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30. *Some New Problems of Seismic Vibrations of a Structure. Part 3. (Application to Design).*

By Kiyoshi KANAI,

Earthquake Research Institute.
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Abstract

The idea of the multiple reflection problem of waves in an elastic layer is applied to the vibration problem of a building of which the natural period is considerably larger than the predominant period of ground on which the building stands.

Some examples of numerical calculation are carried out and a new method of design of a resistant earthquake building is presented.

1. Introduction

From the results of the previous investigation[1], we saw that the vibration of an actual building as well as the dam due to earthquake motions should be treated as the multiple reflection phenomena of waves.

The results also showed that the most important part of the vibrational damping of a structure is based on that at the time of an earthquake, the vibration energy of a structure dissipating into the ground again as in a broad sense the elastic waves which start from the foundation. Therefore, the usual way of assuming all the sources of damping to be included in such terms in a differential equation of motion that specify the damping forces, namely, viscous fluid damping, solid friction damping, etc., is not correct, at any rate in the previous or similar problem.

In the present paper, a new method of design of a resistant earthquake structure will be investigated applying the idea of the multiple reflection problem of waves in an elastic layer.

1) K. KANAI and S. YOSHIZAWA, "Some New Problems of Seismic Vibrations of a Structure. Part 1," *Bull. Earthq. Res. Inst.*, **41** (1963), 825-833.

K. KANAI and S. YOSHIZAWA "ditto. Part 2. (Case of a Dam)," *ditto*, **42** (1964), 237-243.

2. Application to a tall building

We shall apply the idea of the multiple reflection problem of waves in an elastic layer to the vibration problem of a building of which the natural period, T_S, is considerably larger than the predominant period of ground, T_G, on which the building stands.

Actually, the delineation of seismic waves is very complicated but we may postulate for the sake of simplicity that the form of waves which yields the largest strain in a building is similar to the simple harmonic motion of a few trains. Two cases of the relation among the period of waves T, T_G and T_S will be dealt with, that is (I) $T_S \gg T_G$, $T = T_G$ and (II) $T_S \gg T_G$, $T = T_S$.

Case (I): $T_S \gg T_G$, $T = T_G$.

In this case, as a first approximation, the effect of the superposition of waves reflected at the top as well as the bottom of a building need not be taken into consideration excepting once, a reflection at the top. Consequently, the maximum displacement occurs at the top of the building and the value becomes as follows:

$$y_{max} = \frac{a}{2} \times m \times \gamma \times 2 = am\gamma \ , \qquad (1)$$

in which a is the displacement amplitude of seismic waves at the outcrop, twice that of incident waves in the bed rock, m is the magnification factor of the ground, γ is the transmission coefficient from the ground to the building and 2 of the last term is the effect of the reflection at the top of the building. Therefore, the maximum strain occurs at a position $1/4 \times$(wave length) distant from the top, and the value becomes as follows:

$$\left(\frac{\partial y}{\partial z}\right)_{max} = am\gamma \times \frac{2\pi}{VT} \ , \qquad (2)$$

in which V is the transmission velocity of waves in the building.

On the other hand, there exists the following relations, that is

$$V = \frac{4H}{T_S} \ , \quad v = \frac{2\pi a}{T} \ , \qquad (3)$$

in which H is the height of the building and v represents the velocity amplitude of seismic waves at the outcrop and takes a constant value depending on both the magnitude and the hypocentral distance of an earthquake regardless of the period of waves[2].

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2) K. KANAI and others, "An Empirical Formula for the Spectrum of Strong Earthquake Motions. II," *Bull, Earthq. Res. Inst.*, **41** (1963), 269. Figs. 9 and 10.

Substituting (3) by (2), we get

$$\left(\frac{\partial y}{\partial z}\right)_{\max}=\frac{m\gamma T_S v}{4H}\,. \tag{4}$$

Equation (4) tells us that the maximum strain in the building caused by seismic waves is proportional to the velocity amplitude of seismic waves at the outcrop, twice that of incident waves in the bed rock.

