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35. *Tsunami in an L-shaped Canal* [I].

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1. Introduction.

In the preceding paper[1] entitled "The Effects of Coastlines on the Tsunami," we obtained the solutions of the waves plunging into a right-angled canal. These solutions are derived under the approximations that

$$\left.\begin{aligned} &\sin a_0 d \simeq a_0 d\,, \\ &\cos a_0 d \simeq 1\,, \end{aligned}\right\} \qquad \text{(I)}$$

where a_0 stands for the wave-number of the incoming waves and d the width of the canal.

In the present purview, the solutions are obtained under the more relaxed approximations that

$$\left.\begin{aligned} &\sin a_0 d \simeq a_0 d - \frac{1}{6}(a_0 d)^3\,, \\ &\cos a_0 d \simeq 1 - \frac{1}{2}(a_0 d)^2\,. \end{aligned}\right\} \qquad \text{(II)}$$

Then the applicability of the theory is considerably extended to include the range of the medium wave-length.

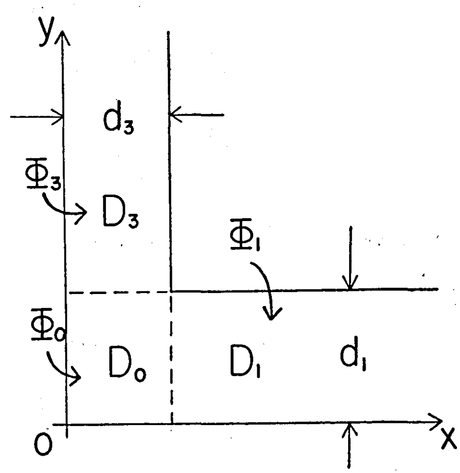

Fig. 1.

2. Theory.

Using the same notations and definitions as in the preceding paper[1] (Fig. 1), the velocity potentials are expressed as follows:

in the domain D_1,

1) T. MOMOI, *Bull. Earthq. Res. Inst.*, **40** (1962), 719.

$$\Phi_1 = A_1^{(1)} e^{-ia_0 x} \cosh a_0(H+z) + \sum_{m=0}^{\infty} A_1^{(2)m} \cos \frac{m\pi}{d_1} y \cdot e^{+ik_x^{(1)m} \cdot x} \cdot \cosh a_0(H+z) \; ; \quad (1)$$

in the domain D_3,

$$\Phi_3 = \sum_{m=0}^{\infty} A_3^m \cos \frac{m\pi}{d_3} x \cdot e^{+ik_y^{(3)m} \cdot y} \cdot \cosh a_0(H+z) \; ; \quad (2)$$

in the domain D_0,

$$\Phi_0 = \sum_{f_0} A_0(f_0) \cos k_x^{(0)} x \cdot \cos k_y^{(0)} y \cdot \cosh a_0(H+z) \; , \quad (3)$$

where

$$k_x^{(1)m} = \sqrt{a_0^2 - \left(\frac{m\pi}{d_1}\right)^2} \; ;$$

$$k_y^{(3)m} = \sqrt{a_0^2 - \left(\frac{m\pi}{d_3}\right)^2} \; ;$$

$$(k_x^{(0)})^2 + (k_y^{(0)})^2 = a_0^2 \; ;$$

$A_1^{(2)m}$, A_3^m and $A_0(f_0)$ are the arbitrary constants;
$\sum_{f_0}$ stands for integration over the range permitted by the relation $(k_x^{(0)})^2 + (k_y^{(0)})^2 = a_0^2$.

