frac{K_BH^2}{EI}\right)\sinh p_2\xi\right\} + p_1p_2(p_1\sin p_1 + p_2\sinh p_2) \\ &\times\left\{p_1\left(p_1^2 + \frac{K_BH^2}{EI}\right)\sin p_1\xi - p_2\left(p_2^2 - \frac{K_BH^2}{EI}\right)\sinh p_2\xi\right\}\Bigg]\left\{\frac{EI}{H^3}\right\}Ce^{int}\,. \end{aligned} \tag{15}$$

Bending moment of the core wall: —

$$M_W=[p_1p_2(p_1^2\cos p_1+p_2^2\cosh p_2)(p_1\sin p_1\xi+p_2\sinh p_2\xi)$$
$$-p_1p_2(p_1\sin p_1+p_2\sinh p_2)(p_1^2\cos p_1\xi+p_2^2\cosh p_2\xi)]\left\{\frac{EI}{H^2}\right\}Ce^{int}. \quad (16)$$

Bending moment of the boundary beam: —

$$M_B=[p_1p_2(p_1^2\cos p_1+p_2^2\cosh p_2)(\cos p_1\xi-\cosh p_2\xi)$$
$$+p_1p_2(p_1\sin p_1+p_2\sin hp_2)(p_1\sin p_1\xi+p_2\sin hp_2\xi)]\left\{\frac{K_B}{H}\right\}Ce^{int}. \quad (17)$$

These modal values in 1st to 3rd mode for $\gamma=20$ are illustrated in Fig. 5.

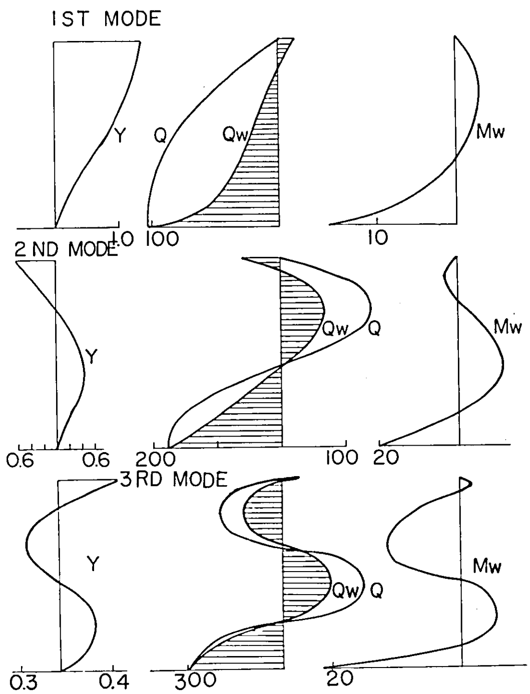

Fig. 5. Illustration of mode shapes, shears and moments.

5. Earthquake response of some ideal buildings

5.1 Sample buildings

Six different buildings are considered in the response analysis. The number of stories and the column sections of these buildings are shown in Table 1. These dimensions are chosen as having simple integer values of α and γ. The natural periods of buildings are also shown in Table 1. The fraction of critical damping is assumed to be 0.05 for the first mode in every building.

Table 1.

	Number of stories	α^2	γ	Column Section (cm)	Natural Period (Fundamental) (Sec)
BUILDING A	12	1	5	70× 70	0.876
〃 B	12	1	10	85× 85	0.686
〃 C	20	10	20	75× 75	1.66
〃 D	20	10	50	110×110	1.125
〃 E	40	100	100	80× 80	2.61
〃 F	40	100	100	110×110	1.89

5.2 *Method of computation*

The analog computer "SERAC" has been used to integrate equation (8) and to superpose the modal values. Each modal value for displacement, shear and moment were obtained by digital computer according to equations in the preceding section. The computed shears in core-wall of Building D are illustrated in Fig. 6.

