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24. *Tsunami in a T-shaped Canal.*

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Introduction

In the preceding papers (1),[1] (2)[2] and (3),[3] the author has treated the tsunamis in a right-angled canal and bay and compared the theory with the data of the Chilean Tsunami of 1960.

To obtain the wave heights in each branch of the canal or bay, a very intriguing method was introduced such that the long wave approximation and the wave number relation derived from the basic equation as a result of separation of the variables are ingeniously used for the reduction of equations. Hereafter the author call this method as "Momoi's method". In this paper he also demonstrates the effectiveness of this method for the analysis of a tsunami in a canal. The present purview consists of two parts:

Part I : the case where periodic waves surge from a lower side of "T" character,

Part II: the case where periodic waves invade from one of the horizontal branches of the canals.

Part I.

I. 1. Theory.

Referring to Fig. (I. 1), the Cartesian co-ordinates (x, y) are centered at the conjunction point of three branches, x- and y-axis being fixed at the rims of the canals.

Suppose that (Fig. (I. 1))

D_1 : the domain in the range $(x>d_2,\ d_1>y>0)$,

D_2 : the domain in the range $(d_2>x>0,\ y>d_1)$,

D_3 : the domain in the range $(d_3>x>0,\ 0>y)$,

1) T. MOMOI, *Bull. Earthq. Res. Inst.*, **40** (1962), 719.

2) T. MOMOI, *ditto*, 733.

3) T. MOMOI, *ditto*, 747.

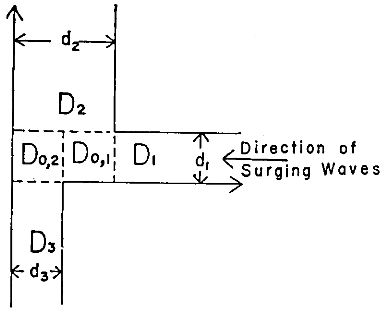

Fig. (I. 1).

$D_{0,1}$: the domain in the range $(d_2>x>d_3,\ d_1>y>0)$,
$D_{0,2}$: the domain in the range $(d_3>x>0,\ d_1>y>0)$,
ζ_j $(j=1, 2, 3)$: the wave heights in the domains $D_j\,(j=1, 2, 3)$,
$\zeta_{0,1}$: the wave height in the domain $D_{0,1}$,
$\zeta_{0,2}$: the wave height in the domain $D_{0,2}$,
c : the velocity of long wave, viz., $\sqrt{gH}$ (H being the depth of water),
t : a variable of time,

then we have, as basic equations,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\zeta_j=\frac{1}{c^2}\frac{\partial^2\zeta_j}{\partial t^2}\quad (j=1;\ 2;\ 3;\ 0{,}1 \text{ and } 0{,}2)\,, \tag{1.1}$$

for the domains D_j $(j=1;\ 2;\ 3;\ 0{,}1$ and $0{,}2)$.

(i) *The Solution in the Domain* D_1

Since the boundary conditions in this domain are

$$\frac{\partial\zeta_1}{\partial y}=0 \quad (x>d_2,\ y=0 \text{ and } d_1)\,,$$

the wave height is expressed as[4)]

$$\zeta_1=\zeta_0 e^{-ikx}+\sum_{m=0}^{\infty}\zeta_1^{(m)}\cos\frac{m\pi}{d_1}y\cdot e^{+ik_1^{(m)}x}\,, \tag{1.2}$$

where a time factor $\exp(-i\omega t)$ is omitted as usual (ω: the angular frequency of the surging waves); the first term the surging periodic wave; the second group of terms the reflected waves; ζ_0 and $\zeta_1^{(m)}$ $(m=0, 1, 2, 3,\cdots)$ the amplitudes of the surging and the reflected waves; $k_1^{(m)}=+\sqrt{k^2-\left(\frac{m\pi}{d_1}\right)^2}$ $(k=\omega/c)$.

(ii) *The Solution in the Domain* D_2

Since the conditions at the boundary are

$$\frac{\partial\zeta_2}{\partial x}=0 \quad (y>d_1,\ x=0 \text{ and } d_2)\,,$$

4) T. MOMOI, *loc. cit.*, 1).

the solution in this domain becomes

$$\zeta_2 = \sum_{m=0}^{\infty} \zeta_2^{(m)} \cos \frac{m\pi}{d_2} x \cdot e^{+ik_2^{(m)} y}, \tag{1.3}$$

where $\zeta_2^{(m)}$ is the amplitude of the m-th mode of waves, and

$$k_2^{(m)} = +\sqrt{k^2 - \left(\frac{m\pi}{d_2}\right)^2}.$$

(iii) *The Solution in the Domain D_3*

The conditions at the boundary are given by

$$\frac{\partial \zeta_3}{\partial x} = 0 \quad (y<0,\ x=0, \text{ and } d_3)\ .$$

Hence it follows that the wave height in this domain has the following form:

$$\zeta_3 = \sum_{m=0}^{\infty} \zeta_3^{(m)} \cos \frac{m\pi}{d_3} x \cdot e^{-ik_3^{(m)} y}\ , \tag{1.4}$$

$\zeta_3^{(m)}$ being the amplitude of the m-th mode of waves and $k_3^{(m)} = +\sqrt{k^2 - \left(\frac{m\pi}{d_3}\right)^2}$.

(iv) *The Solution in the Domain $D_{0,1}$*

In consideration of the condition $\frac{\partial \zeta_{0,1}}{\partial y} = 0$ $(y=0,\ d_2>x>d_3)$, a particular solution for the equation (1.1) has the form

$$(A_{0,1}(f_{0,1}) \cos k_x^{(1)} x + B_{0,1}(f_{0,1}) \sin k_x^{(1)} x) \cos k_y^{(1)} y\ ,$$

where $A_{0,1}(f_{0,1})$ and $B_{0,1}(f_{0,1})$ are arbitrary constants: $f_{0,1}$ denotes a pair of $k_x^{(1)}$ and $k_y^{(1)}$ permissible by the relation $(k_x^{(1)})^2 + (k_y^{(1)})^2 = k^2$.

