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8. *Coastal Effects upon Tsunamis and Storm Surges.*

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Abstract

The effect of the continental slope is mathematically treated, and a numerical example is given.

1. Introduction

As is well known, a tsunami is a train of long waves, almost, non-dispersive in the leading part, almost inperceptible in the open ocean masked by wind waves. It comes to our notice only when it approaches the coast or enters a harbor, hence it is called *tsu*(harbor)-*nami*(wave) in Japanese. The amplification of the wave height is sometimes due to the squeezing of wave energy into a shortened wave-length and crest-width, and at other times due to the resonance of bay water. The tsunami waves must, however, pass through the continental slope beforehand, where they suffer some deformation in the wave form.

The pressure set-up is the main cause of the sea-level change in the case of a storm surge in the deep open ocean. The level change due to wind stress would be very small in the deep ocean far distant from the shore. Although the pressure set-up water hump is a forced wave, which moves with the depression, it suffers the effect of bottom topography as tsunami waves do, since the diameter of the depression is generally very large compared with the sea depth.

As a first step to the study on coastal effects we will consider the effect of the continental slope on these waves in the following section.

2. The effect of the continental slope

We will first study the effect of continental slopes upon incoming long waves. We assume the wave-length to be very long compared with the depth at all places under consideration so that long wave approximation holds good everywhere. If we consider only waves with periods longer than 5 min, this condition will be approximately fulfilled for seas shallower than 5,000 m.

We assume a continental shelf of a constant depth h_1, an ocean of a constant depth h_2, and a continental slope where the depth is taken to be proportional to the square of the distance x, that is, the long wave velocity is assumed to be proportional to the distance. To avoid abrupt changes of depth at x_1 and x_2, we make $h_1=ax_1^2$ and $h_2=ax_2^2$, *i.e.* we have

$$\begin{aligned} h=h_1=ax_1^2 \quad & \text{for} \quad x\leqq x_1 \\ h=ax^2 \quad & \text{for} \quad x_1\leqq x\leqq x_2 \\ h=h_2=ax_2^2 \quad & \text{for} \quad x_2\leqq x\,. \end{aligned}$$

This assumption of sea depths makes mathematics very simple. Lord Rayleigh[1] treated the case of reflection of sound waves when the medium density varies as an inverse square of the abscissa. Kajiura[2] calculated the reflection coefficient for long water waves when the depth is supposed to vary as the square of the distance. We will calculate in the following the transmission coefficient of the continental slope when the depth is given as above.

The sea surface elevation due to waves in the oceanic region $(x>x_2)$ may be written as follows:

$$\eta=Ae^{ik_2x}+Be^{-ik_2x}$$

the time factor $e^{i\sigma t}$ being understood. A is the amplitude of incoming wave and B that of the reflected wave. We have $k_2^2=\sigma^2/agx_2^2$.

The wave on the continental shelf $(x<x_1)$ may be written as

$$\eta=Ee^{ik_1x}$$

where $k_1^2=\sigma^2/agx_1^2$, because we do not consider at present any reflected wave in this region.

As the differential equation of the wave for the region $x_1\leqq x\leqq x_2$, we obtain

$$\frac{\partial}{\partial x}\left(x^2\frac{\partial\eta}{\partial x}\right)+n^2\eta=0$$

where $n^2=\sigma^2/ag$. The solution of this equation is given by

1) LORD RAYLEIGH, *Theory of Sound* (1962), § 148 b.

2) Kinjiro KAJIURA, "On the partial reflection of water waves passing over a bottom of variable depth". Xth Pacific Science Congress, 1962, Tsunami Symposium.

$$\eta = Cx^{-1/2+im} + Dx^{-\frac{1}{2}-im} ,$$

where

$$m^2 = n^2 - 1/4 .$$

The boundary conditions that the elevation of the sea surface and the quantity of flow of the sea water must be continuous at x_1 and x_2 gives the following four expressions:

$$Ae^{in} + Be^{-in} = Cx_2^{-1/2+im} + Dx_2^{-1/2-im} ,$$

$$Ae^{in} - Be^{-in} = C\frac{in}{\frac{1}{2}+im} x_2^{-1/2+im} + D\frac{in}{\frac{1}{2}-im} x_2^{-1/2-im} ,$$

$$Ee^{in} = Cx_1^{-1/2+im} + Dx_1^{-1/2-im} ,$$

$$Ee^{in} = C\frac{in}{\frac{1}{2}+im} x_1^{-1/2+im} + D\frac{in}{\frac{1}{2}-im} x_1^{-1/2-im} .$$

In the above expression the relations $k_1^2x_1^2 = k_2^2x_2^2 = n^2$ were used.

