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25. *Tsunami in a Canal of Varying Width.*

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Introduction

In the preceding papers[1)-3)] the author demonstrated the effectiveness of Momoi's method for treating the tsunamis in the canals. In this paper he has also applied this method to a tsunami in a canal of varying width and further study is made towords developing a theory to produce a new formula. For convenience of reference Momoi's method is outlined here:

(1) the first reduction is an application of the long wave approximation,

(2) the second one is to eliminate the "buffer domain" by the relation of the wave number components (for the meaning of "buffer domain" refer to Part I.

Having outlined the method, it remains for the author to solve a particular problem following the principles of this method. The present purview is composed of two parts:

Part I: the case where two canals of different widths are connected (Fig. 1),

Part II: the case where three canals of different widths are connected (Fig. 2).

Part I.

The case where two canals of different widths are connected.

I. 1. Theory

Assuming that (refer to Fig. 1),

D_1 : the domain in the range $x>d_4$, $d_1>y>0$;

D_2 : the domain in the range $x<0$, $d_2>y>0$;

D_{01}: the domain in the range $d_4>x>0$, $d_1>y>d_2$;

1) T. MOMOI, *Bull. Earthq. Res. Inst.*, **40** (1962), 719.
2) T. MOMOI, *ditto*, **40** (1962), 747.
3) T. MOMOI, *ditto*, **41** (1963), 357.

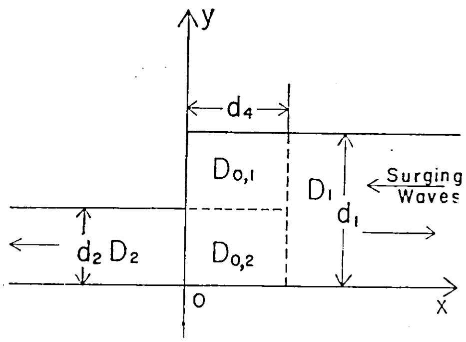

Fig. 1.

$D_{0,2}$: the domain in the range $d_4>x>0$, $d_2>y>0$;

$\zeta_1, \zeta_2, \zeta_{0,1}, \zeta_{0,2}$: the wave heights in the domains D_1, D_2, $D_{0,1}$, $D_{0,2}$;

t : time variable;

ω : the angular frequency of the surging waves;

H : the depth of water;

g : the acceleration of gravity;

c : the velocity of long wave, i.e., $\sqrt{gH}$;

k : the wave number of the surging waves, i.e., ω/c;

then we have, as basic equations,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\zeta_j=0 \quad (j=1;\ 2;\ 0{,}1;\ 0{,}2), \tag{I.1}$$

and, as boundary conditions,

$$\left.\begin{aligned}
&\frac{\partial\zeta_1}{\partial y}=0 \quad (x>d_4,\ y=0 \text{ and } d_1)\ ,\\
&\frac{\partial\zeta_2}{\partial y}=0 \quad (x<0,\ y=0 \text{ and } d_2)\ ,\\
&\frac{\partial\zeta_{0,1}}{\partial y}=0 \quad (d_4>x>0,\ y=d_1)\ ,\\
&\frac{\partial\zeta_{0,1}}{\partial x}=0 \quad (d_1>y>d_2,\ x=0)\ ,\\
&\frac{\partial\zeta_{0,2}}{\partial y}=0 \quad (d_4>x>0,\ y=0)\ ,
\end{aligned}\right\} \tag{I.2}$$

where the time factor exp $(-i\omega t)$ is omitted as usual; the length of d_4 is of order such as the relation

$$kd_4\ll 1\ , \tag{I.3}$$

holds.

The domains $D_{0,1}$ and $D_{0,2}$ have temporarily been made in order that the conditions at the crooked part are satisfied. Using Momoi's method, the expressions of the wave heights in these domains are later eliminated.

These domains, therefore, have been named "buffer domain" in the introduction of this paper.

When the periodic waves proceed from the domain D_1, the solutions of (I.1) satisfying the boundary conditions (I.2) are as given below:

for the domain D_1,

$$\zeta_1 = \zeta_0 e^{-ikx} + \sum_{m=0}^{\infty} \zeta_1^{(m)} \cos \frac{m\pi}{d_1} y \cdot e^{+ik_1^{(m)}x} , \tag{I.4}$$

where ζ_0 and $\zeta_1^{(m)}$ are arbitrary constants; $k_1^{(m)} = +\sqrt{k^2 - \left(\frac{m\pi}{d_1}\right)^2}$,

for the domain D_2,

$$\zeta_2 = \sum_{m=0}^{\infty} \zeta_2^{(m)} \cos \frac{m\pi}{d_2} y \cdot e^{-ik_2^{(m)}x} , \tag{I.5}$$