Integrating the above-mentioned particular solution over the range to be permitted by $(k_x^{(1)})^2 + (k_y^{(1)})^2 = k^2$, we have a general solution, i.e.,

$$\zeta_{0,1} = \sum_{f_{0,1}} (A_{0,1}(f_{0,1}) \cos k_x^{(1)} x + B_{0,1}(f_{0,1}) \sin k_x^{(1)} x) \cos k_y^{(1)} y\ . \tag{1.5}$$

(v) *The Solution in the Domain $D_{0,2}$*

In a manner similar to the fore-going paragraph, the solution satisfying the condition $\frac{\partial \zeta_{0,2}}{\partial x} = 0$ $(x=0,\ d_1>y>0)$ becomes

$$\zeta_{0,2}=\sum_{f_{0,2}}\cos k_x^{(2)}x(A_{0,2}(f_{0,2})\cos k_y^{(2)}y+B_{0,2}(f_{0,2})\sin k_y^{(2)}y)\ , \tag{1.6}$$

where $A_{0,2}(f_{0,2})$ and $B_{0,2}(f_{0,2})$ are arbitrary constants: $f_{0,2}$ a pair of $k_x^{(2)}$ and $k_y^{(2)}$: $\sum_{f_{0,2}}$ the integration under the condition $(k_x^{(2)})^2+(k_y^{(2)})^2=k^2$.

(vi) *Condtions for Determining the Arbitrary Constants*

At $x=d_2$,

$$\left.\begin{aligned}\zeta_{0,1}&=\zeta_1\ ,\\ \frac{\partial\zeta_{0,1}}{\partial x}&=\frac{\partial\zeta_1}{\partial x}\ ,\end{aligned}\right\}\quad \text{for } (d_1>y>0)\ . \tag{1.7}$$

At $x=d_3$,

$$\left.\begin{aligned}\zeta_{0,2}&=\zeta_{0,1}\ ,\\ \frac{\partial\zeta_{0,2}}{\partial x}&=\frac{\partial\zeta_{0,1}}{\partial x}\ ,\end{aligned}\right\}\quad \text{for } (d_1>y>0)\ . \tag{1.8}$$

At $y=d_1$,

$$\left.\begin{aligned}\zeta_2&=\begin{cases}\zeta_{0,1} & \text{for } (d_2>x>d_3)\ ,\\ \zeta_{0,2} & \text{for } (d_3>x>0)\ ,\end{cases}\\ \frac{\partial\zeta_2}{\partial y}&=\begin{cases}\dfrac{\partial\zeta_{0,1}}{\partial y} & \text{for } (d_2>x>d_3)\ ,\\ \dfrac{\partial\zeta_{0,2}}{\partial y} & \text{for } (d_3>x>0)\ .\end{cases}\end{aligned}\right\} \tag{1.9}$$

At $y=0$,

$$\left.\begin{aligned}\zeta_3&=\zeta_{0,2}\ ,\\ \frac{\partial\zeta_3}{\partial y}&=\frac{\partial\zeta_{0,2}}{\partial y}\ ,\end{aligned}\right\}\quad \text{for } (d_3>x>0)\ . \tag{1.10}$$

(vii) *Determination of the Arbitrary Constants*

Substituting from (1.2)−(1.6) for (1.7)−(1.10), we have:

$$\left.\begin{aligned}&\sum_{f_{0,1}}(A_{0,1}\cos k_x^{(1)}d_2+B_{0,1}\sin k_x^{(1)}d_2)\cos k_y^{(1)}y\\ &\qquad=\zeta_0e^{-ikd_2}+\sum_{m=0}^{\infty}\zeta_1^{(m)}\cos\frac{m\pi}{d_1}y\cdot e^{+ik_1^{(m)}d_2}\ ,\\ &\sum_{f_{0,1}}k_x^{(1)}(-A_{0,1}\sin k_x^{(1)}d_2+B_{0,1}\cos k_x^{(1)}d_2)\cos k_y^{(1)}y\\ &\qquad=-ik\zeta_0e^{-ikd_2}+\sum_{m=0}^{\infty}(+ik_1^{(m)})\zeta_1^{(m)}\cos\frac{m\pi}{d_1}y\cdot e^{+ik_1^{(m)}d_2}\ ,\\ &\qquad\qquad\text{for } (d_1>y>0):\end{aligned}\right\} \tag{1.7'}$$

$$\left.\begin{aligned}&\sum_{f_{0,2}} \cos k_x^{(2)} d_3 (A_{0,2} \cos k_y^{(2)} y + B_{0,2} \sin k_y^{(2)} y) \\ &\qquad = \sum_{f_{0,1}} (A_{0,1} \cos k_x^{(1)} d_3 + B_{0,1} \sin k_x^{(1)} d_3) \cos k_y^{(1)} y \ , \\ &\sum_{f_{0,2}} (-k_x^{(2)}) \sin k_x^{(2)} d_3 (A_{0,2} \cos k_y^{(2)} y + B_{0,2} \sin k_y^{(2)} y) \\ &\qquad = \sum_{f_{0,1}} k_x^{(1)} \{-A_{0,1} \sin k_x^{(1)} d_3 + B_{0,1} \cos k_x^{(1)} d_3\} \cos k_y^{(1)} y \ , \\ &\qquad\qquad \text{for } (d_1 > y > 0) :\end{aligned}\right\} \tag{1.8'}$$

