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37. On Microtremors. IX.

(Multiple Reflection Problem)

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1. Introduction

It has been ascertained from the previous investigations[1] that microtremor observation is useful as a means not only for anticipating the features of destructive earthquake motions but also for determining the seismic force coefficient to be considered in earthquake-proof construction design.

In the present investigations, firstly, we are going to carry out the spectral analysis as well as the period distribution analysis of microtremor records obtained simultaneously at the surface of the ground and underground. Secondly, mathematical interpretations based on the theory of the multiple reflection of elastic waves in a layer will be made from the above results.

2. Observations of microtremors

Simultaneous observations of microtremors on the surface and under the ground were carried out at two locations on a selected building site near Nihon-bashi, Tokyo. The depths of the bore-holes were 18.35 m and 23.01 m, respectively, and the horizontal distance between them about 5 m.

The soil structure and the result of the standard penetration test for the holes are illustrated in Fig. 1. As seen in Fig. 1, there is abrupt change in N-values at the depth of 22.5 m which corresponds to the boundary between the Upper Tokyo Formation and the Tokyo Gravel Bed.

For observations, two horizontal bore-hole transducers of the so-called

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[1] K. KANAI and T. TANAKA, "On Microtremors. VIII," *Bull. Earthq. Res. Inst.*, **39** (1961), 97-114.

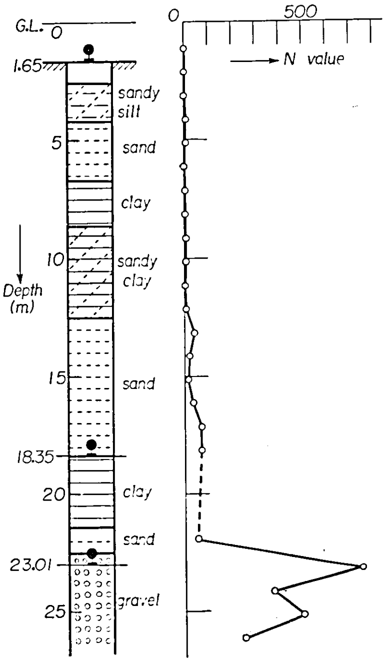

Fig. 1. Subsoil structure and N-values in standard penetration test at a observation site.

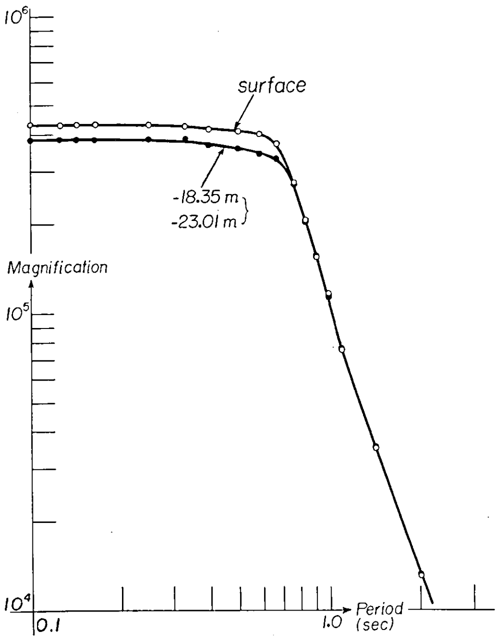

Fig. 2. Overall period response curves of the bore-hole vibrographs.

self-levelling type[2] with a natural period of 0.8 sec were used. One was set near the surface of ground and the other at the bottom of a hole.

The output signals from the transducers were recorded on a magnetic tape by the P.W.M. system after passed through amplifiers with an integrating circuit. For analysing the waves of microtremors, the recorded signals were reproduced on a photographic paper by using an electromagnetic oscillograph. The paper speed and the time marks of the record were chosen to be 10 cm/sec and 1/100 sec, respectively. The overall period response curves of the bore-hole vibrographs used here are shown in Fig. 2. The magnification of the instruments in the present investigation was 25,000 or so at a period of 0.5 sec. The observations were made at midnight and the record was taken for about 5 minutes for each observation.

