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# 2. A Computer Program for Precise Determination of Focal Mechanism of Local Earthquakes by Revising Focal Depths and Crust-Mantle Structure.

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## Introduction

It has been emphasized since early days of the first motion studies[1,2] that for a precise determination of focal mechanism of an earthquake by the use of data from a local station network, the depth of focus and the crust-mantle structure in the area must be accurately known. This paper describes a computer program for revising focal depth on the basis of the best available crust-mantle model, and determining the emergence angle of the ray at the focus for waves which show up as the first arrival.

Several papers have been written on the computer program for determining coordinates of earthquake focus and origin time. In Flinn's[3] program, it is assumed that seismic waves travel along a staight line with a constant velocity. Nordquist[4] took into account layered crustal structure and used arrival times of refracted waves as well as direct waves. In Cisternas' program[5] the arrival times are corrected for variations in crustal structure by the use of residual term assigned to each station. There is another group of programs in which standard travel time-tables are used instead of the structure of wave medium. Bolt[6] uses the Jeffreys-Bullen table and the Japan Meteorological Agency[7] uses the Wadati-Sagisaka-Masuda's table.

The program described here determines coordinates of earthquake focus and origin time in a wave medium which consists of spherical shells,

1) K. WADATI, *Geophys. Mag.*, **1** (1927), 89-96.
2) H. KAWASUMI, *Bull. Earthq. Res. Inst.*, **12** (1934), 660-705.
3) E. A. FLINN, *Bull. Seism. Soc. Amer.*, **50** (1960), 467-470.
4) J. M. NORDQUIST, *Bull. Seism. Soc. Amer.*, **52** (1962), 431-437.
5) A. CISTERNAS, *Bull. Seism. Soc., Amer.*, **53** (1963), 1075-1083.
6) B. A. BOLT, *Geophys. J.*, **3** (1960), 433-440.
7) Technical Report of the Japan Meteological Agency, No. 22 (1963).

in each of which the velocity of wave varies with the distance from the earth's centre according to the law $v=v_i(r/r_i)^{\zeta_i}$.

We used and extended the result of Bullen's[8] detailed study of seismic rays in the above medium. In locating an earthquake, the travel time equation is linearized and the method of least squares is applied as in the other programs introduced above. Our program is written for the *IBM* 7090.

## Input data

1. *Structure parameter* In the present form of our program, the wave medium consists of two spherical shells, one of which represents the crust and the other upper mantle. In each shell, the velocity is expressed as $v=v_i(r/r_i)^{\zeta_i}$, where $r_i$ is the radius of the outer surface of the shell and $v_i$ is the velocity at $r_i$. Except for the radius of the earth's surface, there are five structure parameters which may by specified arbitrarily. It is, therefore, in principle possible to choose the best parameters which minimize the arrival time residuals over many stations and earthquakes. In most applications to local earthquakes in Japan, however, we used the parameters for the crust which are determined to agree with the observed arrival times of $P$ waves from the artificial explosions, and those for the upper mantle which fit the Jeffreys-Bullen travel times.

2. *Earthquake data* The date of earthquake, number of stations, and the first approximations to coordinates and origin time are punched on a single card.

3. *Station data* The latitude, longitude and height of a station are punched on a single card with the arrival times of $P$ and $S$ waves and the sense of first motion.

## Computation

Since our program is designed primarily for the study of local earthquakes in Japan, the radius $r_0$ of our spherical medium is taken as that of the curvature of the spheroid earth in the meridian plane at 36°N. In the same way as done by Nordquist,[9] the geographic coordinates of stations and sources are transformed to those on the above sphere which is in contact with the spheroid at (36°N, 138°E). Then, the epicentral distances are computed for the source of the first approximation.

8) K. E. BULLEN, *Geophys. J.*, 4 (1961), 93-105.

9) J. M. NORDQUIST, *loc. cit.*, 4).

