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22. *Compaction of Sedimentary Rock—A Consideration by the Theory of Porous Media.*

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1. Introduction

In the previous paper[1], effect of pore for the deformation and failure of porous media was examined by introducing the pore compressibility and pore shear compliance in the theory of porous media. As an application of the theory, compaction of sedimentary rock is analyzed in this paper. According to Krumbein and Sloss[2], compaction is defined as a reduction in bulk volume of sediment, caused mainly by vertical forces exerted by an increasing overburden. Further, compaction is conveniently expressed as a change in porosity brought about by the tighter packing of the grains. Therefore it will be supposed that the process of compaction will be equated by that of porosity change in the porous media due to external stress. The external stress will be caused by the own weight of rock strata and tectonic stress if the latter exists.

The density-depth relation of sedimentary rock has been studied by Athy[3], Hedberg[4], A. Matsuzawa[5], and others. It has been empirically known that the logarithm of porosity is proportional to depth of burial. This relationship is derived from the data of oil wells whose depths are about 2000 m. In recent years, several deep wells have been drilled up to 3000 m to 3800 m deep by the Geological Survey of Japan for the stratigraphic investigation at the central part of sedimentary basins. Many geological and geophysical measurements have been made both for drilled core samples and for the well itself. Among these tests, data of density and porosity to the deep horizon enable us to examine the functional form of density-depth relation or porosity-depth relation in a wider range of burial depth.

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1) S. NAGUMO, *Bull. Earthq. Res. Inst.*, **43** (1965), 317.
2) KRUMBEIN and SLOSS, *Stratigraphy and Sedimentation* (Freeman, 1953).
3) L. F. ATHY, *Am. Assoc. Petroleum Geologists Bull.*, **14** (1930), 1.
4) H. D. HEDBERG, *Am. Jour. Sci.*, **31** (1936), 241.
5) A. MATSUZAWA, *Butsuri-Tankô (Geophysical Exploration)*, **15** (1962), 1.

In 2, compaction equation of porous media is derived. The compaction formula is obtained by integrating the equation. Compaction coefficients are evaluated in 3, by comparing the well data with the theoretical formula. In 4, mechanism of compaction is discussed with respect to development of framework, and to coefficient of deformation in geological time scale during the process of compaction.

2. Compaction equation

Let us see the change of porosity due to stress in the theory of porous media. Porosity n is defined as

$$n = V_p / V_b \ , \tag{1}$$

where V_p, V_b denote the pore volume and bulk volume respectively. Since n is functions of V_p and V_b, the total differential of n is given by

$$\frac{dn}{n} = \frac{dV_p}{V_p} - \frac{dV_b}{V_b} \ . \tag{2}$$

By substituting the stress-strain relationship of the form of

$$\begin{cases} dV_p/V_p = c_p d\bar{\sigma} + c_s d\tilde{\sigma} \\ dV_b/V_b = c_b d\bar{\sigma} + c_s d\tilde{\sigma} \end{cases} \tag{3}$$

into equation (2), we have the relation

$$dn/n = (c_p - c_b) d\bar{\sigma} = \kappa d\bar{\sigma} \tag{4}$$

where

$$\kappa \equiv (c_p - c_b) \ .$$

c_s, c_b, c_p are the compressibilities of solid material, framework and pore respectively. $d\bar{\sigma}$, $d\tilde{\sigma}$ are the differential stress of framework and liquid respectively. Positive stress is taken as dilatative. In the discussion of compaction, we treat the compressive stress. Let us denote the compressive stress by p, with the definition of $p = -\sigma$. Then the equation (4) becomes

$$dn/n = -(c_p - c_b) d\bar{p} = -\kappa d\bar{p} \ . \tag{5}$$

This is a form of compaction equation. It is seen that the porosity change depends only upon the framework stress, not upon the liquid stress. Since the total stress, i.e., overburden stress in the case of sedimentary strata, is partitioned into framework stress and liquid stress, equation (5) is expressed by the total stress in the form of

$$dn/n=\kappa\frac{d\bar{\sigma}}{d\sigma}d\sigma = \kappa(1-\beta)d\sigma$$
$$=-\kappa(1-\beta)dp\ , \qquad (6)$$

where

$$(1-\beta)=d\bar{\sigma}/d\sigma\ .$$

β is a coefficient of stress partition into liquid.

