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37. *The Fracture of a Semi-infinite Body Caused by an Inner Stress Origin and Its Relation to the Earthquake Phenomena (Second Paper).*

—The Case of the Materials Having Some Heterogeneous Structures—

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1. Introduction.

This paper is a continuation of a preceding paper[1] in which fracturing of a semi-infinite body caused by an inner stress origin was experimentally investigated. According to previous experiments[2][3] on the uniform stress states, the mechanical structure of materials gives such a decisive influence on some important characteristics in the fracture occurrence. Therefore, in this series of experiments, the influences of the mechanical structure of materials are also studied. In the preceding paper[4], fractures of a completely homogeneous material and an extremely heterogeneous one have been investigated. In a homogeneous material, a large fracture suddenly occurs and many small fractures follow. The initial fracture is predominantly large. On the other hand, when the material is extremely heterogeneous in structure, small fractures begin to increase at a low stress state and the magnitude of fracture increases gradually with time. In the case, we cannot find a single predominantly large fracture.

In this paper, an intermediate case between the above mentioned two extreme cases is investigated. This case is most important for us, because the earth's crust is neither completely homogeneous nor extremely heterogeneous like a clastic material, but it is heterogeneous in some degree. The concrete is used as a suitable model material which seems to be analogous to the earth's crust in structure. On this

1) K. MOGI, *Bull. Earthq. Res. Inst.*, **40** (1962), 815-829.
2) K. MOGI, *Bull. Earthq. Res. Inst.*, **40** (1962), 125-173.
3) K. MOGI, *Bull. Earthq. Res. Inst.*, **40** (1962), 831-853.
4) K. MOGI, *loc. cit.*, 1).

model material, elastic shocks accompanying fractures and the deformation of the surface of a semi-infinite body under the above mentioned stress state are investigated, and the pattern of fracture occurrence in this case is compared with the previous two extreme cases. Magnitude-frequency relations of the elastic shocks are also investigated and their analogy to those of earthquakes is discussed.

2. Experimental procedure.

The schematic view of the laboratory set is shown in Fig. 1.

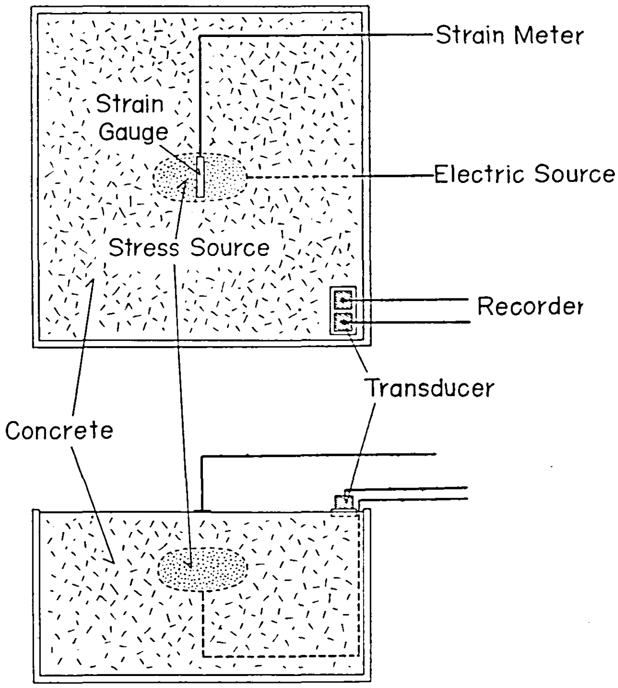


Fig. 1. Schematic view of experimental arrangement.

Concrete of various kinds was used as model materials. The stress was applied by the volume change of a paraffin ball buried at some depth from the surface of the concrete. The elastic shocks accompanying fractures in the concrete were picked up by piezo-electric type transducers and were recorded by a tape recorder and re-recorded by an electro-magnetic oscillograph. The strain at the surface of the medium was measured by an electric resistance strain meter. The experimental procedure is similar to that of the previous experiment[5].

Stress source. Two types of stress sources were used. The one was a prolate spheroid and the other was an oblate spheroid. Their dimensions are shown in Fig. 2 (a). The structure of the stress source is shown in Fig. 2 (b). The stress source is a paraffin ball, including

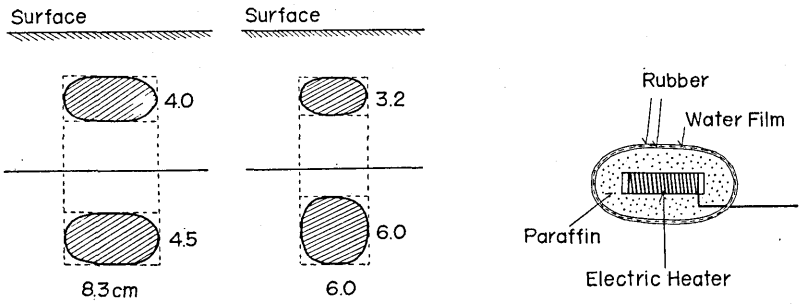


Fig. 2. (a) Dimensions of stress sources; (b) Internal structure of stress source.

a small electric heater, which is covered doubly by thin rubber film. When the electric heater is switched on, the solid paraffin begins to melt partially and melting proceeds gradually with time. The volume of paraffin increases about 10% on melting. The total volume of paraffin increases monotonically with time. Then the rubber cover prevents leakage of the melted paraffin to the concrete medium. The concrete was set in a wooden box of 23 cm(height) $\times$ 30 cm $\times$ 30 cm(bottom) and the paraffin ball was buried at 3$\sim$5 cm depth from the surface.

Thus, the increasing hydrostatic pressure was applied to the surrounding body. This noiseless stress application has a great advantage for high sensitive measurement of elastic shocks. The strain gauge was pasted at the center of the surface.

Model materials. As a moderately heterogeneous material, the concrete was used. As seen in a later section, the magnitude-frequency relations of elastic shocks in this material is nearly similar to that of general earthquakes. Therefore, from the previous experimental results, the heterogeneous structure of the concrete is deduced to be approximately similar to that of the earth's crust. Model materials of various structures were obtained by mixing of the following materials. The structure of these materials is shown in photographs of Fig. 13.

5) K. MOGI, *loc. cit.*, 1).

(1) *Sand* : Fine sand (75%)+Cement (25%). Grain size of fine sand is 0.2~0.7 mm.

(2) *C. Sand*: Coarse sand (75%)+Cement (25%). Grain size of coarse sand distributes on the wide range (0.3~3 mm).

(3) *Pumice* : Pumice (50%)+Sand (25%)+Cement (25%). Grain size of pumice particles fluctuates (0.3~4 mm).

(4) *Pebble* : Pebble (50%)+Sand (25%)+Cement (25%). Size of pebbles is nearly constant (3~5 mm). Therefore, the structure of this mixture is very regular.

3. Experimental results.

When the electric heater inside the stress source is switched on, the pressure at the source increases monotonically with time, as mentioned above. Although the pressure was not directly measured in all cases, it may be approximately estimated from a pressure-strain relation shown in Fig. 3. When the pressure increases gradually, small

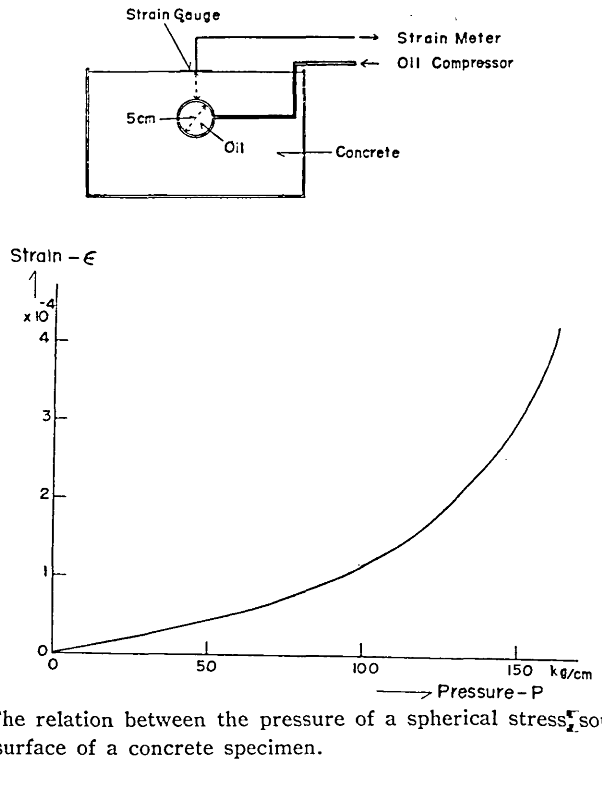


Fig. 3. The relation between the pressure of a spherical stress source and the strain at the surface of a concrete specimen.