In order to formulate the linearized normal equation[10] of the least-squares determination of corrections to the first approximation values of origin time and coordinates, we must compute the values of $t$, $\partial t/\partial \Delta$ and $\partial t/\partial h$ for each station for the source of the first approximation, where $t$ is travel time, $\Delta$ epicentral distance, and $h$ focal depth. Different formulas must be used to compute these values when the focus is located in different shells.

As mentioned before, in the present from of our program, the medium consists of two shells; one for the crust, the other for the upper mantle. Following Bullen's notation,[11] we define the following variables by

$$\eta = r/v \ ,$$
$$\eta_0 = r_0/v_0 \ ,$$
$$\eta_1 = r_1/v_1 \ ,$$
$$\mu = \eta/\eta_0 \ ,$$
$$p = \eta \cos e \ ,$$
$$\lambda = p/\eta_0 \ ,$$
$$T = t/\eta_0 \ ,$$
$$\alpha_0 = \frac{2}{1-\zeta_0} \ ,$$
$$\alpha_1 = \frac{2}{1-\zeta_1} \ ,$$

where $v$ denotes the seismic wave velocity, $r$ the distance from the centre of the spherical Earth model, $e$ the emergence (angle between a ray and a level surface of radius $r$), and $t$ the travel time. The velocity varies as $v = v_0(r/r_0)^{\zeta_0}$ in the crust and as $v = v_1(r/r_1)^{\zeta_1}$ in the upper mantle.

When the focus is in the crust, we compute the travel time and its derivatives for direct and refracted waves, and choose the values corresponding to the wave with shorter travel time.

The formulas for the direct wave are written in terms of the emergence angle $e_h$ at the focus.

$$\Delta = \frac{\alpha_0}{2}(\cos^{-1}\lambda - e_h) \ ,$$

$$T = \frac{\alpha_0}{2}\{(1-\lambda^2)^{1/2} - \mu_h \sin e_h\} \ ,$$

10) B. A. BOLT, *loc. cit.*, 6).
11) K. E. BULLEN, *loc. cit.*, 8).

$$\frac{d\varDelta}{de_h}=\frac{\alpha_0}{2}\left\{\frac{\mu_h \sin e_h}{(1-\lambda^2)^{1/2}}-1\right\},$$

$$\left(\frac{\partial T}{\partial \varDelta}\right)_h=\lambda ,$$

$$\left(\frac{\partial T}{\partial h}\right)_{\varDelta}=-\frac{\alpha_0}{2}\frac{d\mu_h}{dh}\sin e_h$$

where $\mu_h$ denotes the value of $\mu$ at the focus. In order to determine the travel time and its derivatives for a given $\varDelta$, we used an iteration method, in which a trial value of $e_h$ is corrected by $\{\varDelta-\varDelta(e_h)\}de_h/d\varDelta$. The iteration is repeated until $\varDelta(e_h)$ agrees with $\varDelta$ within prescribed limits.

The formulas for the refracted waves are the following.

$$\varDelta=\frac{\alpha_0}{2}\left\{\cos^{-1}\lambda+\cos^{-1}\frac{\lambda}{\mu_h}-2\cos^{-1}\frac{\lambda}{\mu_{01}}\right\}+\alpha_1\cos^{-1}\frac{\lambda}{\mu_1},$$

$$T=\frac{\alpha_0}{2}\{(1-\lambda^2)^{1/2}+(\mu_h^2-\lambda^2)^{1/2}-2(\mu_{01}^2-\lambda^2)^{1/2}\}+\alpha_1(\mu_1^2-\lambda^2)^{1/2},$$

$$\frac{d\varDelta}{d\lambda}=\frac{\alpha_0}{2}\{-(1-\lambda^2)^{-1/2}-(\mu_h^2-\lambda^2)^{-1/2}+2(\mu_{01}^2-\lambda^2)^{-1/2}\}-\alpha_1(\mu_1^2-\lambda^2)^{-1/2},$$

$$\left(\frac{\partial T}{\partial \varDelta}\right)_h=\lambda ,$$

$$\left(\frac{\partial T}{\partial h}\right)_{\varDelta}=\frac{\alpha_0}{2\mu_h}(\mu_h^2-\lambda^2)^{1/2}\frac{d\mu_h}{dh},$$

where $\mu_{01}$ denotes the value of $\mu$ at the base of the crust, $\mu_1$ that at the top of the mantle. The iteration in this case proceeds with $\lambda$ as variable instead of the emergence angle. Care must be taken in the process so that $\lambda$ may not exceed $\mu_1$.