The compaction formula will be obtained by the integration of equation (6), i.e.

$$\int_{n_0}^{n} dn/n=-\int_{p_0}^{p}\kappa(1-\beta)dp\ , \qquad (7)$$

where n_0, p_0 indicate the initial values. In order to evaluate the integration of equation (7), we must know the functional form of κ and β with respect to stress. In general, κ and β may not be constants but functions of overburden stress. However, for simplicity, if we assume κ and β as constants, we can integrate the equation (7), and we then have

$$n=n_0e^{-\kappa(1-\beta)(p-p_0)}\ . \qquad (8)$$

This is a form of compaction formula with respect to compressive stress p. In the next section, we will examine the validity of this assumption, κ and β being constants.

On the other hand, one may suppose that κ is proportional to the reciprocal of compressive stress. In the term of bulk modulus, one may suppose that bulk modulus becomes larger as overburden stress increases. Under this assumption, the integral of equation (7) becomes

$$\log(n/n_0)=-\alpha(1-\beta)\log(p/p_0)\ , \qquad (9)$$

where

$$\kappa=\alpha/p\ .$$

In the next section, the validity of the above assumptions will be examined by the well data. The overburden stress p is expressed with respect to depth by the relation

$$p=\rho_m gz\ , \qquad (10)$$

where ρ_m, g, z are mean density of rock strata, gravity constant, and depth respectively. Then compaction formulas of (8) and (9) become

$$n = n_0 e^{-\kappa(1-\beta)\rho_m g(z-z_0)} \tag{11}$$

$$\log (n/n_0) = -\alpha(1-\beta) \log (z/z_0) \; . \tag{12}$$

3. Well data—the determination of compaction coefficient

In order to examine the validity of the assumption used in the preceding section, i.e. whether or not κ is kept constant through compaction, let us see the porosity-depth relation by the well data. We will use the porosity data obtained by the wells, Kambara GS-1[6] and Kasukabe GS-1[7].

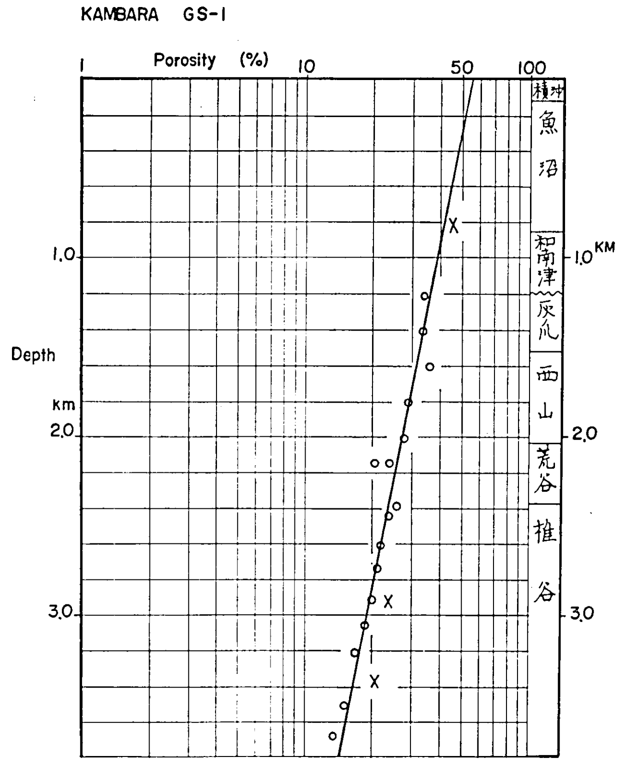

Fig. 1. Porosity-depth relation in semi-logarithmic plot in the well Kambara GS-1, in Niigata Prefecture. ○ : Mudstone × : Sandstone

6) S. NAGUMO and K. INAMI, "*Velocity measurement of drilled core sample, Kambara GS-1*," *Bull. Geological Survey of Japan* (*in press*).

7) S. NAGUMO and K. INAMI, "*Velocity measurement of drilled core sample, Kasukabe GS-1*," *Bull. Geological Survey of Japan* (*in press*).