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[2] K. KANAI and T. TANAKA, "Self-levelling Vibrograph," *Bull. Earthq. Res. Inst.*, **36** (1958), 359-368.

3. Comparison between the period distribution curves and the spectra of microtremors

Fig. 3 shows the period distribution curves of microtremors obtained from analysis of two minutes of the simultaneous records in the cases (I) and (II). The work was done by the period distribution analyser[3].

Figs. 4 and 5 show the spectra of microtremors derived from the Fourier analysis of the same records. For the analysis, a 5 seconds portion was taken from a record and amplitudes at every 1/100 second were read. The calculations of Fourier transform were made with the IBM 7090 computer. On refering to Figs. 3–5 we may find some of the following noticeable facts.

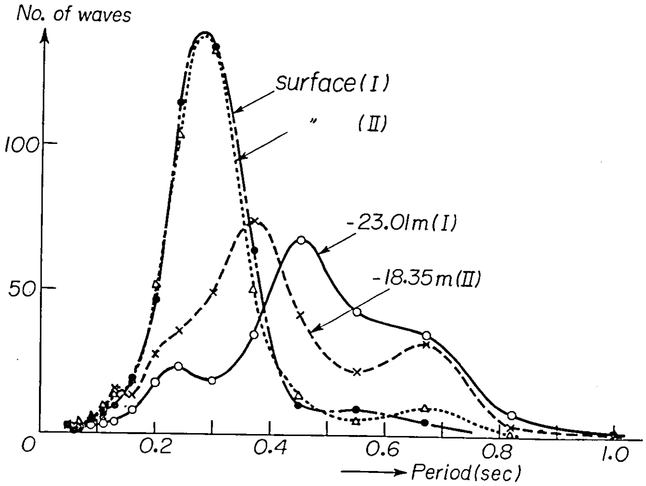

Fig. 3. Period distribution curves of microtremors observed simultaneously on the surface and under the ground in the cases (I) and (II).

i) A peak appeared at 0.255 sec in the period distribution curves for the ground surface.

A remarkable peak is also found at 0.255 sec in the spectrum for the surface. As for the spectrum for the underground, 18.35 m depth, the amplitude of the peak at the period was approximately one quarter of that for the surface, while in the spectrum for 23.01 m depth no significant peak was seen in the neighbourhood of the period. From these facts, it may be said that the vibrations having a period 0.255 sec indicate the vibrational characteristics of the subsoil. Discussions in detail

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[3] T. TANAKA, "Period Distribution Analyser for Irregular Motions," *Bull. Earthq. Res. Inst.*, **40** (1962), 861-871.

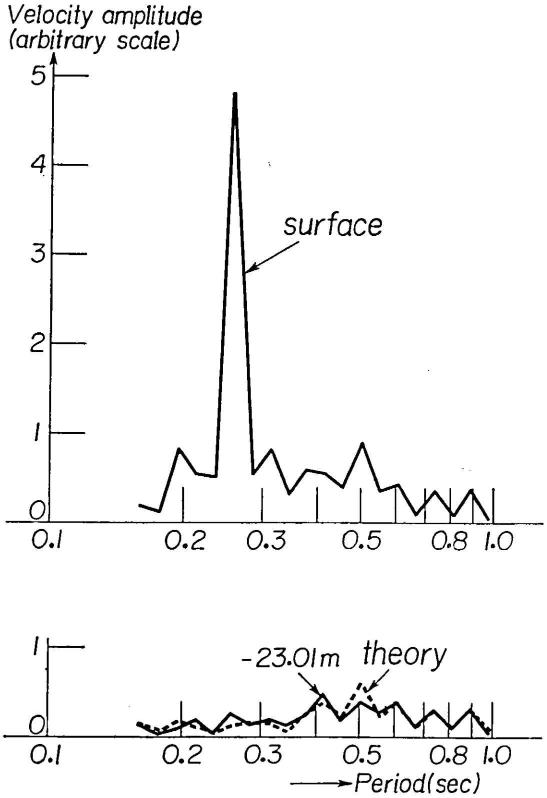

Fig. 4. Spectra of microtremors observed simultaneously at the surface and the underground, 23.01 m depth.