$$\left.\begin{aligned}&\sum_{m=0}^{\infty} \zeta_2^{(m)} \cos \frac{m\pi}{d_2} x \cdot e^{+ik_2^{(m)} d_1} \\ &\qquad \begin{cases} = \sum_{f_{0,1}} (A_{0,1} \cos k_x^{(1)} x + B_{0,1} \sin k_x^{(1)} x) \cos k_y^{(1)} d_1 \ , \\ \qquad\qquad \text{for } (d_2 > x > d_3) ; \\ = \sum_{f_{0,2}} \cos k_x^{(2)} x (A_{0,2} \cos k_y^{(2)} d_1 + B_{0,2} \sin k_y^{(2)} d_1) \ , \\ \qquad\qquad \text{for } (d_3 > x > 0) : \end{cases} \\ &\sum_{m=0}^{\infty} (+ik_2^{(m)}) \zeta_2^{(m)} \cos \frac{m\pi}{d_2} x \cdot e^{+ik_2^{(m)}} d_1 \\ &\qquad \begin{cases} = \sum_{f_{0,1}} (A_{0,1} \cos k_x^{(1)} x + B_{0,1} \sin k_x^{(1)} x)(-k_y^{(1)}) \sin k_y^{(1)} d_1 \ , \\ \qquad\qquad \text{for } (d_2 > x > d_3) ; \\ = \sum_{f_{0,2}} \cos k_x^{(2)} x (-k_y^{(2)} A_{0,2} \sin k_y^{(2)} d_1 + k_y^{(2)} B_{0,2} \cos k_y^{(2)} d_1) \ , \\ \qquad\qquad \text{for } (d_3 > x > 0) : \end{cases}\end{aligned}\right\} \tag{1.9'}$$

$$\left.\begin{aligned}&\sum_{m=0}^{\infty} \zeta_3^{(m)} \cos \frac{m\pi}{d_3} x = \sum_{f_{0,2}} A_{0,2} \cos k_x^{(2)} x \ , \\ &\sum_{m=0}^{\infty} (-ik_3^{(m)}) \zeta_3^{(m)} \cos \frac{m\pi}{d_3} x = \sum_{f_{0,2}} B_{0,2} k_y^{(2)} \cos k_x^{(2)} x \ , \\ &\qquad \text{for } (d_3 > x > 0) \ .\end{aligned}\right\} \tag{1.10'}$$

Applying the operator $\int_0^{d_1} dy$ to (1.7′) and (1.8′), using long wave approximation (the first reduction of Momoi's method),

$$\left.\begin{aligned}&\text{i.e.,} \quad kd_l \ll 1 \ , \quad |k_j^{(i)}| d_l \ll 1 \ , \quad (i=1, 2 : j=x, y : l=1, 2, 3) \\ &\text{or} \quad \cos kd_l \simeq 1 \ , \ \cos k_j^{(i)} d_l \simeq 1 \ , \ \sin kd_l \simeq kd_l \ , \ \sin k_j^{(i)} d_l \simeq k_j^{(i)} d_l\end{aligned}\right\} \tag{1.11}$$

(this approximation denotes that the wave length of the surging periodic waves is long enough as compared with the width of the canal), we have[5]:

$$\left.\begin{aligned}
&\sum_{f_{0,1}} (A_{0,1}+B_{0,1}k_x^{(1)}d_2)=\zeta_0 e^{-ikd_2}+\zeta_1^{(0)}e^{+ikd_2}\ ,\\
&\sum_{f_{0,1}} k_x^{(1)}(-A_{0,1}k_x^{(1)}d_2+B_{0,1})=-ik\zeta_0 e^{-ikd_2}+ik\zeta_1^{(0)}e^{+ikd_2},\\
&\sum_{f_{0,2}} A_{0,2}=\sum_{f_{0,1}} (A_{0,1}+B_{0,1}k_x^{(1)}d_3)\ ,\\
&-\sum_{f_{0,2}} (k_x^{(2)})^2 d_3 A_{0,2}=\sum_{f_{0,1}} k_x^{(1)}(-A_{0,1}k_x^{(1)}d_3+B_{0,1})\ .
\end{aligned}\right\} \tag{1.12}$$

Applying the operator $\int_0^{d_2} dx$ to (1.9′) and by use of the approximation (1.11), the relation (1.9′) becomes

$$\left.\begin{aligned}
\zeta_2^{(0)}e^{+ikd_1}d_2&=\sum_{f_{0,1}} A_{0,1}(d_2-d_3)+\sum_{f_{0,2}} d_3(A_{0,2}+B_{0,2}k_y^{(2)}d_1)\ ,\\
+ikd_2\zeta_2^{(0)}e^{+ikd_1}&=-\sum_{f_{0,1}} A_{0,1}(k_y^{(1)})^2 d_1(d_2-d_3)+\sum_{f_{0,2}} k_y^{(2)}d_3(-A_{0,2}k_y^{(2)}d_1+B_{0,2})\ .
\end{aligned}\right\} \tag{1.13}$$

Also applying the operator $\int_0^{d_3} dx$ to (1.10′) and by use of (1.11), we have

$$\left.\begin{aligned}
\zeta_3^{(0)}&=\sum_{f_{0,2}} A_{0,2}\ ,\\
-ik\zeta_3^{(0)}&=\sum_{f_{0,2}} k_y^{(2)}B_{0,2}\ .
\end{aligned}\right\} \tag{1.14}$$