where $\zeta_2^{(m)}$ is an arbitrary constant; $k_2^{(m)} = +\sqrt{k^2 - \left(\frac{m\pi}{d_2}\right)^2}$,

for the domain $D_{0,1}$,

$$\zeta_{0,1} = \sum_{f_{0,1}} A_{0,1} \cos k_x^{(1)} x \cos k_y^{(1)} (y - d_1) , \tag{I.6}$$

where $A_{0,1}$ is an arbitrary constant; $k^2 = (k_x^{(1)})^2 + (k_y^{(1)})^2$, $\sum_{f_{0,1}}$ the integration over the range permitted by the relation $k^2 = (k_x^{(1)}) + (k_y^{(1)})^2$,

for the domain $D_{0,2}$,

$$\zeta_{0,2} = \sum_{f_{0,2}} (A_{0,2} \cos k_x^{(2)} x + B_{0,2} \sin k_x^{(2)} x) \cos k_y^{(2)} y , \tag{I.7}$$

where $A_{0,2}$ is an arbitrary constant; $k^2 = (k_x^{(2)})^2 + (k_y^{(2)})^2$; $\sum_{f_{0,2}}$ the integration over the range permissible by the wave number relation.

The derivation of (I.4)—(I.7) should be referred to the previous papers.[4)—6)] Available conditions to determine the arbitrary constants are as follows:

at $x = d_4$,

$$\left.\begin{aligned} &\left.\begin{aligned} &\zeta_{0,1} \quad (\text{for } d_1 > y > d_2) \\ &\zeta_{0,2} \quad (\text{for } d_2 > y > 0) \end{aligned}\right\} = \zeta_1 \quad (\text{for } d_1 > y > 0) , \\ &\left.\begin{aligned} &\frac{\partial \zeta_{0,1}}{\partial x} \quad (\text{for } d_1 > y > d_2) \\ &\frac{\partial \zeta_{0,2}}{\partial x} \quad (\text{for } d_2 > y > 0) \end{aligned}\right\} = \frac{\partial \zeta_1}{\partial x} \quad (\text{for } d_1 > y > 0) ; \end{aligned}\right\} \tag{I.8}$$

4) T. MOMOI, *loc. cit.*, 1).
5) T. MOMOI, *loc. cit.*, 2).
6) T. MOMOI, *loc. cit.*, 3).

at $x=0$,

$$\left.\begin{aligned}\zeta_2&=\zeta_{0,2}\,,\\ \frac{\partial\zeta_2}{\partial x}&=\frac{\partial\zeta_{0,2}}{\partial x}\,,\end{aligned}\right\}\ (\text{for }\ d_2>y>0)\ ; \tag{I.9}$$

at $y=d_2$,

$$\left.\begin{aligned}\zeta_{0,1}&=\zeta_{0,2}\,,\\ \frac{\partial\zeta_{0,1}}{\partial y}&=\frac{\partial\zeta_{0,2}}{\partial y}\,,\end{aligned}\right\}\ (\text{for }\ d_4>x>0)\ . \tag{I.10}$$

Substituting (I.4)—(I.7) for (I.8)—(I.10) and as the result of the following integrations:

$$\left.\begin{aligned}&\left.\begin{aligned}&\int_{d_2}^{d_1}\zeta_{0,1}dy\\ &\int_0^{d_2}\zeta_{0,2}dy\end{aligned}\right\}=\int_0^{d_1}\zeta_1 dy\\ &\left.\begin{aligned}&\int_{d_2}^{d_1}\frac{\partial\zeta_{0,1}}{\partial x}dy\\ &\int_0^{d_2}\frac{\partial\zeta_{0,2}}{\partial x}dy\end{aligned}\right\}=\int_0^{d_1}\frac{\partial\zeta_1}{\partial x}dy\end{aligned}\right\}\ (x=d_4)\ ,$$

$$\left.\begin{aligned}\int_0^{d_2}\zeta_2 dy&=\int_0^{d_2}\zeta_{0,2}dy\\ \int_0^{d_2}\frac{\partial\zeta_2}{\partial x}dy&=\int_0^{d_2}\frac{\partial\zeta_{0,2}}{\partial x}dy\end{aligned}\right\}\ (x=0)\ ,$$

$$\left.\begin{aligned}\int_0^{d_4}\zeta_{0,1}dx&=\int_0^{d_4}\zeta_{0,2}dx\\ \int_0^{d_4}\frac{\partial\zeta_{0,1}}{\partial y}dx&=\int_0^{d_4}\frac{\partial\zeta_{0,1}}{\partial y}dx\end{aligned}\right\}\ (y=d_2)\ ,$$

