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30. *Optimum Distribution of Seismic Observation Points. II.*

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Abstract

Arrival times of the waves at seismic observation points are usually used for the determination of the location of the focus and the occurrence time of an earthquake as well as the velocity of wave propagation.

As the observational errors have large or small effects on the errors of the above mentioned quantities according to the distribution of the observation points, our aim is to find such distribution for which the errors are either small (good distribution) or large (bad distribution) by means of the Monte Carlo method. The problem was previously discussed under the assumption of surface focus. The present paper treats the problem by taking into account the depth of the focus.

1. Problem

At the points $P_i(X_i, Y_i, Z_i)$ arrival times τ_i of a phase of an earthquake are known. Because of observational errors e_i, which are included in τ_i, x, y, h, t and v, errors of the coordinates of the focus (X_0, Y_0, H), origin time (T) and velocity (V) of the wave respectively, will take place. The errors $x, \cdots, v$ depend on the position of the points and we consider such distribution for which the errrors are either very small or very large by means of Monte Carlo method.

To make the calculation simple the following is assumed:

i) – P_i, the observation points, are on the surface $(Z_i=0)$,

ii) – velocity of the waves is constant,

iii) – errors e_i are $\pm\varepsilon$ or 0,

iv) – P_i are uniformly distributed in a square

and

v) – the epicenter is at the center of the square.

2. Calculation of the errors x, y, h, t and v

Assuming the smallness of errors we can make a linear equation for x, y, h, t and v, namely

$$X_j x + Y_j y - Hh - D_j v/V + VC_j t = VC_j e_j \,, \qquad j=1, 2, \cdots, 5 \tag{2.1}$$

where:

$$D_j = -(X_j^2 + Y_j^2 + H^2)$$

and

$$C_j = -(-D_j)^{1/2} \,.$$

A set of values (X_j, Y_j) $(j=1, 2, \cdots, 5)$ was read from a random number table and thus the distribution of observation points was given. The errors $x, \cdots, v$ were calculated for all the possible combinations of errors of e_j $(=\varepsilon, 0$ and $-\varepsilon)$.

To save time a set of 5 solutions

$$x_k, y_k, h_k, t_k \text{ and } v_k \,, \qquad k=1, 2, \cdots, 5 \tag{2.2}$$

of (2.1) was obtained, where

$$e_j = \begin{cases} \varepsilon & j=k \,, \\ 0 & j \neq k \,. \end{cases} \tag{2.3}$$

The solutions of (2.1) for all the remaining combinations of e_j can be calculated by linear combinations of the elements of (2.2) taking the advantage of (2.1) as a linear equation.

For the obtained values of errors $x, \cdots, v$ the standard deviations of the position of the hypocenter, origin time and the velocity were calculated.

3. Result

In Fig. 1, 96 distributions of observation points are drawn. The black circle denotes the epicenter and the open circles the observation points. By capital letters A, B, C, D and E it is indicated whether the distribution is very good, good, average, bad or very bad respectively for the determination of depth, epicenter location, origin time and velocity.

Out of 1000 distributions we picked up certain remarkable ones, which are represented at 12 columns denoted by the numbers $1, 2, \cdots, 12$. Their characteristics are as follows:

1) When three observation points form a triangle and the focus and the other two are inside, the result is generally good. Especially when 3 stations form a triangle, the 4th one is on one side and the 5th is close to the epicenter.

2) When four observation points form a flat rhombus with the epicenter and the 5th point inside, the result is also good.

3) When four observation points form a flat parallelogram and the focus and the remaining point are inside, the result is also good.

4) When the observation points form a lens-shaped polygon with the focus inside, determination of the velocity is good.

5) When the focus is on a side of a quadrilateral which is not flat, the result is not good.

6) When the points are distributed on one direction of the focus, the result is generally not good.

7) The result is not good when two points are too close to each other.

8) When the points are on a semi-circle around the focus, the result is generally not good.

9) When the epicenter is near the center of the polygon which is not flat, the result is bad.

10) Good determination of the depth can be obtained when one station is near the epicenter.

11) Distribution with the focus outside the polygon, the result is still good.

12) Distribution neither good nor bad.

4. Remarks

The results can be applied to the cases in which the position of the foci is roughly given in advance (aftershocks, volcanic earthquakes).

Comparison between obtained results and the case of surface focus[1] indicates the similarity of both cases.

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1) Y. SATÔ, "Optimum Distribution of Seismic Observation Points," *Zisin* [ii], **18** (1965), 9-14, (in Japanese).