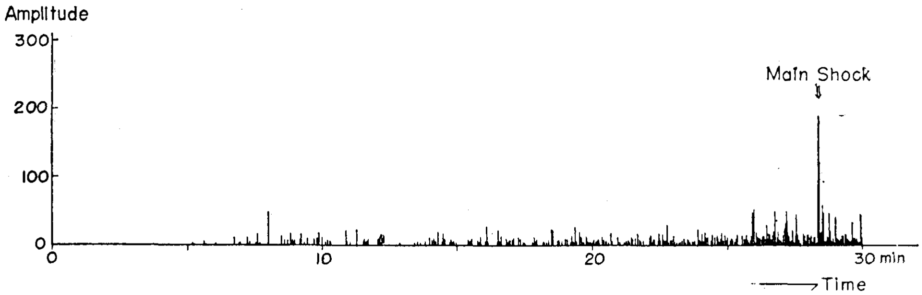


Fig. 4. Typical example of shock occurrence under increasing stress.

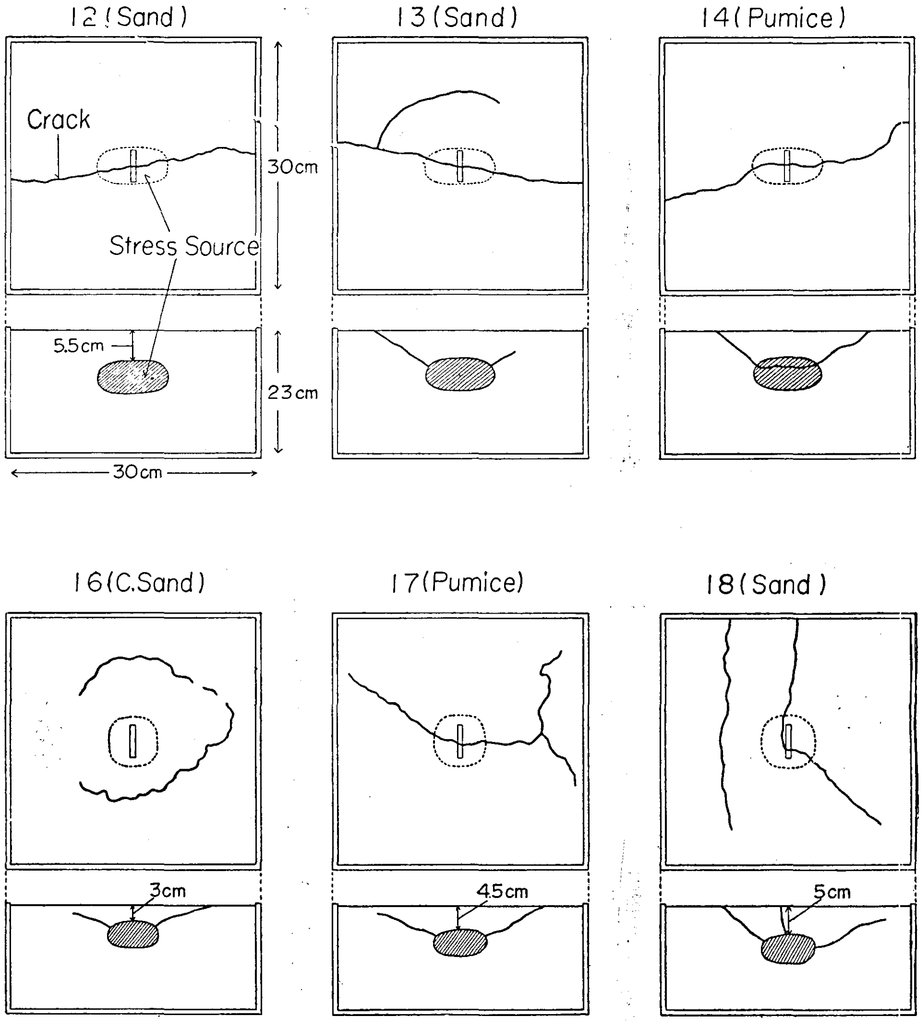


Fig. 5. Patterns of large cracks. upper: prolate source; lower: oblate source.

elastic shocks accompanying local fractures in the material begin to occur and their number and intensity increase gradually with time. After a while, a predominantly large shock occurs and many small shocks follow it. A typical pattern of the shock occurrence is shown in Fig. 4. Visible cracks appear at the time of occurrence of the main shock. Some examples of crack patterns are represented in Fig. 5 (or Fig. 14). In this figure, numerous minor cracks which are considered to have occurred at the stressed region are omitted. In the case of a prolate source, a large crack always appears on a vertical plane containing a long axis of the source and another large crack appears on a conical surface. In the case of an oblate source, the latter type seems to be most ordinary.

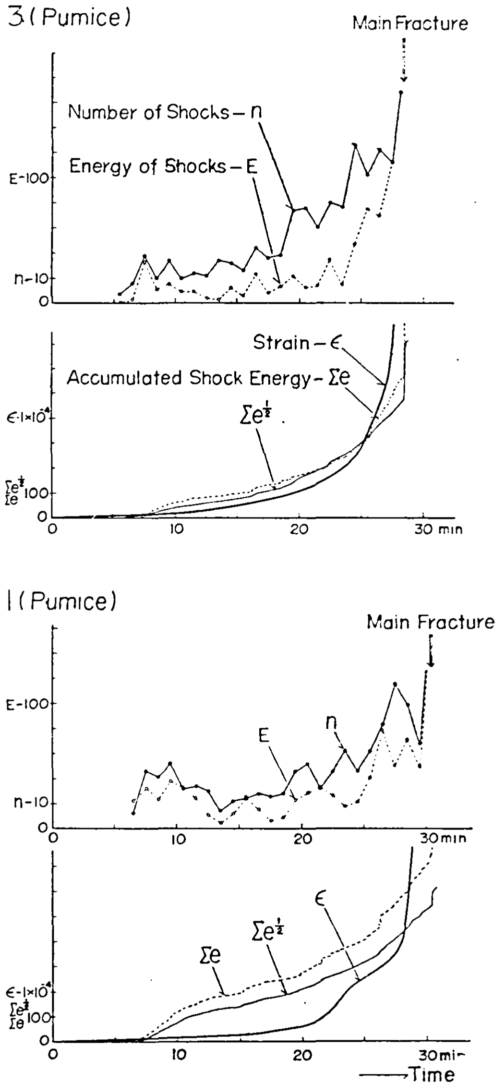


Fig. 6 (a).

The pattern of the successive occurrence of elastic shocks.