Connecting the equations (1)–(3) by the conditions between the adjacent domains and applying the operators,

$$\left. \begin{aligned} &\int_0^{d_1} \cos \frac{m\pi}{d_1} y \cdot dy \\ &\int_0^{d_3} \cos \frac{m\pi}{d_3} x \cdot dx \end{aligned} \right\} \; (m = 0, 1, 2, \cdots) \; ,$$

we have already obtained the following relations (see the preceding paper[1]):

$$A_1^{(1)} e^{-ia_0 d_3} \cdot d_1 + A_1^{(2)0} e^{+ia_0 d_3} \cdot d_1 = \sum_{f_0} A_0(f_0) \cos k_x^{(0)} d_3 \cdot \frac{1}{k_y^{(0)}} \cdot \sin k_y^{(0)} d_1 \; , \quad (4)$$

$$A_1^{(2)m} e^{+ik_x^{(1)m} d_3} \cdot \frac{d_1}{2} = \sum_{f_0} A_0(f_0) \cos k_x^{(0)} d_3 \cdot \frac{k_y^{(0)} d_1 \cdot d_1}{(k_y^{(0)} d_1)^2 - (m\pi)^2} \cdot \sin k_y^{(0)} d_1 \cdot \cos m\pi \; , \quad (5)$$

$$\begin{aligned} &(-ia_0 d_1) e^{-ia_0 d_3} A_1^{(1)} + ia_0 d_1 e^{+ia_0 d_3} A_1^{(2)0} \\ &\qquad = \sum_{f_0} A_0(f_0) k_x^{(0)} (-1) \sin k_x^{(0)} d_3 \cdot \frac{1}{k_y^{(0)}} \cdot \sin k_y^{(0)} d_1 \; , \end{aligned} \quad (6)$$

$$ik_x^{(1)m}\cdot A_1^{(2)m}\cdot e^{+ik_x^{(1)m}d_3}\cdot\frac{d_1}{2}$$

$$=\sum_{f_0}A_0(f_0)k_x^{(0)}(-1)\sin k_x^{(0)}d_3\cdot\frac{(k_y^{(0)}d_1)^2\cdot d_1}{(k_y^{(0)}d_1)^2-(m\pi)^2}\cdot\cos m\pi\ , \quad (7)$$

(The relations described above are derived by use of the conditions between the domains D_0 and D_1):

$$A_3^0e^{+ia_0d_1}\cdot d_3=\sum_{f_0}A_0(f_0)\cos k_y^{(0)}d_1\cdot\frac{1}{k_x^{(0)}}\cdot\sin k_x^{(0)}d_3\ , \quad (8)$$

$$A_3^m e^{+ik_y^{(3)m}d_1}\cdot\frac{d_3}{2}=\sum_{f_0}A_0(f_0)\cos k_y^{(0)}d_1\cdot\frac{k_x^{(0)}d_3\cdot d_3}{(k_x^{(0)}d_3)^2-(m\pi)^2}\cdot\sin k_x^{(0)}d_3\cdot\cos m\pi\ , \quad (9)$$

$$ia_0d_3e^{+ia_0d_1}A_3^0=\sum_{f_0}A_0(f_0)k_y^{(0)}(-1)\sin k_y^{(0)}d_1\cdot\frac{1}{k_x^{(0)}}\cdot\sin k_x^{(0)}d_3\ , \quad (10)$$

$$ik_y^{(3)m}\cdot A_3^m\cdot e^{+ik_x^{(3)m}d_1}\cdot\frac{d_3}{2}$$

$$=\sum_{f_0}A_0(f_0)k_y^{(0)}(-1)\sin k_y^{(0)}d_1\cdot\frac{(k_x^{(0)}d_3)^2\cdot d_3}{(k_x^{(0)}d_3)^2-(m\pi)^2}\cdot\cos m\pi\ , \quad (11)$$

(The relations (8)–(11) are obtained by use of the conditions between the domains D_0 and D_3).

In the preceding paper,[1] we used the approximation (I) described in the introduction of this paper on elimination of the expressions in the buffer domain D_0 from the above relations (4)–(11). In order to expand the applicability of the theory, the approximation (II) mentioned in the introduction is used in this paper.