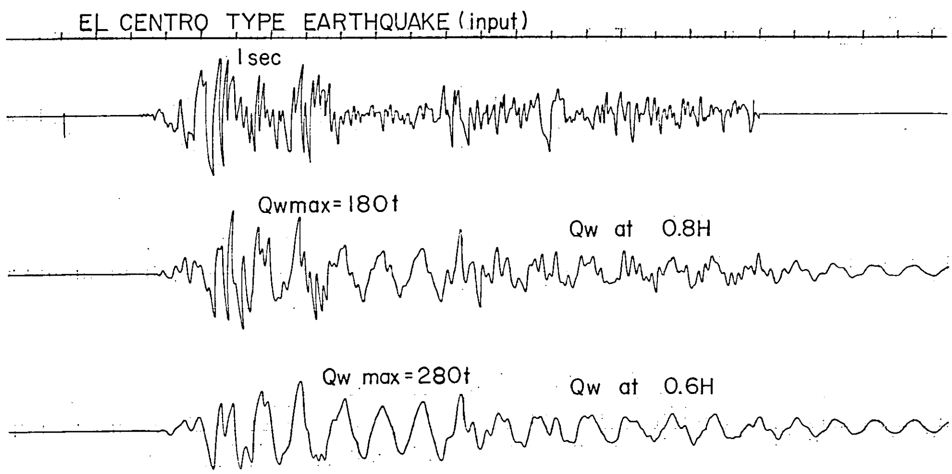

Fig. 6. Illustration of responses obtained by analog computer.

5.3 *Earthquake ground motion*

The following two types of earthquake motions are used.

EL Centro 1940 NS type
Tokyo 101 1956 NS type

5.4 Results

The results are obtained for displacement, shear and moment responses as illustrated in Fig. 6. The maximum values in each response are plotted in Figs. 7-18 by the (⊛) mark.

The (×) mark in the same figures shows so-called "root mean square" values, which are defined as

$$Y_{\max}=\sqrt{\sum_{s} {}_sY_{\max}^2}\,, \quad \text{etc.,} \tag{18}$$

the solid lines representing the maximum values when the first mode only is considered.

Comparing these maximum values it can be said that

(1) "Root mean square" values are always very close to the exact maximum values except in a very few cases.

(2) In most cases the first mode maxima are nearly equal to the exact maxima, and this means the effect of higher modes is very small. The exceptional case is seen in Building F, for which the maximum values of the first and second modes are comparable.

6. Concluding remarks

The method of earthquake response analysis for core-wall buildings has not been well established. The preceding results of the earthquake response of some ideal buildings show that the first mode maxima are nearly equal to the exact maxima with a few exceptional cases. Although these results are obtained for the buildings with constant section along the height, this suggests the possibility of approximate method of analysis in which the building is reduced to the shear type multi-mass vibratory system.

It can be expected that in the core-wall building the vibrational energy be absorbed by partial failure of wall element for severe earthquakes. In such a case non-linear response analysis must be employed and this will be discussed in the succeeding paper.

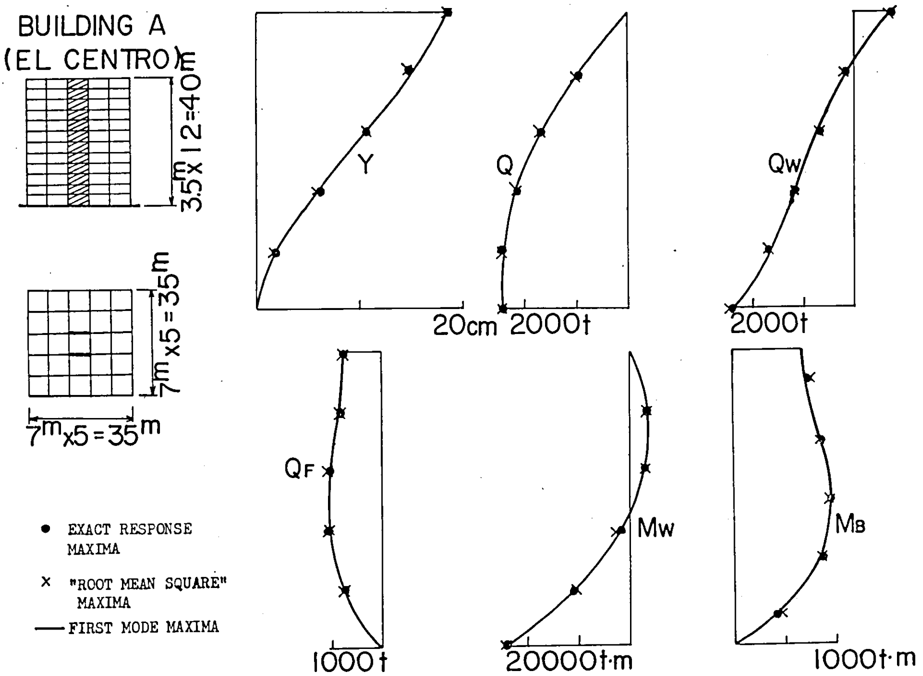

Fig. 7. Comparison of maximum response values (1).