$$\left.\begin{aligned}&\sum_{f_{0,1}}A_{0,1}\cos k_x^{(1)}d_4\cdot\frac{1}{k_y^{(1)}}\cdot\sin k_y^{(1)}(d_1-d_2)\\ &\qquad+\sum_{f_{0,2}}(A_{0,2}\cos k_x^{(2)}d_4+B_{0,2}\sin k_x^{(2)}d_4)\cdot\frac{1}{k_y^{(2)}}\cdot\sin k_y^{(2)}d_2\\ &\quad=(\zeta_0 e^{-ikd_4}+\zeta_1^{(0)}e^{+ikd_4})d_1\ ,\\ &\sum_{f_{0,1}}A_{0,1}(-k_x^{(1)})\sin k_x^{(1)}d_4\cdot\frac{1}{k_y^{(1)}}\cdot\sin k_y^{(1)}(d_1-d_2)\\ &\qquad+\sum_{f_{0,2}}(-A_{0,2}\,k_x^{(2)}\sin k_x^{(2)}d_4+B_{0,2}k_x^{(2)}\cos k_x^{(2)}d_4)\cdot\frac{1}{k_y^{(2)}}\cdot\sin k_y^{(2)}d_2\\ &\quad=(-\zeta_0 e^{-ikd_4}+\zeta_1^{(0)}e^{+ikd_4})(+ikd_1)\ ,\end{aligned}\right\} \tag{I.8$'$}$$

$$\left.\begin{aligned} \zeta_2^{(0)} d_2 &= \sum_{f_{0,2}} A_{0,2} \cdot \frac{1}{k_y^{(2)}} \cdot \sin k_y^{(2)} d_2 \,, \\ \zeta_2^{(0)}(-ikd_2) &= \sum_{f_{0,2}} B_{0,2} k_x^{(2)} \cdot \frac{1}{k_y^{(2)}} \cdot \sin k_y^{(2)} d_2 \,, \end{aligned}\right\} \qquad \text{(I.9}'\text{)}$$

$$\left.\begin{aligned} &\sum_{f_{0,1}} A_{0,1} \cdot \frac{1}{k_x^{(1)}} \sin k_x^{(1)} d_4 \cos k_y^{(1)}(d_2 - d_1) \\ &= \sum_{f_{0,2}} \left\{ A_{0,2} \cdot \frac{1}{k_x^{(2)}} \cdot \sin k_x^{(2)} d_4 + B_{0,2} \cdot \frac{1}{k_x^{(2)}} \cdot (1 - \cos k_x^{(2)} d_4) \right\} \cos k_y^{(2)} d_2 \,, \\ &\sum_{f_{0,1}} A_{0,1}(-k_y^{(1)}) \cdot \frac{1}{k_x^{(1)}} \cdot \sin k_x^{(1)} d_4 \sin k_y^{(1)}(d_2 - d_1) \\ &= \sum_{f_{0,2}} \left\{ A_{0,2} \cdot \frac{1}{k_x^{(2)}} \cdot \sin k_x^{(2)} d_4 + B_{0,2} \cdot \frac{1}{k_x^{(2)}} \cdot (1 - \cos k_x^{(2)} d_4) \right\} (-k_y^{(2)}) \sin k_y^{(2)} d_2 \,. \end{aligned}\right\} \qquad \text{(I.10}'\text{)}$$

Following the principles of Momoi's method, i. e.,

(1) the application of the long wave approximation: $k_j d_j \ll 1$ $(j=1, 2)$, $k_l^{(i)} d_p \ll 1$ $(i=1, 2;\ p=1, 2;\ l=x, y)$, and $kd_4 \ll 1$, the last of which is from (I.3),

(2) the elimination of the expressions in the buffer domains $D_{0,1}$ and $D_{0,2}$ by use of the wave number relations $k^2 = (k_x^{(j)})^2 + (k_y^{(j)})^2$ $(j=1, 2)$,

the next two equations are obtained from (I.8′)–(I.10′):

$$\left.\begin{aligned} d_1 e^{+ikd_4} \zeta_1^{(0)} + (-ikd_4 d_1 + d_2) \zeta_2^{(0)} &= d_1 e^{-ikd_4} \zeta_0 \,, \\ -d_1 e^{+ikd_4} \zeta_1^{(0)} + (-ikd_4 d_2 + d_1) \zeta_2^{(0)} &= d_1 e^{-ikd_4} \zeta_0 \,, \end{aligned}\right\} \qquad \text{(I.11)}$$

where the actual reductions to attain the results mentiond above should be referred to in the preceding papers[7)–9)]; the equations (I.11) are two simultaneous equations in terms of two unknowns $\zeta_1^{(0)}$ and $\zeta_2^{(0)}$, i. e. the first modes of the expressions of the waves in the domains D_1 and D_2.

