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36. *Diffraction of Tsunami Invading a Semi-circular Peninsula.*

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1. Introduction.

In the Chilean Tsunami of May 24, 1960, the nearby places of Inubo-zaki in the Kanto district and Erimo-misaki in Hokkaido, Japan, experienced very huge wave heights. The author has considered that these huge wave heights are due to the following reasons:

(1) the convergence of energy by refraction due to the submarine topography,

(2) the diffraction of waves along the wedge-shaped coast.

At an appropriate point off the tip of the headland, maximum wave heights occurred in the Chilean Tsunami. This phenomenon is thought to be due to the second reason described above. In order to ascertain this possibility, when the tsunami attacked the semi-circular peninsula, we computed the wave heights along the coast. As a result of this calculation, it turns out that the maximum wave heights take place at the root of the peninsula, and are caused by the diffraction of waves along the coast.

2. Theoretical Analysis.

Referring to Fig. 1, the polar coordinates (r, θ) are centered at the center of the semi-circle of the peninsula, the basal line being taken in the direction of the open sea: the radius of the peninsula is r_0: the surging waves are directed perpendicular to the straight coastline.

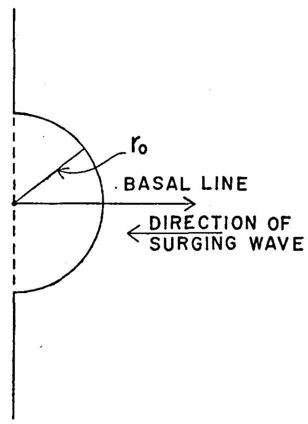

Fig. 1.

Then the equation for the periodic waves is

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}+k^2\right)\zeta=0\ , \qquad (1)$$

where ζ is the wave height and k the wave number of the surging waves, which is related to the angular frequency (ω) and the long wave velocity ($c=\sqrt{gH}$: g; the acceleration of the gravity: H; the depth of water).

The boundary condition at the straight coast ($x=0$: x is the x-component of the Cartesian coordinates) becomes

$$\frac{\partial \zeta}{\partial x}=\frac{\partial \zeta_r}{\partial x}+\frac{\partial \zeta_{in}}{\partial x}=0 \quad (x=0)\,, \tag{2}$$

when ζ_{in} and ζ_r are the incident and the reflected waves respectively.

The condition at the rigid boundary of the peninsula is

$$\frac{\partial \zeta}{\partial r}=0 \quad (r=r_0)\,. \tag{3}$$

Let the incident wave (ζ_{in}) be

$$\zeta_{in}=\zeta_0 e^{+ikx}\,,$$

where the vibrating time factor $\exp(+i\omega t)$ is omitted as usual, the only real part has physical meaning and ζ_0 is the amplitude.

Then the condition (2) is reduced to

$$\frac{\partial \zeta_r}{\partial x}=-ik\zeta_0 \quad (x=0)\,. \tag{2$'$}$$

The particular soultion ($\zeta_r^{(p)}$) of the equation (1) satisfying the condition (2$'$) is given by

$$\zeta_r^{(p)}=\zeta_0 e^{-ikx}\,.$$

Suppose that $\zeta_r^{(h)}$ is the general solution of (1) satisfying the *homogeneous* condition $\partial\zeta_r^{(h)}/\partial x=0$, which is put as the right-hand side of (2$'$) becomes zero, the general solution for the *inhomogeneous* condition (2$'$) is given by

$$\zeta_r=\zeta_r^{(h)}+\zeta_r^{(p)}\,. \tag{5}$$

$\zeta_r^{(h)}$is readily obtained by use of Hankel function as follows:

$$\zeta_r^{(h)}=\sum_{m=0}^{\infty} A_{2m}^{(2)}\cos 2m\theta\cdot H_{2m}^{(2)}(kr)\,, \tag{6}$$

where $A_{2m}^{(2)}$ is the arbitrary constant.

In the expression (6), only the second kind of Hankel function is retained, allowing for the outgoing damping waves, and the azimuthal modes are selected for the condition $\partial\zeta_r^{(h)}/=\mp(1/r)(r\zeta_r^{(h)}/\partial\theta)=0$ $(\theta=\mp\pi/2)$ to be satisfied.