After some reductions of (1.12), we have:
from the first and the third relations of (1.12),

$$\left.\begin{aligned}
\sum_{f_{0,1}} B_{0,1}k_x^{(1)}&=\frac{1}{d_2-d_3}\{(\zeta_0^{-ikd_2}+\zeta_1^{(0)}e^{+ikd_2})-\sum_{f_{0,2}} A_{0,2}\}\ ,\\
\sum_{f_{0,1}} A_{0,1}&=\frac{1}{d_2-d_3}\{-d_3(\zeta_0 e^{-ikd_2}+\zeta_1 e^{+ikd_2})+d_2\sum_{f_{0,2}} A_{0,2}\}\ ,
\end{aligned}\right\} \tag{1.15}$$

where $d_2 \neq d_3$ $(d_2 > d_3)$ (referring to Fig. (I. 1));
from the second and the fourth of (1.12),

$$\left.\begin{aligned}
\sum_{f_{0,1}} A_{0,1}(k_x^{(1)})^2&=\frac{-1}{d_2-d_3}\{(-ik\zeta_0 e^{-ikd_2}+ik\zeta_1^{(0)}e^{+ikd_2})+\sum_{f_{0,2}} A_{0,2}(k_x^{(2)})^2 d_3\}\ ,\\
\sum_{f_{0,1}} B_{0,1}k_x^{(1)}&=\frac{-1}{d_2-d_3}\{d_3(-ik\zeta_0 e^{-ikd_2}+ik\zeta_1^{(0)}e^{+ikd_2})+\sum_{f_{0,2}} A_{0,2}(k_x^{(2)})^2 d_2 d_3\}\ .
\end{aligned}\right\} \tag{1.16}$$

On substitution of the first expressions of (1.15) and (1.16) for the

5) T. MOMOI, *loc. cit.*, 1).

fourth of (1.12),

$$\sum_{f_{0,2}} A_{0,2} = d_3(-ik\zeta_0 e^{-ikd_2} + ik\zeta_1^{(0)} e^{+ikd_2}) + (\zeta_0 e^{-ikd_2} + \zeta_1^{(0)} e^{+ikd_2}) \,, \tag{1.17}$$

where the following reduction is used, i. e.,

$$\sum_{f_{0,2}} A_{0,2}\{1-(k_x^{(2)})^2 d_3 d_2\} \simeq \sum_{f_{0,2}} A_{0,2} \,, \tag{1.18}$$

and hence

$$\sum_{f_{0,2}} A_{0,2}(k_x^{(2)})^2 \simeq 0 \,.$$

Putting (1.18) into (1.16),

$$\left.\begin{aligned} \sum_{f_{0,1}} A_{0,1}(k_x^{(1)})^2 &= \frac{-ik}{d_2-d_3}(-\zeta_0 e^{-ikd_2} + \zeta_1^{(0)} e^{+ikd_2}) \,, \\ \sum_{f_{0,1}} B_{0,1} k_x^{(1)} &= \frac{-ikd_3}{d_2-d_3}(-\zeta_0 e^{-ikd_2} + \zeta_1^{(0)} e^{+ikd_2}) \,. \end{aligned}\right\} \tag{1.19}$$

From the latter of (1.15) and (1.17),

$$\begin{aligned} \sum_{f_{0,1}} A_{0,1} = \frac{1}{d_2-d_3}\{&(d_2-d_3)(\zeta_0 e^{-ikd_2} + \zeta_1^{(0)} e^{+ikd_2}) \\ &+ ikd_3 d_2(-\zeta_0 e^{-ikd_2} + \zeta_1^{(0)} e^{+ikd_2})\} \,. \end{aligned} \tag{1.20}$$

Substituting (1.17), (1.14) and (1.20) for the first of (1.13), we have

$$\begin{aligned} -\{kd_2 + ik^2 d_3(d_2+d_3)\} e^{+ikd_2}\zeta_1^{(0)} + kd_2 e^{+ikd_1}\zeta_2^{(0)} + ik^2 d_1 d_3 \zeta_3^{(0)} \\ = \{kd_2 - ik^2 d_3(d_2+d_3)\} e^{-ikd_2}\zeta_0 \,. \end{aligned} \tag{1.21}$$

Following the principle of Momoi's method (the second reduction of this method) (refer to the introduction of this paper), (1.18) becomes, in consideration of the wave number relation $k^2 = (k_x^{(2)})^2 + (k_y^{(2)})^2$,

$$\sum_{f_{0,2}} A_{0,2}(k_x^{(2)})^2 = \sum_{f_{0,2}} A_{0,2} k^2 - \sum_{f_{0,2}} A_{0,2}(k_y^{(2)})^2 \simeq 0 \text{ or } \sum_{f_{0,2}} A_{0,2}(k_y^{(2)})^2 \simeq k^2 \sum_{f_{0,2}} A_{0,2} \,. \tag{1.22}$$

By use of (1.22), the second equation of (1.13) becomes

$$+ikd_2\zeta_2^{(0)} e^{+ikd_1} = -\sum_{f_{0,1}} A_{0,1}(k_y^{(1)})^2 d_1(d_2-d_3) - d_1 d_3 k^2 \sum_{f_{0,2}} A_{0,2} + \sum_{f_{0,2}} k_y^{(2)} d_3 B_{0,2} \,. \tag{1.23}$$

Putting (1.14) into (1.23),

$$+ikd_2\zeta_2^{(0)} e^{+ikd_1} = -\sum_{f_{0,1}} A_{0,1}(k_y^{(1)})^2 d_1(d_2-d_3) - (k^2 d_1 d_3 + ikd_3)\zeta_3^{(0)} \,. \tag{1.24}$$