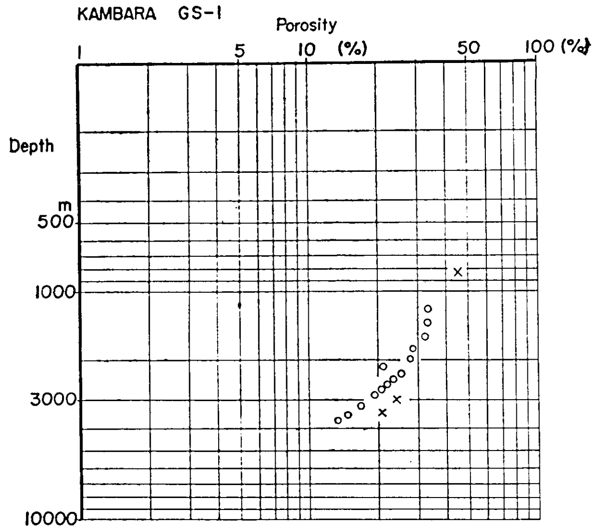

Fig. 2. Porosity-depth relation in log-log plot in the same well of Fig. 1.

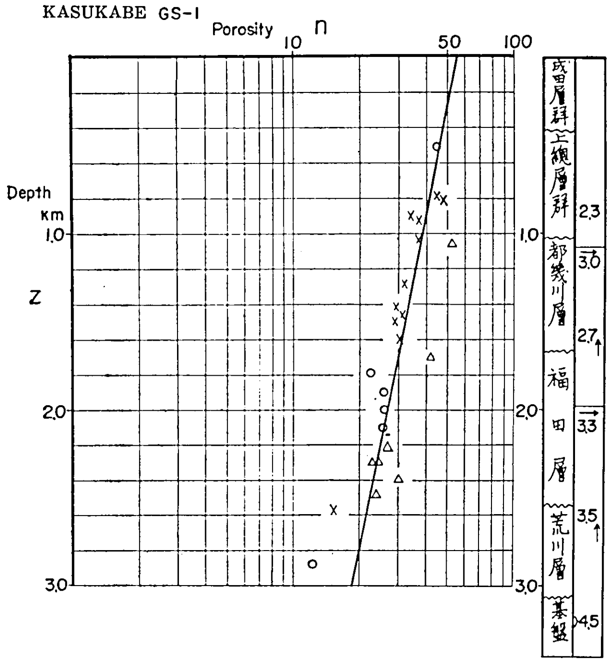

Fig. 3. Porosity-depth relation in semi-logarithmic plot in the well Kasukabe GS-1, in Kanto plain.
○: Mudstone, ×: Sandstone, △: Tuff.

In Fig. 1 and 2, the porosity-depth relations are illustrated in semi-logarithmic plot and log-log plot for the well Kambara GS–1. In Fig. 3, semi-logarithmic plot for the well Kasukabe GS–1. Porosities are measured by the drilled core samples. From these illustrations, it is seen that there is nearly linear relationship between log n and depth z with respect to mudstone, and that there is no linear relation between log n and log z. From these data, it is concluded that (1) compaction formula of the form of equations (11) and (8) suits for mudstone, and that (2) the assumption of κ being constant holds during the process of compaction.

Next, let us determine the value of κ. By fitting a line for the log $n-z$ relation as is shown in Figs. 1 and 3, the values of $\kappa(1-\beta)$ and n_0 are determined by the equation

$$\log_{10} n - \log_{10} n_0 = -\kappa(1-\beta)\rho_m g z \log_{10} e \,. \tag{13}$$

The values are

$$\kappa(1-\beta) = 0.166\times10^{-8} \text{ C.G.S.} \qquad n_0 = 0.567 \text{ (Kambara)}$$
$$\kappa(1-\beta) = 0.168\times10^{-8} \quad '' \qquad n_0 = 0.570 \text{ (Kasukabe)}\,,$$

where $\rho_m = 2.3(gr/\text{cm}^3)$ is used. When the value of $\kappa(1-\beta)$ is expressed in the unit kg/cm², we have

$$\frac{1}{\kappa(1-\beta)} = 603(\text{kg/cm}^2) \qquad \text{(Kambara GS–1)}$$
$$\frac{1}{\kappa(1-\beta)} = 595(\text{kg/cm}^2) \qquad \text{(Kasukabe GS–1)}$$

since $1\text{ kg/cm}^2 = 10^{-6}\text{ dyne/cm}^2$.