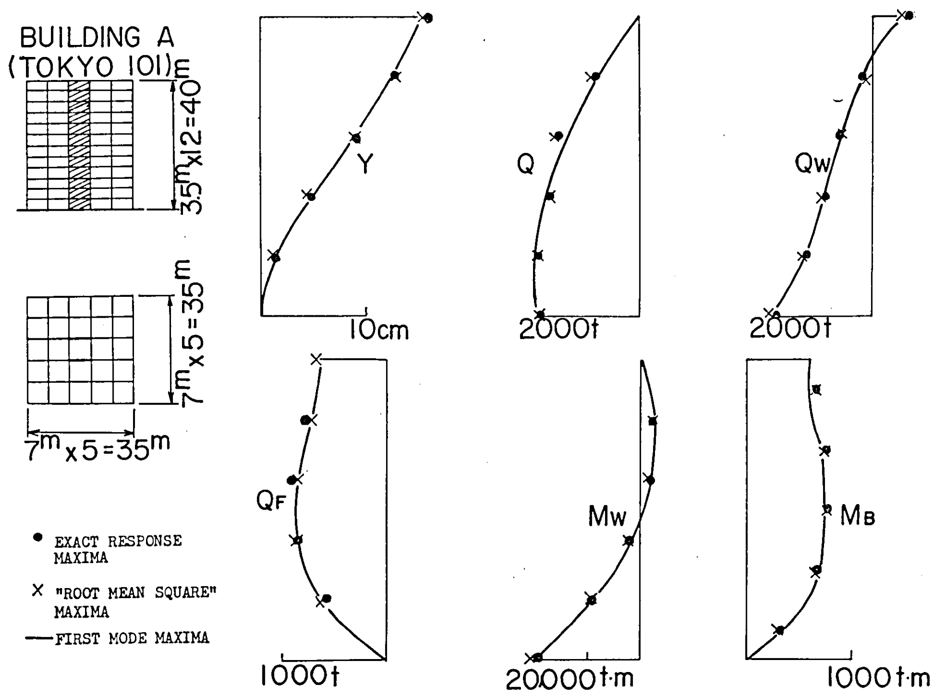

Fig. 8. Comparison of maximum response values (2).