As mentioned above, the pattern of shock occurrence in the moderately heterogeneous structure is remarkably different from those both in the homogeneous and the extremely heterogeneous. A predominantly large shock occurs and numerous aftershocks follow it, as in the homogeneous case, but in the present case, small elastic shocks increase remarkably preceding the main shock. The occurrence of these foreshocks depends on the heterogeneous structure of materials in the

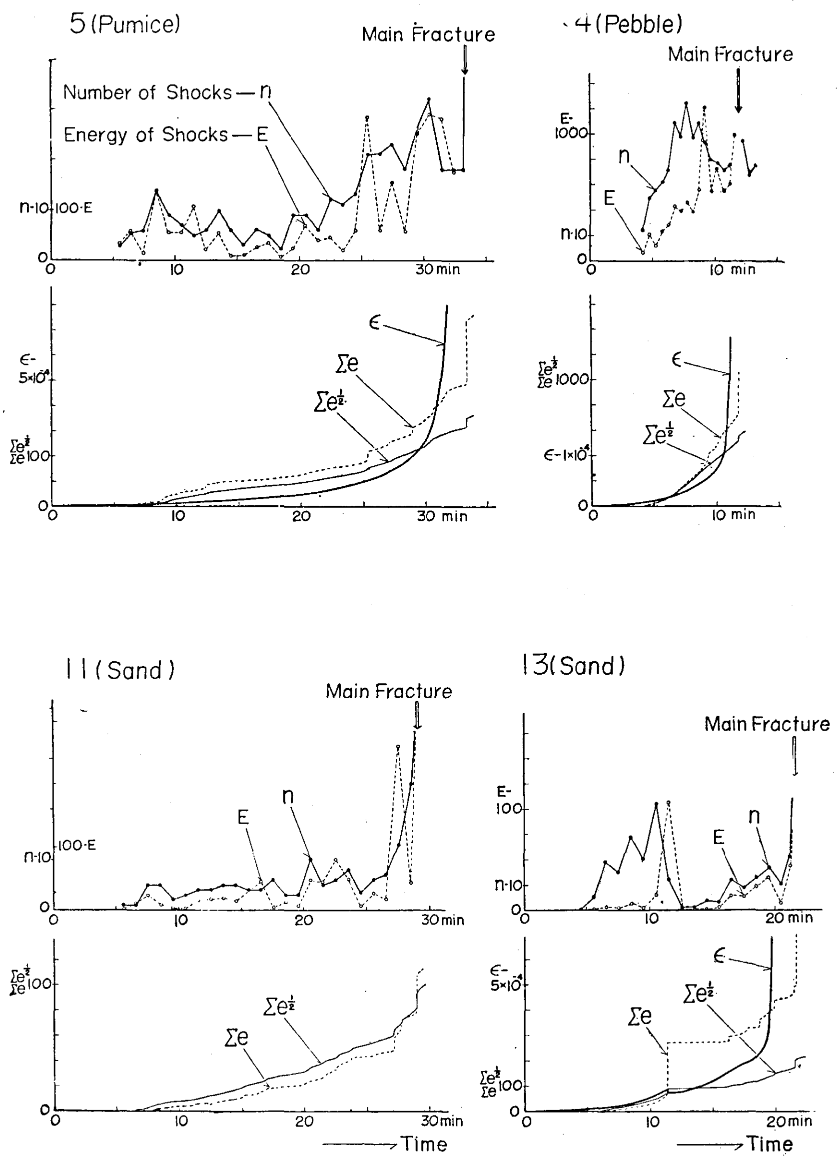


Fig. 6 (b).

Fig. 6. Changes in the number of elastic shocks (n), the energy of elastic shocks per min (E), accumulated shock energy ($\sum e$), accumulated values of the root of the energy of each elastic shock ($\sum e^{1/2}$) and the strain at the surface of the specimen (ϵ) before a main fracture.

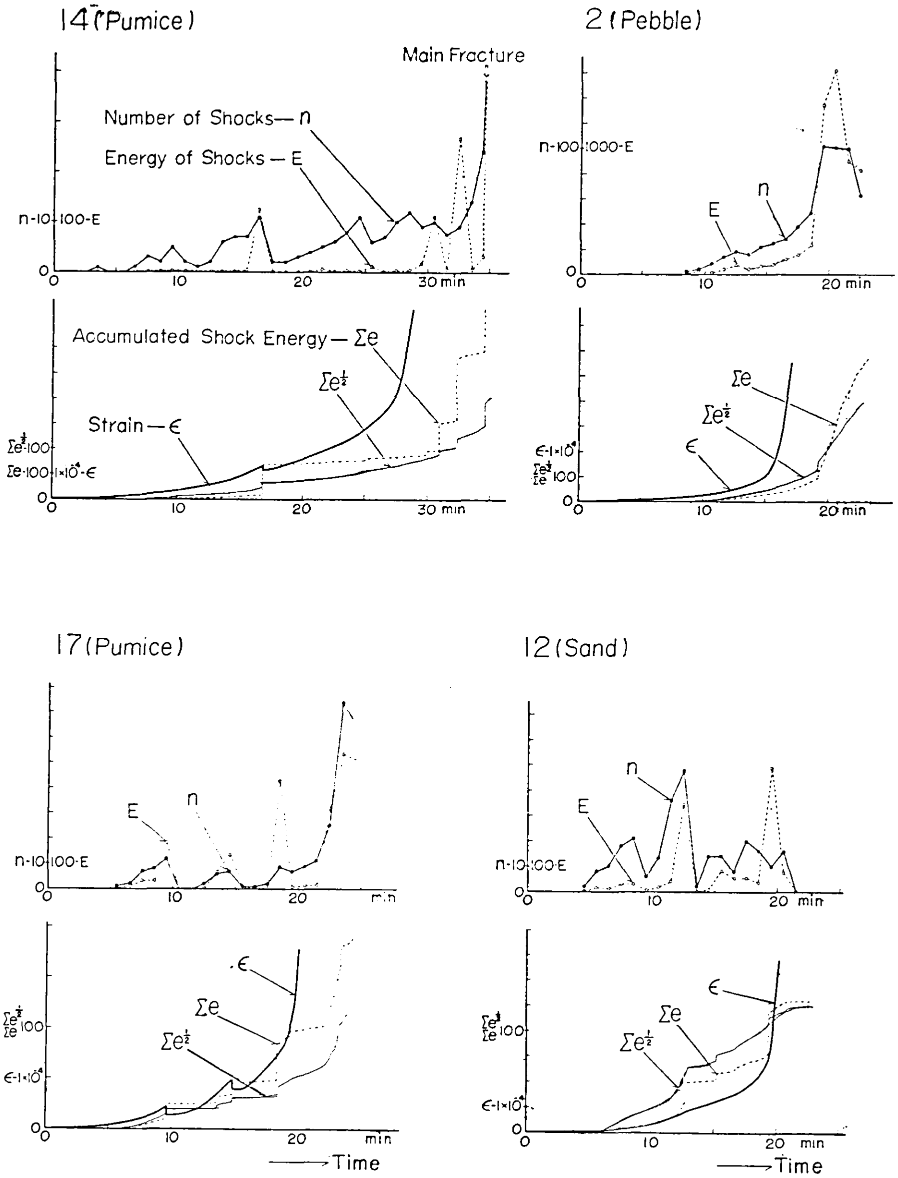


Fig. 6 (c).

following way. That is, when a stress is applied to the material, stress concentration takes place at many structurally irregular points in the body and local fractures at such points occur at a stress lower than the normal strength of the material. Fig. 6 (a) shows typical cases of shock occurrence before a main fracture. Upper curves show monotonical increases of number (n) of elastic shocks and of energy (E) of elastic shocks per min. The lower curves indicate changes in the accumulated energy ($\sum e$) of elastic shocks, the accumulated value ($\sum\sqrt{e}$) of the root of energy of an elastic shock, and the strain (ϵ) at the surface of the central part. $\sum(\sqrt{e})$ is named *strain characteristics* after Benioff[6]. Thus, both the energy of elastic shocks and the strain increase gradually with time and increase more rapidly just before a main fracture. Other various examples of shock occurrence are represented in Figs. 6 (b) and (c). Although they are frequently similar to the above mentioned cases, the number of elastic shocks and their energy in some cases do not always increase monotonically. Especially, the pattern of shock occurrence in a few cases (2 (Pebble), 12 (Sand)) is rather similar to a swarm type and a single predominantly large shock does not occur so distinctly. Such a type seems to occur in the materials which have a comparatively low strength resulting from structural irregularities or poor content of a cementing material. Therefore, the transition from the intermediate type of shock occurrence to the swarm type is gradual in proportion to the degree of structural heterogeneity.

Thus, in a moderately heterogeneous structure, foreshocks, a main shock and aftershocks are clearly distinguished. Figs. 7 (a) and (b) show some examples of the frequency distributions of shocks and the amplitude of each shock before and after a main shock which corresponds to the generation of a visible large crack. The shock occurrence in No. 5 (Pumice) is typical, but the occurrence of aftershocks in some cases is not remarkable, as seen in No. 13 (Sand).