It is very interesting that the values of $\kappa(1-\beta)$ and n_0 are about the same for both wells, even though the locations of well are in different sedimentary basins. The well Kambara GS–1 is located nearly in the center of Niigata sedimentary basin, where sedimentation has developed with monotonous and uniform depression of geosyncline. Therefore, the compaction coefficient obtained here will be thought of as a standard compaction coefficient.

β is a coefficient of partition of stress into liquid stress. During the process of compaction, β will be thought of as very small if liquid within pore space would finally flow out of the pore space. Therefore, the value of compaction coefficient κ is about the order of 0.17×10^{-8} C.G.S., and $1/\kappa \doteqdot 600\text{ kg/cm}^2 = 6.0\times10^{8}$ C.G.S. The value of compaction coefficient κ is very large compared with the value of liquid or common rock samples. This characteristic will be enhanced if the stress partition into liquid has a finite value.

4. A consideration of the mechanism of compaction

Ratio of elastic constant of sedimentary rock and compaction coefficient

The values of compressibility of sedimentary rock are presented in Table 1 of the previous paper[8]. In sedimentary rocks whose P-wave velocity is 1.4 km/sec–5.0 km/sec, density 2.1–2.4, and Poisson's ratio 0.25, value of compressibility ranges from about 4.5 to 0.3×10^{-11} C.G.S. Since the value of κ is about 0.17×10^{-8} C.G.S., the compressibility of common sedimentary rocks are smaller than the compaction coefficient by the factor of about $10^{-2}-10^{-3}$. Namely, the ratio is approximately given by

$$\frac{\text{Compressibility of sedimentary rock}}{\text{Compaction coefficient}}=O\,(10^{-2}\sim10^{-3})$$

This will mean that sedimentary rock is more easily compressed in the compaction process of geological time scale than in the case of laboratory experiment. On the other hand, c_b and c_p are those which are defined as elastic constants of porous media. Therefore, let us examine next where such a great difference takes place in the process of compaction. Since $\kappa=c_p-c_b$, let us examine the development of c_p and c_b due to compaction.

Development of framework during compaction

From the relations,

$$\begin{cases} c_b=c_s+nc_p \\ c_p-c_b=\kappa\ , \end{cases} \tag{14}$$

we have the relations

$$\begin{cases} c_p=\dfrac{c_s}{1-n}+\dfrac{\kappa}{1-n}=\dfrac{\kappa}{(1-n)}(1+c_s/\kappa) \\ c_b=\dfrac{c_s}{1-n}+\dfrac{n\kappa}{1-n}=\dfrac{n\kappa}{(1-n)}(1+c_s/n\kappa)\ . \end{cases} \tag{15}$$

Since $c_s=0.185\times10^{-11}$ for $v_p=6.0$ km/sec, $\rho=2.70$, $\nu=0.25$, and $\kappa=$

8) S. NAGUMO, *loc. cit.*, 1).

0.167×10^{-8}, the value of $c_s/\kappa_k=0(10^{-3})$. Therefore, for the ordinary value of porosity, say $n>10^{-2}$, equation (15) is approximated in the form of

$$\begin{cases} c_p=\kappa/(1-n) \\ c_b=n\kappa/(1-n)\ , \quad \text{for } n>10^{-2} \end{cases} \tag{16}$$

or

$$\begin{cases} k_p=(1-n)k_c \\ k_b=(1-n)/n\cdot k_c\ , \end{cases} \tag{17}$$

where $k_c=1/\kappa$, and k_p, k_b are bulk modulus of pore and framework respectively. The values of c_p, c_b and their reciprocals as a function of porosity are illustrated in Fig. 4.

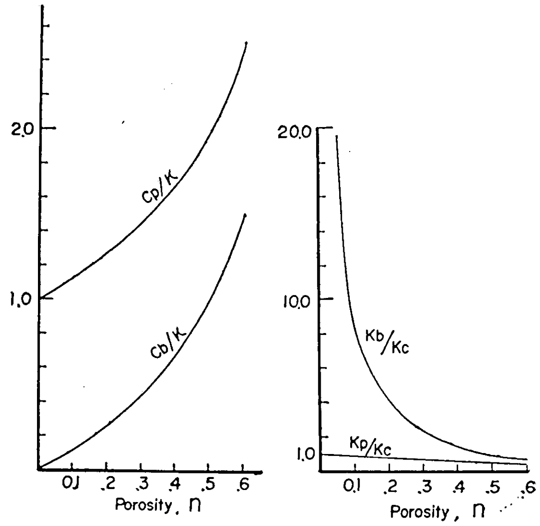

Fig. 4 Development of compressibility and bulk modulus with respect to porosity.
c_p, c_b : compressibility of pore and framework
k_p, k_b : bulk modulus 〃 〃
κ : compaction coefficient, $k_c=1/\kappa$

