BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 43 (1965), pp. 161-166

11. *Electromagnetic Induction in a Semi-infinite Conductor having an Undulatory Surface.*

By Tsuneji RIKITAKE,

Earthquake Research Institute.

(Read Oct. 27, 1964.—Received Oct. 29, 1964.)

Summary

A theory of electromagnetic induction in a semi-infinite conductor having an undulatory surface is advanced. Perfect conductivity is assumed in the theory. It transpires that the horizontal induced field becomes intense above the upheaved portion of the conductor. Judging from the distribution of the induced fields, it appears difficult to determine the detailed shape of an underground conductor.

1. Introduction

A number of anomalies in geomagnetic variation would appear to be explained by electromagnetic induction in upheavals of the conducting mantle. Theories of electromagnetic induction in a circular or elliptical cylinder have been studied by Kertz,[1] Wait,[2] Rikitake and Whitham[3] and others in the hope of applying them to the interpretation of variation anomalies. Rikitake[4] also advanced a theory of electromagnetic induction in a nearly spherical conductor.

It is aimed in this paper at developing a theory of electromagnetic induction in a semi-infinite conductor having an undulatory surface in order to see the effect of underground non-flat conductors on geomagnetic variations.

2. Theory and discussion

A uniform inducing field H_0 is assumed to be in a direction parallel to the x axis as can be seen in Fig. 1. Outside the conductor of which

1) W. KERTZ, *Gerl. Beitr. Geophys.*, **69** (1960), 4.
2) J. R. WAIT, *J. Res. Nat. Bur. Stand.*, **64B** (1960), 15.
3) T. RIKITAKE and K. WHITHAM, *Can. J. Earth Sci.*, **1** (1964), 35.
4) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **42** (1964), 621.

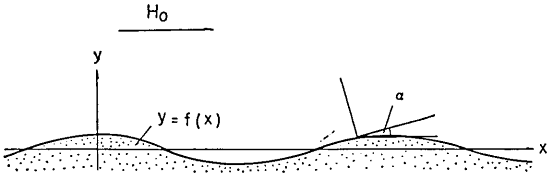

Fig. 1 Model of the conductor and the inducing field.

the surface is denoted by $y=f(x)$, we can define a potential of the induced magnetic field W_i which satisfies $\nabla^2 W_i=0$. When everything is uniform in the direction perpendicular to the x-y plane, we see that W_i can be written in a form

$$W_i=\sum_n C_n e^{-ny}\sin nx\ , \tag{1}$$

in which only harmonic terms of sine function are taken into account because of the symmetry of $f(x)$ about the y axis.

Let us assume perfect conductivity for the conductor. It has been ascertained that such an assumption is a good approximation for most problems of induction by short-period geomagnetic variations. In that case the normal component of the magnetic field at the boundary of the conductor should vanish. The boundary condition is therefore given by

$$-H_{ix}\sin\alpha+H_{iy}\cos\alpha+H_0\sin\alpha=0 \quad \text{on} \quad y=f(x)\ , \tag{2}$$

where α is the angle between the tangent at a point on $y=f(x)$ and

Table 1. Coefficients and right-hand members (in units

1	2	3	4	5	6
−2.2594210	0.2061760	−0.0115950	−0.0075900	0.0057610	−0.0029560
0.8469480	−2.3742610	0.5826600	−0.1315100	0.0256170	−0.0026840
−0.1052370	1.2292510	−2.6604060	1.0418430	−0.3387560	0.1016820
−0.0075630	−0.2701230	1.6827530	−3.0930960	1.6088370	−0.6552010
−0.0004060	0.0364510	−0.5078170	2.2369160	−3.7036170	2.3295010
0.0000530	−0.0035790	0.0977980	−0.8373300	2.9332950	−4.5386180
−0.0000440	0.0003720	−0.0138240	0.2060070	−1.2864190	3.8249660
0.0000520	−0.0001310	0.0017320	−0.0376140	0.3804040	−1.8938040
−0.0000550	0.0001450	−0.0003900	0.0057860	−0.0844110	0.6473050
0.0000640	−0.0001440	0.0003180	−0.0011390	0.0155100	−0.1678580
−0.0000670	0.0001760	−0.0003150	0.0006560	−0.0030770	0.0358470
0.0000770	−0.0001720	0.0003690	−0.0006250	0.0013410	−0.0075720

the x axis. $\cos\alpha$ and $\sin\alpha$ are given by

$$\left.\begin{aligned} \cos\alpha &= (1+f')^{-1/2}\,, \\ \sin\alpha &= f'(1+f')^{-1/2}\,, \end{aligned}\right\} \tag{3}$$

where f' denotes df/dx. It is therefore required for C_n's to satisfy

$$\left(H_0 - \sum_n nC_n e^{-nf(x)}\cos nx\right)f'(x) + \sum_n nC_n e^{-nf(x)}\sin nx = 0\,. \tag{4}$$

