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50. *Some New Problems of Seismic Vibrations of a Structure. Part 1.*

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Abstract

The idea of the multiple reflection problem of waves in an elastic layer is applied to the problem of seismic vibration of a structure.

The theoretical results by using a simple formula obtained here, in which the form of seismic motions at the lower boundary of a structure is able to get from that at the top of it, are compared with the observational results of the actual buildings.

It is concluded that the problem of seismic vibration of a building should be treated as the multiple reflection phenomena of waves.

1. Introduction

The problem of vibrations of structures due to seismic waves has been studied by many researchers.

But the standpoints, on which these researchers based their theories are not theoretically rigorous, as their methods of treatment are mainly to apply approximately the solution of the equation of motion of materials or to neglect certain boundary conditions.

Indeed, the exact calculation of the complicated structure is, in general, very difficult.

However, the vibration problem of a structure still seems worth while to investigate in some way further because of the fact that we are now in a position, owing to its importance in earthquake-proof constructions, to re-examine several points which were not adopted in the observational results.

Recently, we had an occasion to study the problem of behaviour contained in this question.

In the present investigation, we shall first deal theoretically with the multiple reflection problem of waves[1] in a structure, and then

1) K. SEZAWA and K. KANAI, "Decay Constants of Seismic Vibrations of a Surface Layer," *Bull. Earthq. Res. Inst.*, **13** (1935), 251-265.

compare the theoretical results with the observational ones.

Regarding the principle of the methods in question the following results will be taken into consideration.

We have calculated rigorously the natural periods of the framed structures with one, two and an infinite number of spans, and the distributions of the deflections as well as bending moments in the structures due to forced oscillations[2]. It has been found that the case of three or more spans far approximates to that of an infinite number of spans, so that it seems that the treatment of the case of an infinite number of spans covers practically all the problems of horizontal vibrations of framed structures with any number (larger than two) of spans in general.

Another important conclusion to be noticed which may be deduced from the results of our previous investigations[3] is that the period of vibrations of a framed structure with clamped conditions at the floors increases linearly with the number of its floors.

On the other hand, the results of our previous investigations[4] with respect to the decaying of seismic vibrations of framed structures tell us that the dissipation of vibrational energy in the form of seismic waves transmitted into the ground seems to be a very important aspect of the problem.

2. Vibration of a structure treated by a wave problem

The multiple reflection problem of waves[5] in a structure is treated rather analytically in the present investigation.

If the incident waves arriving at the foundation of a structure, $z=0$, be of the type:

$$u_0=F(t) , \tag{1}$$

2) K. SEZAWA and K. KANAI, "Vibrations of a Singled-storyed Framed Structure," *Bull. Earthq. Res. Inst.*, **10** (1932), 767-802.

3) K. SEZAWA and K. KANAI, "Some New Problems of Free Vibrations of a Structure," *Bull. Earthq. Res. Inst.*, **12** (1934), 804-822.

4) K. SEZAWA and K. KANAI, "Improved Theory of Energy Dissipation in Seismic Vibrations of a Structure," *Bull. Earthq. Res. Inst.*, **14** (1936), 164-188. K. KANAI and S. YOSHIZAWA, "Relation between the Earthquake Damage of Non-Wooden Buildings and the Nature of the Ground. II," *ditto*, **29** (1951), 209-214. K. KANAI, T. SUZUKI and S. YOSHIZAWA, "Relation between the Property of Building Vibration and the Nature of the Ground. III," *ditto*, **34** (1956), 61-86.

5) *loc. cit.*, 1)

the transmitted waves in a structure, u_1, at $z=0$ and $z=H$ in Fig. 1 are expressed by

$$\left.\begin{aligned} u_{1z=0} &= \gamma F(t) , \\ u_{1z=H} &= \gamma F\left(t-\frac{H}{V}\right) , \end{aligned}\right\} \tag{2}$$

where γ is the transmission coefficient of waves from the ground to the structure and V the apparent transmission velocity of waves in that structure.

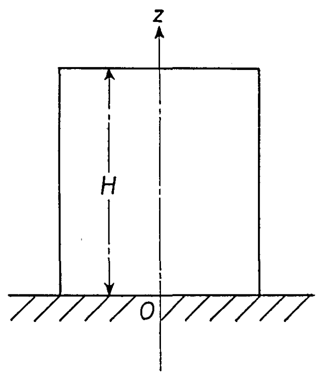

Fig. 1.

If the assumptions are made that the reflection coefficients of waves at the free surface, $z=H$, and the lower boundary, $z=0$, in a structure are 1 and β, respectively, and the attenuation of waves in that structure is negligibly small, the expression for the resulting motions at $z=H$ and $z=0$, as influenced by the multiple reflection of waves in that structure can be written by an infinite series, as follows[6]:

$$\begin{aligned} u_{z=0}(t) &= \gamma F(t) + \left\{\gamma F\left(t-\frac{2H}{V}\right) + \gamma\beta F\left(t-\frac{2H}{V}\right)\right\} + \left\{\gamma\beta F\left(t-\frac{4H}{V}\right)\right. \\ &\quad \left. + \gamma\beta^2 F\left(t-\frac{4H}{V}\right)\right\} + \cdots \\ &= \gamma\left[F(t) + \left\{F\left(t-\frac{2H}{V}\right) + \beta F\left(t-\frac{2H}{V}\right)\right\}\right. \\ &\quad \left. + \left\{\beta F\left(t-\frac{4H}{V}\right) + \beta^2 F\left(t-\frac{4H}{V}\right)\right\} + \cdots\right] , \end{aligned} \tag{3}$$

$$\begin{aligned} u_{z=H}(\tau) &= 2\gamma F\left(t-\frac{H}{V}\right) + 2\gamma\beta F\left(t-\frac{3H}{V}\right) + 2\gamma\beta^2 F\left(t-\frac{5H}{V}\right) + \cdots \\ &= 2\gamma\left[F\left(t-\frac{H}{V}\right) + \beta F\left(t-\frac{3H}{V}\right) + \beta^2 F\left(t-\frac{5H}{V}\right) + \cdots\right] . \end{aligned} \tag{4}$$

6) When a seismic wave of a purely plane type propagated vertically upwards in a semi-infinite elastic medium is partly transmitted through the bottom boundary of the superficial elastic layer, the transmission coefficient, γ, and the reflection coefficient, β, become $\gamma=2/(\alpha+1)$ and $\beta=(\alpha-1)/(\alpha+1)$, respectively, in which, $\alpha=\rho_1 V_1/\rho_2 V_2$ and ρ_1, ρ_2; V_1, V_2 are the densities and the velocities of the surface layer and the lower medium, respectively.