k_b and k_p increase as porosity decreases. k_p increases linearly with porosity, whereas k_b increases very rapidly in the range of small porosity, being proportional to $1/n$. As is seen in Figs. 4 and 5, the orders of c_p, c_b and κ are the same. It is also seen that k_p and k_b develop during compaction even though the difference $c_p - c_b$ is kept constant. However, the values of c_p and c_b in the process of compaction are still larger than the value of sedimentary rock in the laboratory experiment.

The difference of the values of compressibility between compaction and laboratory deformation will be interpreted as follows. Namely, the difference may imply that the value of deformation coefficient in geological time scale is smaller than the value obtained in ordinary laboratory experiment. The analysis in this paper shows that the order of difference in compressibility will be about $10^{-2}-10^{-3}$.

The value of deformation coefficient during the development of geological structure is one of the basic unknown problems in the mechanics of structural geology and tectonics. One way of estimation has been tried by the experiment of creep phenomena in rocks. The analysis in this paper, however, will give another method of estimation. Since deformation is to be described by at least two fundamental coefficients, compressibility and rigidity, we have to look for another way of estimation for the rigidity of rock strata.

5. Summary and conclusion

Compaction of sedimentary rock is analyzed by the process of porosity change due to external stress in the theory of porous media. Compaction equation is derived for the differential change of porosity. The integration of the compaction equation gives the compaction formula. By comparing the theoretical curve with data of deep well, values of compaction coefficients are obtained. Main results obtained are as follows:

(1) The linear relationship between log n (n : porosity) and depth or stress is derived from the theory of porous media.

(2) The above result agrees with porosity depth relation obtained by deep wells, which were drilled in the center of sedimentary basins to a depth of about 3000 m–3800 m.

(3) The value of compaction coefficient is about 0.17×10^{-8} C.G.S.

(4) The linear relationship between log n and depth is due to such a mechanism that the difference $c_p - c_b$ is kept constant during the process of compaction.

(5) However, both pore compressibility and framework compressibility develop during the process of compaction.

(6) The ratio of compressibility of common sedimentary rock at laboratory test and compaction coefficient is about the order of $10^{-2}-10^{-3}$.

(7) This result may imply that the deformation constant of geological deformation during the geological time scale is smaller than the values of those elastic constants which are obtained by common laboratory experiment. A factor of reduction is about the order of $10^{-2}-10^{-3}$.

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22. 堆積岩の圧密—多孔性媒質理論による一考察

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多孔性媒質における外部応力による孔隙率変化の過程によつて，堆積岩の圧密現象を解析した．まず孔隙率の微小変化に対する圧密方程式を導き，それを積分することによつて圧密公式を求めた．理論曲線を深井戸の資料と比較することによつて圧密係数を求めた．主な結果は次の通りである．

(1) 多孔性媒質理論において孔隙率の対数と深度あるいは応力が直線的関係にあることが導かれる．

(2) 上記の結果は，堆積盆地のほぼ中央部に掘削された深度約 3,000 m～3,800 m の深井戸の資料から得られる孔隙率—深度関係に一致する．

(3) 圧密係数の値は約 0.17×10^{-8} C.G.S. である．

(4) $\log n$ と深度との直線的関係は，孔隙圧縮率 c_p と Framework 圧縮率 c_b との差 c_p-c_b が，圧密過程において一定に保たれるという機巧に基づく．

(5) しかしながら，孔隙の圧縮率，Framework の圧縮率はいずれも圧密過程で発達する．

(6) 実験室で得られる普通の堆積岩の圧縮率と圧密係数との比はほぼ 10^{-2}～10^{-3} の程度である．

(7) この結果は地質的時間尺度において行われる地質構造生成における変形係数は，通常の室内実験で得られる弾性係数よりはるかに小さいことを意味するものと思われる．