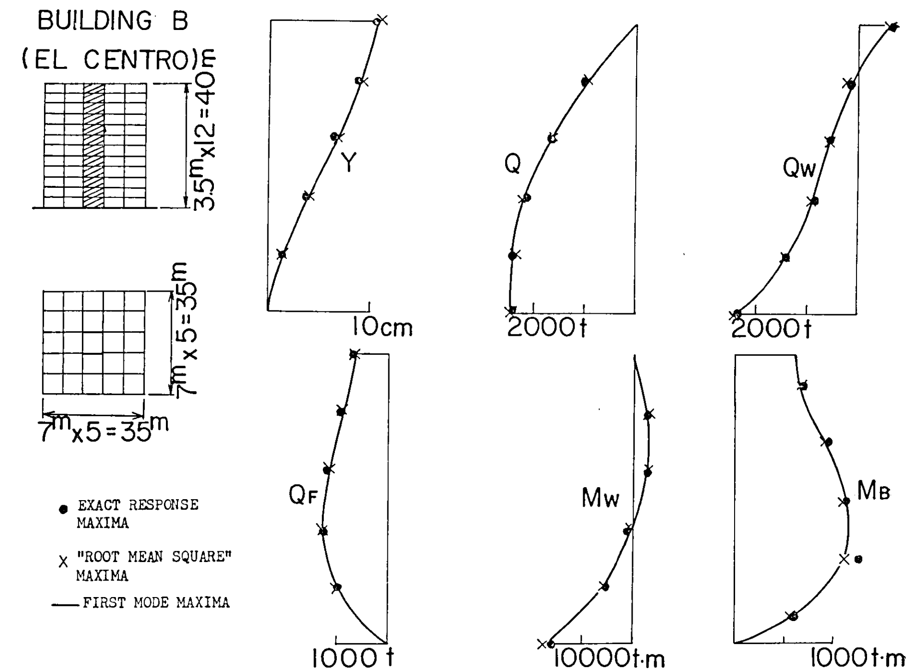

Fig. 9. Comparison of maximum response values (3).

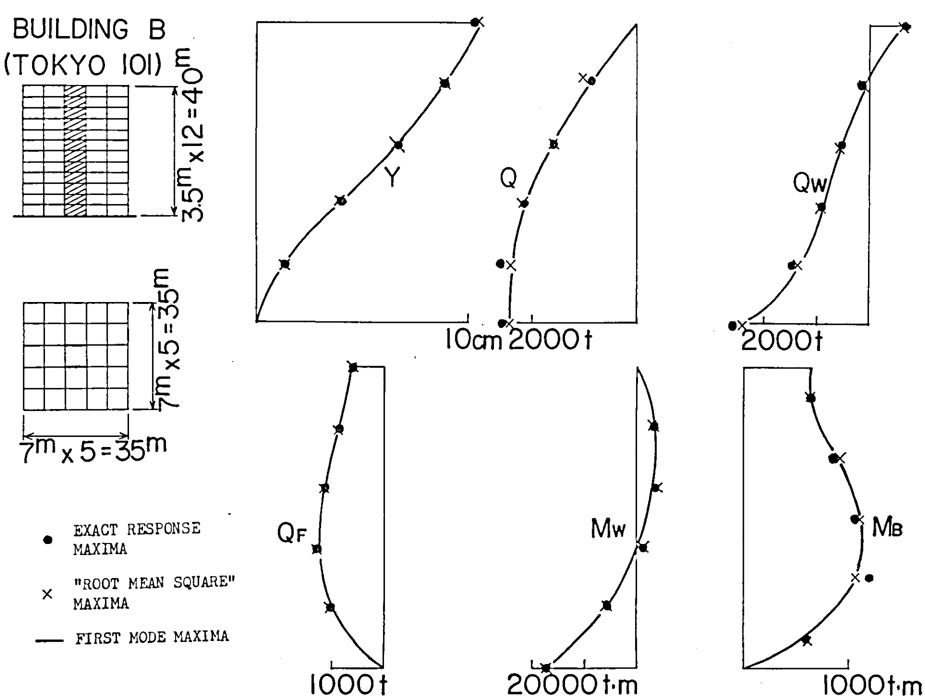

Fig. 10. Comparison of maximum response values (4).