It is possible to obtain the next relation by modifing (4), that is,

$$\frac{1}{2\gamma}\left\{u_{z=H}\left(\tau+\frac{H}{V}\right)+u_{z=H}\left(\tau-\frac{H}{V}\right)\right\}=\left\{F(t)+\beta F\left(t-\frac{2H}{V}\right)+\beta^2F\left(t-\frac{4H}{V}\right)+\cdots\right\}$$
$$+\left\{F\left(t-\frac{2H}{V}\right)+\beta F\left(t-\frac{4H}{V}\right)+\beta^2F\left(t-\frac{6H}{V}\right)+\cdots\right\}. \quad (5)$$

From (3) and (5), a simple relation between the resulting motions at $z=H$ and $z=0$ can be obtained as follows:

$$\frac{1}{2}\left\{u_{z=H}\left(\tau+\frac{H}{V}\right)+u_{z=H}\left(\tau-\frac{H}{V}\right)\right\}=u_{z=0}(t)+\beta^n F\left(t-\frac{\overline{2n+1}H}{V}\right), \quad (6)$$

in which $\tau=t-H/V$, and $\tau=0$ and $t=0$ correspond to the arrival time of waves at $z=H$ and $z=0$, respectively.

If we take the case in which

$$|\beta|<1, \quad (7)$$

the last term of (6) becomes as follows:

$$\beta^n F\left(t-\frac{\overline{2n+1}H}{V}\right)\longrightarrow 0, \quad (8)$$

because (3) as well as (4) are an infinite series. Substituting (8) in (6), we obtain

$$u_{z=0}(t)=\frac{1}{2}\left\{u_{z=H}\left(\tau+\frac{H}{V}\right)+u_{z=H}\left(\tau-\frac{H}{V}\right)\right\}. \quad (9)$$

Another expression of (9) is

$$u_{z=0}\left(t-\frac{H}{V}\right)=\frac{1}{2}\left\{u_{z=H}(\tau)+u_{z=H}\left(\tau-\frac{2H}{V}\right)\right\}. \quad (9')$$

Equations (9) as well as (9′) tell us that if only we know the value of $2H/V$ in a structure, even if we know neither the absolute values of the thickness, velocity and other constants of that structure or anything about the ground on which that structure stands, the earthquake motions at the foundation of that structure will be ascertained easily by utilizing the earthquake records obtained at the top of it.

One way of estimating the value of $2H/V$ is from the equation $T=4H/V$, in which T is the natural period of a structure.

3. Comparison of the theoretical and observational results

The constants of the buildings adopted in the present investigation are shown in Table 1.

The seismographs installed at the buildings are SMAC type[7]. The data of the earthquakes used in the present investigation is listed in Table 2.

In order to estimate the natural periods of the buildings, period distribution curves are obtained from the seismograms recorded at the upper part of the buildings as shown in Figs. 2–6. The natural periods of the buildings adopted in the theoretical study are shown as vertical strips in Figs. 2–6.

The final results of the theoretical study by means of equation (9) or (9′) are illustrated in Figs. 7–11. In each figure, the uppermost curve, (a), represent the actual record of earthquake motions obtained at the upper part of each building and the second, (b), and the third, (c), curves, respectively are the actual record of the lower part of that

Table 1. Constants of the buildings.

Symbol	Name	Use	Location	Constr.	Height (m)	Height No. of stories	Orient.	Breadth	Natural period T(sec)
M	Maibara-Station Bldg.	Office	Maibara	R.C.	+10.0	3	NS	60.8	0.27
							EW	19.8	0.26
K	Kanden-Sangyo Bldg.	〃	Osaka	S.R.C.	+44.5	12	NS	93.4	0.52
					−12.0	3	EW	26.1	0.84
S	Shin-Sumitomo Bldg.	〃	〃	〃	+45.0	12	NS	80.6	0.55
					−18.1	4	EW	65.1	0.62
N	Nippon Itagarasu Bldg.	〃	〃	〃	+31.0	8	NS	35.2	0.48
					−11.0	2	EW	N27.5 S31.8	〃
I	Ikebukuro-Station Bldg.	Stores	Tokyo	〃	+30.8	8	NS	94.0	0.46
					−12.0	3	EW	30.3	0.60

R.C.: reinforced concrete,
S.R.C.: steel framed reinforced concrete.

7) The constants of the SMAC seismograph used here are as follows: natural period of pendulum; 0.1 sec, mechanical magnification; 16 times, recording range; 10–1,000 gals, recording speed; 1 cm/sec, time marking; every 1 sec.

Table 2. Data of the earthquakes.

Earthq.	Bldg.	Epic. dist. (km)	Orient.	Position	Largest accel. (gal)
Mar. 27, 1963: 35.8°N, 135.8°E, 20 km depth	M	73	NS	RF	50
				1F	30
			EW	RF	89
				1F	45
	K	140	NS	12F	50
				B3F	18
			EW	12F	69
				B3F	16
	S	140	NS	RF	58
				B3F	23
			EW	RF	50
				B3F	19
	N	140	NS	RF	40
				B2F	19
			EW	RF	40
				B2F	19
Jan. 14, 1960: 36.0°N, 140.1°E, 80km depth.	I	46	NS	RF	30
				B3F	7.4
			EW	RF	57
				B3F	6.4

building and the theoretical result obtained by using the equation (9) or (9′).

In addition to these, the lowest two curves are presented in each figure, in order to support more readily the comparison of the observational result indicated by the full line, (b), to the theoretical one represented by the dotted line, (c). As will be seen from Figs. 7–11, the agreement of the observational result and the theoretical one in each building is well beyond expectation.

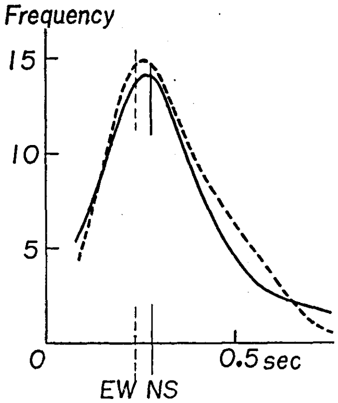

Fig. 2. Period distribution curve of the earthquake motion obtained at the roof floor of M Building.