Substituting (4) and (6) for (5), ζ_r becomes

$$\zeta_r=\zeta_0 e^{-ikx}+\sum_{m=0}^{\infty} A_{2m}^{(2)} \cos 2m\theta \cdot H_{2m}^{(2)}(kr) .$$

The general solution in the open sea is given by the sum of the incident (ζ_{in}) and the reflected waves (ζ_r), *i. e.*,

$$\zeta=\zeta_{in}+\zeta_r=2\zeta_0 \cos kx+\sum_{m=0}^{\infty} A_{2m}^{(2)} \cos 2m\theta \cdot H_{2m}^{(2)}(kr) . \tag{7}$$

Since $\cos kx$ is expressed by the Bessel series, (7) becomes as follows:

$$\zeta=2\zeta_0 \sum_{m=0}^{\infty}(-1)^m \varepsilon_m \cos 2m\theta \cdot J_{2m}(kr)+\sum_{m=0}^{\infty} A_{2m}^{(2)} \cos 2m\theta \cdot H_{2m}^{(2)}(kr) . \tag{8}$$

Here the arbitrary constant $A_{2m}^{(2)}$ is left undetermined and

$$\varepsilon_m=\begin{cases}1 & (m=0) ,\\ 2 & (m\geqq 1) .\end{cases}$$

In order to determine $A_{2m}^{(2)}$, substituting (8) for (3) and integrating (3) from $-\pi/2$ to $+\pi/2$ in terms of θ, we have

$$A_{2m}^{(2)}=(-1)^{m+1}\cdot 2\zeta_0\varepsilon_m\cdot\frac{J_{2m}'(kr_0)}{H_{2m}^{(2)\prime}(kr_0)} . \tag{9}$$

Putting (9) into (8), we finally obtain

$$\zeta=2\zeta_0\sum_{m=0}^{\infty}(-1)^m\cdot\varepsilon_m\cos 2m\theta\cdot\left\{J_{2m}(kr)-\frac{J_{2m}'(kr_0)}{H_{2m}^{(2)\prime}(kr_0)}H_{2m}^{(2)}(kr)\right\} . \tag{10}$$

In the following section, the numerical analysis and the discussions are made.

3. Numerical Analysis and Discussion.

In order to calculate the wave heights at the coasts the expression (10) is transformed into the forms convenient for calculation.

Firstly, a simplified expression at the coast of the semi-circular peninsula is obtained;

Putting $r=r_0$ in (10),

$$\zeta=2\zeta_0\sum_{m=0}^{\infty}(-1)^m\cdot\varepsilon_m\cdot\cos 2m\theta\cdot\left\{J_{2m}(kr_0)-\frac{J'_{2m}(kr_0)}{H^{(2)\prime}_{2m}(kr_0)}\cdot H^{(2)}_{2m}(kr_0)\right\}. \tag{11}$$

Using Lommel's formula:

$$J_\nu(z)H^{(2)\prime}_\nu(z)-J'_\nu(z)H^{(2)}_\nu(z)=-2i/(\pi z),$$

(11) becomes

$$\zeta=\zeta_0\cdot\frac{4}{i\pi kr_0}\cdot\sum_{m=0}^{\infty}(-1)^m\cdot\varepsilon_m\cdot\cos 2m\theta\cdot\frac{1}{H^{(2)\prime}_{2m}(kr_0)}, \tag{12}$$

which is used only when the calculation is made at the coast of the peninsula.

Secondly, a simplified expression at the straight coast is considered: Putting $\theta=\pi/2$ in (10),

$$\zeta=2\zeta_0\sum_{m=0}^{\infty}\varepsilon_m\cdot\left\{J_{2m}(kr)-\frac{J'_{2m}(kr_0)}{H^{(2)\prime}_{2m}(kr_0)}\cdot H^{(2)}_{2m}(kr)\right\}. \tag{13}$$

The actual computation is made only for the case $kr_0=4$, which is sufficient to explain the reason why the maximum wave height takes place in the vicinity of the root of semi-circular peninsula. The wave heights at the coast are computed using the expressions (12) and (13).

In the calculation, the terms are retained from 0 to 4 in terms of m. The result of the calculation is tabulated in the following. For convenience of expression, a new notation is introduced, *i.e.*,

$$\zeta^{(R)}(kr,\theta)=\zeta(kr,\theta)/\zeta_0.$$

The values of the wave heights at the coast.

$$\left.\begin{array}{ll}|\zeta^{(R)}(4,\ 0)|=2.1539, & |\zeta^{(R)}(4,\ \pi/16)|=1.9383,\\ |\zeta^{(R)}(4,\ \pi/8)|=1.6680, & |\zeta^{(R)}(4,\ 3\pi/16)|=2.0469,\\ |\zeta^{(R)}(4,\ \pi/4)|=2.5188, & |\zeta^{(R)}(4,\ 5\pi/16)|=2.0540,\\ |\zeta^{(R)}(4,\ 3\pi/8)|=0.68764, & |\zeta^{(R)}(4,\ 7\pi/16)|=1.5726,\\ |\zeta^{(R)}(4,\ \pi/2)|=2.6402, & \end{array}\right\} \tag{14}$$

$$|\zeta^{(R)}(5,\ \pi/2)|=2.6772, \qquad |\zeta^{(R)}(6,\ \pi/2)|=2.4660. \tag{15}$$