Figure 8 shows another type of shock occurrence. That is, the successive occurrence of elastic shocks suddenly stops immediately after a large shock. The large shock is not a main shock, but it is one of the largest shocks among the above mentioned foreshock. This is a phenomenon contrasted with aftershock occurrence. In the previous paper, the mechanism of aftershock occurrence was deduced to be as follows. A large part of the accumulated strain energy is liberated at the time of a main shock and a fractured region appears around the origin of the main shock. If the residual part of the accumulated strain energy is not so small, many local fractures occur successively following the main shock, because the stress concentrates at many

6) H. BENIOFF, *Bull. Seis. Soc. Amer.*, **41** (1951), 31-62.

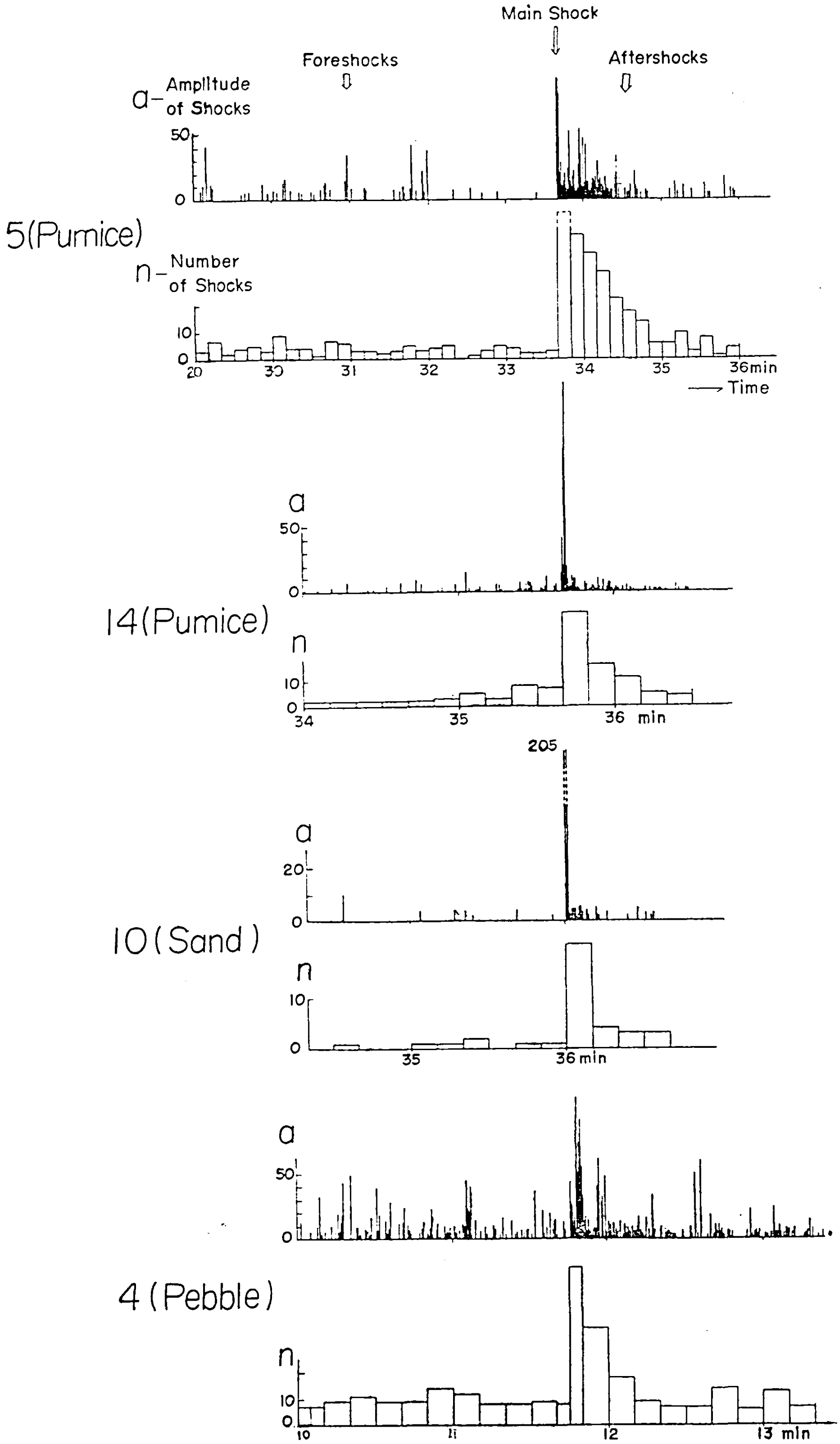

Main Shock
Foreshocks
Aftershocks
a – Amplitude of Shocks
5(Pumice)
n – Number of Shocks
36min
Time
14(Pumice)
10(Sand)
205
36 min
4 (Pebble)
13 min

Fig. 7 (a).

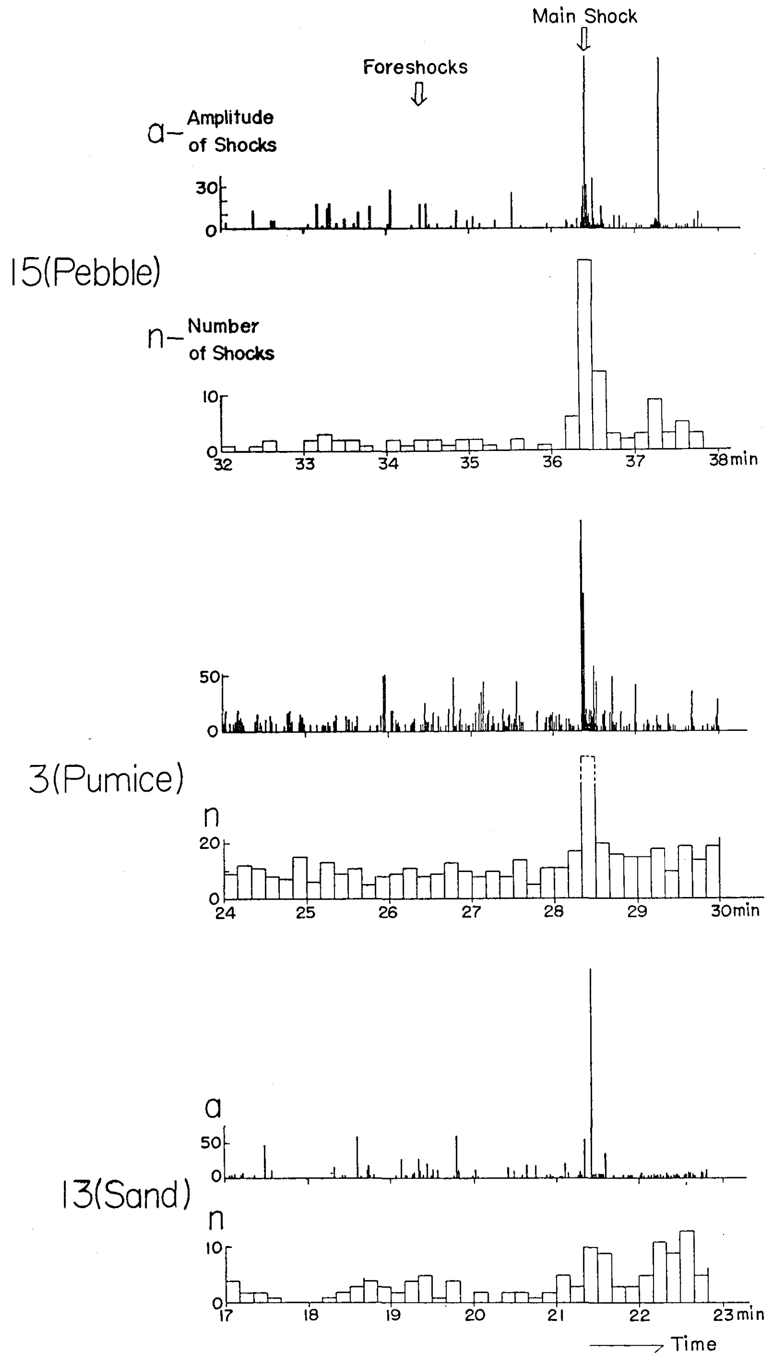


Fig. 7 (b).