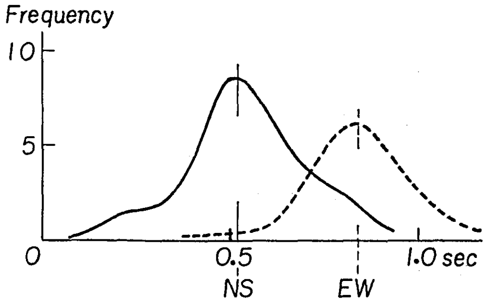

Fig. 3. Period distribution curve of the earthquake motion obtained at the twelfth floor of K Building.

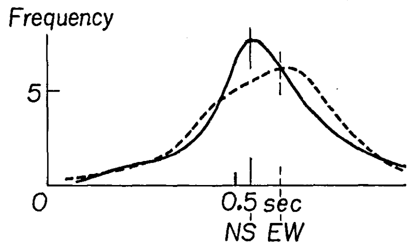

Fig. 4. Period distribution curve of the earthquake motion obtained at the roof floor of S Building.

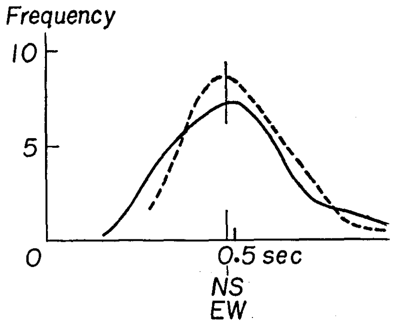

Fig. 5. Period distribution curve of the earthquake motion obtained at the roof floor of N Building.

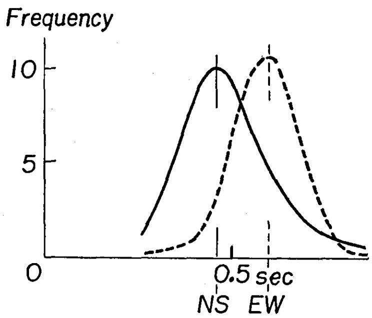

Fig. 6. Period distribution curve of the earthquake motion obtained at the roof floor of I Building.

From the results of the present investigation, we know that the vibration of a building due to earthquake motions should be treated as the multiple reflection phenomena of waves.

It is a noteworthly fact that the absorption of vibration energy or inner damping in a building is negligibly small, because there is no consideration of the attenuation of waves in that building as seen in equations (3) and (4). Further, the vibrational damping of a building relates to the so-called outer damping and depends mostly on the boundary conditions between the building and the ground, that is, β in equations (3) and (4). It may be said that the most important part of the vibrational damping of a building mentioned above is based on that at the time of an earthquake, the vibration energy of a building dissipating into the ground again as in a wide sense the elastic waves which start from the foundation.

4. Conclusion

We may complete the present investigation by a brief note on the results of the theoretical treatment.

It has been shown in the comparative diagrams of Figs. 7–11 that the vibration problem of a building should be treated as the multiple reflection phenomena of waves in that building, and a building alone cannot be regarded as a conservative system, but the vibration in that building should be considered as taking place in a dissipative system.

It is equally important fact that, as neither the solid viscosity nor the other feature of attenuation in a building has been assumed from the start of the mathematical treatment, the decaying character of the oscillation in the present problem depends entirely upon the fact that the partial emission of the waves into the ground during their multiple reflection gradually diminishes the energy of them.

In addition, it may be said that there are many observational results obtained in Japan as well as in the U. S. A. in which the ratios of the maximum acceleration of the top to the basement of the buildings took approximately two to one when the predominant periods of seismic waves were fairly shorter than the natural periods of those buildings. Although the fact mentioned above have been explained at some length on the same idea of the present investigation, details of the discussion concerning this problem will be published later.