When the focus is located in the mantle the following formulas are applicable.

$$\varDelta=\frac{\alpha_0}{2}\{\cos^{-1}\lambda-\cos^{-1}(\lambda/\mu_{01})\}+\frac{\alpha_1}{2}\cos^{-1}(\lambda/\mu_1)-\frac{\alpha_1}{2}e_h ,$$

$$T=\frac{\alpha_0}{2}\{(1-\lambda^2)^{1/2}-(\mu_{01}^2-\lambda^2)^{1/2}\}+\frac{\alpha_1}{2}(\mu_1^2-\lambda^2)^{1/2}-\frac{\alpha_1}{2}\mu_h\sin e_h ,$$

$$\frac{d\varDelta}{de_h}=\frac{\alpha_0}{2}\left\{\frac{\mu_h\sin e_h}{(1-\lambda^2)^{1/2}}-\frac{\mu_h\sin e_h}{(\mu_{01}^2-\lambda^2)^{1/2}}\right\}+\frac{\alpha_1}{2}\frac{\mu_h\sin e_h}{(\mu_1^2-\lambda^2)^{1/2}}-\frac{\alpha_1}{2},$$

$$\left(\frac{\partial T}{\partial \Delta}\right)_h=\lambda ,$$

$$\left(\frac{\partial T}{\partial h}\right)_\Delta=-\frac{\alpha_1}{2}\frac{d\mu_h}{dh}\sin e_h .$$

| 1963 JAN 11 14H02M STATION NUMBER 13 | LATITUDE | LONGITUDE | DEPTH | ORIGIN TIME |
|---|---|---|---|---|
| FIRST APPROXIMATION | 35.900 | 140.383 | 60.0 | 3.6 |
| TRANSFORMED COORDINATE | 35.900 | 140.394 | | |

| STRUCTURE PARAMETER | V0 | R0 | Z0 | V1 | R1 | Z1 |
|---|---|---|---|---|---|---|
| | 5.78 | 6356.8 | -24.4 | 7.75 | 6324.8 | -2.3 |
| | 3.34 | 6356.8 | -24.4 | 4.35 | 6324.8 | -2.3 |

| STATION | | DELTA(DEG) | AZIMUTH(DEG) | ANGLE(DEG) | WULFF GRID | WEIGHT | TIME(SEC) | L.SQ.RESIDUAL | O-C.RESIDUAL |
|---|---|---|---|---|---|---|---|---|---|
| KAKIOKA | U | 0.366 | -24.9 | 50.8 | 0.356 | 1.0 | 10.606 | -0.088 | 0.994 |
| CHOSHI | D | 0.414 | 114.9 | 46.9 | 0.395 | 1.0 | 11.059 | -0.187 | 0.941 |
| MITO | U | 0.484 | 8.5 | 42.0 | 0.445 | 1.0 | 11.723 | 0.654 | 1.177 |
| TOKYO | U | 0.550 | 247.3 | 37.7 | 0.491 | 1.0 | 12.433 | 0.293 | 2.167 |
| UTSUNOMIYA | D | 0.768 | -32.5 | 26.8 | 0.616 | 1.0 | 15.061 | -0.523 | 0.039 |
| YOKOHAMA | U | 0.754 | 232.3 | 27.3 | 0.609 | 1.0 | 14.866 | 0.420 | 2.234 |
| KUMAGAYA | D | 0.849 | -72.8 | 23.7 | 0.653 | 1.0 | 16.098 | 0.145 | 1.302 |
| MAEBASHI | D | 1.181 | -64.5 | 15.3 | 0.763 | 1.0 | 20.476 | -0.690 | 0.124 |
| AJIRO | U | 1.357 | 231.2 | 12.4 | 0.805 | 1.0 | 22.898 | -1.621 | -0.098 |
| KOFU | U | 1.506 | 261.6 | 10.5 | 0.831 | 1.0 | 24.922 | 0.054 | 1.278 |
| SHIZUOKA | U | 1.864 | 240.8 | 7.1 | 0.884 | 1.0 | 29.876 | -0.220 | 1.124 |
| MATSUSHIRO | D | 1.874 | -69.4 | 7.0 | 0.884 | 1.0 | 30.058 | 0.254 | 0.942 |
| HAMAMATSU | U | 2.485 | 242.1 | 3.3 | 0.944 | 1.0 | 38.554 | 1.509 | 2.746 |