In like manner, applying Momoi's method to $\sum_{f_{0.1}}$-term of (1.24) by use of the first expression of (1.19) and the wave number relation $(k^2=(k_x^{(1)})^2+(k_y^{(1)})^2)$, we have

$$(+ikd_2)\zeta_2^{(0)}e^{+ikd_1}+ikd_1(-\zeta_0 e^{-ikd_2}+\zeta_1^{(0)}e^{+ikd_2})$$
$$=-d_1(d_2-d_3)k^2\sum_{f_{0,1}}A_{0,1}-(k^2d_1d_3+ikd_3)\zeta_3^{(0)} . \quad (1.25)$$

Since the expression $\sum_{f_{0,1}}A_{0,1}$ is given in (1.20), (1.25) becomes, to the approximation of the order of $k^2d_j^2$ $(j=1, 2, 3)$,

$$\{-kd_1+ik^2d_1(d_2-d_3)\}e^{+ikd_2}\zeta_1^{(0)}-kd_2e^{+ikd_1}\zeta_2^{(0)}+(ik^2d_1d_3-kd_3)\zeta_3^{(0)}$$
$$=-\{kd_1+ik^2d_1(d_2-d_3)\}e^{-ikd_2}\zeta_0 . \quad (1.26)$$

On equating the first of (1.14) to (1.17), we have, as the third equation with respect to $\zeta_1^{(0)}$; $\zeta_2^{(0)}$; $\zeta_3^{(0)}$ (the first and the second equations are given in (1.21) and (1.26) respectively),

$$(ikd_3+1)e^{+ikd_2}\zeta_1^{(0)}-\zeta_3^{(0)}=(ikd_3-1)e^{-ikd_2}\zeta_0 . \quad (1.27)$$

The reductions made so far are to connect the amplitudes in the straight parts of the canal by use of the relations in the conjunction part (which is characteristic of Momoi's method).

Thus we have three equations (1.21), (1.26) and (1.27) available to determine $\zeta_1^{(0)}$, $\zeta_2^{(0)}$ and $\zeta_3^{(0)}$. Solving these equations, the following results are obtained:

$$\zeta_1^{(0)}=\frac{d_1-d_2-d_3}{d_1+d_2+d_3}\cdot e^{-i\cdot 2kd_2}\zeta_0 , \quad (1.28)$$

$$\zeta_2^{(0)}=\frac{2d_1}{d_1+d_2+d_3}\cdot e^{-ik(d_1+d_2)}\zeta_0 , \quad (1.29)$$

$$\zeta_3^{(0)}=\frac{2(d_1-ikd_2d_3)}{d_1+d_2+d_3}\cdot e^{-ikd_2}\zeta_0 , \quad (1.30)$$

where $d_3<d_2$ (refer to Fig. (I. 1)).

Though we must determine, as a next step, the amplitudes of the higher modes of the waves, viz., $\zeta_1^{(m)}$, $\zeta_2^{(m)}$, $\zeta_3^{(m)}$ $(m=1, 2, 3, \cdots)$, it is probable that the orders of these amplitudes are of $k^2d_j^2/(m\pi)^2$ $(m=1, 2, 3, \cdots ; j=1, 2, 3)$ under the long wave approximation $(kd_j \ll 1)$ (refer to the preceding paper[6] of the present author). On the basis of this fact,

6) T. MOMOI, *loc. cit.*, 1).

no consideration is made of the higher modes of the waves (*These higher modes, if necessary, may be obtained in the same manner as the first mode*).

So far as the conjunction part of the canal (D_j ($j=0, 1;\ 0, 2$)) is concerned, the dimension of this part falls within the first order of the long wave approximation. Although the solution in this part is not obtained, the variation of the wave may be considered to be very small to the extent that the wave in this part can be approximated to those in the adjacent canals.

(viii) *Consideration of Particular Cases*

Here the consideration of particular cases is made in the following:

(1) when $d_3 \to 0$;

$$\left.\begin{aligned} \zeta_1^{(0)} &\to \frac{d_1-d_2}{d_1+d_2}\cdot e^{-i\cdot 2kd_2}\zeta_0\,, \\ \zeta_2^{(0)} &\to \frac{2d_1}{d_1+d_2}\cdot e^{-ik(d_1+d_2)}\zeta_0\,, \\ \zeta_3^{(0)} &\to \frac{2d_1}{d_1+d_2}\cdot e^{-ikd_2}\zeta_0\,. \end{aligned}\right\} \tag{1.31}$$

The first two expressions are identical in form with those obtained for the case of the right-angled canal, which can be regarded as a particular case for the present study.

Our concern is with the third expression. As d_3 decreases, the amplitude in the domain D_3 becomes equal to that in the domain D_2, while the former is different in phase with the latter by kd_1. In spite of d_3 tending to zero ($d_2 \not\equiv 0$), $\zeta_3^{(0)}$ is still finite. Then it may be interpreted that the amplitude at the corner of the crooked part of the right-angled canal is expressed by $\frac{2d_1}{d_1+d_2}\cdot e^{-ikd_2}\zeta_0$.

(2) when $d_1=d_2=d_3=d$;

$$\left.\begin{aligned} \zeta_1^{(0)} &= -\frac{1}{3}\cdot e^{-i\cdot 2kd}\zeta_0 \quad \text{(from (1.28))}\,, \\ \zeta_2^{(0)} &= \frac{2}{3}\cdot e^{-i\cdot 2kd}\zeta_0 \quad \text{(from (1.29))}\,, \\ \zeta_3^{(0)} &= \frac{2}{3}\cdot e^{-i\cdot 2kd}\zeta_0 \quad \text{(from (1.30))}\,, \end{aligned}\right\} \tag{1.32}$$

the third expression being derived by use of the following reduction:

$$d_1 - ikd_2d_3 = d(1-ikd)$$
$$\simeq de^{-ikd} \quad (kd \ll 1)\ .$$

The amplitudes in the domains D_2 and D_3 become equal to each other, as would be expected. The amplitude of the reflected wave in the domain D_1 is one-third that of the incident wave and late in phase by $(\pi+2kd)$. Since $kd \ll 1$, these two waves are nearly in inverse phase, while those of the progressive waves in the domains D_2 and D_3 being two-third the incident one.