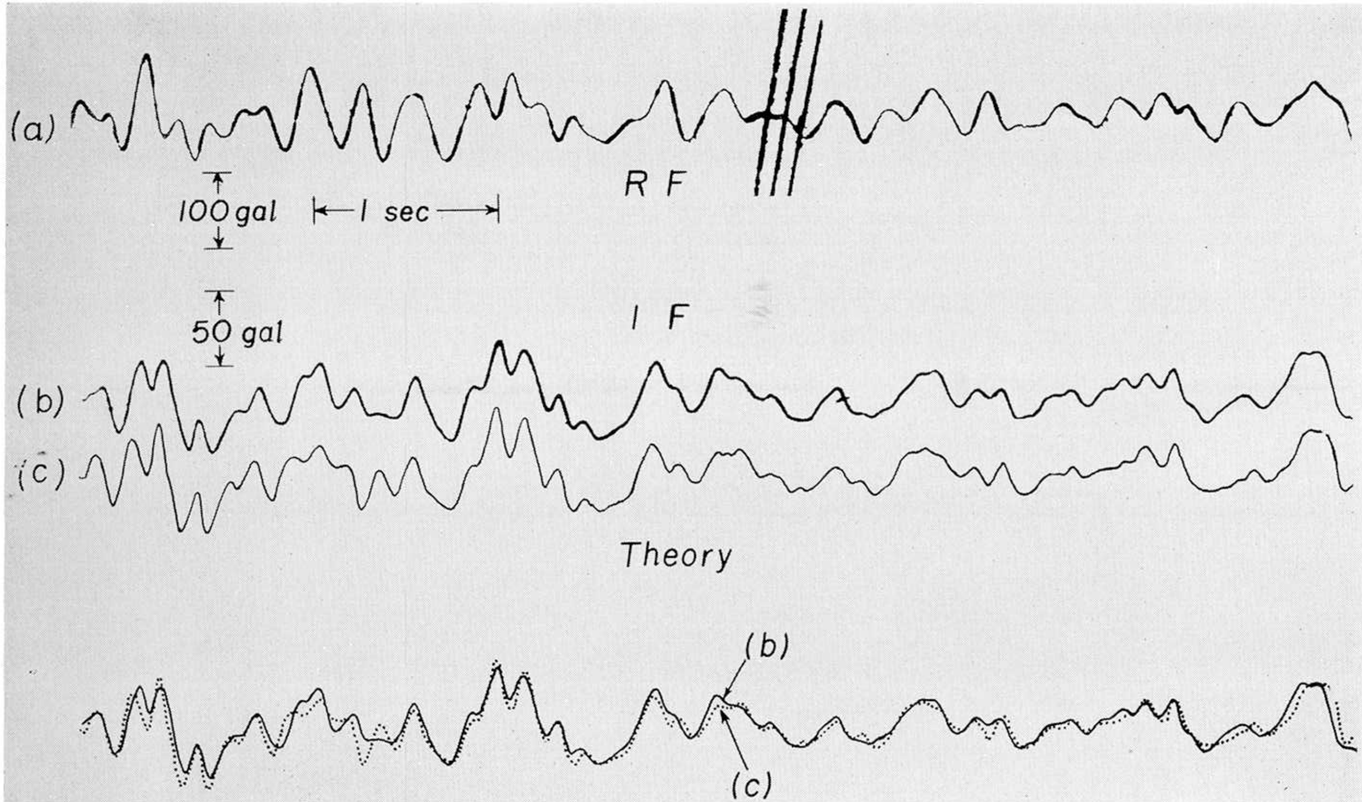

Fig. 7a. M Building. NS component. Original ×2.5. RF; roof floor, 1F; first floor.

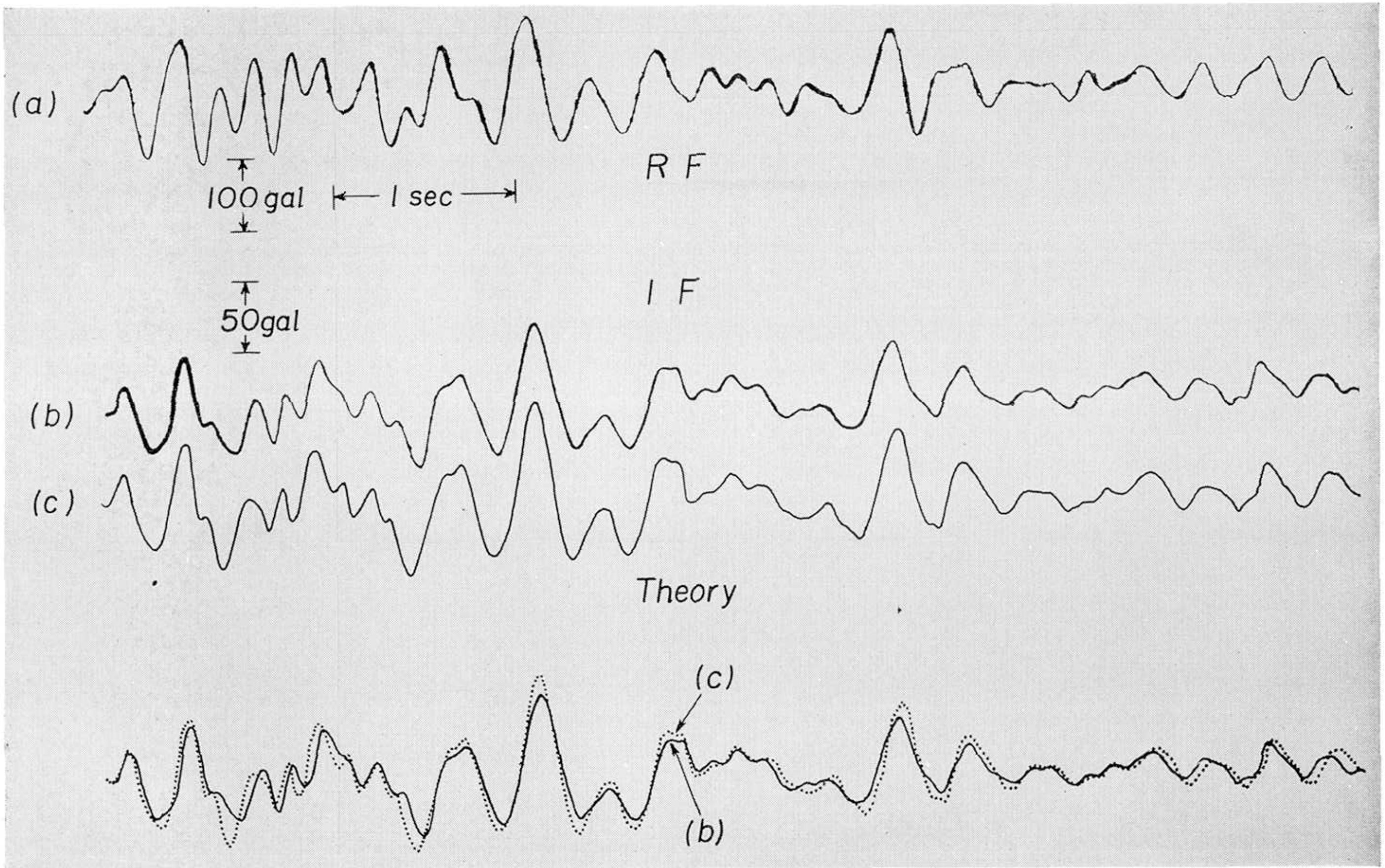

Fig. 7b. M Building. EW component. Original ×2.5. RF; roof floor, 1F; first floor.