Fig. 7. Changes in frequency and amplitude of elastic shocks before and after a main shock.

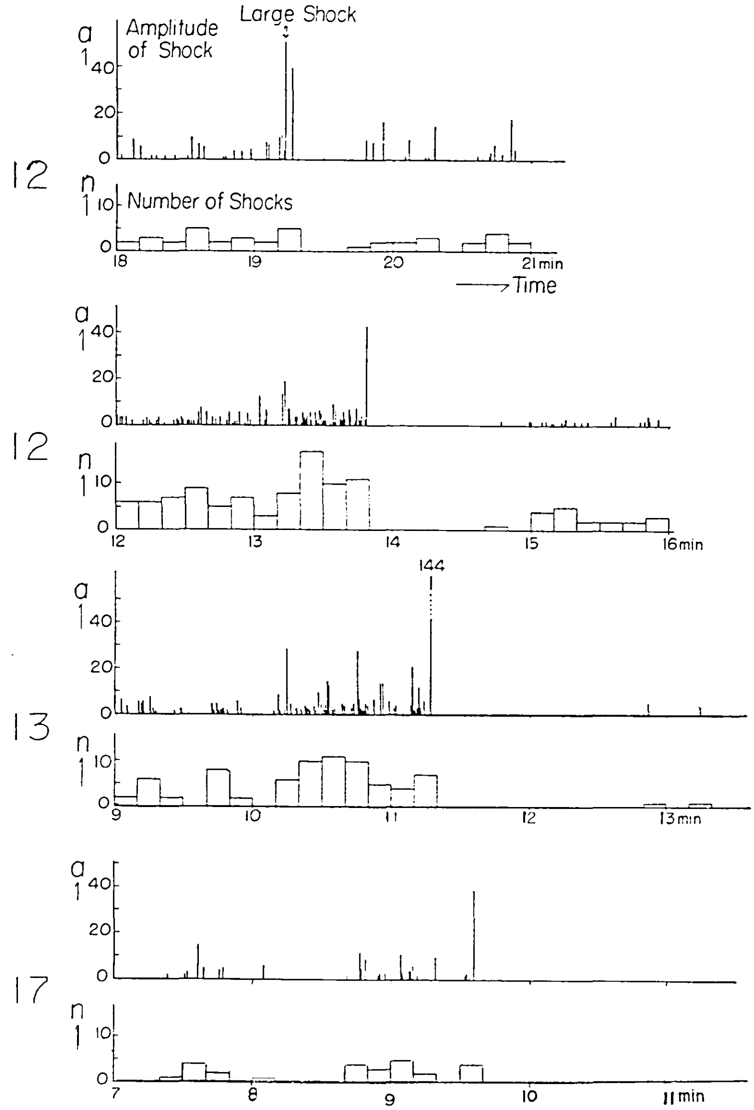


Fig. 8. A special type of changes in frequency and amplitude of elastic shocks before and after some large shocks.

weak points in the fractured region. Therefore, the pattern of shock occurrence shown in Fig. 8 may correspond to a case in which the residual stress after a large shock is very low or the remarkably fractured region does not appear around the origin. An analogous

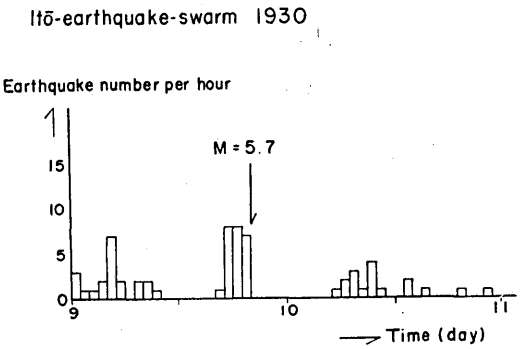


Fig. 9. An example of an earthquake sequence corresponding to the shock occurrence shown in Fig. 8.

pattern is seen in earthquake occurrence. An example in Fig. 9 is a frequency distribution before and after a large earthquake ($M=6$) in the Itō earthquake-swarm in 1930.

4. Magnitude-frequency relation of elastic shocks.

Recently[7], it was found that the magnitude-frequency relation has also a close connection with the mechanical structures of model materials. That is, the following statistical equation is satisfied for many heterogeneous materials.

$$n(a)da = n_0 a^{-m} da \ , \qquad (1)$$

where a is the maximum trace amplitude of an elastic shock, $n(a)da$ the number of elastic shocks having a maximum trace amplitude a to $a+da$, and n_0 and m are both constants chosen to fit the data. The exponent m which characterizes magnitude-frequency distribution has a relation to the degree of heterogeneity of material. The $\log n - \log a$ relations in the present case are shown in Figs. 10 (a) and (b). Closed circles and open circles indicate data of the elastic shocks before and after the main shock respectively. As mentioned before, the exponent m of these model materials before the main fracture is nearly identical to that of the general earthquakes (~ 1.8). Therefore, the degree of heterogeneity of the materials may be roughly similar to that of the earth's crust. From this experimental result, the following noteworthy relations on the magnitude distribution are obtained.

(1) The $\log n - \log a$ relation for foreshocks deviates in some cases

7) K. MOGI, *loc. cit.*, 2) 3).

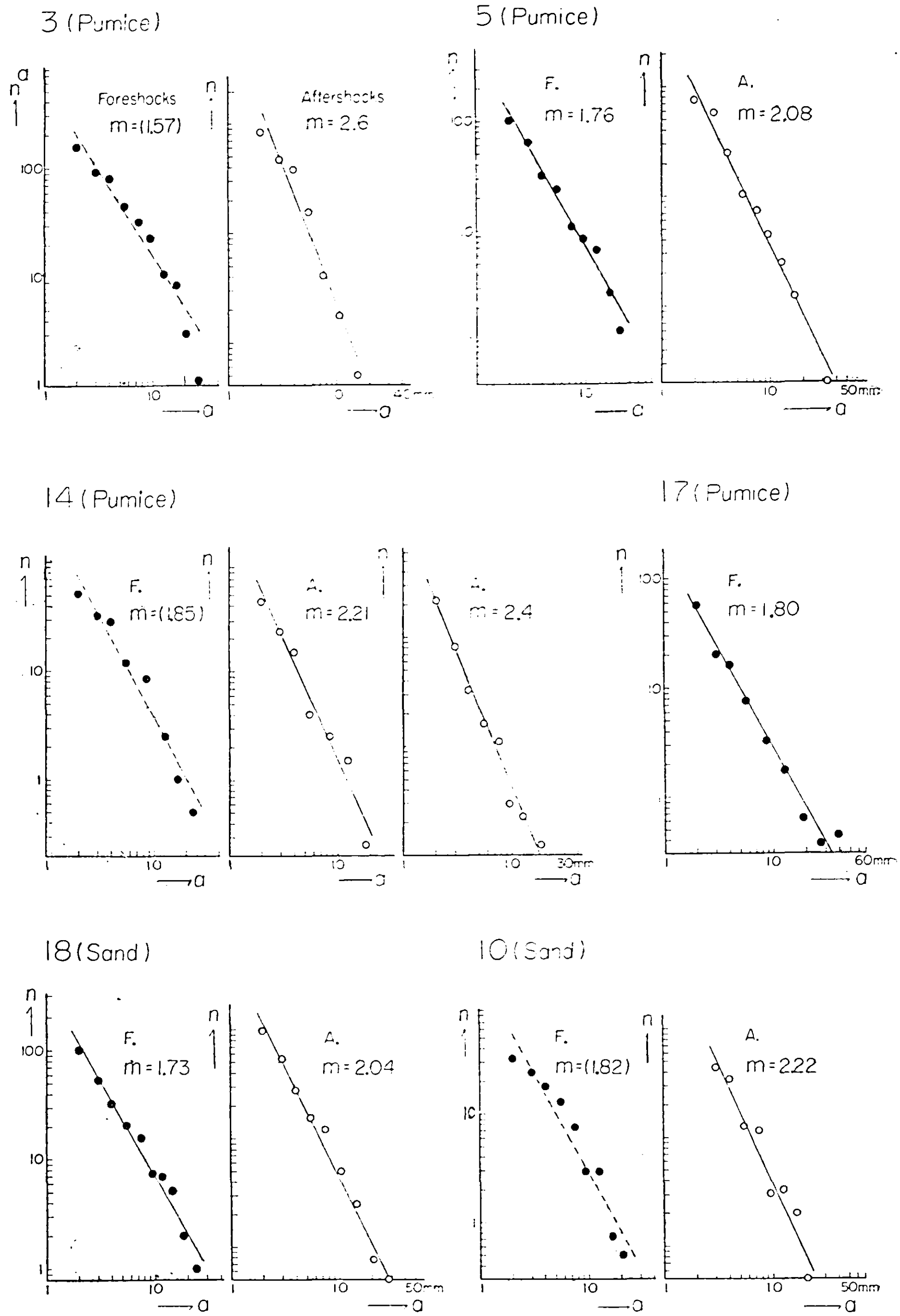


Fig. 10 (a).