Assuming, for example

$$f(x) = h\cos x\,, \tag{5}$$

we obtain

$$\sum_n nC_n e^{-nh\cos x}(h\sin x\cos nx - \sin nx) = H_0 h\sin x\,. \tag{6}$$

If (6) is multiplied by $\sin Nx$ and integrated from $-\pi$ to $+\pi$ with respect to x, we obtain

$$\left.\begin{aligned} &\sum_n nC_n(hT_{n1} - S_{n1}) = H_0 h\pi \qquad \text{for} \quad N=1\,, \\ &\sum_n nC_n(hT_{nN} - S_{nN}) = 0 \qquad \text{for} \quad N=2, 3, \cdots, \end{aligned}\right\} \tag{7}$$

where

$$\left.\begin{aligned} S_{nN} &= \int_{-\pi}^{\pi} e^{-nh\cos x}\sin nx \sin Nx dx\,, \\ T_{nN} &= \int_{-\pi}^{\pi} e^{-nh\cos x}\sin x\cos nx \sin Nx dx\,. \end{aligned}\right\} \tag{8}$$

of H_0) of the simultaneous equation for $h=0.1\pi$.

7	8	9	10	11	12	Right-hand member
0.0012500	−0.0003140	−0.0002500	0.0007030	− 0.0011930	0.0018370	0.986959
−0.0009820	0.0006540	0.0002770	− 0.0013570	0.0022920	− 0.0036240	0
−0.0284420	0.0075440	−0.0025340	0.0022210	− 0.0033960	0.0053490	0
0.2411280	−0.0839730	0.0288760	− 0.0112770	0.0070470	− 0.0077920	0
−1.1174610	0.4742670	−0.1886650	0.0735710	− 0.0309520	0.0176370	0
3.2637640	−1.7772390	0.8436360	− 0.3712380	0.1583950	− 0.0706740	0
−5.6626270	4.4903780	−2.7062350	1.4088360	− 0.6737170	0.3093180	0
4.9808620	--7.1634549	6.1138350	− 4.0031680	2.2532600	− 1.1570740	0
−2.7126000	6.4915950	−9.1596589	8.2736330	− 5.8033990	3.4933680	0
1.0427830	−3.8150540	8.4771689	−11.8106750	11.1566960	− 8.2920749	0
−0.3074580	1.6163800	−5.2990350	11.0972540	−15.3305310	15.0140901	0
0.0746270	−0.5309500	2.4361500	− 7.2968450	14.5649630	−20.0063341	0

Table 2. Numerical values of C_n in units of H_0.

n	C_n	n	C_n
1	−0.452630	7	−0.000268
2	−0.089130	8	−0.000100
3	−0.024832	9	−0.000039
4	−0.007229	10	−0.000017
5	−0.002268	11	−0.000007
6	−0.000760	12	−0.000002

(7) provides a set of simultaneous equations from which we can solve C_n's. As an example, the coefficients and the right-hand members of the simultaneous equations for $n \leqq 12$ and $h=0.1\pi$ are calculated and shown in Table 1. The simultaneous equations are solved by an IBM 7090 computer and the coefficients are shown in Table 2. Although no exact theory of convergence is attempted, the smallness of the coefficients for higher degree terms indicates that the approximation with 12 harmonics is satisfactory. But if we take a much larger value of h, it is doubtful whether the theory works well.

With these coefficients solved in the above, the x and y components of the induced magnetic field are obtained at a number of levels above the conductor as shown in Fig. 2.

The x component of the induced field (H_x) takes its maximum immediately above the upraised portion of the conductor while its minimum appears immediately above the subsiding portion. The absolute value of the maximum amplitude is larger than that of the minimum one. The shape of the distribution of H_x along an x direction differs therefore from a simple sinusoidal curve even though the surface of the conductor is sinusoidal. Such differences from a purely sinusoidal form become smaller as the height above the conductor gets larger.

The y component of the induced field (H_y) shows an antisymmetric distribution about a point immediately above the upheaved part of the conductor. The maximum intensity of H_y is smaller than that of H_x.

Rikitake and Whitham[3] have calculated the magnetic field components over underground conductors of various shapes. It was found that the maximum of H_x and zero of H_y are always found directly above the top of the conductor. This is also the case for a conductor with an undulatory surface. Roughly speaking, the shape of the distribution curves of H_x and H_y are more or less the same for all the con-