| SOLUTION FROM THE ABOVE TRIAL | LATITUDE | LONGITUDE | DEPTH(KM) | ORIGIN TIME | L.SQ.RES.VAR. | O-C.SQ.SUM | D.FR.NO. |
|---|---|---|---|---|---|---|---|
| | 35.960 | 140.454 | 72.41 | 3.752 | 0.725 | 25.99 | 9.0 |
| PROBABLE ERROR | 0.025 | 0.049 | 8.69 | 0.728 | | | |

LONGITUDE TRANSFORMED BACK TO 140.443

| STATION | | DELTA(DEG) | AZIMUTH(DEG) | ANGLE(DEG) | WULFF GRID | WEIGHT | TIME(SEC) | L.SQ.RESIDUAL | O-C.RESIDUAL |
|---|---|---|---|---|---|---|---|---|---|
| KAKIOKA | U | 0.340 | -36.6 | 59.2 | 0.275 | 1.0 | 11.681 | -0.056 | -0.233 |
| CHOSHI | D | 0.402 | 125.6 | 54.5 | 0.320 | 1.0 | 12.162 | -0.200 | -0.315 |
| MITO | U | 0.420 | 3.2 | 53.2 | 0.332 | 1.0 | 12.308 | 0.620 | 0.439 |
| TOKYO | U | 0.619 | 244.0 | 41.1 | 0.454 | 1.0 | 14.182 | 0.358 | 0.266 |
| UTSUNOMIYA | D | 0.748 | -38.1 | 35.0 | 0.521 | 1.0 | 15.613 | -0.516 | -0.665 |
| YOKOHAMA | U | 0.829 | 231.1 | 31.7 | 0.558 | 1.0 | 16.552 | 0.453 | 0.396 |
| KUMAGAYA | D | 0.881 | -77.4 | 29.7 | 0.580 | 1.0 | 17.176 | 0.177 | 0.072 |
| MAEBASHI | D | 1.202 | -68.0 | 20.8 | 0.689 | 1.0 | 21.230 | -0.687 | -0.782 |
| AJIRO | U | 1.432 | 230.6 | 16.5 | 0.747 | 1.0 | 24.279 | -1.617 | -1.632 |
| KOFU | U | 1.563 | 259.7 | 14.6 | 0.773 | 1.0 | 26.057 | 0.032 | -0.009 |
| SHIZUOKA | U | 1.936 | 240.0 | 10.3 | 0.834 | 1.0 | 31.129 | -0.275 | -0.282 |
| MATSUSHIRO | D | 1.900 | -71.6 | 10.7 | 0.828 | 1.0 | 30.679 | 0.234 | 0.169 |
| HAMAMATSU | U | 2.556 | 241.5 | 5.8 | 0.903 | 1.0 | 39.667 | 1.477 | 1.481 |

| SOLUTION FROM THE ABOVE TRIAL | LATITUDE | LONGITUDE | DEPTH(KM) | ORIGIN TIME | L.SQ.RES.VAR. | O-C.SQ.SUM | D.FR.NO. |
|---|---|---|---|---|---|---|---|
| | 35.965 | 140.453 | 71.17 | 3.740 | 0.718 | 6.60 | 9.0 |
| PROBABLE ERROR | 0.029 | 0.060 | 9.20 | 0.923 | | | |