Fig. 10. Relation between the maximum trace amplitude a of an elastic shock an dits number $n(a)$. closed circle: foreshocks; open circle: aftershoks.

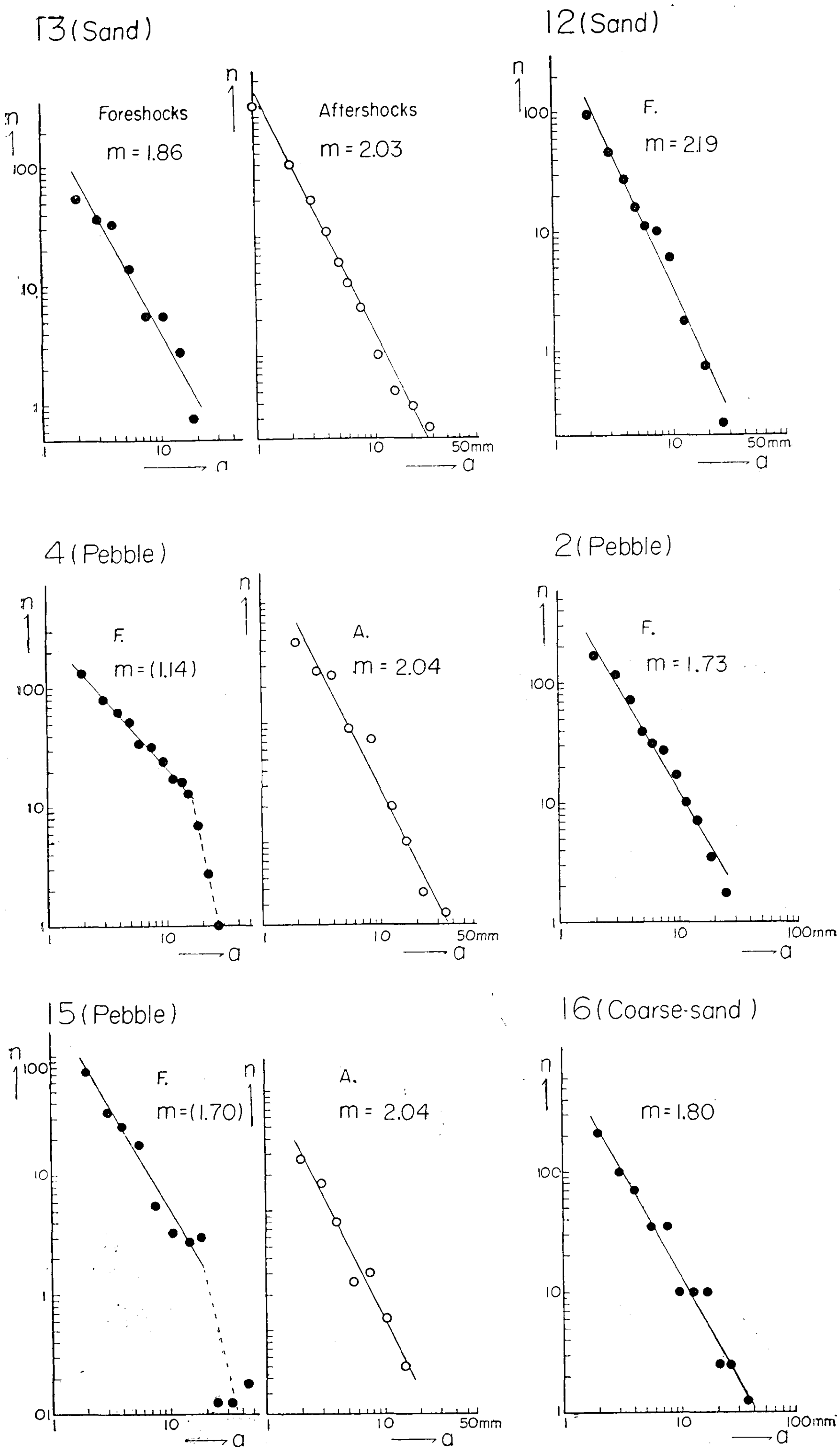

13 (Sand)
Foreshocks
m = 1.86
Aftershocks
m = 2.03
12 (Sand)
F.
m = 2.19
4 (Pebble)
F.
m = (1.14)
A.
m = 2.04
2 (Pebble)
F.
m = 1.73
15 (Pebble)
F.
m = (1.70)
A.
m = 2.04
16 (Coarse-sand)
m = 1.80
n
a
50mm
100mm

Fig. 10 (b).

from a linear relation. This deviation is caused by structural regularities in the materials. The material which includes pebbles as an aggregate has an appreciably regular structure and so its $\log n - \log a$ relation is represented by two straight lines linking at a point (Fig. 10 (b)). On the other hand, $\log n - \log a$ relations for aftershocks are always represented by a straight line. This may be understood from the fact that the structure at a fractured region where there was a main fracture is quite irregular. Thus, the original regular structure of the material is not seen in the aftershock region.

(2) The exponent m of foreshocks and aftershocks is given in

Table 1. The constant m of elastic shocks before and after a main shock.

Specimen	Foreshocks $m(F)$	Aftershocks $m(A)$	Difference $m(A)-m(F)$
5 (Pumice)	1.76	2.08	0.32
18 (Sand)	1.73	2.04	0.31
13 (Sand)	1.86	2.03	0.17
10 (Sand)	(1.82)	2.22	0.40
14 (Pumice)	(1.85)	2.21	0.36
3 (Pumice)	(1.57)	2.6	1.03
15 (Pebble)	(1.70)	2.04	0.34
4 (Pebble)	(1.14)	2.04	0.90

Table 1. It is a very interesting fact that the m value of aftershocks is always larger than that of foreshocks. The mean value of difference between them is about 0.3. Such change in the m value is attributed to the increase of crack density caused by a main fracture.

Such magnitude-frequency relations in elastic shocks may be similar to those of earthquakes. However, we have only very few data to establish such magnitude-frequency relations for earthquakes. In Fig. 11, a magnitude-frequency relation of major earthquakes ($M \geq 6$) in and near Japan[8] (left) and that of aftershocks (right) are represented. M is Richter's instrumental magnitude, n is the number of earthquakes having magnitude M, and the constant b is obtained by application of Gutenberg-Richter's relation[9] $\log n = a' + b\ (8 - M)$. There is the following relation between b and m.

8) Japan Meteorological Agency, "Catalogue of Major Earthquakes Which Occurred in and near Japan (1926-1956)".

9) B. GUTENBERG and C. F. RICHTER, Seismicity of the Earth, (1954).