If we assume that the natural period of the building is proportional to the number of storeys, N, that is

$$T_S=cN\,, \tag{5}$$

and the strain distributes uniformly in the maximum strained storey, the maximum value of the relative displacement between the neighbouring floors, $D_{\max}$, becomes approximately as follows

$$D_{\max}=\left(\frac{\partial y}{\partial z}\right)_{\max}\times\frac{H}{N}=\frac{m\gamma cv}{4}\,. \tag{6}$$

In obtaining the value of m, the following empirical formula[3] may be available in the ground where the predominant period of seismic waves appears clearly,

$$m=1+\frac{\sqrt{T_G}}{0.3}\,. \tag{7}$$

In the cases of the San Francisco earthquake of 1906 and the Kantō earthquake of 1923 the severest damage to the buildings occurred at certain middle storeys which produced several kinds of the explanations by many different researchers concerning the cause of this phenomenon[4]. The result of the present investigation seems to be applicable for interpreting naturally the above-mentioned fact. Some examples of the practical application of the problem are tabulated in Table 1. Nevertheless, as seen in Table 1, the condition $T_S \gg T_G$ does not hold satisfactorily in such buildings, the agreement between the storeys that suffered the most severe damage and those of the calculated largest strain seems very good. We hope to ascertain more sufficiently

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3) K. KANAI, "An Empirical Formula for the Spectrum of Strong Earthquake Motion," *Bull. Earthq. Res. Int.*, **39** (1961), 87, Eq. (6).

4) T. TANIGUTI, "Seismic Action and Damage in Relation to Character of Building," *World Engg. Congress, Tokyo* (1929), paper No. 645.

Table 1. Seismic damage in the Kantō earthquake of 1923.

Name of bldg.	No. of Storeys	Natural period of bldg. (before Kantō earthq.)	Predominat period of ground	Storeys of most severe damage by Kantō earthq.)	Storeys of largest strain (VT/4)
Kaijō	7	0.45 sec	0.4 sec	2 to 3	2
Maru-no-uchi	8	0.67, 0.71	0.4	3 to 5	4

the feasibility of the present idea by obtaining such-like data for the taller buildings.

Case (II): $T_S \gg T_G,\ T=T_S$.

In this case, as a practical approximation, the magnification of the amplitude of waves in the ground can be neglected[5] and the magnification factor ξ in a building should be taken into consideration. The maximum displacement occurs at the top of the building and the value may be written as follows:

$$y_{\max}=\frac{a}{2}\times\gamma\times\xi\times 2=a\gamma\xi . \tag{8}$$

Consequently, the maximum strain occurs at the base of the building and the value becomes as follows

$$\left(\frac{\partial y}{\partial z}\right)_{\max}=\frac{\gamma\xi T_S v}{4H} . \tag{9}$$

Subsequently, the maximum relative displacement between the neighbouring floors takes on a value as follows:

$$D_{\max}=\frac{\gamma\xi c v}{4} , \tag{10}$$

$$\xi=\frac{1-|\beta|^{2n}}{1-|\beta|} , [n=1/2,\ 1,\ 3/2,\ \cdots .] \tag{11}$$

where β is the reflection coefficient of waves at the bottom of the

5) Strictly speaking, the value of magnification becomes $2/(1+\alpha_0)$, in which, α_0 is the impedance ratio of the ground to the bed rock.

building and $2n$ is the succession number of waves in which the period is equal to the natural period of the building[6]. It way be said, from (6) and (10), that when the natural period of a building is fairly larger than the predominant period of the ground on which the building is standing, the maximum strain occurs at some considerable height in the case where the magnification factor of ground, m, is larger than that of the building, ξ; on the contrary, it occurs at the base in the case where $m<\xi$.

Strictly speaking, it is a very difficult problem to decide the height where the stress reaches its maximum, because the boundary conditions between building and ground are not so simple. Nevertheless, the results just mentioned and those of the theoretical studies based upon the idea that at the time of earthquake the vibration energy of buildings dissipates to the ground again as the elastic waves which start from the foundation[7] do not contradict each other, and that both results agree qualitatively with the results of the statistical studies of the damage of buildings due to large earthquakes in the past.

In the special case of $T=T_G=T_S$, the maximum displacement, strain and the relative displacement between the neighbouring floors become as follows:

$$y_{\max}=am\gamma\xi\ , \tag{12}$$

$$\left(\frac{\partial y}{\partial z}\right)_{\max}=\frac{m\gamma\xi T_S v}{4H}\ , \tag{13}$$

$$D_{\max}=\frac{m\gamma\xi cv}{4}\ , \tag{14}$$

and it is a matter of course that (12) appears at the top and (13) as well as (14) take place at the base.

It should be borne in mind that the strain in a structure caused by an earthquake is proportional to the velocity amplitude of seismic waves. The height itself of a structure has only a slight effect on the problem of resistant earthquake, and reduction of the value of c is very important for the design of a resistant earthquake building.