Eliminating B, C and D from these expressions we can obtain after some reduction

$$\frac{E}{A} = \sqrt{\frac{x_2}{x_1}} \frac{m}{m\cos(m\log\mu) + in\sin(m\log\mu)}$$

where

$$\mu = x_2/x_1 .$$

Then transmission coefficient is then

$$|E/A| = \sqrt{\frac{x_2}{x_1}} \frac{2m}{\sqrt{4m^2 + \sin^2(m\log\mu)}} .$$

If $n^2 < 1/4$, m becomes imaginary. Then putting $m = im'$, we have

$$\frac{E}{A} = \sqrt{\frac{x_2}{x_1}} \frac{m'}{m'\cosh(m'\log\mu) + in\sinh(m'\log\mu)}$$

and

$$|E/A| = \sqrt{\frac{x_2}{x_1}} \frac{2m'}{\sqrt{4m'^2 + \sinh(m'\log\mu)}} .$$

If $n^2 = 1/4$, $m = 0$, and we have

$$\frac{A}{E} = \sqrt{\frac{x_2}{x_1}} \frac{1}{1 + in\log\mu}$$

and

$$|E/A|=\frac{2}{\sqrt{4+(\log\mu)^2}}.$$

In Fig. 2.1, the values of $|E/A|$ are given for a topography as shown in the same Figure.

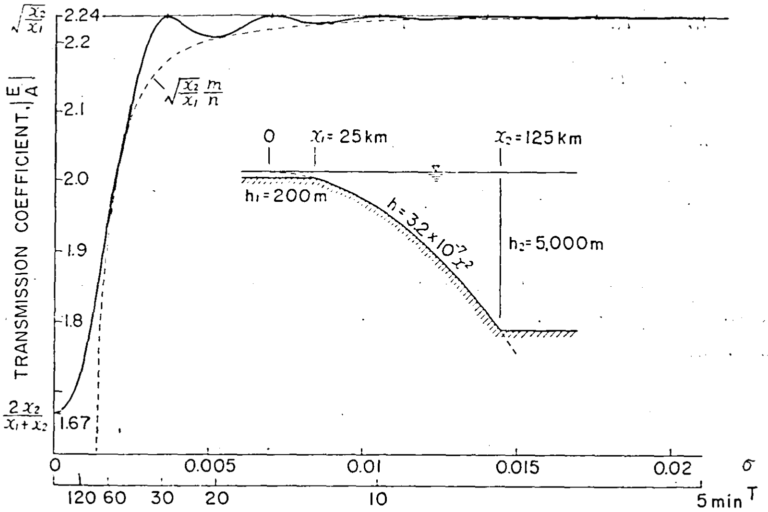

Fig. 2.1. Transmission Coefficient of a Continental Shelf.

When σ is very large, that is, when the wave length λ is short compared with x_2-x_1, m becomes also very large and $|E/A|$ approaches to $\sqrt{x_2/x_1}$, that is, to $\sqrt[4]{h_2/h_1}$, the value which is given by Green's law.

When σ is very small, m' approaches to 1/2, and $|E/A|$ reduces to $2x_2/(x_1+x_2)$. This is exactly the same with the value obtainable in the case of abrupt change of depth, if we notice that the wave velocities on the shelf and in the ocean are respectively proportional to x_1 and x_2 in our case. When $m\log\mu=\pi/2,\ 3\pi/2,\ \cdots$, $|E/A|$ becomes equal to $\sqrt{x_2/x_1}$. $|E/A|$ curve therefore starts from the value $2x_2/(x_1+x_2)$, oscillates between the curve $\sqrt{x_2/x_1}\ m/n$ and $\sqrt{x_2/x_1}$, and finally approaches to the latter value. The maxima and minima correspond approximately to the cases when $2s\cdot\lambda/4$ and $(2s\text{-}1)\cdot\lambda/4$ are contained between x_2 and x_1, where λ is the local wave-length and $s=1, 2, 3, \cdots$.

Energy density transmission coefficient will be given by $|E/A|^2$. From the law of conservation of energy, the reflection coefficient can be obtained as

$$\left|\frac{B}{A}\right|=\sqrt{1-|E/A|^2/\mu}\,.$$

The phase angle θ of E/A can be written as

$$\theta=\arctan\left[\frac{n}{m}\tan(m\log\mu)\right],$$

but may be substituted by $\theta=-\sigma(\log\mu)/\sqrt{ag}$ in a rough approximation. When plotted against σ, θ gives a wavy curve, starting from the origin, and then passing through the values, $-\pi/2$, $-\pi$, $-3\pi/2$, $\cdots$ successively corresponding to the successive values of σ which makes $\frac{\cos}{\sin}(m\log\mu)=0$. The amplitude of the wavy fluctuation decreases rapidly with increasing σ, finally becoming a straight line.