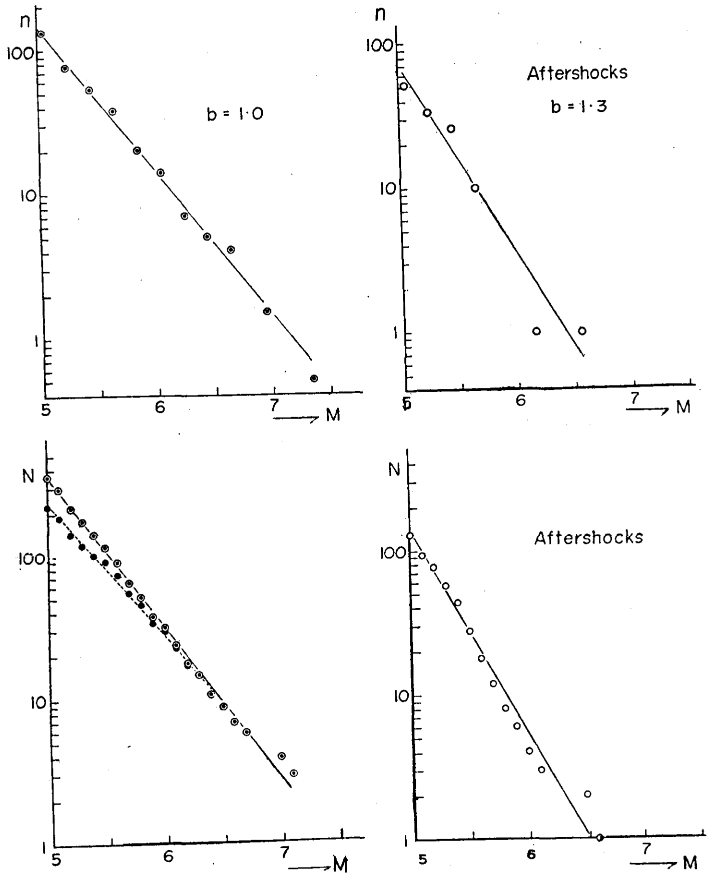


Fig. 11. Relation between the magnitude of earthquake M and its number $n(M)$. left: all earthquakes ($M \geq 6$), right: aftershocks only ($M \geq 6$). N: accumulated frequency of earthquakes.

$$b = m - 1 .$$

According to the result, the b value of aftershocks is also larger than that of all earthquakes.

The m values for small earthquakes have been obtained for many cases by Japanese seismologists[10)11)12)13)14)], but most of these results were

10) M. ISHIMOTO and K. IIDA, *Bull. Earthq. Res. Inst.*, **17** (1939), 443-478.

11) T. ASADA and Z. SUZUKI, *Geophysical Notes*, **16** (1949), 1-14.

12) Z. SUZUKI, *Sci. Rep. Tōhoku Univ.*, Ser. 5, **5** (1953), 177-182; **6** (1955), 105-118; **10** (1958), 15-27; **11** (1959), 10-54.

13) Y. TOMODA, *Zisin (Journ. Seis. Soc. Japan)*, [ii], **7** (1954), 155-169.

14) T. ASADA, *Journ. Phys. Earth*, **5** (1957), 83-113.

obtained from aftershock data. The linear relation between $\log n$ and $\log a$ is satisfied in many cases and the m value is nearly constant ($\sim$1.8). However, on the basis of the above mentioned experimental result, the present author wants to suggest that aftershocks are a special case in magnitude-frequency relations, that the magnitude-frequency relation of other ordinary earthquakes may give important information about the structural states of seismic regions and that the m value in this case may be smaller than that of aftershocks in the corresponding regions. A few data which are available at present seem to be consistent with the above mentioned suggestion (Table 2)[15][16][17][18][19].

Table 2. The constant m in Ishimoto-Iida's equation of earthquakes. (Magnitude M: 1$\sim$3)

General earthquakes			Aftershocks		
Region	m	Author	Region	m	Author
Kwanto	1.51	Suzuki[15]	Tango	1.95	Suzuki[15]
Kinugawa	1.58	Suzuki[15]	Nankai	1.82	Suzuki[15]
Kwanto	1.6	Ichikawa[16]	Imaichi	1.79	Suzuki[15]
Kwanto	1.74	Ishimoto[17] Iida	Fukui	1.86	Suzuki[15]
			Tottori	1.83	Omote[18]
			Teshikaga	1.909	Matsumoto[19]

5. Conclusion.

By this series of experimental investigation on the fracturing of various model materials, the mechanism of the successive occurrence of elastic shocks accompanying fractures has been worked out as follows. Under a gradually increasing stress, there are three typical patterns of shock occurrence, and they are closely connected with the mechanical structures of the materials and/or the spatial distribution of the applied stress (Fig. 12). They are:

First type: When the material is homogeneous and the stress is uniformly applied, a main shock occurs suddenly without any preceding

15) Z. SUZUKI, *loc. cit*,. 12).
16) M. ICHIKAWA, *Quarterly Journ. Seis.*, **21** (1956), 114-123.
17) M. ISHIMOTO and K. IIDA, *loc. cit.*, 10).
18) S. OMOTE, *Bull. Earthq. Res. Inst.*, **33** (1955), 641-661.
19) T. MATSUMOTO, *Bull. Earthq. Res. Inst.*, **37** (1959), 531-544.

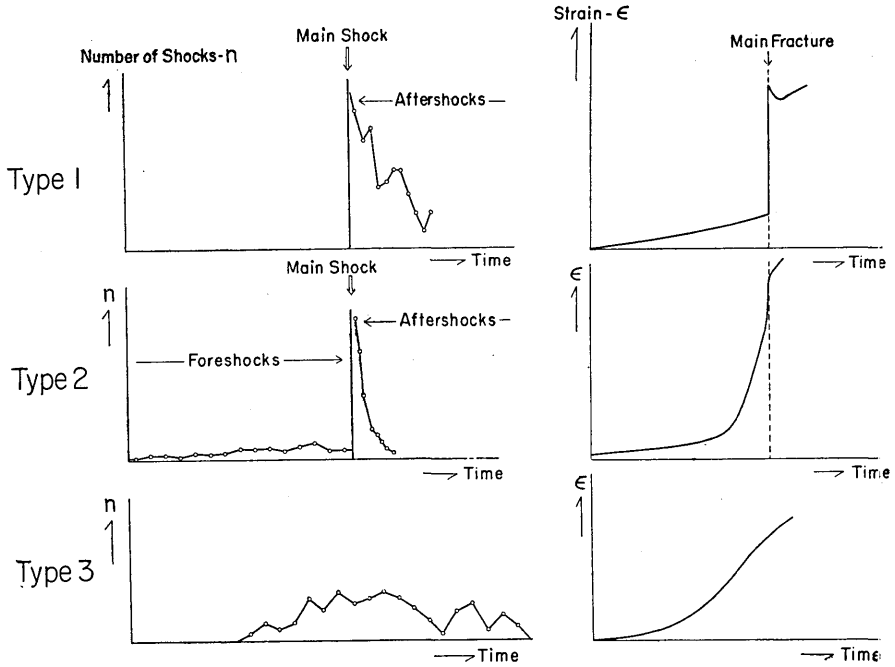


Fig. 12. Three types of the successive occurrence of elastic shocks (left) and the deformations of the surface of the semi-infinite body in each case (right).

shocks and many elastic shocks follow the main shock. Then, the deformation of the material increases linearly to the occurrence of the main shock.

Second type: When the material has a rather heterogeneous structure and/or the space distribution of the applied stress is not so uniform, small elastic shocks occur prior to a main shock which in turn is accompanied by many aftershocks. In this case, the deformation increases abnormally just before the main fracture.

Third type: When the structure of the material is extremely heterogeneous and/or the concentration of an applied external stress is very considerable, elastic shocks begin to occur as soon as the stress is applied. Their number and magnitude increase gradually and they decrease after some fluctuations. This sequence of the elastic shocks is a swarm type and has no one outstanding principal shock.

The three types are also seen in earthquake occurrence. Therefore, it may be deduced from the above mentioned experimental results that various patterns of earthquake series depend on the mechanical structures and the applied stress states. From this point of view, earthquakes in and near Japan will be systematically investigated and the relations between the pattern of earthquake sequences and the structure of the Japanese islands will be discussed in a next paper[20].