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6) K. KANAI and others, "Observational Study of Earthquake Motion in the Depth of the Ground. IV," *Bull. Earthq. Res. Inst.*, **31** (1953), 232, Eq. (2).

7) K. KANAI, "Relation between the Earthquake Damage of Non-Wooden Buildings and the Nature of the Ground," *Bull. Earthq. Res. Inst.*, **27** (1949), 98, Table II and ditto, **29** (1951), 213, Fig. 7.

3. Examples of numerical calculation

When the size of the plane of a building is so large as to be treated as a wave problem of a single stratified layer, the transmission coefficient, γ, and the reflection coefficient, β, can be written as follows:

$$\gamma=\frac{2}{1+\alpha}\,, \qquad \beta=\frac{\alpha-1}{\alpha+1}\,, \tag{15}$$

in which α is the impedance ratio of building to ground, that is, $\alpha=\rho V/\rho' V'$ and ρ, ρ' and V, V' are the densities and velocities, respectively, of the building and the ground.

Let h be the fraction of critical damping, then β can be expressed as

$$|\beta|=e^{-\pi h}\ ^{8)}\ . \tag{16}$$

From (15) and (16), we obtain

$$\gamma=1+e^{-\pi h}\ . \tag{17}$$

Accordingly, if we know the values of h we can obtain the values of ξ and γ by using equations (11) and (16) and (17), respectively.

The values of the velocity amplitude of seismic waves at the outcrop, v, obtained from the following empirical formula[9] are shown in Fig. 1.

$$v=10^{0.61M-1.73\log x-0.67}\,, \tag{18}$$

in which M and x are the magnitude and hypocentral distance of an earthquake, respectively.

In carrying out the numerical calculation of the case (I) by using (6), (7), (17) and Fig. 1, we assume that the larger the predominant period of ground the smaller the damping of structure[10].

In practice, we shall adopt the following values, that is, $h=0.02$ to $T_G=0.2$ sec, $h=0.12$ to $T_G=0.6$ sec and the proper values of $m\gamma$ to the values of T_G between 0.2 sec and 0.6 sec. The results of the numerical calculation are shown in Figs. 2–5.

Next, in carrying out the numerical calculation of the case (II)

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8) K. SEZAWA and K. KANAI, "Decay Constants of Seismic Vibrations of a Surface Layer," *Bull. Earthq. Res. Inst.*, **13** (1935), 256, Eq. (17).

9) K. KANAI, *loc. cit.*, 3), 86, Eq. (3).

10) K. KANAI and others, "Relation between the Property of Building Vibration and the Nature of the Ground. III," *Bull. Earthq. Res. Inst.*, **34** (1956), 77, Fig. 16.

by using (10), (11), (17) and Fig. 1, we assume that the smaller the damping of a structure the larger the succession number of waves of equal period[11]. In practice, we shall adopt the following values, that is, $n=2$ to $h=0.02$, $n=1$ to $h=0.12$ and the proper values of $\gamma\xi$ to the values of h between 0.02 and 0.12. The results of the numerical calculation are shown in Figs. 6–9.

4. Conclusion

We shall summarise briefly the results of the present investigation.

(a) The strain in a structure caused by an earthquake is proportional to the velocity amplitude of seismic waves.

(b) In general, on soft ground, as the magnification of amplitude in a building is relatively small because the damping of it is large and the magnification of amplitude in the ground is large, the greatest damage to tall buildings is liable to take place at some considerable height.

(c) Conversely, the greatest degree of damage in buildings on hard ground is liable to take place near the base, because the damping of a building is relatively small and the magnification of amplitude in the ground is not large.

(d) The value itself of the height of a tall building has only a slight effect on the problem of resistant earthquake. In general, the natural period of a tall building is proportional to the number of storeys and reduction of the proportional number is very important for the design of a resistant earthquake building.

(e) The methods of design of a resistant earthquake building presented by the present investigation are as follows:

(1) To decide the magnitude as well as the hypocentral distance of an expectant earthquake at a construction site by an engineering judgement and to obtain the value of velocity amplitude of seismic waves.

(2) To ascertain the predominant period of the ground at a construction site. The measurement of microtremors will be avairable for this.

(3) To decide the allowable maximum relative displacement between the neighbouring floors which depends on circumstance, design, height, material, etc.

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11) K. KANAI and M. SUZUKI, "Analytical Results of the Acceleration Seismograms obtained at Tokyo and Yokohama," *Bull. Earthq. Res. Inst.*, **32** (1954), 195, Figs. 7 and 8.