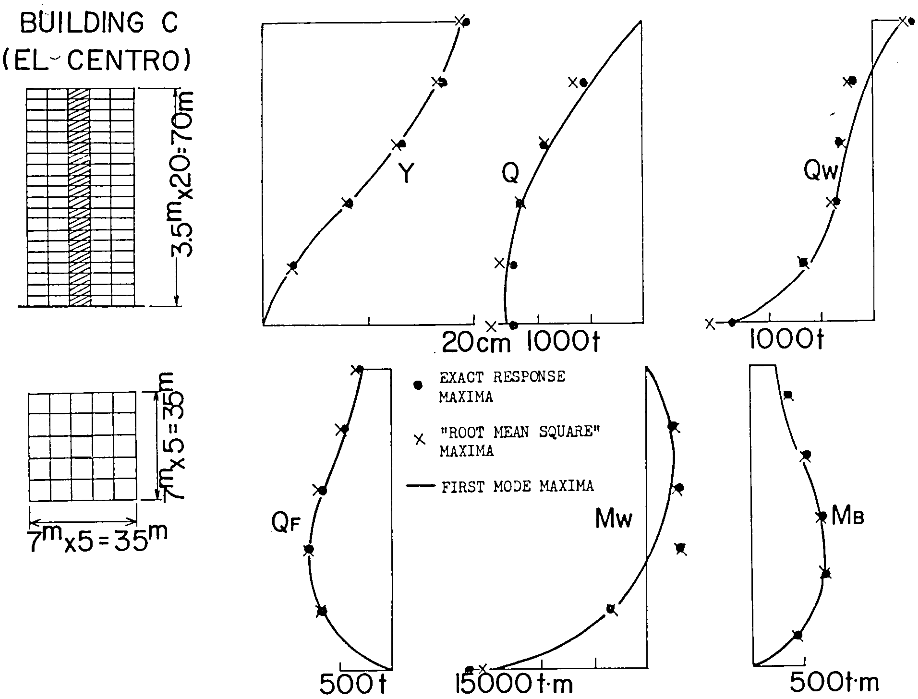

Fig. 11. Comparison of maximum response values (5).

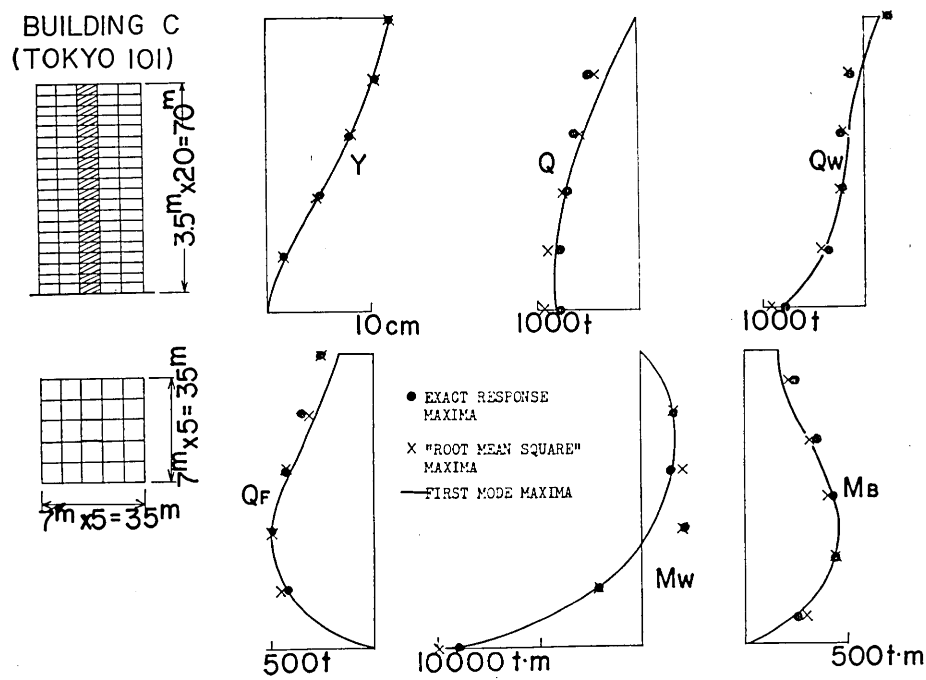

Fig. 12. Comparison of maximum response values (6).