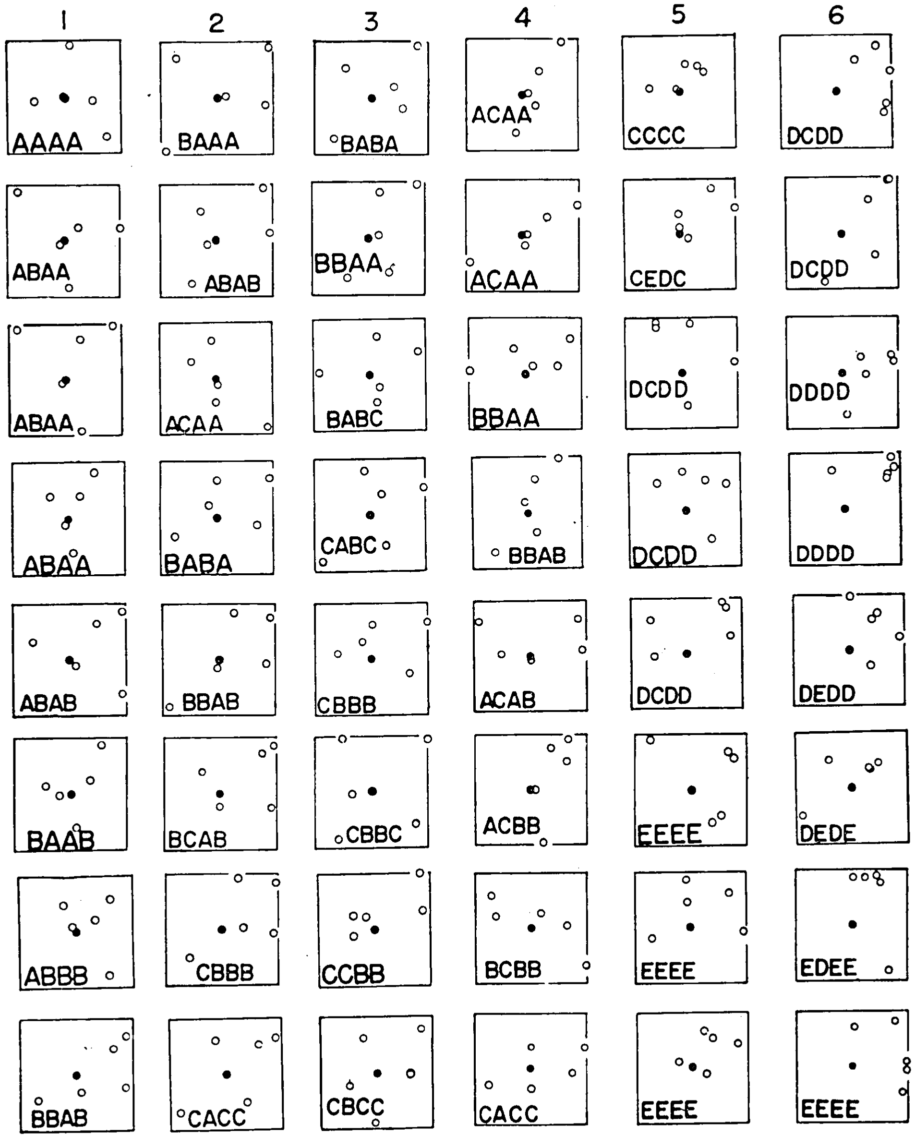

Fig. 1 (a)

The errors of *depth*, *epicenter location*, *origin* letters at the bottom of the squares, where average, bad or very bad distribution respec-