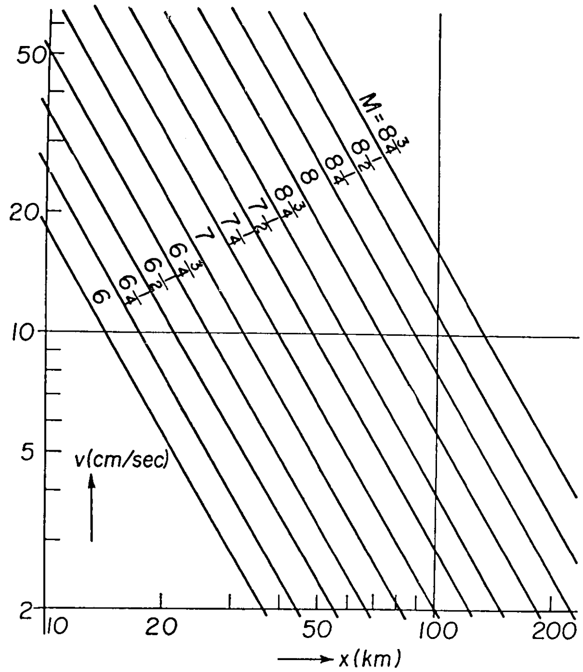

Fig. 1. M=magnitude, x=hypocental distance, v=velocity amplitude at the outcrop.

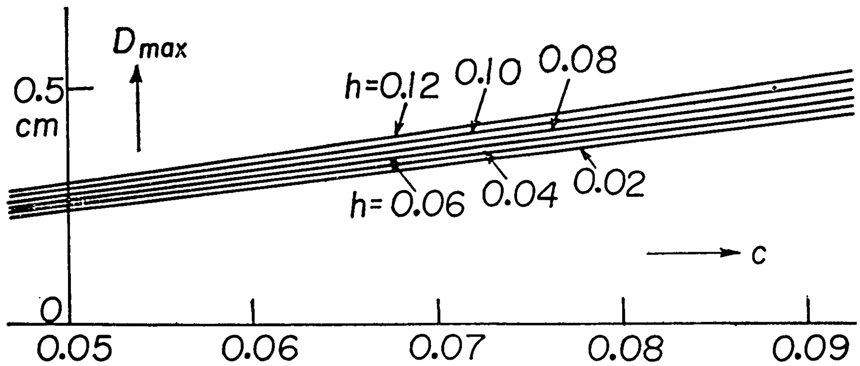

Fig. 2. The case of $T_S \gg T_G$, $T=T_G$ and v=4 cm/sec. T_S=natural period of a building, T_G=predominant period of ground, T=period of seismic waves, v=velocity amplitude at the outcrop, h=fraction of critical damping, D_{max}= maximum relative displacement between the neighbouring floors, c=proportional number of natural period to the number of storeys.

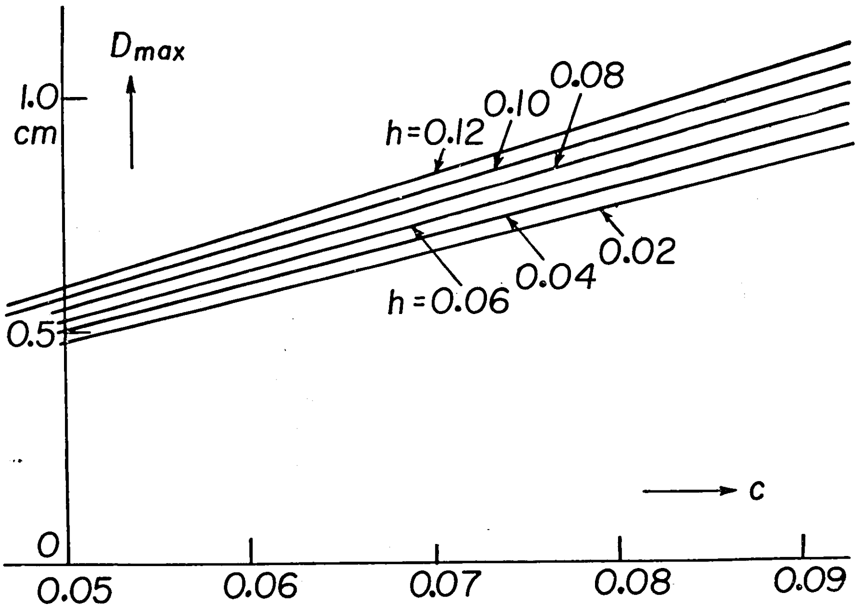

Fig. 3. Case of $T_S \gg T_G$, $T=T_G$ and $v=8$ cm/sec. Notatios are the same to those of Fig. 2.