Applying the approximation (II) and retaining the terms to the order of $(kd)^2$, (4), (6), (8) and (10) become as follows (see Appendix):

$$A_1^{(1)}e^{-ia_0d_3}+A_1^{(2)0}e^{+ia_0d_3}=\sum_{f_0}A_0(f_0)-\frac{1}{2}\sum_{f_0}A_0(f_0)(k_x^{(0)}d_3)^2-\frac{1}{6}\sum_{f_0}A_0(f_0)(k_y^{(0)}d_1)^2, \quad (4')$$

$$(-ia_0d_1)e^{-ia_0d_3}A_1^{(1)}+ia_0d_1e^{+ia_0d_3}A_1^{(2)0}=\sum_{f_0}A_0(f_0)(-1)(k_x^{(0)})^2d_1d_3\ , \quad (6')$$

$$A_3^0e^{+ia_0d_1}=\sum_{f_0}A_0(f_0)-\frac{1}{2}\sum_{f_0}A_0(f_0)(k_y^{(0)}d_1)^2-\frac{1}{6}\sum_{f_0}A_0(f_0)(k_x^{(0)}d_3)^2\ , \quad (8')$$

$$ia_0d_3e^{+ia_0d_1}A_3^0=\sum_{f_0}A_0(f_0)(-1)(k_y^{(0)})^2d_1d_3\ . \quad (10')$$

Adding (4') to (8'), we have

$$A_1^{(1)}e^{-ia_0d_3}+A_1^{(2)0}e^{+ia_0d_3}+A_3^0e^{+ia_0d_1}$$
$$=2\sum_{f_0}A_0(f_0)-\frac{2}{3}\sum_{f_0}A_0(f_0)(k_x^{(0)}d_3)^2-\frac{2}{3}\sum_{f_0}A_0(f_0)(k_y^{(0)}d_1)^2 . \quad (12)$$

Subtracting (8′) from (4′), we have

$$A_1^{(1)}e^{-ia_0d_3}+A_1^{(2)0}e^{+ia_0d_3}-A_3^0e^{+ia_0d_1}$$
$$=-\frac{1}{3}\sum_{f_0}A_0(f_0)(k_x^{(0)}d_3)^2+\frac{1}{3}\sum_{f_0}A_0(f_0)(k_y^{(0)}d_1)^2 . \quad (13)$$

After some reductions, (6′) and (10′) become

$$(-ia_0d_3)e^{+ia_0d_3}A_1^{(1)}+ia_0d_3e^{+ia_0d_3}A_1^{(2)0}=\sum_{f_0}A_0(f_0)(-1)(k_x^{(0)}d_3)^2 , \quad (6'')$$
$$ia_0d_1e^{+ia_0d_1}A_3^0=\sum_{f_0}A_0(f_0)(-1)(k_y^{(0)}d_1)^2 . \quad (10'')$$

Adding (6′) to (10′), we have

$$(-ia_0d_1)e^{-ia_0d_3}A_1^{(1)}+ia_0d_1e^{+ia_0d_3}A_1^{(2)0}+ia_0d_3e^{+ia_0d_1}A_3^0=-a_0^2d_1d_3\cdot\sum A_0(f_0) . \quad (14)$$

Eliminating

$$\sum_{f_0}A_0(f_0)(k_x^{(0)}d_3)^2 , \quad \sum_{f_0}A_0(f_0)(k_y^{(0)}d_1)^2 ,$$

and

$$\sum_{f_0}A_0(f_0) ,$$

from (12), (6″) and (10″), we have

$$\left(-1-\frac{2i}{a_0d_3}+\frac{2}{3}\cdot ia_0d_3\right)e^{+ia_0d_3}A_1^{(2)0}+\left(-1-\frac{2i}{d_0d_1}+\frac{2}{3}\cdot ia_0d_1\right)e^{+ia_0d_1}A_3^0$$
$$=\left(1-\frac{2i}{a_0d_3}+\frac{2}{3}\cdot ia_0d_3\right)e^{-ia_0d_3}A_1^{(1)} . \quad (15)$$