LONGITUDE TRANSFORMED BACK TO 140.443

| STATION | | DELTA(DEG) | AZIMUTH(DEG) | ANGLE(DEG) | WULFF GRID | WEIGHT | TIME(SEC) | L.SQ.RESIDUAL | O-C.RESIDUAL |
|---|---|---|---|---|---|---|---|---|---|
| KAKIOKA | U | 0.335 | -37.1 | 59.1 | 0.277 | 1.0 | 11.515 | -0.061 | -0.055 |
| CHOSHI | D | 0.405 | 126.2 | 53.7 | 0.328 | 1.0 | 12.062 | -0.199 | -0.202 |
| MITO | U | 0.414 | 3.2 | 53.0 | 0.334 | 1.0 | 12.140 | 0.621 | 0.620 |
| TOKYO | U | 0.621 | 243.6 | 40.4 | 0.462 | 1.0 | 14.102 | 0.352 | 0.358 |
| UTSUNOMIYA | D | 0.744 | -38.3 | 34.5 | 0.526 | 1.0 | 15.477 | -0.515 | -0.517 |
| YOKOHAMA | U | 0.832 | 230.8 | 30.9 | 0.567 | 1.0 | 16.508 | 0.449 | 0.451 |
| KUMAGAYA | D | 0.879 | -77.8 | 29.1 | 0.587 | 1.0 | 17.084 | 0.174 | 0.175 |
| MAEBASHI | D | 1.199 | -68.2 | 20.3 | 0.696 | 1.0 | 21.147 | -0.684 | -0.687 |
| AJIRO | U | 1.436 | 230.5 | 16.0 | 0.754 | 1.0 | 24.279 | -1.615 | -1.619 |
| KOFU | U | 1.564 | 259.5 | 14.1 | 0.780 | 1.0 | 26.030 | 0.034 | 0.030 |
| SHIZUOKA | U | 1.938 | 239.8 | 9.9 | 0.840 | 1.0 | 31.142 | -0.276 | -0.283 |
| MATSUSHIRO | D | 1.899 | -71.8 | 10.4 | 0.834 | 1.0 | 30.632 | 0.235 | 0.228 |
| HAMAMATSU | U | 2.558 | 241.4 | 5.5 | 0.908 | 1.0 | 39.684 | 1.484 | 1.476 |

| SOLUTION FROM THE ABOVE TRIAL | LATITUDE | LONGITUDE | DEPTH(KM) | ORIGIN TIME | L.SQ.RES.VAR. | O-C.SQ.SUM | D.FR.NO. |
|---|---|---|---|---|---|---|---|
| | 35.965 | 140.455 | 71.41 | 3.717 | 0.718 | 6.46 | 9.0 |
| PROBABLE ERROR | 0.029 | 0.059 | 9.17 | 0.912 | | | |