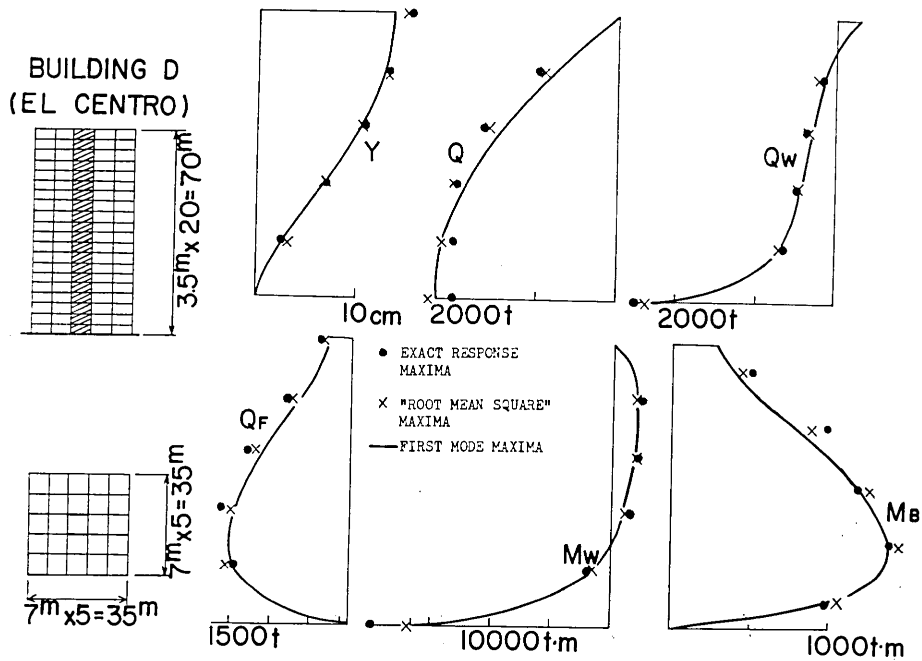

Fig. 13. Comparison of maximum response values (7).

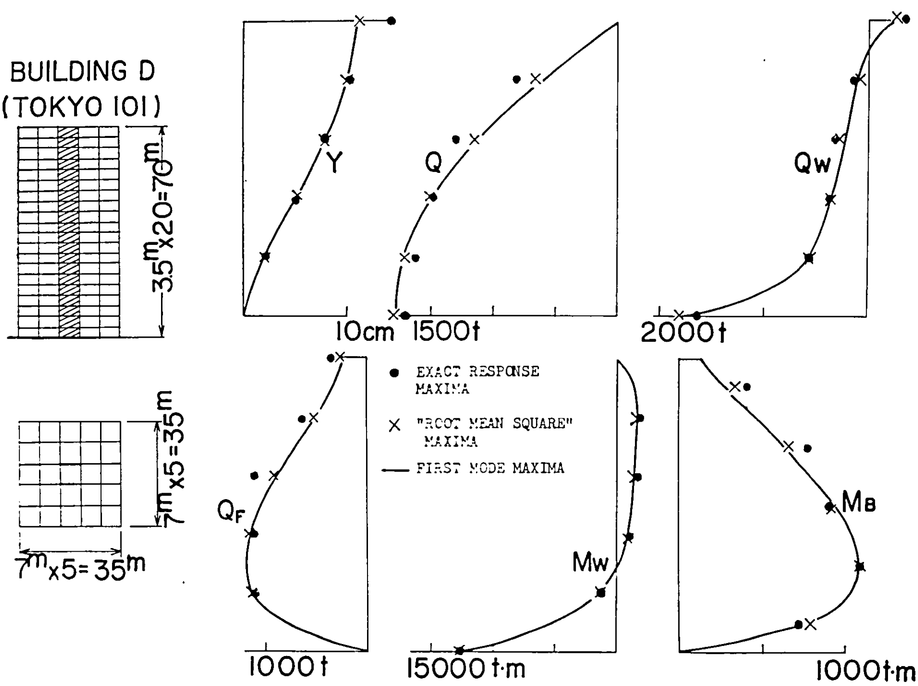

Fig. 14. Comparison of maximum response values (8).