Solving (I.11), we obtain, to the first order of kd_j $(j=1, 2, 4)$,

$$\left.\begin{aligned} \zeta_1^{(0)} &= \frac{d_1 - d_2}{d_1 + d_2} \cdot \zeta_0 \,, \\ \zeta_2^{(0)} &= \frac{2d_1}{d_1 + d_2} \cdot \zeta_0 \,, \end{aligned}\right\} \qquad \text{(I.12)}$$

where the reduction

7) T. Momoi, *loc. cit.*, 1).
8) T. Momoi, *loc. cit.*, 2).
9) T. Momoi, *loc. cit.*, 3).

$$1-ikd_4 \simeq e^{-ikd_4}$$

is used.

Here it should be noted that the expressions (I.12) (the first modes of the waves in the domains D_1 and D_2) are, to the first order of kd_j ($j=1, 2, 4$), independent of d_4 (the width of the buffer domain), if the widths of the buffer domains are taken within the long wave approximation.

The nature of the buffer domain is detailed in the section (1.3).

As a next step, the higher modes of the waves are considered in the following.

In a manner similar to the analysis described in the fore-going part, applying the operators:

$$\int_0^{d_1} \cos \frac{m'\pi}{d_1} \cdot y dy \quad (m'=1, 2, 3, \cdots),$$

to the condition (I.8), and

$$\int_0^{d_2} \cos \frac{m'\pi}{d_2} \cdot y dy \quad (m'=1, 2, 3, \cdots),$$

to (I.9), and by use of the long wave approximation (the first reduction of Momoi's method), we have:

$$-\sum_{f_{0.1}} A_{0,1}\left\{(k_y^{(1)})^2 \cdot \frac{d_1^2(d_1-d_2)}{(m'\pi)^2} \cdot \cos m'\pi\frac{d_2}{d_1} + \frac{d_1}{m'\pi} \cdot \sin m'\pi\frac{d_2}{d_1}\right\}$$
$$+\sum_{f_{0.2}} (A_{0,2}+B_{0,2}k_x^{(2)}d_4)\left\{-(k_y^{(2)})^2 \cdot \frac{d_1^2 d_2}{(m'\pi)^2} \cdot \cos m'\pi\frac{d_2}{d_1} + \frac{d_1}{m'\pi} \cdot \sin m'\pi\frac{d_2}{d_1}\right\}$$
$$=\zeta_1^{(m')} \cdot \frac{d_1}{2} \cdot e^{+ik_1^{(m')}d_4}, \tag{I.13}$$

$$\sum_{f_{0.1}} A_{0.1}(k_x^{(1)})^2 d_4\left\{(k_y^{(1)})^2 \cdot \frac{d_1^2(d_1-d_2)}{(m'\pi)^2} \cdot \cos m'\pi\frac{d_2}{d_1} + \frac{d_1}{m'\pi} \sin m'\pi\frac{d_2}{d_1}\right\}$$
$$+\sum_{f_{0.2}}\{-A_{0,2}(k_x^{(2)})^2 d_4 + B_{0,2}k_x^{(2)}\}\left\{-(k_y^{(2)})^2 \cdot \frac{d_1^2 d_2}{(m'\pi)^2} \cdot \cos m'\pi\frac{d_2}{d_1} + \frac{d_1}{m'\pi} \cdot \sin m'\pi\frac{d_2}{d_1}\right\}$$
$$=\zeta_1^{(m')} \cdot (+ik_1^{(m')}) \cdot \frac{d_1}{2} \cdot e^{+ik_1^{(m')}d_4}, \tag{I.14}$$

$$\zeta_2^{(m')} \cdot \frac{d_2}{2} = \sum_{f_{0.2}} A_{0,2}(-1)^{m'+1} \cdot \frac{(k_y^{(2)}d_2)^2}{(m'\pi)^2} \cdot d_2, \tag{I.15}$$

$$\zeta_2^{(m')}\cdot(-ik_2^{(m')})\cdot\frac{d_2}{2}=\sum_{f_{0,2}}B_{0,2}k_x^{(2)}\cdot(-1)^{m'+1}\cdot\frac{(k_y^{(2)}d_2)^2}{(m'\pi)^2}\cdot d_2\,, \tag{I.16}$$

where $m'=1, 2, 3, \cdots$.

In order to reduce the above equations, three more relations are needed.

So long as the problem is confined to the case of the long wave compared with the width of the canal, these relations are supplied by the latter of (I.9′) and the two of (I.10′) after applying the long wave approximation, i. e.,

$$\zeta_2^{(0)}(-ikd_2)=\sum_{f_{0,2}}B_{0,2}k_x^{(2)}d_2\,, \tag{I.17}$$

$$\sum_{f_{0,1}}A_{0,1}=\sum_{f_{0,2}}A_{0,2}\,, \tag{I.18}$$

$$\sum_{f_{0,1}}A_{0,1}(k_y^{(1)})^2(d_2-d_1)=\sum_{f_{0,2}}A_{0,2}(k_y^{(2)})^2d_2\,. \tag{I.19}$$

As the second reduction of Momoi's method, let us eliminate the expressions in the buffer domains using the relations (I.17), (I.18) and (I.19).