LONGITUDE TRANSFORMED BACK TO 140.444

| STATION | | DELTA(DEG) | AZIMUTH(DEG) | ANGLE(DEG) | WULFF GRID | P TIME(SEC) | RESIDUAL | WT | S TIME | RESIDUAL | WT |
|---|---|---|---|---|---|---|---|---|---|---|---|
| KAKIOKA | U | 0.336 | -37.3 | 59.1 | 0.276 | 11.54 | -0.06 | 1.0 | 20.27 | -1.19 | 1.0 |
| CHOSHI | D | 0.405 | 126.3 | 53.9 | 0.326 | 12.08 | -0.20 | 1.0 | 21.22 | 0.86 | 1.0 |
| MITO | U | 0.414 | 3.1 | 53.2 | 0.333 | 12.16 | 0.62 | 1.0 | 21.36 | 2.92 | 1.0 |
| TOKYO | U | 0.622 | 243.6 | 40.5 | 0.461 | 14.13 | 0.35 | 1.0 | 24.84 | 0.24 | 1.0 |
| UTSUNOMIYA | D | 0.744 | -38.4 | 34.6 | 0.525 | 15.50 | -0.51 | 1.0 | 27.26 | -1.08 | 1.0 |
| YOKOHAMA | U | 0.833 | 230.8 | 31.0 | 0.566 | 16.53 | 0.45 | 1.0 | 29.09 | -1.21 | 1.0 |
| KUMAGAYA | D | 0.880 | -77.8 | 29.2 | 0.586 | 17.11 | 0.17 | 1.0 | 30.11 | 0.17 | 1.0 |
| MAEBASHI | D | 1.200 | -68.3 | 20.4 | 0.695 | 21.17 | -0.68 | 1.0 | 37.40 | -41.12 | 0. |
| AJIRO | U | 1.436 | 230.5 | 16.0 | 0.753 | 24.30 | -1.61 | 1.0 | 42.88 | -3.10 | 1.0 |
| KOFU | U | 1.565 | 259.5 | 14.2 | 0.779 | 26.05 | 0.03 | 1.0 | 45.99 | -1.21 | 1.0 |
| SHIZUOKA | U | 1.939 | 239.8 | 10.0 | 0.839 | 31.16 | -0.28 | 1.0 | 55.07 | -58.79 | 0. |
| MATSUSHIRO | D | 1.899 | -71.8 | 10.4 | 0.833 | 30.65 | 0.24 | 1.0 | 54.16 | -1.47 | 1.0 |
| HAMAMATSU | U | 2.559 | 241.4 | 5.6 | 0.907 | 39.70 | 1.48 | 1.0 | 70.30 | -74.01 | 0. |

Fig. 1 Example of compter output

## Output

Fig. 1 shows an example of the printed output of our computation.

First, the date of earthquake and the number of stations used in computation are shown with the first approximations to the epicentre, focal depth and origin time. The epicentre coordinates transformed to the spherical earth model are also shown. The structure parameters are then given for $P$ and $S$ waves. The parameters for $S$ wave are used only in computation of $S$ time residuals after the hypocentre and origin time are determined from $P$ wave data.

Next, some useful informations relevant to each station produced in the process of the first correction are given. The first column shows the name of station, the second shows whether the first motion at the station is *down* or *up*. The epicentral distance, azimuth angle (clockwise from north), emergence angle at the focus, $P$ travel time and residual $(O-C)$ are shown in the Columns 3, 4, 5, 8 and 10 respectively for the origin of the first approximation. In Column 6, the radial length on the stereographic projection corresponding to given emergence angle is computed as $\tan\{(\pi/2-|e_h|)/2\}$. This value is conveniently used in the fault plane study of earthquake with the aid of the Wulff's grid[12] as will be illustrated later. The weight shown in Column 7 takes the unity when the first motion is sharp ($iP$) and zero when it is blunt ($eP$). In the first correction, all stations with $iP$ are used, but in the second correction stations are neglected if the least squares-residual of the station exceeds 3 seconds. This residual is the error of the least-squares determination and is shown in Column 9. The variance of this residual is used to estimate the probable errors of the solutions, which are shown below the corresponding solution. The solutions are given in degree for latitude and longitude, in km for focal depth and in sec for origin time. The longitude is transformed back to the spheroid earth; the transformation produces negligible effect on latitude.

The least-squares determination is repeated 3 times, each time taking the solution of the preceding determination as the trial solution. Last station listing is based on the final solution. In this list, computed travel time and residual $(O-C)$ are shown for $S$ waves as well as for $P$ waves. The $S$ times are computed for the model structure given in the 6th line of the output sheet. The weight for $S$ waves (indicated as $WT$) takes unity when the onset is sharp ($iS$) and zero when it is blunt ($eS$). The

12) E. N. BESSONOVA et. al., *Investigation of the Mechanism of Earthquakes* (1960), p. 151.

value of $S$ residual for $WT=0$ has no physical meaning and must be disregarded. $P$ residual of the station for which the weight of $P$ time is zero must also be neglected.