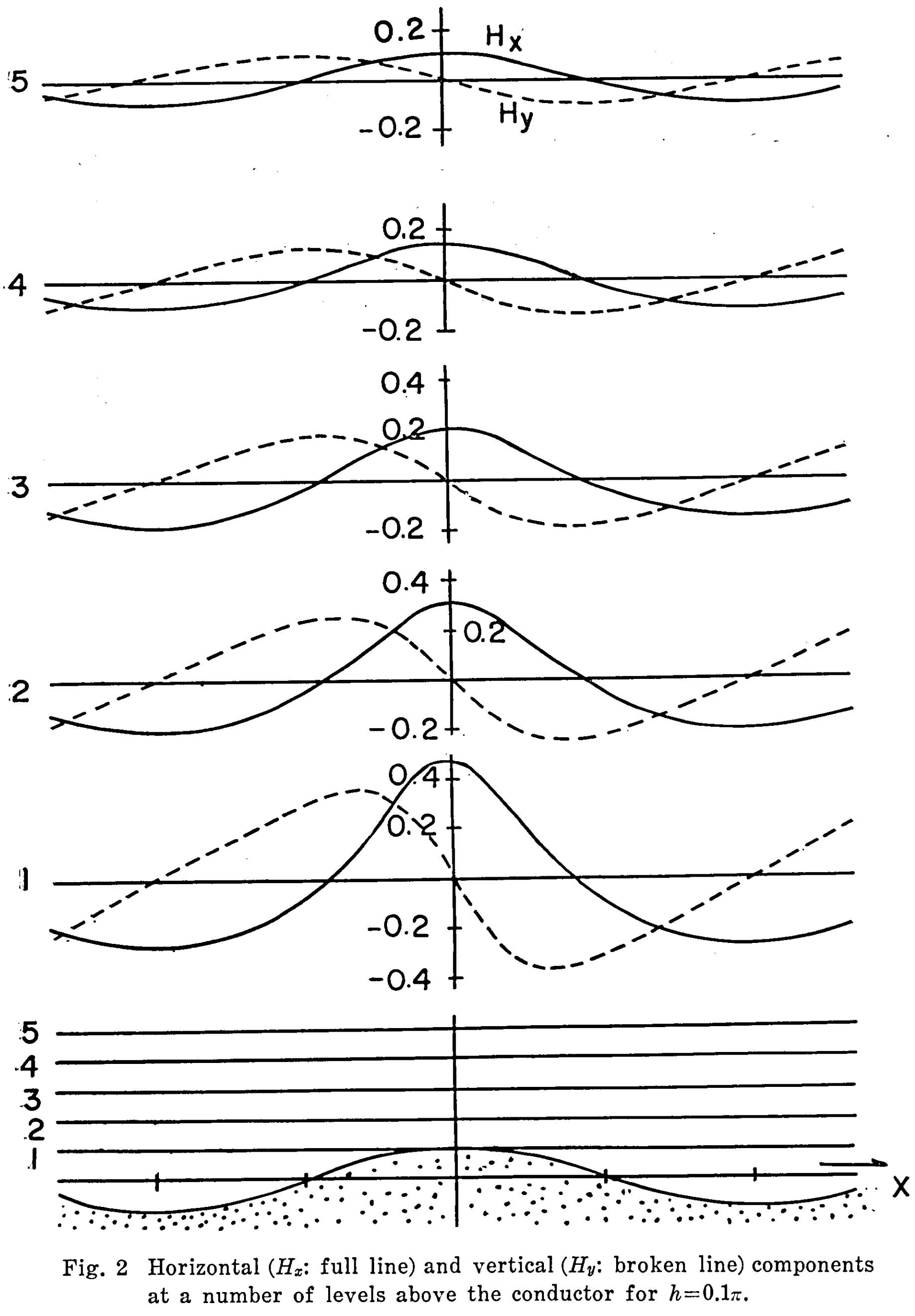

Fig. 2 Horizontal (H_x: full line) and vertical (H_y: broken line) components at a number of levels above the conductor for $h=0.1\pi$.

ductors. Judging from the accuracy of magnetic observation, it would be difficult to determine the exact size and shape of an underground conductor from results of a geomagnetic variation survey. Such a survey is useful, however, for locating the top of an underground conductor. The ratio H_x/H_0 is particularly useful for estimating the depth from the earth's surface to the top of the conductor. Although the present theory is developed only for a perfect conductor, the theory can readily be extended to a case of finite conductivity.

In conclusion the writer thanks Professor Y. Satô who kindly made available an IBM computer programme for solving simultaneous equations.

---

11. 表面に起伏のある半無限導体中の電磁感応

地震研究所　力　武　常　次

地下の導体の短周期地磁気変化におよぼす影響の一般的性質をみるために，起伏が正弦波の形をしている表面をもつ半無限導体中の一様水平磁場による電磁感応を論じた.

地磁気水平成分は導体の盛上つた部分の直上で最大となり，鉛直成分はそこでゼロ，そのまわりに反対称の分布となる．これらの分布は，従来研究された円筒形，楕円形，その他の場合とよく似ていて，地表の変化磁場分布から地下の導体形を正確に求めることは困難である．ただし地表に近接した導体の凸出部の位置を求めるためには，変化磁場分布を調べる方法は有力であろう.