Substituting (I.18) and (I.19) for (I.13) and after some reductions, we have

$$\sum_{f_{0,2}}B_{0,2}k_x^{(2)}d_4\cdot\frac{d_1}{m'\pi}\cdot\left\{\sin\, m'\pi\frac{d_2}{d_1}+O((kd_1)^2)\right\}=\zeta_1^{(m')}\cdot\frac{d_1}{2}\cdot e^{+ik_1^{(m')}d_4}\,. \tag{I.20}$$

Again putting (I.12) and (I.17) into (I.20), we finally obtain the higher modes of the waves in the domain D_1, to the first order of kd_j $(j=1, 2, 4)$,

$$\zeta_1^{(m')}=\zeta_0\cdot(-ikd_4)\cdot\frac{4d_1}{d_1+d_2}\cdot\frac{1}{m'\pi}\cdot\sin\, m'\pi\frac{d_2}{d_1}\cdot e^{+(m'\pi/d_1)d_4}\ (m'=1, 2, 3, \cdots)\,, \tag{I.21}$$

where the following reduction is used:

$$-ik_1^{(m')}=-i\sqrt{k^2-\left(\frac{m'\pi}{d_1}\right)^2}\quad (m'\geqq 1)\,,$$

$$\simeq +\frac{m'\pi}{d_1}\quad \text{(by long wave approximation).}$$

From (I.15), the order of the higher modes of the waves in the domain D_2 is as follows:

$$\zeta_2^{(m')}\sim\sum_{f_{0,2}}A_{0,2}\cdot\frac{1}{(m'\pi)^2}\cdot O((kd_1)^2)\,. \tag{I.22}$$

On the other hand, the following relation is obtained from (I.9′) after an application of the long wave approximation:

$$\zeta_2^{(0)} = \sum_{f_{0,2}} A_{0,2} \,. \tag{I.23}$$

From (I.12), (22) and (I.23), we have

$$\zeta_2^{(m')} \sim \zeta_0 \cdot \frac{2d_1}{d_1+d_2} \cdot \frac{1}{(m'\pi)^2} \cdot O((kd_1)^2) \quad (m'=1, 2, 3, \cdots) \,.$$

More rigorous expression for $\zeta_2^{(m')}$, if one needs, can be derived from (I.13)–(I.16) and the relations for the zeroth modes.

When the consideration is limited to the first order of kd_j ($j=1, 2, 4$), the higher modes of the waves in the narrower canal (D_2) may be regarded as null and those in the wider canal (D_1) only remain in magnitde of the first order of kd_4.

I. 2. Particular Cases

In this paragraph the considerations for particular cases are made.

(1) when $d_1=d_2$;

$$\left.\begin{aligned} \zeta_1^{(0)} &= 0 \,, \\ \zeta_2^{(0)} &= \zeta_0 \,, \end{aligned}\right\} \quad \text{(from (I.12))} \,,$$

$$\zeta_1^{(m')} = 0 \quad (m'=1, 2, 3, \cdots) \ \text{(from (I.21))} \,.$$

These are very trivial results for the waves in a straight canal. One cannot expect any reflected waves. Only a progressive wave is expected.

(2) when $d_2 \to 0$;

$$\zeta_1^{(0)} \to \zeta_0 \,,$$
$$\zeta_2^{(0)} \to 2\zeta_0 \,,$$
$$\zeta_1^{(m')} \to 0 \quad (m'=1, 2, 3, \cdots) \ \text{(from (I.21))} \,.$$

In this case, the canal of greater width (domain D_1) eventually becomes a semi-closed canal. The surging waves are perfectly reflected at the conjunction part of the two canals. In the very narrow canal the progressive wave becomes twice the wave height of the surging one. The higher modes of the reflected waves tend to zero, as would be expected.

(3) when $d_1 \simeq 2d_2$;

$$\left.\begin{aligned} \zeta_1^{(0)} &\simeq \frac{1}{3}\zeta_0 \,, \\ \zeta_2^{(0)} &\simeq \frac{4}{3}\zeta_0 \,, \end{aligned}\right\} \quad \text{(form (I.12))} \,,$$

$$\zeta_1^{(m')} \simeq \zeta_0 \cdot (-ikd_4) \cdot \frac{8}{3} \cdot \frac{1}{m'\pi} \cdot \sin\frac{m'\pi}{2} \cdot e^{+(m'\pi/a_1)a_4} \,.$$

$$(m'=1, 2, 3, \cdots) \ \text{(from (I.21))} \,.$$

When the width of the narrow canal is approximately half that of the wide one, the disturbance in the vicinity of the conjunction part becomes most remarkable as shown in the last expression mentioned above.