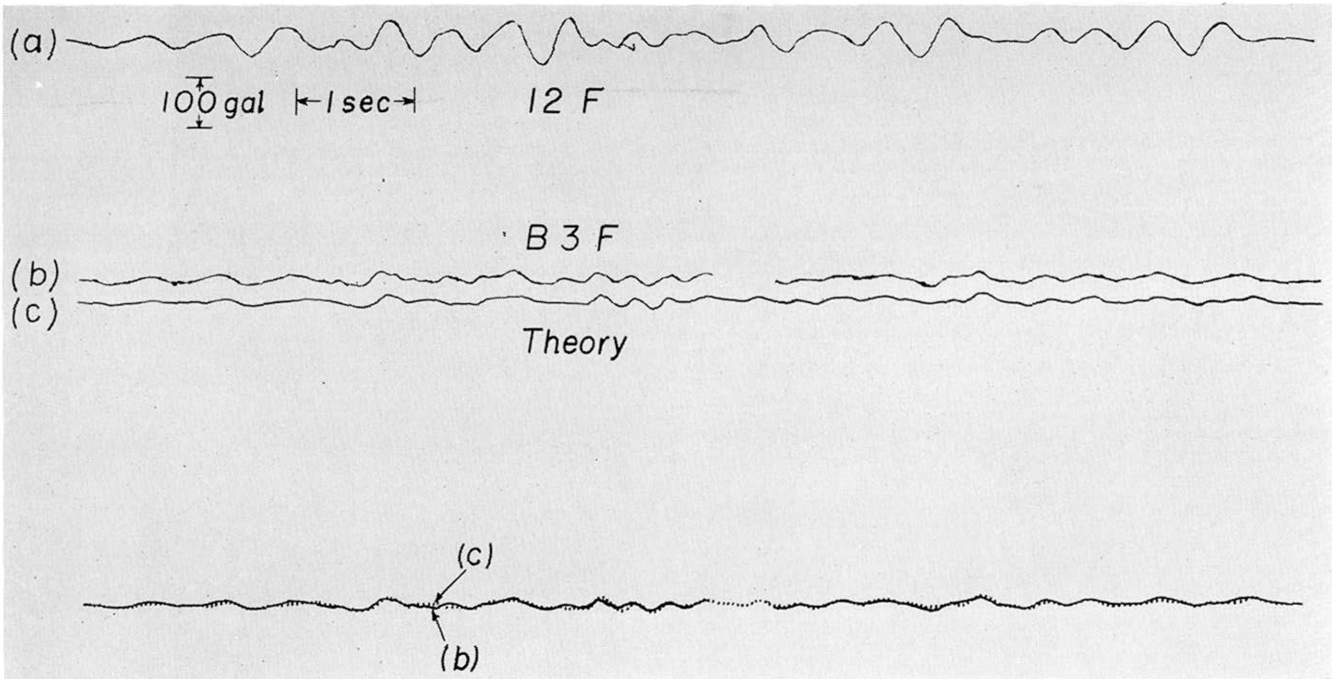

Fig. 8a. K Building. NS component. Original ×1.5. 12F; twelfth floor, B3F; basement third floor.

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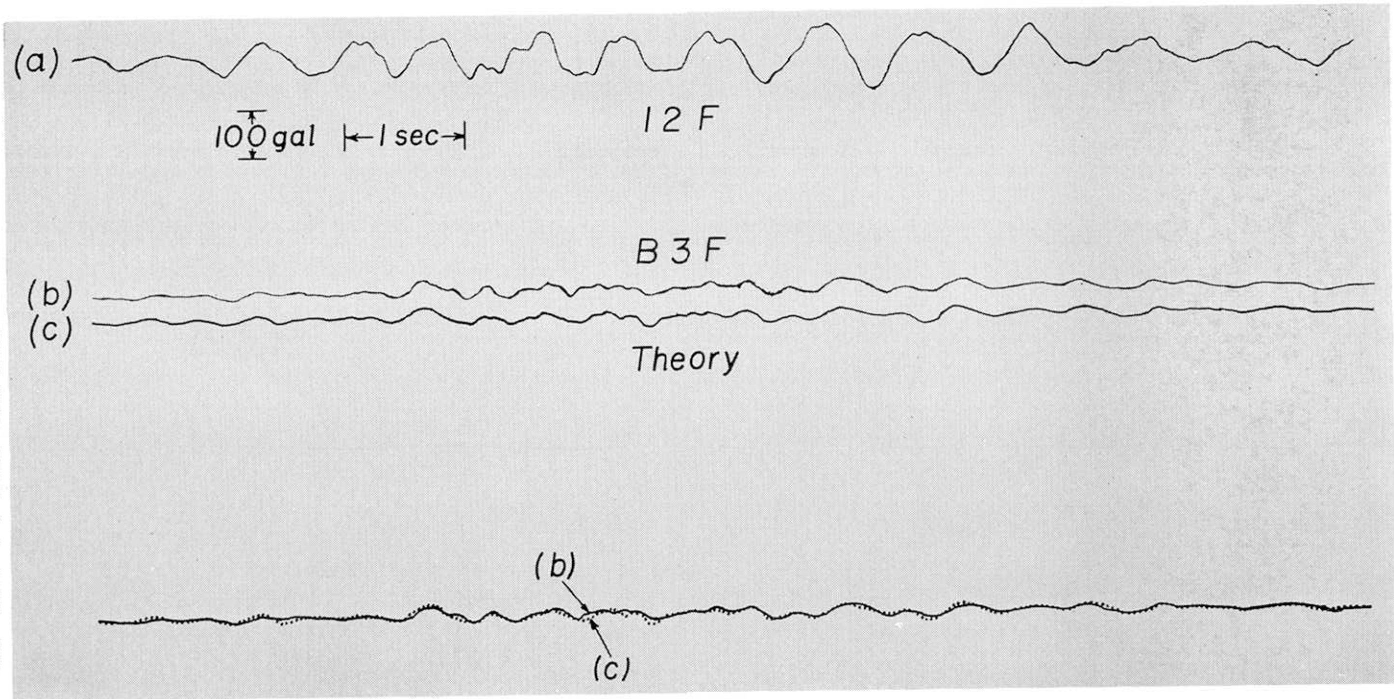

Fig. 8b. K Building. EW component. Original ×1.5. 12F; twelfth floor, B3F; basement third floor.