(3) when $d_j \to 0$ $(j=2, 3)$;

$$\zeta_1^{(0)} \to \zeta_0 \qquad \text{(from (1.28))}\ ,$$
$$\zeta_2^{(0)} \to 2e^{-ikd_1}\zeta_0 \qquad \text{(from (1.29))}\ ,$$
$$\zeta_3^{(0)} \to 2\zeta_0 \qquad \text{(from (1.30))}\ ,$$

that is to say, the reflected wave in the domain D_1 tends to the incident wave in magnitude and both waves are in phase; the amplitudes of the progressive waves in the domains D_2 and D_3 approach twice that of the incident wave (the phase difference is a consequence of the assumption that $d_2 > d_3$ (refer to Fig. (I. 1))).

I. 2. Numerical Results.

To see the variations of the wave heights in each canal for the ratios of the canal widths, the following expressions are introduced:

$$\left.\begin{aligned} |\zeta_1^{(0)}/\zeta_0| &= \frac{|1-R_{2,1}-R_{3,1}|}{1+R_{2,1}+R_{3,1}} \quad \text{(from (1.28))}\ , \\ |\zeta_2^{(0)}/\zeta_0| &= \frac{2}{1+R_{2,1}+R_{3,1}} \quad \text{(from (1.29))}\ , \\ |\zeta_3^{(0)}/\zeta_0| &= \frac{2}{1+R_{2,1}+R_{3,1}} \quad \text{(from (1.30))}\ , \end{aligned}\right\} \qquad (1.33)$$

where $d_2/d_1 = R_{2,1}$, $d_3/d_1 = R_{3,1}$.

The variations of the ratios $|\zeta_2^{(0)}/\zeta_0|$ and $|\zeta_1^{(0)}/\zeta_0|$ for $R_{2,1}$ and $R_{3,1}$ are plotted in Fig. (I. 2), where $R_{2,1} > R_{3,1}$ by $d_2 > d_3$.

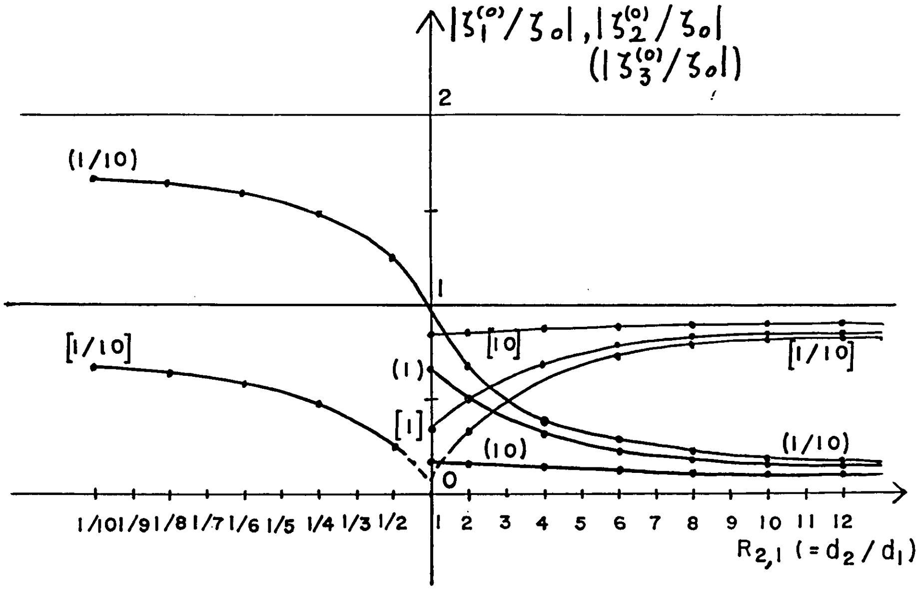

Fig. (I. 2). The values stated in round and square brackets denote the ratios of d_3 and d_1 or $R_{3,1}$. The drawn curves are the variations of $|\zeta_2^{(0)}/\zeta_0|$ and $|\zeta_1^{(0)}/\zeta_0|$ versus $R_{2,1}$ ($=d_2/d_1$) for given $R_{3,1}$ respectively, the former and the latter of which are marked with round and square brackets respectively.

Part II.

This part is composed of two cases, i. e., the incident waves come from *a*) the domain D_3 and *b*) the domain D_2 (refer to Figs. (II. a1 and b1)), on the supposition that the width of the canal in D_2 is larger than that in D_3. Firstly, the former case is treated.

a) *The case where the periodic waves come from the domain* D_3.

II. *a*1. Theory.

Using the same notations and definitions as in Part I, the basic equation and the boundary conditions are identical with those in Part I, except for the difference of the directions of the surging periodic waves. Hence the solutions are given as below:

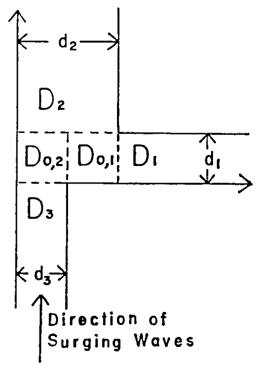

Fig. (II. *a*1).