In like manner, substituting (13) for (6″) and (10″), (13) becomes

$$\left(-1+\frac{1}{3}\cdot ia_0d_3\right)e^{+ia_0d_3}A_1^{(2)0}+\left(1-\frac{1}{3}\cdot ia_0d_1\right)e^{+ia_0d_1}A_3^0$$
$$=\left(1+\frac{1}{3}\cdot ia_0d_3\right)e^{-ia_0d_3}A_1^{(1)} . \quad (16)$$

Solving the simultaneous equations (15) and (16), the expressions for the zeroth modes of the waves become as follows:

$$A_1^{(2)0}=\frac{A_1^{(1)}}{\Delta}e^{-ia_0d_3+ia_0d_1}\cdot$$

$$\cdot\left[2+\frac{4}{9}a_0^2d_1d_3-\frac{2}{3}\left(\frac{d_1}{d_3}+\frac{d_3}{d_1}\right)+i\cdot\left\{a_0(d_3-d_1)+2\left(\frac{1}{a_0d_1}-\frac{1}{a_0d_3}\right)\right\}\right], \tag{17}$$

$$A_3^0=\frac{A_1^{(1)}}{\Delta}\cdot 2i\cdot\left(-\frac{2}{a_0d_3}+\frac{1}{3}a_0d_3\right), \tag{18}$$

where

$$\Delta=e^{+ia_0(d_1+d_3)}\cdot$$

$$\cdot\left[-2+\frac{4}{9}a_0^2d_1d_3-\frac{2}{3}\left(\frac{d_1}{d_3}+\frac{d_3}{d_1}\right)+i\cdot\left\{a_0(d_1+d_3)-2\left(\frac{1}{a_0d_3}+\frac{1}{a_0d_1}\right)\right\}\right]. \tag{19}$$

As a_0d_j ($j=1, 3$) tending to zero,

and

$$\left.\begin{aligned} A_1^{(2)0}&\rightarrow\frac{d_1-d_3}{d_1+d_3}\cdot A_1^{(1)} \\ A_3^0&\rightarrow\frac{2d_1}{d_1+d_3}\cdot A_1^{(1)}\,. \end{aligned}\right\} \tag{20}$$

From the consideration of flux introduced by Lamb,[2] the same results as in (20) are easily obtained. From this fact, *Lamb's flux consideration is understood to be valid for waves long enough to be put as* $a_0d_j\simeq 0$ ($j=1, 3$).

In the same manner as in the preceding paper,[1] the higher modes of waves are obtained as follows:

$$\left.\begin{aligned} A_3^m&=A_3^0e^{+ia_0d_1}\cdot e^{-ik_y^{(3)m}d_1}\cdot\left\{-\frac{\cos m\pi}{(m\pi)^2}\right\}\cdot\frac{a_0^2d_1d_3^2\cdot k_x^{(1)m}}{(d_3k_y^{(3)m}+d_1k_x^{(1)m})}\,, \\ A_1^{(2)m}&=A_3^0e^{+ia_0d_1}\cdot e^{-ik_x^{(1)m}d_3}\cdot\left\{-\frac{\cos m\pi}{(m\pi)^2}\right\}\cdot\frac{a_0^2d_1^2d_3\cdot k_y^{(3)m}}{(d_3k_y^{(3)m}+d_1k_x^{(1)m})}\,, \end{aligned}\right\} \tag{21}$$

where the expression A_3^0 is given by (18) and (19) instead of

$$A_3^0=\frac{2d_1}{(d_1+d_3)-ia_0d_1d_3}\cdot A_1^{(1)}\cdot e^{-ia_0(d_1+d_3)}\,,$$

in the foregoing paper.[1]

3. Comparison.

In this section, the comparison is made between the results derived from Lamb's flux consideration[2] and from the approximations (I) and

2) H. LAMB, *Hydrodynamics*, (Cambridge, 1932).