on this problem will be given in the next chapter.

ii) Low peaks appearing at about 0.67 sec in almost all the period distribution curves seem to have close relation to the remarkable peaks at 0.5–0.55 sec in the spectra. The chain line in Fig. 8 shows the estimated distribution curve of the amplitudes for the periods of 0.5–0.55 sec. From these results, it seems somewhat natural to consider that the feature of the waves of 0.5–0.55 sec indicate the vibrational characteristics of a layer between the surface and the Tertiary Bed of about 100 m depth.

iii) A peak appeared at 0.37 sec in the period distribution curve for the underground, 18.35 m depth.

Low peaks exist at a period 0.37 sec in both spectra for the surface and the underground, 18.35 m depth, while there is no peak in the period distribution curve for the surface in the case (II). On the other hand, no peak is seen at the period 0.37 sec in both spectra for the surface and the

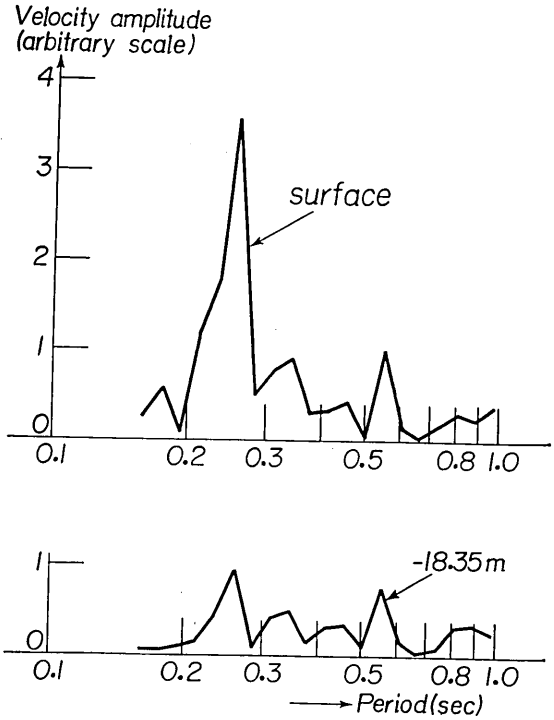

Fig. 5. Spectra of microtremors observed simultaneously at the surface and the underground, 18.35 m depth.

underground, 23.01 m depth. Therefore, it will be natural to consider that the vibrations of a period 0.37 sec have no connection with the vibrational characteristics of the subsoil.

iv) A peak appeared at 0.47 sec in the period distribution curve for the underground, 23.01 m depth.

In all of the spectra of the surface and the underground, the amplitudes at 0.5–0.6 sec do not especially predominate compared with those in other ranges of periods. So that we may consider that the vibrations of this period have also no connection with the vibrational characteristics of the subsoil.

4. Interpretation by the theory of multiple reflections of waves

In this chapter, we will try to interpret the present observational results by the theory of multiple reflections of waves in an elastic layer.

i) Comparison of the observational records of microtremors of the underground, 23.01 m depth, and the wave form at the bottom of ground obtained theoretically.

If the incident waves arriving at the lower boundary of ground, $z=0$, be of the type:

$$u_0 = F(t)\,, \tag{1}$$

the expression for the resulting motions at the surface, $z=H$, and the bottom, $z=0$, as influenced by the multiple reflection of waves in that ground can be written by an infinite series, as follows;

$$u_{z=0}(t)=\gamma F(t)+\left\{\gamma F\left(t-\frac{2H}{V_1}\right)+\gamma\beta F\left(t-\frac{2H}{V_1}\right)\right\}+\cdots\,, \tag{2}$$

$$u_{z=H}(\tau)=2\gamma F\left(t-\frac{H}{V_1}\right)+2\gamma\beta F\left(t-\frac{3H}{V_1}\right)+\cdots\,, \tag{3}$$

in which $\beta=(\alpha-1)/(\alpha+1)$, $\gamma=2/(\alpha+1)$ and $\alpha=\rho_1 V_1/\rho_2 V_2$, and ρ_1, ρ_2, V_1, V_2 are the densities and the velocities of the ground and the bed rock, respectively, and $\tau=t-H/V_1$ and $\tau=0$ and $t=0$ correspond to the arrival time of waves at the surface and the bottom of ground, respectively.