For the domain D_1,

$$\zeta_1=\sum_{m=0}^{\infty}\zeta_1^{(m)}\cos\frac{m\pi}{d_1}y\cdot e^{+ik_1^{(m)}x}, \tag{2.1}$$

which is an expression excluding the surging wave term in (1.2);

for the domain D_2, the solution is given by (1.3), i. e.,

$$\zeta_2=\sum_{m=0}^{\infty}\zeta_2^{(m)}\cos\frac{m\pi}{d_2}x\cdot e^{+ik_2^{(m)}y}; \tag{2.2}$$

for the domain D_3, adding the surging wave term to (1.4),

$$\zeta_3=\zeta_0 e^{+iky}+\sum_{m=0}^{\infty}\zeta_3^{(m)}\cos\frac{m\pi}{d_3}x\cdot e^{-ik_3^{(m)}y}; \tag{2.3}$$

for the domains $D_{0,1}$ and $D_{0,2}$, the solutions are described by (1.5) and (1.6), i. e.,

$$\zeta_{0,1}=\sum_{f_{0,1}}(A_{0,1}\cos k_x^{(1)}x+B_{0,1}\sin k_x^{(1)}x)\cos k_y^{(1)}y \tag{2.4}$$

and

$$\zeta_{0,2}=\sum_{f_{0,2}}\cos k_x^{(2)}x(A_{0,2}\cos k_y^{(2)}y+B_{0,2}\sin k_y^{(2)}y), \tag{2.5}$$

respectively.

Available conditions for determining the arbitrary constants are the same as those given in section (vi) of Part I.

Following the procedure in section (vii) of Part I (Momoi's Method), the first modes of the waves in each canal are obtained as follows:

$$\zeta_1^{(0)}=\frac{2d_3}{d_1+d_2+d_3}\cdot e^{-ik(d_1+d_2)}\zeta_0, \tag{2.6}$$

$$\zeta_2^{(0)}=\frac{2d_3}{d_1+d_2+d_3}\cdot e^{-ikd_1}\zeta_0, \tag{2.7}$$

$$\zeta_3^{(0)}=\frac{-d_1-d_2+d_3}{d_1+d_2+d_3}\cdot\zeta_0, \tag{2.8}$$

where $d_3<d_2$ (refer to Fig. (II. a1).

By the same reason as in part I, the solutions of the higher modes of the waves in the straight parts of the canals and the wave in the conjunction part are left untouched.

As the next step, let us consider the particular cases.

(a. 1) when $d_3 \to 0$;

$$\zeta_1^{(0)} \to 0 ,$$
$$\zeta_2^{(0)} \to 0 ,$$
$$\zeta_3^{(0)} \to -\zeta_0 ,$$

The progressive waves in the domains D_1 and D_2 tend to zero, while the reflected wave in the domain D_3 approches in amplitude the incident one with inverse phase.

(a. 2) when $d_1 = d_2 = d_3 = d$;

$$\left.\begin{aligned} \zeta_1^{(0)} &= \frac{2}{3}\cdot e^{-i\cdot 2kd}\zeta_0 , \\ \zeta_2^{(0)} &= \frac{2}{3}\cdot e^{-i\cdot kd}\zeta_0 , \\ \zeta_3^{(0)} &= \frac{-1}{3}\cdot\zeta_0 . \end{aligned}\right\} \tag{2.9}$$

The amplitudes in the domains D_1 and D_2 are equal to each other, which is an unexpectedly interesting result. At least, the author has anticipated larger wave in the domain D_2 than in the domain D_1 so far. The amplitudes of these waves are two-third that of the incident one, while the reflected wave amplitude is one-third.

A comparison of the same cases in Part I and II is made in a later section.

(a. 3) when $d_1 \to 0$;

$$\left.\begin{aligned} \zeta_1^{(0)} &\to \frac{2d_3}{d_2+d_3}\cdot e^{-ikd_2}\zeta_0 , \\ \zeta_2^{(0)} &\to \frac{2d_3}{d_2+d^3}\cdot\zeta_0 , \\ \zeta_3^{(0)} &\to \frac{-d_2+d_3}{d_2+d_3}\cdot\zeta_0 . \end{aligned}\right\} \tag{2.10}$$

The wave height in the domain D_1 is completely dominated by the ratio of the widths in the domains D_2 and D_3. When $d_2 = d_3$, the model becomes a straight canal.

II. a2. Numerical Results

For convenience of graphical expression, expressions similar to Part

I are used, i. e.,

$$\left.\begin{aligned} |\zeta_1^{(0)}/\zeta_0| &= \frac{2}{R_{1,3}+R_{2,3}+1} \qquad \text{(from (2.6))}\,, \\ |\zeta_2^{(0)}/\zeta_0| &= \frac{2}{R_{1,3}+R_{2,3}+1} \qquad \text{(from (2.7))}\,, \\ |\zeta_3^{(0)}/\zeta_0| &= \frac{|-R_{1,3}-R_{2,3}+1|}{R_{1,3}+R_{2,3}+1} \qquad \text{(from (2.8))}\,, \end{aligned}\right\} \tag{2.11}$$

where $d_1/d_3=R_{1,3}$, $d_2/d_3=R_{2,3}$ and $R_{2,3}>1$ by $d_2>d_3$.

The expression of the reflected wave in (2.11), viz., the last of (2.11) is identical in form with the corresponding one in (1.33). Likewise, the first two of (2.11) are of the same forms as the last two of (1.32), which are the expressions of the progressive waves. Therefore, the following substitutions of $\zeta_1^{(0)}$, $\zeta_2^{(0)}$ (or $\zeta_3^{(0)}$), $R_{2,1}$ and $R_{3,1}$ stated in Fig. (I. 2) give rise to the use of Fig. (I. 2) for this case:

$$\zeta_1^{(0)} \to \zeta_3^{(0)}\,, \quad \zeta_2^{(0)} \text{ (or } \zeta_3^{(0)}) \to \zeta_1^{(0)} \text{ (or } \zeta_2^{(0)})\,,$$
$$R_{2,1} \to R_{1,3}\,, \quad R_{3,1} \to R_{2,3}\,,$$

where the restriction $R_{2,3}>1$ for this case, instead of $R_{2,1}>R_{3,1}$ for the former case.

b) The case where the periodic waves come from the domain D_2.