I. 3. Property of Buffer Domain

Our primary concern is with the limitations of the dimension of the buffer domain. The considerations for these points are described in the following:

(1) *the upper limit*;

In order to apply the long wave approximation (the first reduction of Momoi's method), the dimension of the buffer domain must be taken as

$$\text{(the wave number of the surging wave)} \times \text{(the width of the buffer domain)} \ll 1 \,,$$

(2) *the lower limit*;

The existence of the buffer domain is for the correction of the irregular boundary. Therefore, the dimension of this domain is desired to be taken as large as possible, so that all kinds of modes of waves are contained in this domain.

The condition (I.3) corresponds to the first limitation described above.

Part II.

The case where three canals of different widths are connected.

Referring to Fig. 2, let us consider the case where three canals are connected as stairs, the widths of which monotonically diminish from d_0 to d_2.

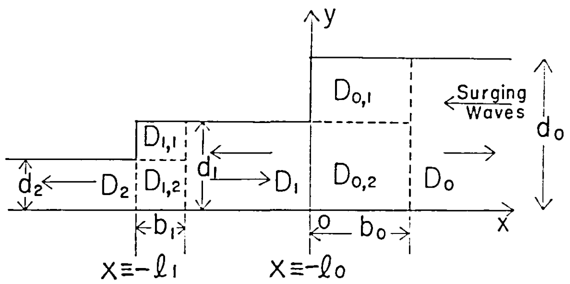

Fig. 2.

The basic equations and boundary conditions are as follows:

$$\left.\begin{aligned}
&\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\zeta_j=0 \quad (j=0,1,2)\,,\\
&\frac{\partial\zeta_0}{\partial y}=0 \quad (b_0<x,\ y=0 \text{ and } d_0)\,,\\
&\frac{\partial\zeta_1}{\partial y}=0 \quad (-l_1+b_0<x<-l_0,\ y=0 \text{ and } d_1)\,,\\
&\frac{\partial\zeta_2}{\partial y}=0 \quad (x<-l_1,\ y=0 \text{ and } d_2)\,;
\end{aligned}\right\} \tag{II.1}$$

$$\left.\begin{aligned}
&\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\zeta_{q,1}=0 \quad (q=0,1)\,,\\
&\frac{\partial\zeta_{q\,1}}{\partial y}=0 \quad (-l_q<x<-l_q+b_q,\ y=d_q\ (q=0,1))\,,\\
&\frac{\partial\zeta_{q\,1}}{\partial x}=0 \quad (x=-l_q,\ d_q>y>d_{q+1}\ (q=0,1))\,;
\end{aligned}\right\} \tag{II.2}$$

$$\left.\begin{aligned}
&\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k^2\right)\zeta_{q,2}=0 \quad (q=0,1)\,,\\
&\frac{\partial\zeta_{q\,2}}{\partial y}=0 \quad (-l_q<x<-l_q+b_q,\ y=0\ (q=0,1))\,.
\end{aligned}\right\} \tag{II.3}$$

As shown in Fig. 2, the co-ordinates are centered at the first step of the stairs and $-l_0$ is identically taken to be zero. The widths of the buffer domains $D_{q,1}$ and $D_{q\,2}$ $(q=0, 1)$ are taken in such a way that the first reduction of Momoi's method is possible, i.e.,

$$kb_q \ll 1 \quad (q=0, 1) . \tag{II.4}$$

The defined domains D_j $(j=0, 1, 2)$, $D_{q,1}$ and $D_{q\,2}$ $(q=0, 1)$ should be referred to in Fig. 2, and ζ_j, $\zeta_{q\,1}$ and $\zeta_{q\,2}$ are the wave heights in the domains D_j, $D_{q\,1}$ and $D_{q,2}$. Other definitions and notations are to be referred to in Part I.

Then the solutions are:
in the domain D_0, (from (II.1))

$$\zeta_0 = \zeta e^{-ikx} + \sum_{m=0}^{\infty} \zeta_0^{(m)} \cos \frac{m\pi}{d_0} y \cdot e^{+ik_0^{(m)}x} ; \tag{II.5}$$

in the domain D_1, (from (II.1))

$$\zeta_1 = \sum_{m=0}^{\infty} \{\zeta_1^{(m)} e^{+ik_1^{(m)}x} + \bar{\zeta}_1^{(m)} e^{-ik_1^{(m)}x}\} \cos \frac{m\pi}{d_1} y ; \tag{II.6}$$

in the domain D_2, (from (II.1))