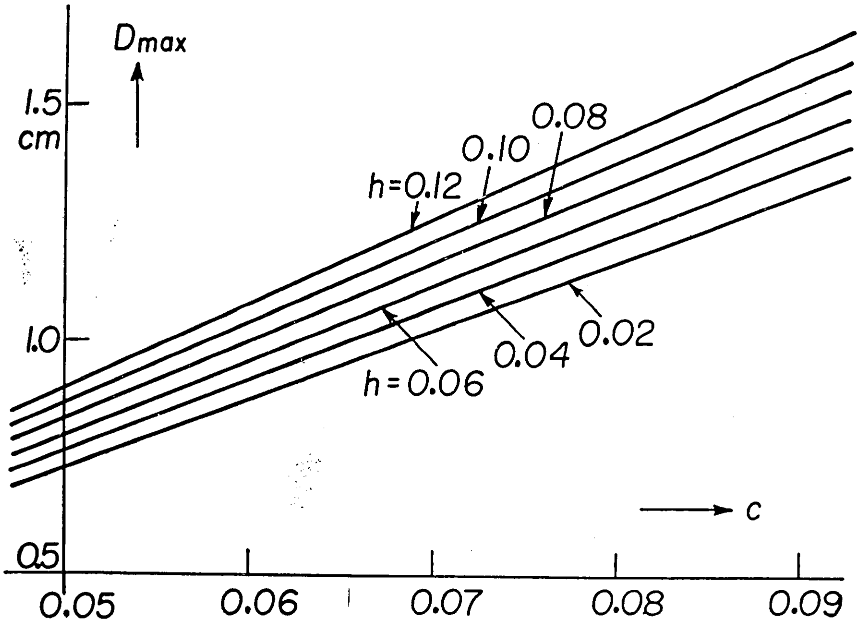

Fig. 4. Case of $T_S \gg T_G$, $T=T_G$ and $v=12$ cm/sec, Notations are the same to those of Fig. 2.

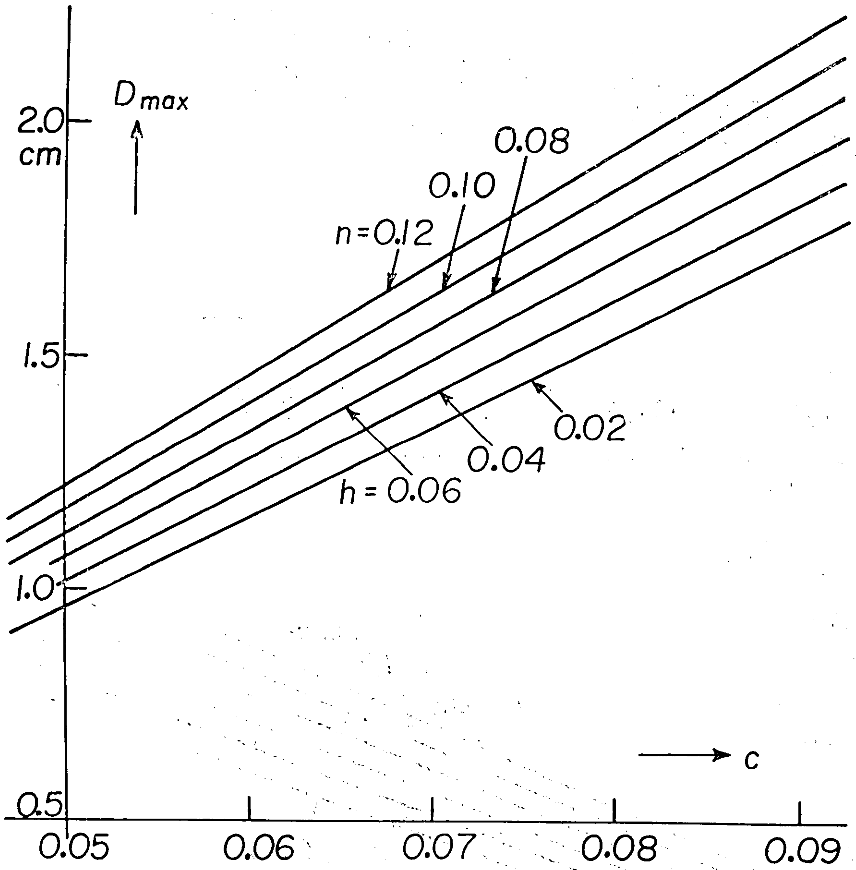

Fig. 5. Case of $T_S \gg T_G$, $T=T_G$ and $v=16$ cm/sec. Notations are the same to those of Fig. 2.