From (2) and (3), a simple relation between the resulting motions at $z=H$ and $z=0$ have been obtained as follows[4]:

$$u_{z=0}\left(t-\frac{H}{V_1}\right)=\frac{1}{2}\left\{u_{z=H}(\tau)+u_{z=H}\left(\tau-\frac{2H}{V_1}\right)\right\}\,. \tag{4}$$

Now, we try to apply this method to the results of the observations of microtremors. For applying the method, the value of $2H/V_1$ in a layer must have been known. In this case it is estimated from the predominant period (T_s) of microtremors obtained at the surface by using the relation $T_s=4H/V_1$.

The final result of the calculation is illustrated in Fig. 6. In Fig. 6, the upper curve represents the actual record of microtremors at the surface and the middle and the lower curves respectively are the actual record at the depth of 23.01 m, and the calculated wave form at the bottom of ground. The dotted line shown in Fig. 4 represents the spectrum obtained from the calculated curve.

It will be seen in Figs. 4 and 6 that there is considerable agreement between the observational and the theoretical results.

ii) Amplification of wave amplitudes in a surface layer.

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[4] K. KANAI and S. YOSHIZAWA, "Some New Problems of Seismic Vibrations of a Structure. Part 1," *Bull. Earthq. Res. Inst.*, **41** (1963), 825–833.

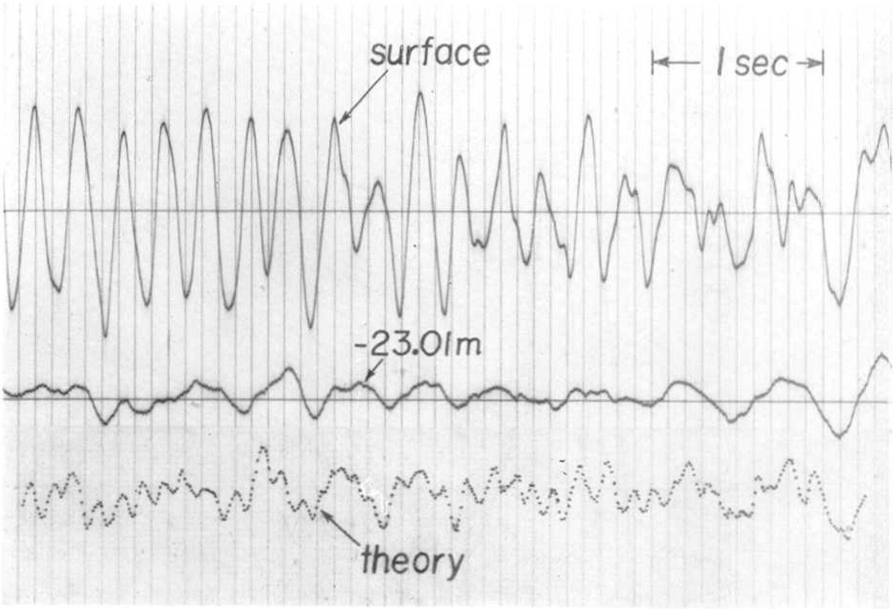

Fig. 6. The actual records of microtremors observed at the surface (above) and the underground, 23.01 m depth, (middle), and the wave form at the bottom obtained theoretically (below).

A full line in Fig. 7 shows the ratios of the amplitudes of microtremors at the surface to those at the 23.01 m depth obtained from Fig. 4.