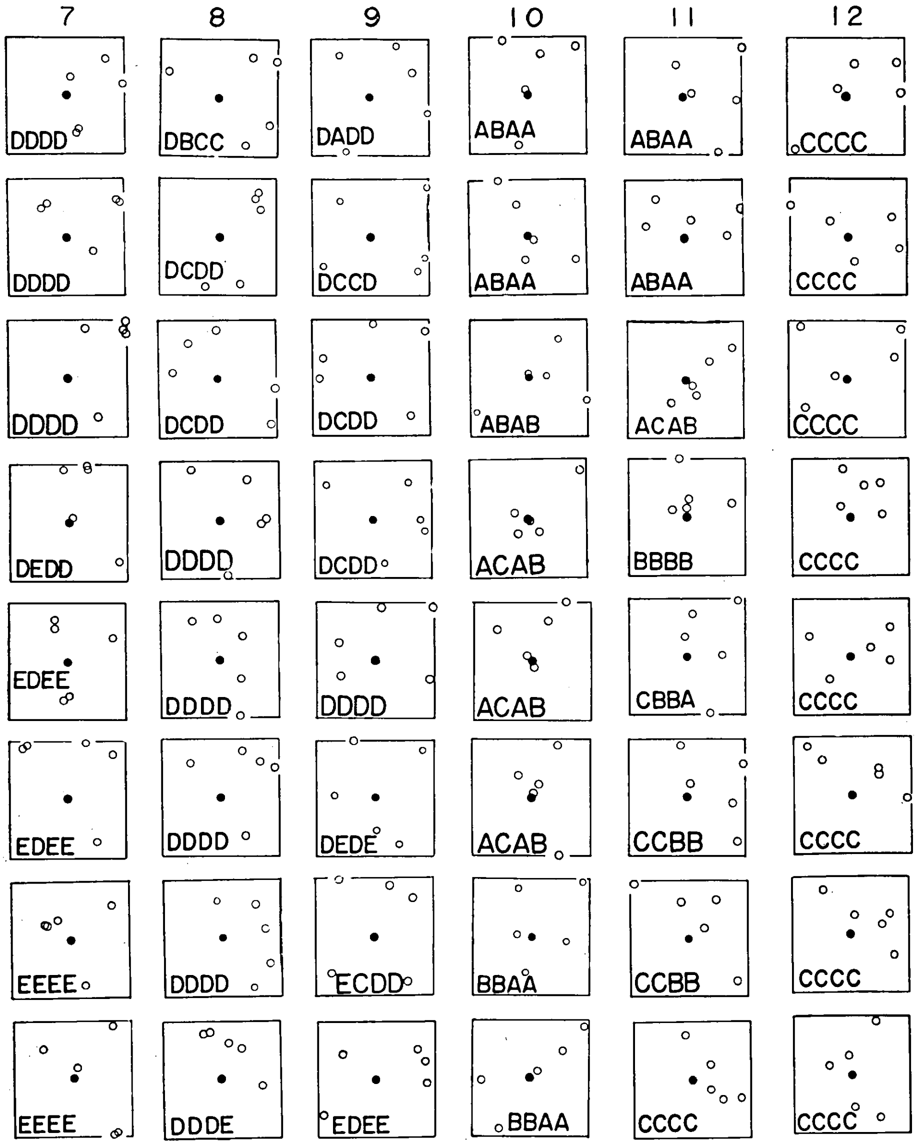

Fig. 1 (b)

time and *velocity* are indicated by the capital
A, B, C, D and E represent very good, good,
tively.

5. Amount of errors

The foregoing discussions hold regardless of the numerical values we assume. If we take the side of the square in which the points are distributed to be the unit of length and the propagation velocity to that of velocity, then the time unit is given as the ratio of the above two quantities.

Using these units the median of standard deviations of depth of the focus, position of the epicenter, origin time and velocity is given in Table 1.

Table 1. Median and minimum values of standard deviations.

Error	Median		Minimum
	(nondimensional)	(dimensional)	
h (depth)	0.40	40 km	3 km
r (epicenter location)	0.031	3 km	0.8 km
t (origin time)	0.28	2.8 sec	0.4 sec
v (propagation velocity)	0.41	4.1 km/sec	0.5 km/sec

The second and third column show dimensional quantities when
unit of length (=side of the square) =100 km,
unit of velocity = 10 km/sec,
unit of time = 10 sec
and
observation error of time ($=\varepsilon$) = 0.1 sec
are assumed.

Table 1 indicates that, except the epicenter location, the calculated values have large errors unless we carefully choose the observation points or use some other method of calculation. Moreover, it should be remarked here, when the epicenter is located outside the polygon of observational points and not near any of its sides, the errors easily become several times larger than favorable cases.

6. Acknowledgements

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30. 地震観測点の最良分布 II

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1. 波の到達時刻から震源の位置，発震時，波の伝播速度を求めることは，地震学において重要な問題の一つであるが，観測点の分布次第によつて，観測誤差の影響が上記の量の決定に，あるいは強く，あるいは弱く表われてくる．

後の場合が観測点の分布として望ましいわけである．

2. 問題を解く方法としてモンテ・カルロ法を採ることとし，次の仮定を設ける．

i) 5 個の観測点を地表面にとる．
ii) 速度は一様とする．
iii) 波の到着時刻の観測誤差を ε, 0, $-\varepsilon$, の 3 種とする．
iv) 観測点は正方形の中一様にランダムに分布する．
v) 震央は正方形の中心にある．

観測誤差 ε, 0, $-\varepsilon$ を 5 個の点に分布させる可能な場合の数は $3^5=243$ であるが，これらすべてに対する，震源の深さ，震央，発震時，伝播速度の誤差の二乗の平均をとり，これのもつとも小さくなる分布を以て，"地震観測点の最良分布" とする．

3. 震源の深さを正方形の一辺の 10 分の 1 として計算した結果，さきに行なつた，震源が深さを持たない場合[1] と似た性質のあることがわかつた．図を参照して説明するなら，

1) 3 点の作る三角形の中に，他の 2 点と震央が入るとき，ことにこの 2 点のうち，一方が三角形の辺の上にほぼ乗り，他の点が震央の近くにあるとき，一般的にみて最もよい結果がえられる．
2) 4 点がひらたい菱形を作り，他の点と震央がその中にあるのもよい分布である．
3) 4 点がひらたい平行四辺形を作るときも上に同じ．
4) 観測点がレンズ形を作り，震央がその中にあるのもよい分布である．
5) ひらたくない四辺形の辺の上に震央が来る分布は一般的によくない．
6) 観測点が震央の一方の側にのみあるのはよくない．
7) 2 点があまり近くにあるのはよくない．
8) 観測点が震央を，半円をなしてとりかこむのはよくない．
9) ひらたくない多角形の中心近くに震央があるのも，良い分布ではない．
10) 震源の深さの決定には，観測点の一が震央に近いことが望ましい．
11) 震央が観測点の作る多角形の外にあつて，しかも比較的結果のよいもの．
12) よくも悪くもない分布．

4. 第 1 表に誤差の中位数と最小値を挙げた．これに見る通り普通の分布と最良の分布では誤差は数倍ないし十倍もの相違があり，また震央が観測点の作る多角形の外にでると，誤差はさらに増加するのが普通である．