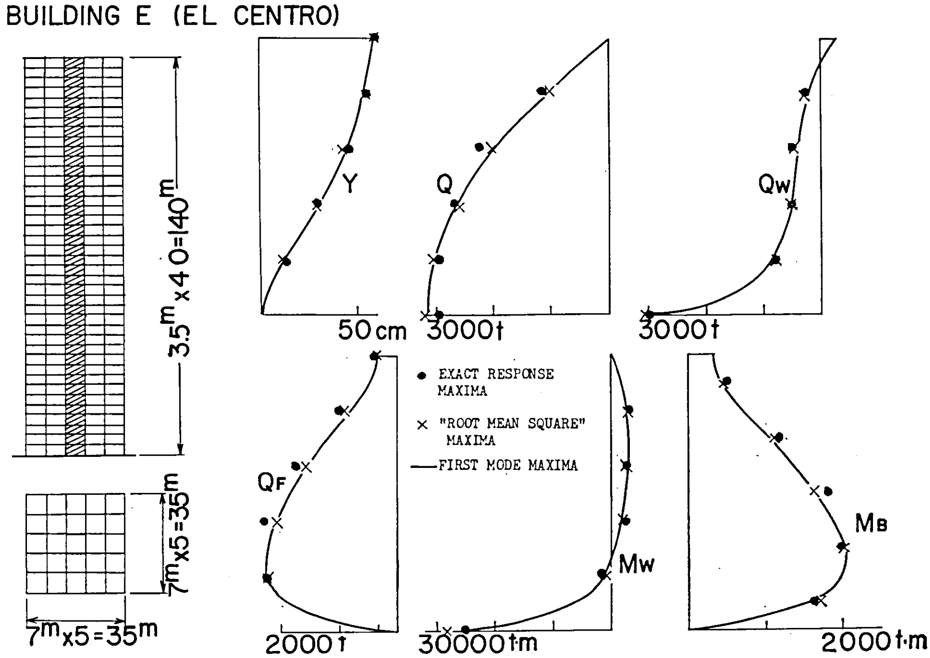

Fig. 15. Comparison of maximum response values (9).

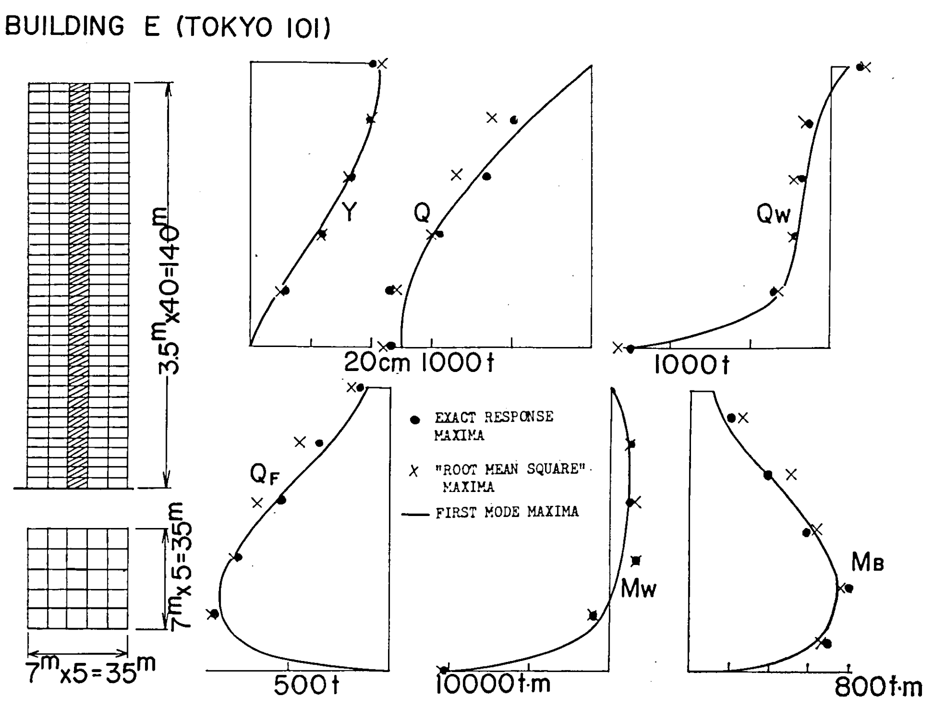

Fig. 16. Comparison of maximum response values (10).