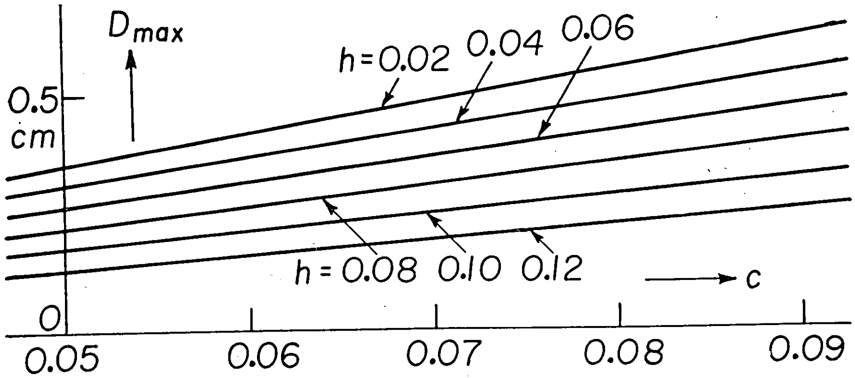

Fig. 6. Case of $T_S \gg T_G$, $T=T_S$ and $v=4$ cm/sec. Notations are the same to those of Fig. 2.

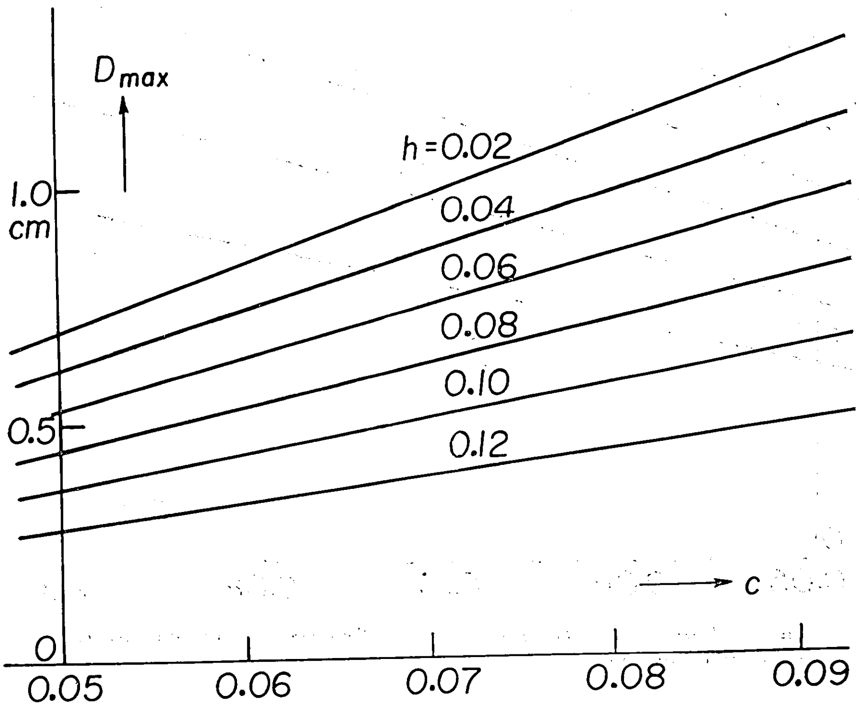

Fig. 7. Case of $T_S \gg T_G$, $T=T_S$ and $v=8$ cm/sec. Notations are the same to those of Fig. 2.