The wave heights are graphically expressed in Fig. 2. At the coast of the semi-circular peninsula, the undulation of the wave heights (14) gradually increases in the sense of θ increasing, until the maximum

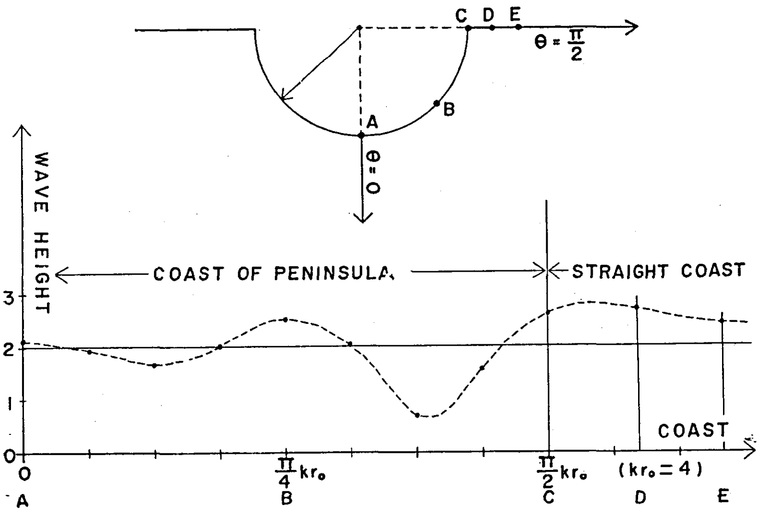

Fig. 2.

occurs in the vicinity of $\theta=\pi/2$; and departing from the peninsula along the straight coast, the wave heights (15) gradually decrease (refer to Fig. 2).

To interpret these undulations, the assumption is made that *the incident waves at the coast of the semi-circular peninsula are all diffracted along the coast.*

Referring to Fig. 3, consider the coupling between the diffracted wave packet (symbolized by *dif. wave*) at the tip of the peninsula ($\theta=0$) and the directly incident wave (symbolized by *dir. wave*).

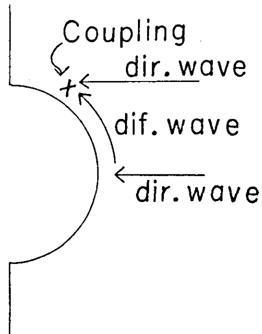

Fig. 3.

In Fig. 4, let the abscissa be the position along the coast of the semi-circular peninsula and the ordinate the wave height. Along the coast, the diffracted wave seems to advance with the same wave length as the incident one, while the directly incident wave has the elongated wave length for the oblique coast. As shown in Fig. 4, the resultant wave height of the dif. and the dir. waves undulates along the coast with gradually increasing amplitude.

The value of $\zeta^{(R)}$ at the tip ($kr=4$, $\theta=0$) of the peninsula is a

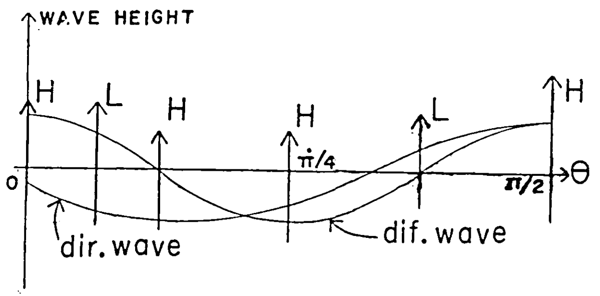

Fig. 4. "*H*" and "*L*" stand for the high and the low resultant wave heights of the dir. and dif. waves.

little larger than 2. It seems to be due to the deficiency of the number of the computed terms. Anyway, the undulation in Fig. 2 is beyond the error due to the deficiency of the terms.

In the Chilean Tsunami of May 24, 1960,[1] which attacked capes throughout Japan, the maximum wave heights are experienced at a little apart from the tip of the headland.

The coupling between the directly incident and the diffracted waves along the coasts is considered to be one of the possible explanations for this phenomenon.

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36. 半円形の半島をおそつた津波の回折

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1960年のチリ津波が日本をおそつたとき，北海道の襟裳岬，関東地方の犬吠岬で岬より少し根元に寄つたところで，津波の波高が最大値をとることを見た．筆者は，この現象を，津波の海岸線にそつての回折であると考え，これを定性的に説明せんとの考えのもとに，半円形の半島をおそった津波の回折を海岸線についてのみ計算した．そして回折波によつて，波高の最大点が実際に起り得ることを知つた．

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1) The Committee for the Field Investigation of the Chilean Tsunami of 1960, *Report on the Chilean Tsunami of May 24, 1960, as observed along the Coast of Japan (1961).*