Next, let us try the multiple reflection problem of sinusoidal waves in a layer. If the incident distortional waves in the lower medium be of the type: $u_0 = A_0 \exp\{i(pt - f_2 z)\}$, the resulting displacements of the subjacent medium and the stratum are expressed by $u_2 = A_0 \exp\{i(pt - f_2 z)\} + A \exp\{i(pt + f_2 z)\}$ and $u_1 = B \exp\{i(pt - f_1 z)\} + C \exp\{i(pt + f_1 z)\}$, respectively, where $p = 2\pi/T$, $f_1 = 2\pi/V_1 T$, $f_2 = 2\pi/V_2 T$ and V_1, V_2 and T are the velocities of the stratum, the subjacent medium and the period, respectively. The boundary conditions at the lower boundary, $z=0$, are $u_2 = u_1$ and $\mu_2(\partial u_2/\partial z) = \mu_1(\partial u_1/\partial z)$, while that at the surface, $z = H$, is $\partial u_1/\partial z = 0$, we get the final solution of the displacement in a layer as follows:

$$u_1 = \frac{2 \cos f_1(H - z)}{\sqrt{\cos^2 f_1 H + \alpha^2 \sin^2 f_1 H}} \cdot \exp\{pt - \tan^{-1}(\alpha \tan f_1 H)\} . \qquad (5)$$

From (5), we obtain the ratio of the absolute values of the displacements at the surface and the lower boundary as follows:

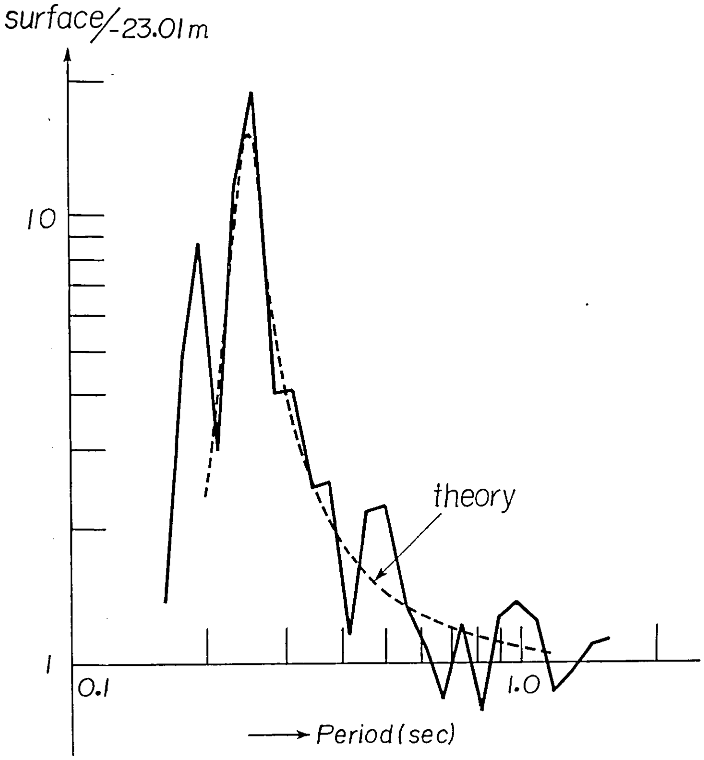

Fig. 7. Amplitude ratios between the spectra obtained at the surface and the underground, 23.01 m depth, and the curve calculated from the theory.

$$\frac{|u_{z=H}|}{|u_{z=0}|} = \frac{1}{\sqrt{\cos^2\left(\frac{2\pi H}{V_1 T}\right) + \alpha^2 \sin^2\left(\frac{2\pi H}{V_1 T}\right)}}, \qquad (6)^{5)}$$

in which α denotes the impedance ratio of the stratum to the subjacent medium.