The E/A curve seems therefore to be approximated well by the following expression

$$\left|\frac{E}{A}\right|=\sqrt{\mu}\left(1-\frac{(1-\sqrt{\mu})}{1+\mu}\frac{\sin^2 f\sigma}{f^2\sigma^2}\right) \qquad (1),$$

where

$$f=\frac{1}{\sqrt{ag}}\log\mu\,.$$

The wavy fluctuation of the transmission coefficient seems to be the result of seiche oscillations provoked on the continental slope owing to the slope discontinuities at x_1 and x_2. In the real sea, the slope would rather be continuous. In such case the wavy fluctuation of $|E/A|$ would disappear and E/A may well be approximated by such an expression as

$$\frac{E}{A}=\sqrt{\mu}\left(1-\frac{(1-\sqrt{\mu})^2}{1+\mu}e^{-4f^2\sigma^2/\pi^2}\right)e^{-if\sigma} \qquad (2),$$

If we write the spectrum of the incoming tsunami or storm surge waves as $W(\sigma)$ in the open ocean, then the wave $E(t,x)$ on the continental shelf would be

$$E(t,x)=\sqrt{\frac{x_2}{x_1}}\int_{-\infty}^{\infty}\frac{mW(\sigma)e^{i\sigma(t+\overline{x-x_1}/c_1)}d\sigma}{m\cos(m\log\mu)+in\sin(m\log\mu)}\,.$$

where

$$c_1^2=agx_1^2\,.$$

If we assume as the incoming wave

$$A_1\, e^{-\nu^2(t+\overline{x-x_2}/c_2)^2} = \int_{-\infty}^{\infty} W(\sigma) e^{i\sigma(t+\overline{x-x_2}/c_2)} d\sigma \;,$$

then writing

$$\tau = t + \overline{x-x_2}/c_2\;, \qquad c_2^2 = agx_2^2$$

$$W(\sigma) = \frac{A_1}{2\pi}\int_{-\infty}^{\infty} e^{-\nu^2\tau^2} e^{-i\sigma\tau} d\tau = \frac{A_1}{2\pi}\frac{\sqrt{\pi}}{p} e^{-\sigma^2/4p^2} \;.$$

Using the approximation (2) for E/A, we have, after reduction,

$$E(t, x) = A_1\sqrt{\mu}\, e^{-p^2(\tau'-f)^2} - A_1\sqrt{\mu}\frac{(1-\sqrt{\mu})^2}{1+\mu} e^{-s^2(\tau'-f)^2} \;,$$

where

$$\tau' = t + \overline{x-x_1}/c_1\;, \quad \frac{1}{4s^2} = \frac{1}{4p^2} + 4f^2/\pi^2 \;.$$

The second term represents a lower and flatter hump than the first term since $s<p$. The term f represents the delay caused by propagating the distance x_2-x_1.

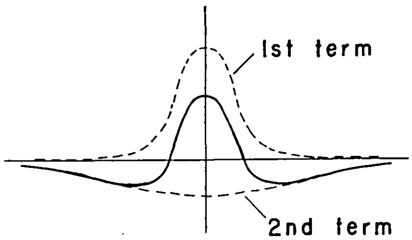

Fig. 2.2. The deformation of wave form due to a continental shelf.

It is somewhat queer that a positive hump of water in the open sea should be superposed by a secondary negative depression when it proceeds into the continental shelf. If, however, the positive hump be succeeded by a negative trough of the same size, the secondary waves, which superpose on the primary ones, will almost compensate to each other, and there will practically remain only the primary ones, the crest and trough being reduced to $2x_2/(x_1+x_2)$ times the original values.

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8. 津波と高潮に対する沿岸効果 (第1報)

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津波とか高潮のような長い波に対する沿岸地形の効果の内，先ず大陸棚斜面の影響を考究した．すなわち外洋から沿岸に押寄せる津波，高潮の勢力の何程の部分が大陸棚に入りこむかを計算し，現実に近い数値を与えた時の結果を本文第 2.1 図に示した．また外洋から孤立波が来る場合陸棚上で変形する可能性をも第 2.2 図に示した．