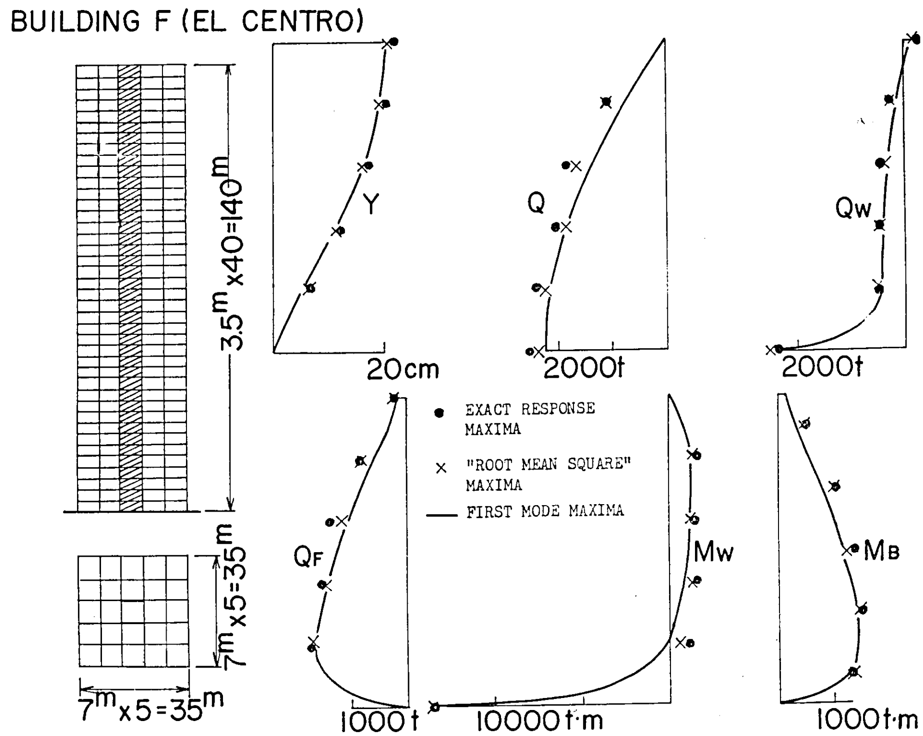

Fig. 17. Comparison of maximum response values (11).

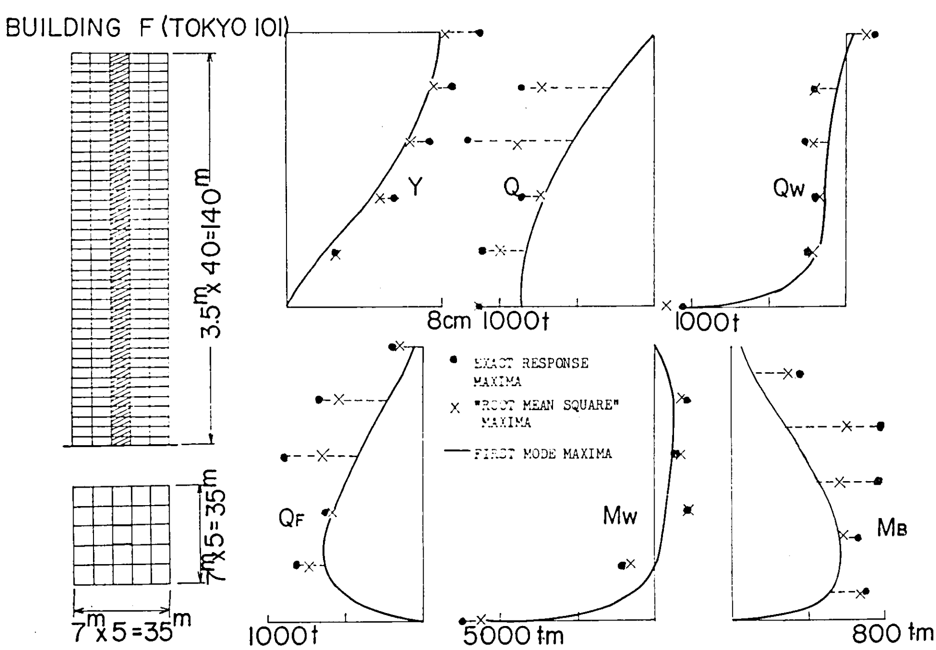

Fig. 18. Comparison of maximum response values (12).

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51. 高層建築物の強震応答解析に関する研究
第1報 コアー式建物の線形応答

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建物の中央部分に壁で囲まれたコアーをもつものは，耐震コアー式建物とよばれるが，これについて弾性範囲の地震応答解析による検討をおこなつた．この種建物の骨組はラーメン部分とコアー部分にわけて考えられるが，ここでは前者を剪断振動体，後者を回転抵抗を有する棒状の曲げ振動体におきかえ，連続体として振動方程式を導き，脚部固定の条件でその解を求めた．この解を用いてラーメン・壁寸法および階数の異なる6種類の場合について2種類の強震記録に対する応答を計算し，その結果を "root mean square" 法による値，1次振動のみ考えた場合と比較し，前者とはいずれもよく一致し，後者とも2，3の例外を除きかなりよく一致することを示した．