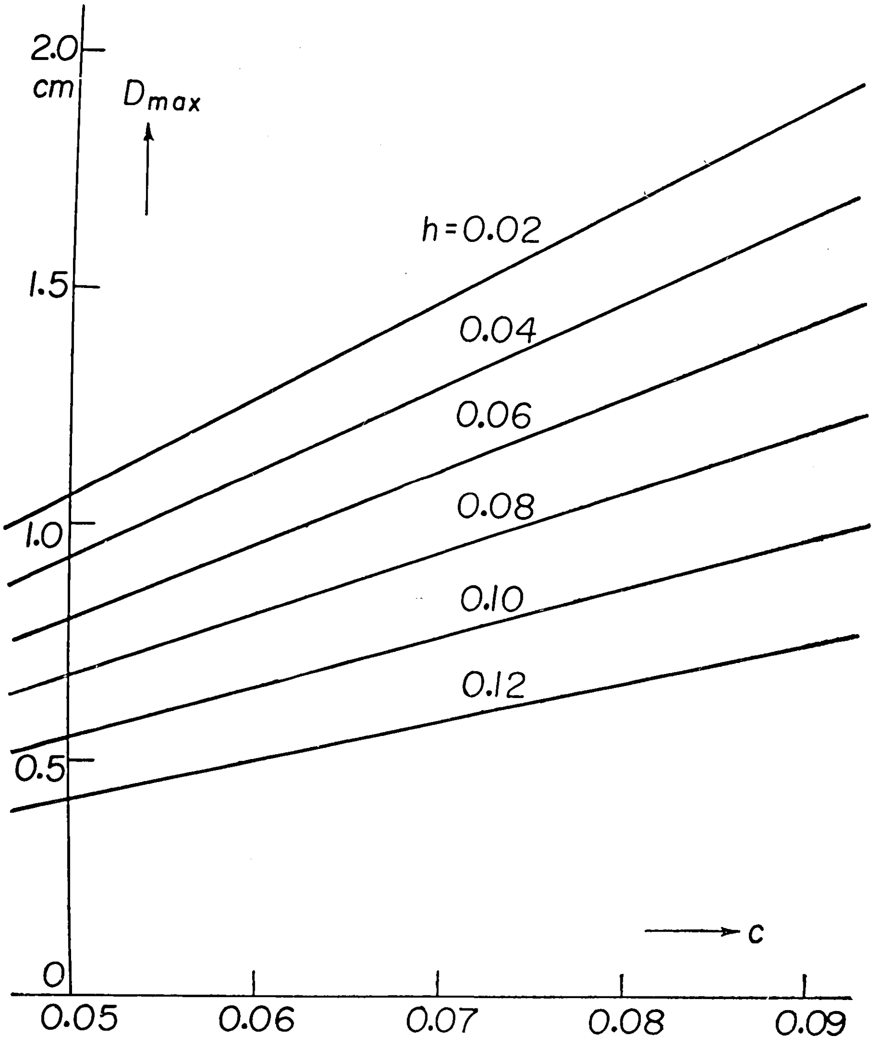

Fig. 8. Case of $T_S \gg T_G$, $T=T_S$ and $v=12$ cm/sec. Notations are the same to those of Fig. 2.