II. *b*1. Theory

The analysis of this case follows exactly the same lines as in section *a*), so that much of the detail should be referred to in the preceding part.

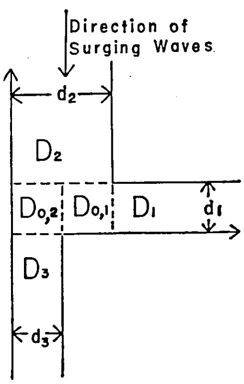

Fig. (II. *b*1).

Using the same notations and definitions as in Part I and the section *a*), the first modes of the waves in the straight canals become

$$\left.\begin{aligned} \zeta_1^{(0)} &= \frac{2d_2}{d_1+d_2+d_3}\cdot e^{-ik(d_1+d_2)}\zeta_0\,, \\ \zeta_2^{(0)} &= \frac{-d_1-d_3+d_2}{d_1+d_2+d_3}\cdot e^{-i\cdot 2kd_1}\zeta_0\,, \\ \zeta_3^{(0)} &= \frac{2d_2}{d_1+d_2+d_3}\cdot e^{-i\cdot kd_1}\zeta_0\,, \end{aligned}\right\} \tag{2.12}$$

where $d_3 < d_2$ (refer to Fig. (II. *b*1)).

The considerations of the particular cases given below:

(*b*. 1) when $d_1 = d_2 = d_3 = d$;

$$\left.\begin{aligned} \zeta_1^{(0)} &= \frac{2}{3} \cdot e^{-i \cdot 2kd} \zeta_0 , \\ \zeta_2^{(0)} &= -\frac{1}{3} \cdot e^{-i \cdot 2kd} \zeta_0 , \\ \zeta_3^{(0)} &= \frac{2}{3} \cdot e^{-i \cdot kd} \zeta_0 , \end{aligned}\right\} \tag{2.13}$$

In the same way as in Part I and the section *a*) of this part, the amplitudes in the domains D_1 and D_3 are equally two-third the incident wave amplitude, while that of the reflected wave in the domain D_2 is one-third. Though the amplitudes in (1.32), (2.9) and (2.13) for the corresponding canals are equal, a clear phase difference of the three cases can be seen.

Firstly, the reflected wave for the case of Part I is in later phase by $\pi + 2kd$ for the incident wave (refer to (1.32)). This suggests that the dimension of the conjunction part affects the phase of the reflected wave. Secondly, the reflected wave for the case of section *a*) in this part is in perfectly inverse phase for the incident wave (refer to (2.9)). This result is due to the assumption that $d_2 > d_3$.

Thirdly, owing to the assumption that $d_2 > d_3$, the phase lag of the reflected wave from the incident one is $\pi + 2kd$ for the case of section *b*) (refer to (2.13)).

Next, as far as the progressive waves in the case of Part I are concerned, the phase lags are $2kd$.

For the cases of Part II, the progressive waves advancing straight through the conjunction part of the canals are later in phase by kd (refer to the second of (2.9) and the last of (2.13)), while those turning in the conjunction part being in later phase by $2kd$ (refer to the first expressions of (2.9) and (2.13)).

II. *b*2. Numerical Results

The amplitude parts of the expressions (2.12) have the same forms as in Part I. By similar changes of the expressions as made in section

(II. $a2$) the use of Fig. (I. 2) for this case is possible.

Concluding Remarks of Part I and II

Although the author's method has only been applied to cases where the widths of the canals are small as compared with the wave-length of the surging waves, this method is also valid for the case where the widths of the canals are comparable with the wave-length of the waves. In the near future, the latter treatment will be made.

Anyway, as far as the problem in the case of the long waves is concerned, the expressions of the first modes of the waves are in exact agreement with those derived from the consideration of the flux in the canals.[7] Our method, however, makes it possible to compute the higher modes of the waves beyond the initial one (refer to the preceding paper[8]), while the method derived by the flux can find no way to a further study of higher modes. When the study of waves of medium wave-length compared with the width of the canal is made, this point will become clearer.

Acknowledgement

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24. T字水路における津波について

地震研究所 桃 井 高 夫

先きに，筆者はL字水路における津波をあつかうに際し（論文題名：津波に対する海岸線の影響），非常に巧妙な方法を導入した．すなわち

(1) 長波近似を行なうこと．

(2) 基本方程式

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\zeta=0,$$

（ここで ζ: 波高　x, y: 直角座標系）

において，変数分離を行なうと，波数関係を表わす式

$$k_x^2+k_y^2=k^2,$$

7) HORACE LAMB, *Hydrodynamics* (Cambridge, 1932), p. 254.

8) T. MOMOI, *loc. cit.*, 1).

$$\left(\begin{array}{ll} \text{ここで} & \zeta=\zeta_x(x)\cdot\zeta_y(y)\,, \\ & \dfrac{d^2\zeta_x}{dx^2}=-k_x{}^2\zeta_x\,, \\ & \dfrac{d^2\zeta_y}{dy^2}=-k_y{}^2\zeta_y\,, \end{array}\right)$$

を得る．この波数関係式を用いて代数的に式の計算を行なつて行くこと．

この二方針のもとに，計算を行なうと今まで解析的に解けなかつた問題，特に，小さな障害をもつた地形における津波，および音波の問題に有力な武器となることが判つた．筆者はこの方法を，今後，桃井の方法と呼称したい．この桃井の方法の優秀性をＴ字水路の場合に適用したのが本論文である．