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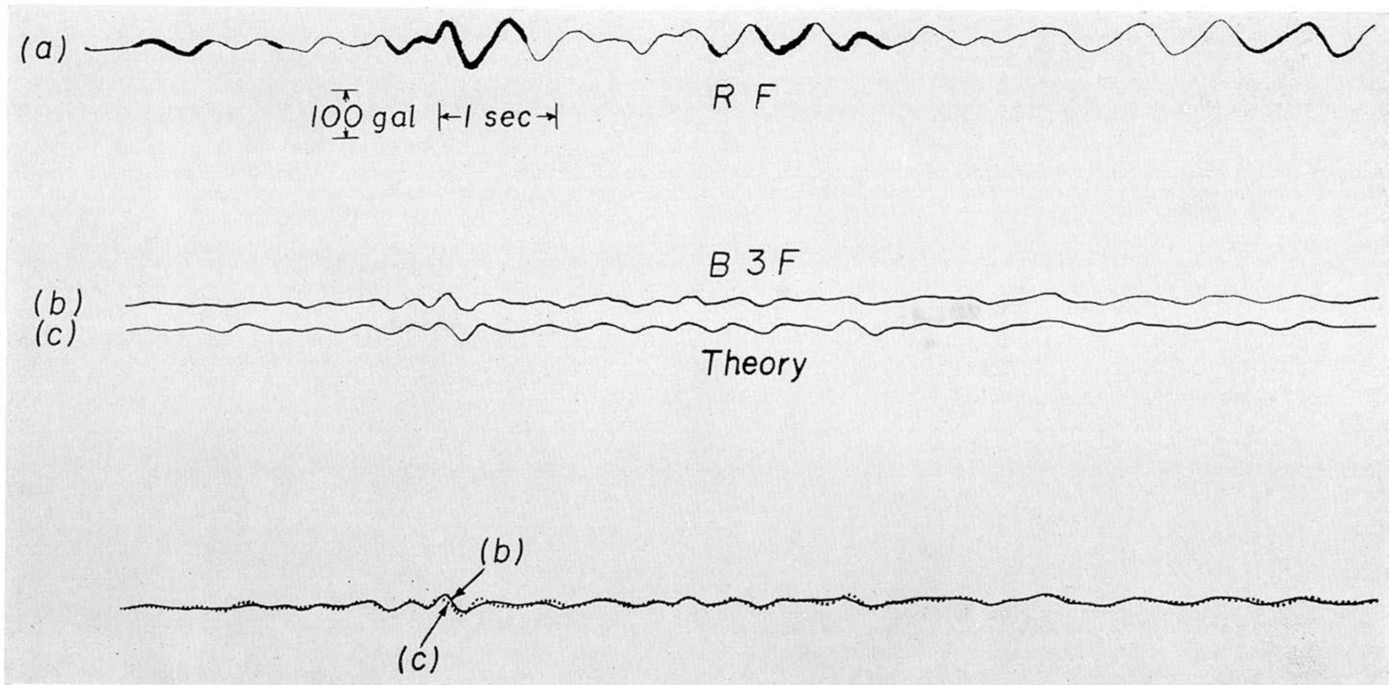

Fig. 9a. S Building. NS component. Original ×1.5. RF; roof floor, B3F; basement third floor.

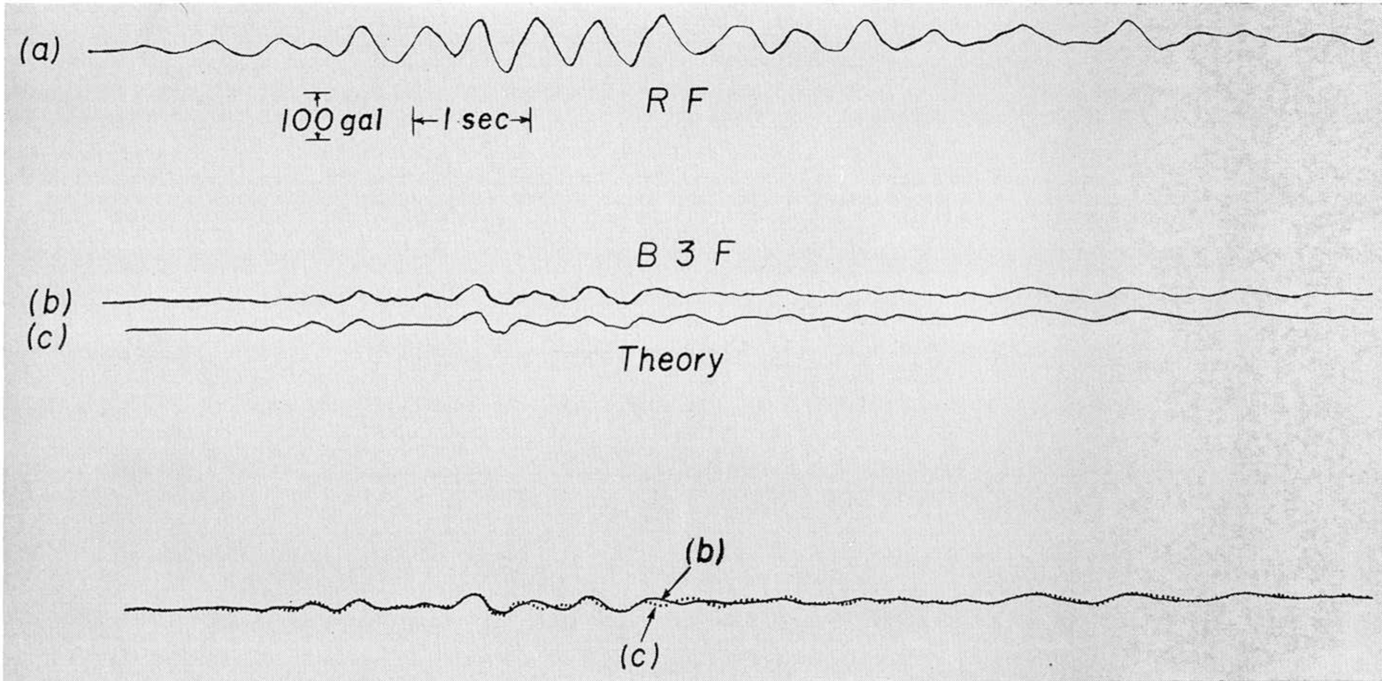

Fig. 9b. S Building. EW component. Original ×1.5. RF; roof floor, B3F; basement third floor.

[Bull. Earthq. Res. Inst., Vol. 41, Pl. 35]