Substituting the synchronous period of the stratum, 0.255 sec, and the value of the amplitude ratio under the synchronous conditions, 15, obtained from Fig. 4, into equation (6), we get the amplitude ratio for each period as represented by the dotted line in Fig. 7. In Fig. 7 we can see a comparatively good agreement between the observational and

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5) Strictly speaking, the expression of the numerator of (6) must be $\cos(2\pi H/V_1T)$. But, in the present case, as a first approximation, the value of it may be considered as 1, because the observation point is not just the bottom of the layer.

theoretical results.

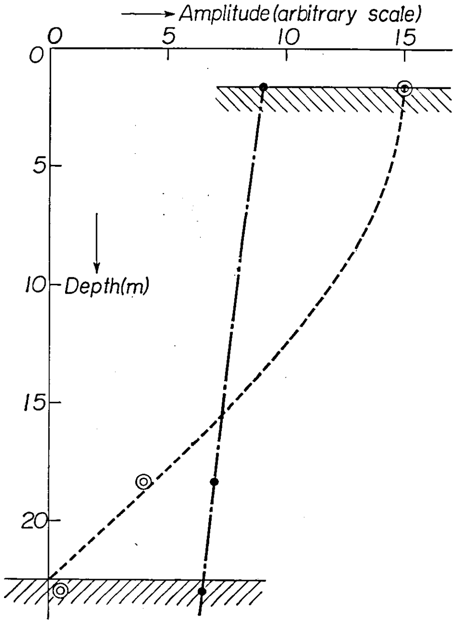

Fig. 8. The distributions of amplitudes of microtremors in a stratum. Double and black circles represent 0.255 and 0.5 sec, respectively, and the dotted line indicates the calculated result from the theory.

iii) Amplitude distributions in the stratum.

Next, we will compare both observational and theoretical results in regard to the amplitude distribution in the stratum. The amplitudes at the periods of 0.255 sec and 0.50–0.55 sec in the spectra of Figs. 4 and 5, in which the two remarkable peaks are existent, are plotted in Fig. 8, respectively by the double and the black circles.

On the other hand, when the period of incident waves synchronizes with the natural period of the stratum, the distribution of amplitudes in that stratum is expressed by the following equation as a special case of (5).

$$\frac{|u_z|}{|u_{z=0}|}=\frac{1}{\alpha}\sin\frac{\pi}{2}\left(\frac{z}{H}\right) \qquad [z=0\sim H] \qquad (7)$$

The result calculated by (7) adopting the values $\alpha=1/15$ and $H=22.5$ m is shown by the broken line in Fig. 8. In Fig. 8, we can also see a considerably good agreement between the observational and theoretical results.

The results of the above three kinds of theoretical investigations tell us that the features of the main parts of microtremors in ground depend mostly on the multiple reflection phenomena of waves transmitted from the bed rock after passing through some considerable depth.[6]

5. Conclusion

From the present investigations, it has been ascertained that the features of the main parts of microtremors in ground depend mostly on the multiple reflection phenomena of waves transmitted from the bed

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[6] See appendix.

rock after passing through some considerable depth. It was also ascertained that the analytical results of the records of microtremors are utilizible enough to know the seismic characteristics of ground which have a great influence upon the spectrum of the strong earthquake motions.

From the previous investigations in which the systematic measurements of microtremors had been carried out at several thousands places in Japan and at several hundreds places in the U.S.A., it was found that the use of values of the predominant, mean and largest periods and the largest amplitude of microtremors is very convenient to make a practical classification of ground even though their physical supports had not yet been satisfactorily made clear⁷⁾. However, if the purpose of the observations of microtremors is to investigate the seismic characteristics of ground precisely at such a site where a very important structure or a structure of considerably long natural period are proposed to be built, the methods of analysis used in the present investigations may be available.

In conclusion, the authors wish to express their sincere gratitude to Prof. Y. Sato of our institute for permission in making the computer program available. Also hearty thanks are due to Prof. H. Tazimi of Nihon University and the members of Takenaka Construction Co., Ltd. for providing facilities and to Mr. T. Morishita for his cooperation in the course of the investigation.