## Application

Our program has been primarily used for the investigation of local earthquakes in Japan from the data supplied by the Japan Meteorological

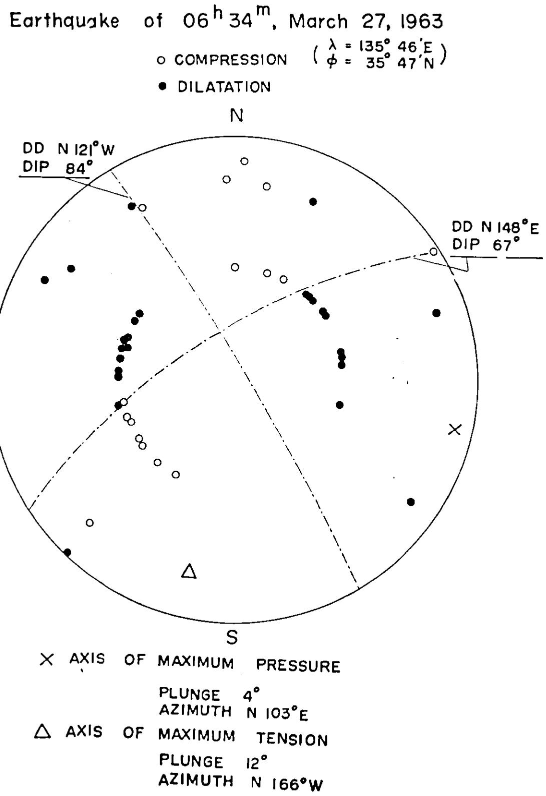


Fig. 2. Example of stereographic projection of upper hemi-focal-sphere

Agency. This investigation includes examination of the accuracy of hypocentre location routinely done by the above agency on the basis of a now obsolete standard travel time table. The main object is, however, to increase accuracy in the determination of focal mechanism of local earthquakes in Japan.

Fig. 2 illustrates the radiation pattern of $P$ waves on the stereographic projection of the upper hemi-focal-shere obtained for the Echizen Oki earthquake of March 27, 1963 ($M$=6.9, revised depth 5.5 km). In this diagram, the polar angle indicates the azimuth to station from epicentre and the radial length is equal to tan $\{(\pi/2-|e_h|)/2\}$, where $e_h$ is the emergence angle of ray at the source. The station for which emergence angle is negative (ray leaves from the focus toward the centre of the earth) is plotted in the opposite azimuth (180° from the azimuth to the station), assuming that the radiation pattern is symmetric with respect to the focal point. Two orthogonal planes are shown which best separates *compressions* from *dilatations*. The axis of the maximum pressure is also indicated.

Another important problem we are investigating is the statistical property of $P$ and $S$ travel time residuals and its bearings on the nature of wave medium.

Separate papers will be published on each of these problems.

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## 2. 発震機構・地下構造の研究に便利な近地地震震源計算プログラム

地震研究所　安　芸　敬　一

初動を用いて近地地震の発震機構を正確にきめるためには，初動が震源を出たときの射出角を正確にきめることができなければならない．このためには，もちろん地殻構造と震源の深さを正しく知る必要がある．特に震源が地殻内部にあるときは，初動が屈折波であるか直接波であるかという判定もからんでくるので，この必要は大きくなる．この論文では，まずある程度任意に与えることのできる媒質内で，震源の深さを決め直し，次に決められた震源から各観測点への初動の射出角を計算するプログラムについて述べる．媒質としては，各殻内で弾性波速度が $v=v_i(r/r_i)^{\zeta_i}$ の形で地心からの距離 $r$ の函数であらわされるような球殻からなるものも考えた．$v_i$, $r_i$, $\zeta_i$ は，人工地震のデータなどから最も適当なものを選ぶ．

震源の深さの決定は，まずその位置と発震時についての第一近似を与え，各観測点での初動時刻のデータをつかって，最小自乗法で第二近似を出すという方法を用いた．普通第四近似まで求めると解は収斂する．

射出角は，Wulff のグリッドが便利に使えるように，ステレオ投影の長さに換算してある．昭和38年3月の越前沖地震について，初動分布の実例を示した．

この他，このプログラムでは，$P$ 波できめた震源に基いて $S$ 波の走時を計算し，理論モデルに対するものからの残差が求められるようになつている．これは $S$ 波速度に関する地殻構造の研究に用いられる．