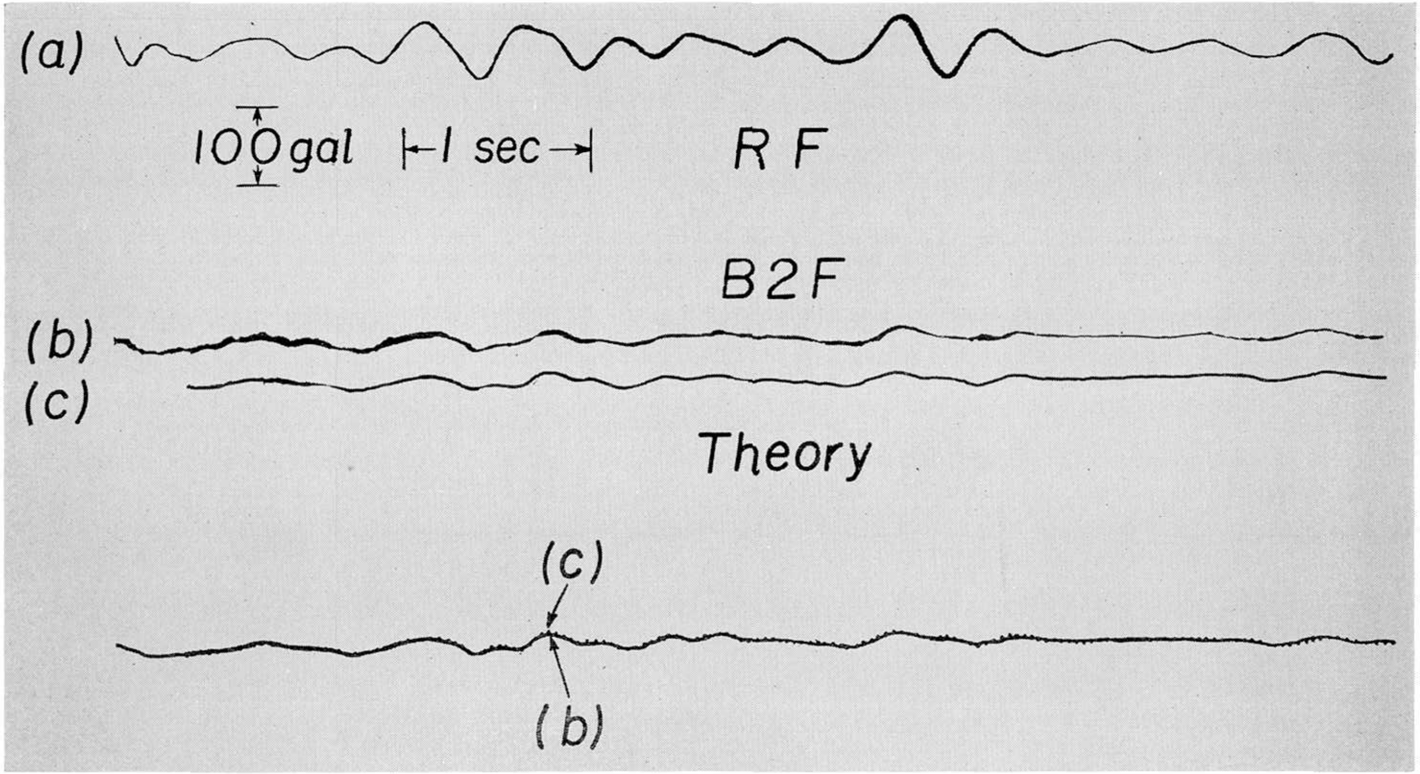

Fig. 10a. N Building. NS component. Original ×2.5. RF; roof floor, B2F; basement second floor.

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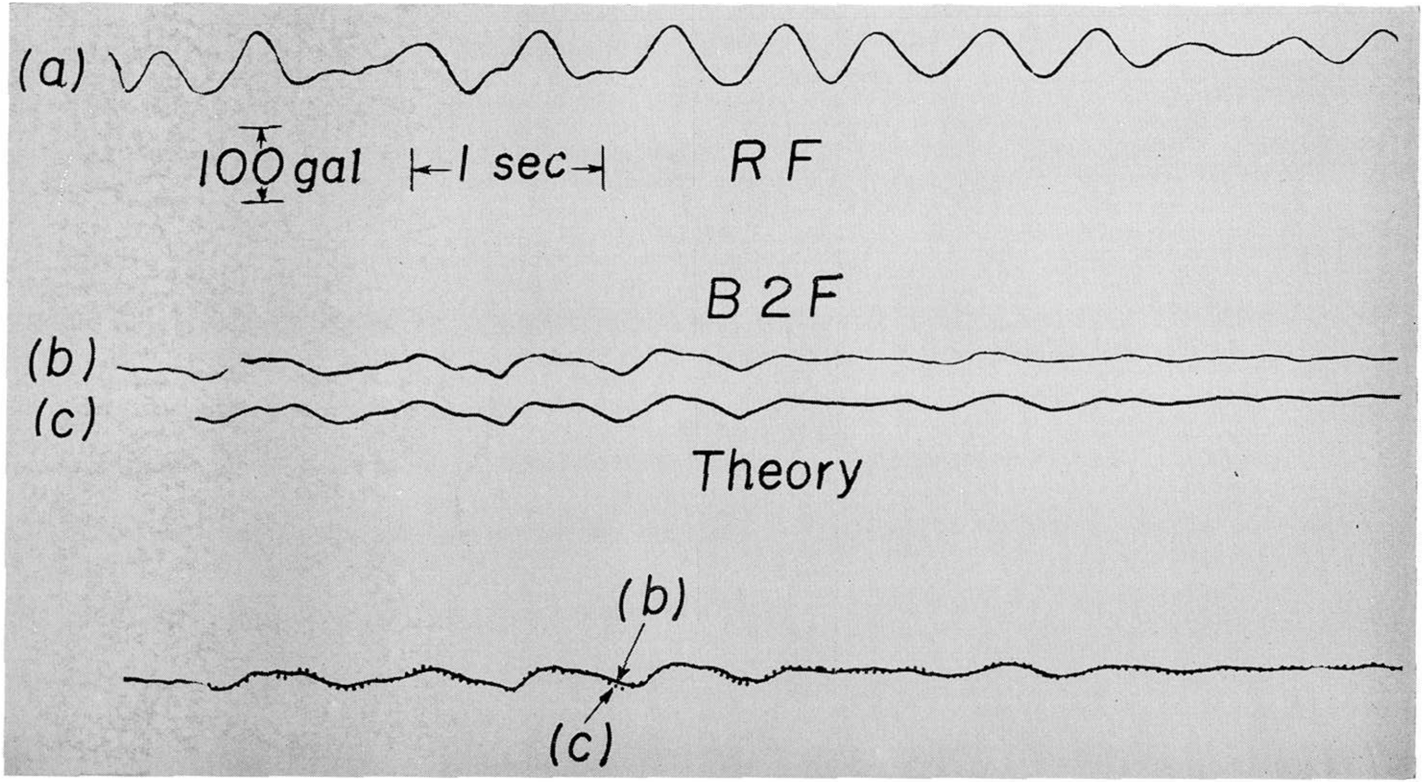

Fig. 10b. N Building. EW component. Original ×2.5. RF; roof floor, B2F; basement second floor.