$$\zeta_2 = \sum_{m=0}^{\infty} \zeta_2^{(m)} \cos \frac{m\pi}{d_2} y \cdot e^{-ik_2^{(m)}x} ; \tag{II.7}$$

in the domain $D_{q,1}$ $(q=0, 1)$, (from (II.2))

$$\zeta_{q\,1} = \sum_{f_{q,1}} A_{q,1} \cos k_x^{(q,1)}(x+l_q) \cos k_y^{(q,1)}(y-d_q) ; \tag{II.8}$$

in the domain $D_{q,2}$ $(q=0, 1)$, (from (II.3))

$$\zeta_{q\,2} = \sum_{f_{q,2}} \{A_{q,2} \cos k_x^{(q,2)}(x+l_q) + B_{q,2} \sin k_x^{(q,2)}(x+l_q)\} \cos k_y^{(q,2)} y . \tag{II.9}$$

In the above-mentioned expressions, ζ is the amplitude of the incident wave surging from the positive side of x in the domain D_0; $\zeta_0^{(m)}$, $\zeta_1^{(m)}$, $\bar{\zeta}_1^{(m)}$ and $\zeta_2^{(m)}$ the arbitrary constants relevant to each mode of the waves in the domains D_0, D_1 and D_2; $k^2=(k_x^{(q,j)})^2+(k_y^{(q,j)})^2$ $(q=0, 1;$ $j=1, 2)$; $\sum_{f_{q,j}}$ $(q=0, 1;\ j=1, 2)$ the integration over the range under the relation $k^2=(k_x^{(q,j)})^2+(k_y^{(q,j)})^2$; $A_{q,j}$, $B_{q,j}$ $(q=0, 1;\ j=1, 2)$ the arbitrary constants, which are the functions of $f_{q,j}$ respectively; $k_j^{(m)} = +\sqrt{k^2 - \left(\frac{m\pi}{d_j}\right)^2}$

$(j=0, 1, 2)$; other undefined expressions are to be referred to in Part I; $\sum\limits_{f_{q,2}}$ the summation under the relation $k^2=(k_x^{(q,2)})^2+(k_y^{(q,2)})^2$.

In the expressions (II.5) to (II.9) mentioned above, the time facter $\exp(-i\omega t)$ is abbreviated as usual.

In order to determaine the arbitrary constants there are the following conditions available:

at $x=-l_q+b_q$ $(q=0, 1)$,

$$\left.\begin{array}{l}\left.\begin{array}{ll}\zeta_{q,1} & (\text{for } d_q>y>d_{q+1})\\ \zeta_{q,2} & (\text{for } d_{q+1}>y>0)\end{array}\right\}=\zeta_p \quad (\text{for } d_q>y>0),\\ \left.\begin{array}{ll}\dfrac{\partial\zeta_{q,1}}{\partial x} & (\text{for } d_q>y>d_{q+1})\\ \dfrac{\partial\zeta_{q,2}}{\partial x} & (\text{for } d_{q+1}>y>0)\end{array}\right\}=\dfrac{\partial\zeta_q}{\partial x} \quad (\text{for } d_q>y>0);\end{array}\right\} \tag{II.10}$$

at $x=-l_q$ $(q=0, 1)$,

$$\left.\begin{array}{l}\zeta_{q+1}=\zeta_{q,2}\,,\\ \dfrac{\partial\zeta_{q+1}}{\partial x}=\dfrac{\partial\zeta_{q,2}}{\partial x}\,,\end{array}\right\} \quad (\text{for } d_{q+1}>y>0)\,; \tag{II.11}$$

at $y=d_{q+1}$ $(q=0, 1)$,

$$\left.\begin{array}{l}\zeta_{q,1}=\zeta_{q,2}\,,\\ \dfrac{\partial\zeta_{q,1}}{\partial x}=\dfrac{\partial\zeta_{q,2}}{\partial y}\,,\end{array}\right\} \quad (\text{for } -l_q<x<-l_q+b_q)\,. \tag{II.12}$$

In a manner similar to Part I, substituting (II.5)—(II.9) for (II.10)—(II.12), integrating (II.10)—(II.12) in the relevant intervals and using Momoi's method (the application of the long wave approximation and the elimination of the buffer domains), we have for the first modes of the waves:

$$\left.\begin{array}{r}d_0e^{+ikb_0}\zeta_0^{(0)}-(d_1+ikb_0d_0)\zeta_1^{(0)}+(d_1-ikb_0d_0)\bar{\zeta}_1^{(0)}=d_0e^{-ikb_0}\zeta\,,\\ -d_0e^{+ikb_0}\zeta_0^{(0)}+(d_0+ikb_0d_1)\zeta_1^{(0)}+(d_0-ikb_0d_1)\bar{\zeta}_1^{(0)}=d_0e^{-ikb_0}\zeta\,,\\ (-d_2+ikb_1d_1)e^{+ikl_1}\zeta_2^{(0)}=d_1e^{+ik(-l_1+b_1)}\zeta_1^{(0)}-d_1e^{-ik(-l_1+b_1)}\bar{\zeta}_1^{(0)}\,,\\ (d_1-ikb_1d_2)e^{+ikl_1}\zeta_2^{(0)}=d_1e^{+ik(-l_1+b_1)}\zeta_1^{(0)}+d_1e^{-ik(-l_1+b_1)}\bar{\zeta}_1^{(0)}\,,\end{array}\right\} \tag{II.13}$$

where the assumption (II.4) has been used in derivation of the above equations.