37. 半無限体の内部力源による破壊およびそれに関連した地震発生の二，三の問題（第2報）やや不均一な構造をもつ媒質の場合

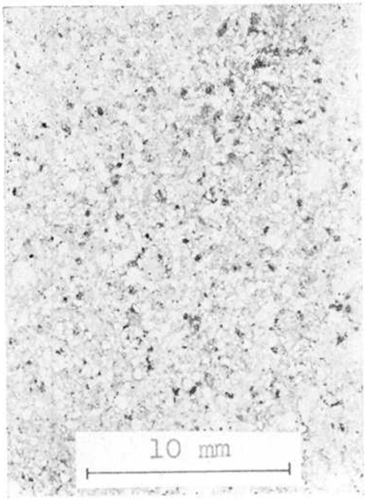
地震研究所　茂 木 清 夫

前回は構造が非常に均一な場合と極度に不均一な場合の両極端の場合について述べたが，今回はその中間のやや不均一な構造をもつ媒質について，内部力源によつて発生する破壊の諸性質，とくに破壊の際に発生する Elastic Shocks の起こり方を実験的に調べた．実際の地殻もこの場合に相当すると考えられる．地殻と類似の不均一構造をもつモデル物質として，適当な混合物をふくんだ4種類のコンクリートを用いて実験を行なつた．

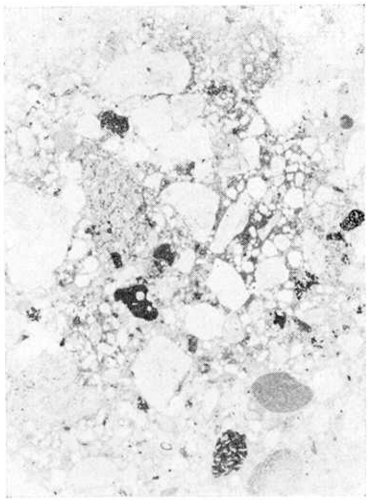
この場合の破壊の発生型として注目されることは，顕著な主破壊の発生に先行して，小破壊群が増加すること（すなわち，Foreshocks の発生）および主破壊にひきつづいて小破壊群が続発すること（Aftershocks）等である．その場合，主破壊前後の Elastic Shocks の大きさ分布に明瞭な差が認められ，石本・飯田の係数 m が，Aftershocks では Foreshocks のそれよりも 0.3 ほど大きくなる．

これまでの実験を綜合すると，破壊乃至 Elastic Shocks の発生型として，(1) 本震にひきつづいて余震が起こる，ただし前震を伴わない型，(2) 前震が先行して，本震，余震と続く型，(3) 群発型，の以上3つの典型的な型があることが結論される．勿論，実際はこれらの発生型の中間的な場合もあり，漸移的なものと考えられる．この発生型の違いは，媒質の構造の均一さまたは不均一さの度合に因ることが明らかにされた．これらの発生型に類似のものが，自然地震においても認められる．

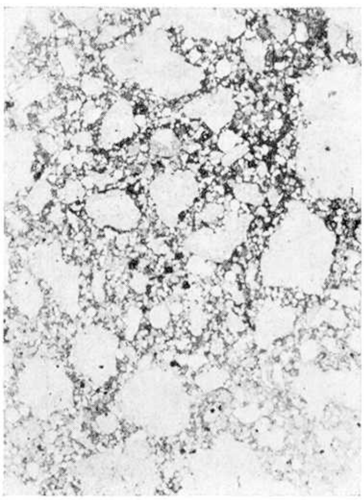
20) K. MOGI, *Bull. Earthq. Res. Inst.*, **41** (1963), 615-657.



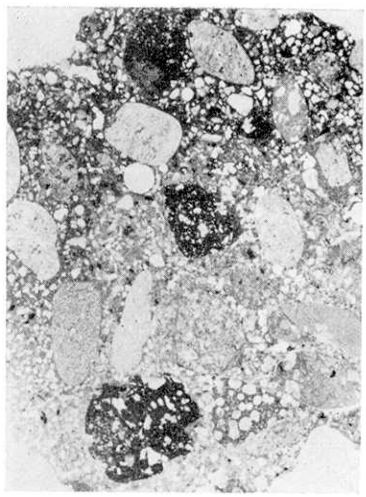
(a) Sand



(b) Coarse sand



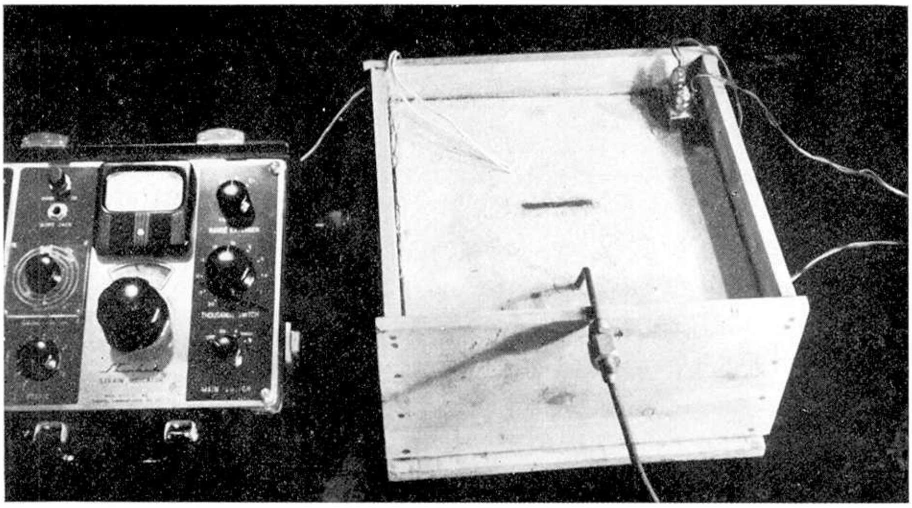
(c) Pumice



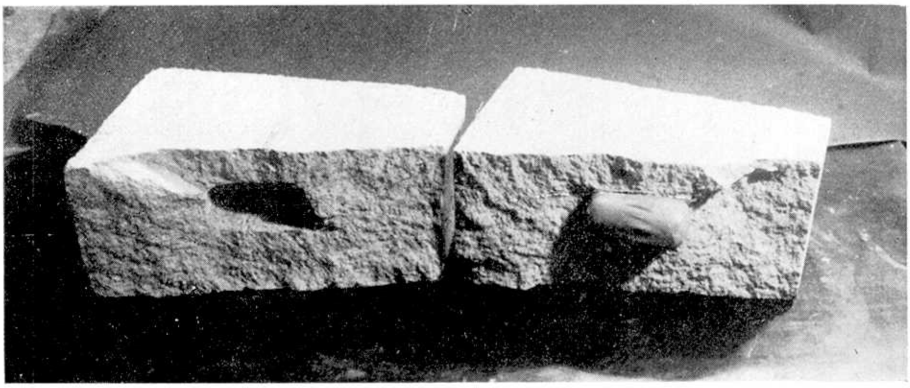
(d) Pebble

Fig. 13. Structure of model materials.

（震研彙報 第四十一号 図版 茂木）



(a)



(b)

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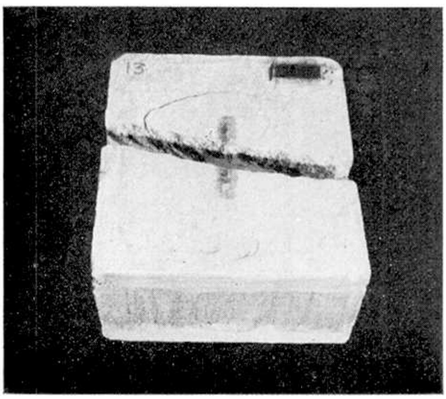
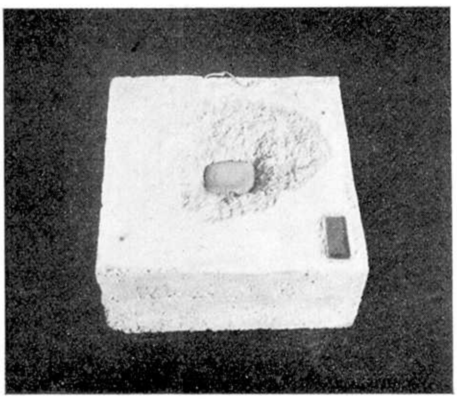


(c)



(d)

Fig. 14. Experimental apparatus and some examples of fracture pattern. (b), (c): prolate spheroid source; (d): oblate spheroid source.