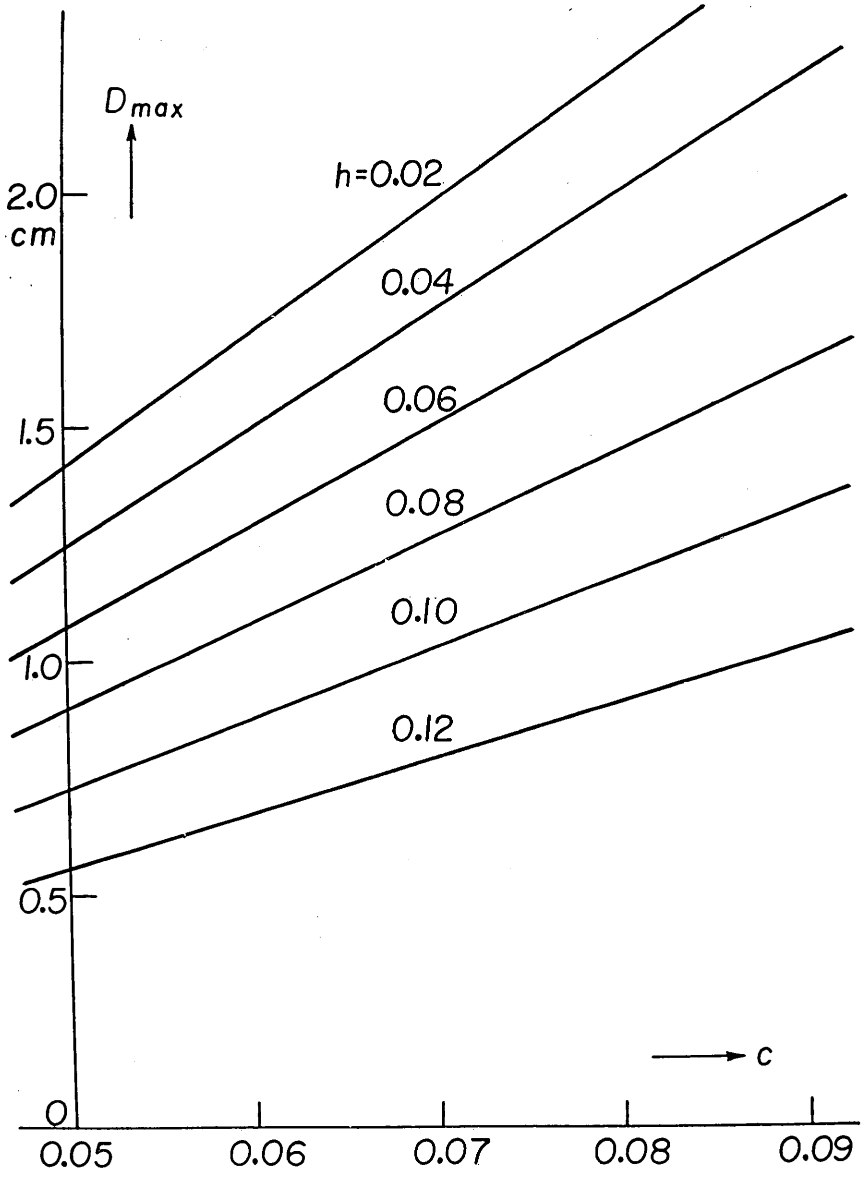

Fig. 9. Case of $T_S \gg T_G$, $T=T_S$ and $v=16$ cm/sec. Notations are the same to those of Fig. 2.

(4) To estimate the value of the fraction of critical damping of a building by well considering the subsoil condition.

(5) To calculate the natural period which corresponds to the proportional number of natural periods to the number of storeys which satisfies Items (1)-(4).

(6) To determine the stiffness, weight and other factors which satisfy the natural period determined by Item (5).

In conclusion I wish to express my sincerest thanks to Miss S. Yoshizawa who assisted me in preparing this paper.

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30. 地震動による構造物振動の新しい問題 第3報（耐震設計への応用）

地震研究所　金井　清

前回までの研究結果で，地震動による実在建物およびダムの振動は，地震波の重複反射の現象という波動問題にほかならないことが明かになつた．また，それらの研究で，建物およびダムの地震時の振動減衰性は，構造物内部では無視できるぐらい小さくて，ほとんどが基礎と地盤との境界面でおこることがよくわかつた．

本研究では，これまでにわかつた性質を，耐震設計の方面に応用して，新しい計算方式を見つけることを試みた．

実際の地震動の性質から離れないように吟味しながら，構造物に対する応答を実用化するために，いくつかの仮定をもうけて計算を進めた結果，次のようなことがわかつた．

(a) 地震動による建物の最大歪は地震動の速度振幅に比例する．

(b) 軟弱地盤上では地盤内での地震動の増幅は大きいが，建物内での増幅が小さいために，最大歪は建物の底部よりも高いところで起る場合が多い．

(c) これに反して堅硬地盤上では，地盤内での増幅は小さいが，建物内での増幅が大きいので，最大歪は建物の底部で起りやすい．

(d) 高層建築物の耐震性を支配するものは，固有周期の値そのものではない．一般に高層建築物の固有周期は層数に比例し，その比例数を小さくすることが，耐震構造上最も大切なことである．

(e) この研究結果にもとづく高層建築物の耐震設計の計算方式は次のようになる．

(1) 先ず，建設予定地で設計上期待すべき地震のマグニチュードと震央距離を，地震工学的判断にもとづいてきめ，地震動の速度振幅を求める．

(2) 建設地地盤の卓越周期を求める．それには，常時微動の測定結果を利用するのも一方法である．

(3) 建築物の周囲の状況，構造様式，高さ，材料などから許容最大層間変位をきめる．

(4) 主として，地盤の性質を考慮に入れて，建築物の減衰係数を推定する．

(5) 第1～4項を満足する．建物の層数と固有周期の比例数を求めてから，固有周期を算出する．

(6) 最後に第5項の固有周期を満足させるような，構造上の諸元の剛性，重量その他をきめる．

これまでの経験によると，設計計算で求めた固有周期の値は完成後の実測値よりも，ほとんどの場合に小さい．すなわち，従来の固有周期の計算法の結果をそのまま本研究に使うと安全側ではあるが，不経済側であることになる．従つて，相当に合理的なはずの本研究の方法を，一層経済的なものに発展させるためには，実際の固有周期が求められる計算法を確立することが先決問題となる．