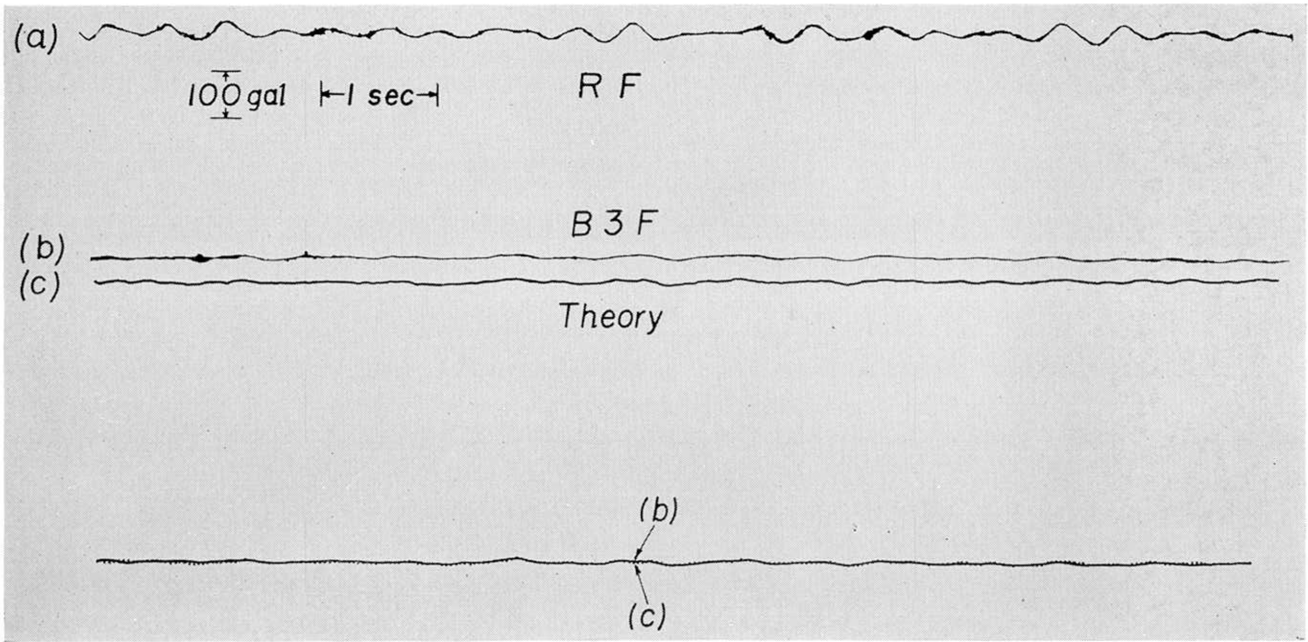

Fig. 11a. I Building. NS component. Original ×1.5. RF; roof floor, B3F; basement third floor.

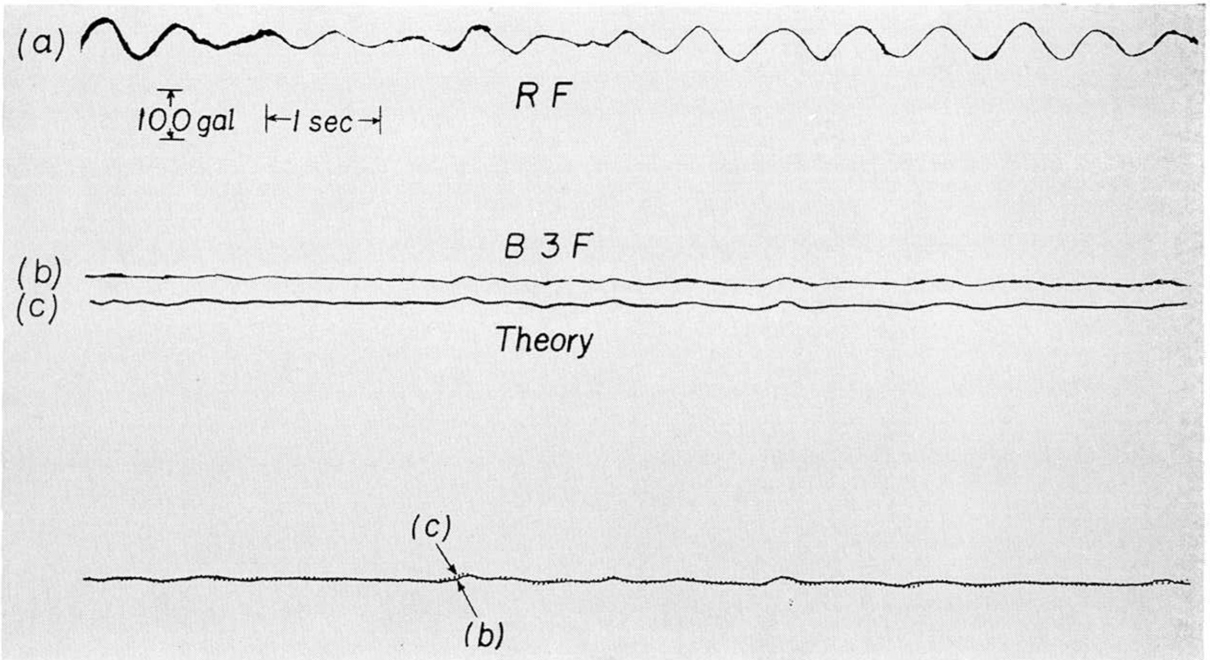

Fig. 11b. I Building. EW component. Original ×1.5. RF; roof floor, B3F; basement third floor.

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50. 地震動による構造物振動の新しい問題 第1報

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地震動による構造物振動は，その観測結果が相当な数にのぼり，いろんな見地からの解析と検討が進むにつれて，波の伝播問題としてとりあつかつた方が，実情に合うのではないかという考えが，段々に深くなつてきた.

そこで，本研究では，地表層内での地震波の重複反射の考えにもとづいて，構造物の上端の地震記録から下端の震動を算出する理論式を非常に簡単な形として導き出し，5つの実在建物に応用してみた.

この理論結果は観測結果と，予想以上に一致した.

したがつて，本研究の結果から，地震動による建物振動は波の重複反射の現象にほかならないことと，建物内部での振動減衰は無視できるぐらい小さいことが，非常にはつきりわかつた.

なお，地震動の卓越周期が建物の固有周期よりも相当に短い場合には，建物の上端と下端の地震動の最大加速度の比が2に近い値になつた実例が，日本とアメリカ合衆国にいくつかあることも，この研究のやり方で説明できることについては，次の機会に詳しく報告したい.