(II) described in the introduction of this paper.

1. *Lamb's Method*[2]

From the flux consideration, we can readily obtain the following expressions for the zeroth mode of waves in the canal:

$$\left.\begin{aligned} A_1^{(2)0} &= \frac{d_1-d_3}{d_1+d_3}\cdot A_1^{(1)}, \\ A_3^0 &= \frac{2d_1}{d_1+d_3}\cdot A_1^{(1)}. \end{aligned}\right\} \tag{22}$$

As described in the foregoing section, the relations (22) obtained as limiting forms in our case are valid for waves of considerably long wave-length.

The weak points of Lamb's method are that: (1) the applicability of the method is considerably restricted by the condion $a_0 d_j \simeq 0$ ($j=1, 3$): (2) no knowledge about the higher modes of waves is obtained.

2. *Momoi's Method (the first order of approximation)*

The approximation (I) in the introduction is called *the first order of approximation in Momoi's method.*

In this approximation, the terms including the factors $(a_0 d_j)^m$ ($m \geqq 2$) are neglected. Hence the obtained theory is only applied under the limitation $(a_0 d_j)^m \simeq 0$ ($m \geqq 2$).

Suppose that the above relation is interpreted to be $(a_0 d_j)^2 \leqq 0.1$, the application of this theory is limited to the range $a_0 d_j \leqq 0.316\cdots$.

Theoretical formulae for the present approximation have already been given in (46), (47), (50) and (51) in the preceding paper.[1]

3. *Momoi's Method (the second order of approximation)*

The approximation (II) in the introduction is called *the second order of approximation in Momoi's method.*

In this approximation, only the terms including the factors $(a_0 d_j)^m$ ($m \leqq 3$) are retained. In the same manner as in the foregoing paragraph, if the relation $(a_0 d_j)^m \simeq 0$ ($m \geqq 4$) is considered to be $(a_0 d_j)^4 \leqq 0.1$, the theory obtained in this paper is applied to the range $a_0 d_j \leqq 0.562\cdots$.

Theoretical formulae for this approximation are given in (17), (18) and (21).

Finally, it is worth while noting that the higher modes of waves can be derived by our method but not by Lamb's.

Appendix.

Although the expanded terms of *sin* and *cos* are retained up to the order of $(kd)^3$ as shown in the introduction, the reason for using the expression "retaining the terms to the order of $(kd)^2$" is as follows:

$\sin k_x^{(0)} d_3$ in the right-hand sides of (8) and (10) is divided by $k_x^{(0)}$. Therefore, the expression $\sin k_x^{(0)} d_3 / k_x^{(0)}$ becomes of the second order of kd, i.e., $1-(1/6)(k_x^{(0)} d_3)^2$.

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35. L字水路における津波 [I]

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前報告 (本文中の脚注 1) 参照) で，著者はすでに直角水路に突入した津波に対する理論公式を次の近似の範囲で求めた．すなわち，

$$\left.\begin{aligned}\cos a_0 d &\simeq 1\,,\\ \sin a_0 d &\simeq a_0 d\,.\end{aligned}\right\}$$

本報告では，さらにこの近似を高めた形，すなわち，

$$\left.\begin{aligned}\cos a_0 d &\simeq 1-\frac{1}{2}(a_0 d)^2\,,\\ \sin a_0 d &\simeq a_0 d-\frac{1}{6}(a_0 d)^3\,.\end{aligned}\right\}$$

の範囲での理論公式を導いた．それらの理論公式は．本文中の (17), (18) および (21) で与えられる．

(17), (18) 式で $a_0 d$ を 0 に収斂させたとき，これらの理論式は (20) で与えられるごとく，Lamb の流量 (*flux*) の概念を用いて求めた結果と全く一致する．したがって，Lamb の流量による概念は波の波長が水路の巾に比して非常に大きいときにのみ適用できることを知る．