Appendix

Observations of microtremors under the ground were made in a long tunnel located on the southwest flank of Mt. Fuji, at a height of about 1,000 m above sea level.

The amplitude distributions of microtremors with depth are shown in Figs. 9 and 10, for the two kinds of predominant period. For drawing the curves, the shortest distance from the surface to the observation point was taken as a depth.

As seen in Fig. 9, the values of the index n of the attenuation formula, $A=A_0x^{-n}$, become $n=0.49$ and 0.21 for 0.25 sec and 0.50 sec, respectively, and then it is natural to adopt the attenuation formula as $A=A_0\exp(-kx)$.

Consequently, from Fig. 10, we obtained $11\times10^{-5}\,\mathrm{cm}^{-1}$ and $2.2\times10^{-5}\,\mathrm{cm}^{-1}$ as the attenuation coefficient k for 0.25 sec and 0.50 sec, respectively.

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7) *loc. cit.*, 1).

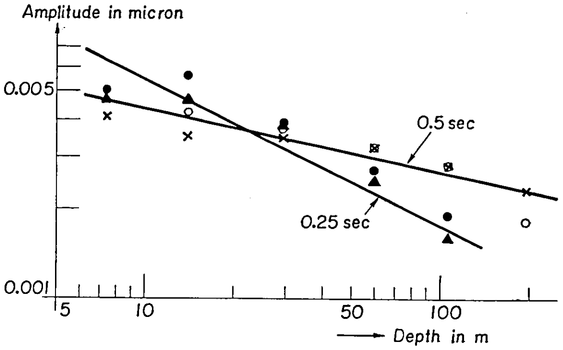

Fig. 9. Attenuation of amplitude of microtremors in a tunnel of Mt. Fuji.

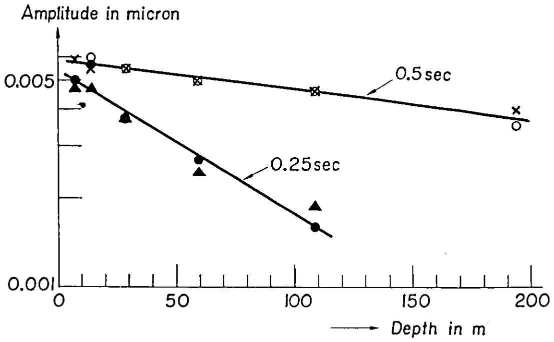

Fig. 10. Attenuation of amplitude of microtremors observed in a tunnel of Mt. Fuji.

The obtained values are nearly inversely proportional to the square of their periods. Namely, it can be said that the values satisfy the relation $k=2\pi^2\xi/\rho V^3T^2$, in which ξ, ρ, V and T represent the viscous coefficient, density, velocity of the medium and the period of waves.

At any rate, it may be said from the result of the present observations that the microtremors transmit into the ground deeply when there are insignificant elastic discontinuities.

37.　常時微動について　第9報

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土層構造が比較的に単純な場所で，常時微動の地上地下の同時観測を行ない，頻度-周期解析のほかにフーリエ解析を行なつた．その結果として，地盤内の常時微動の卓越部分は，基盤から，ほぼ鉛直上方向に伝播してきた波が，地盤内で重複反射の現象を起こす理論で，よく説明できることがわかつた．このことは，一方で，常時微動が，かなり深い所をもぐつてくることになるわけであるが，富士山の中腹の横穴で行なつた常時微動の観測結果は，そのことを裏書きするものであつた．

いずれにしても，常時微動の観測結果が，地盤の振動特性を知る上で，非常に有効な方法であることが，一層はつきりしたものである．

なお，単に，地震工学上の数種の地盤種別を判定するだけならば，これまでやつてきたように，常時微動の卓越，平均，最大周期と最大振幅の値を利用すれば，ほぼその役目が達せられるが，本論文でとりあつかつたような解析検討を行なえば，特に重要な構造物とか，長周期の構造物の耐震設計に使う地震動特性の相当に信頼のおける資料をも，常時微動の観測結果から得ることができそうである．