Solving the equations (II.13), we finally obtain:
as the reflected wave in the domain D_0,

$$\zeta_0^{(0)}=\frac{R_{1,0}+R_{2,1}e^{+i\cdot 2kl_1}}{(1+R_{1,0}\cdot R_{2,1}e^{+i\cdot 2kl_1})}\cdot\zeta\ ; \qquad \text{(II.14)}$$

as the progressive and the retrogressive waves in the domain D_1,

$$\left.\begin{aligned} \bar{\zeta}_1^{(0)}&=\frac{P_{1,0}}{(1+R_{1\,0}\cdot R_{2,1}e^{+i\cdot 2kl_1})}\cdot\zeta \\ &\text{and} \\ \zeta_1^{(0)}&=\frac{P_{1,0}\cdot R_{2,1}e^{+i\cdot 2kl_1}}{(1+R_{1,0}\cdot R_{2\,1}e^{+i\cdot 2kl_1})}\cdot\zeta \\ &\text{respectively}\ ; \end{aligned}\right\} \qquad \text{(II.15)}$$

as the progressive wave in the domain D_2,

$$\zeta_2^{(0)}=\frac{P_{1\,0}\cdot P_{2,1}}{(1+R_{1,0}\cdot R_{2,1}e^{+i\cdot 2kl_1})}\cdot\zeta\ , \qquad \text{(II.16)}$$

where

$$\left.\begin{aligned} R_{p+1,\,p}&=\frac{d_p-d_{p+1}}{d_p+d_{p+1}}\ , \\ P_{p+1,\,p}&=\frac{2d_p}{d_p+d_{p+1}}\ , \end{aligned}\right\} \qquad \text{(II.17)}$$

($d_{p+1}<d_p$ by the assumption (Fig. II)).

In the expressions (II.14)–(II.17), it is noteworthy that the denominators have the term $R_{1\,0}\cdot R_{2\,1}e^{+i\cdot 2kl_1}$ in addition to 1.

For convenience of physical interpretation of (II.15)–(II.16), the denominators are rewritten by infinite series.

Since $|R_{1\,0}\cdot R_{2\,1}e^{+i\cdot 2kl_1}|<1$, (II.15) and (II.16) become:

$$\left.\begin{aligned} \bar{\zeta}_1^{(0)}&=\zeta\cdot[P_{1,0}+\sum_{m=0}^{\infty}P_{1\,0}R_{2,1}^{m}\{(-1)R_{1,0}\}^{m}e^{+i\cdot 2kml_1}]\ , \\ \zeta_1^{(0)}&=\zeta\cdot[P_{1\,0}R_{2,1}e^{+i\cdot 2kl_1}+\sum_{m=0}^{\infty}P_{1\,0}R_{2,1}^{m+1}\{(-1)R_{1,0}\}^{m}e^{+i\cdot 2k(m+1)l_1}]\ , \end{aligned}\right\} \qquad \text{(II.15')}$$

and

$$\zeta_2^{(0)}=\zeta\cdot[P_{1\,0}P_{2\,1}+\sum_{m=0}^{\infty}P_{1\,1}R_{2,1}^{m}\{(-1)R_{1,0}\}^{m}P_{2,1}e^{+i\cdot 2kml_1}]\ . \qquad \text{(II.16')}$$

Using the notations (II.17), (I.12) becomes

$$\left.\begin{aligned}\zeta_1^{(0)}&=R_{2,1}\zeta_0\,,\\ \zeta_2^{(0)}&=P_{2,1}\zeta_0\,,\end{aligned}\right\}\quad \text{for } d_2<d_1\,.$$

If $d_2>d_1$, the first expression becomes, in a manner similar to the fore-going procedure,

$$\zeta_0^{(0)}=(-1)R_{2,1}\zeta_0 \quad \text{for } d_2>d_1\,.$$

Hence $R_{2,1}$ and $(-1)R_{2,1}$ stand for the coefficients of reflection for the case $d_2<d_1$ and $d_2>d_1$, while $P_{2,1}$ that of transmission of the waves for either case. On the basis of this fact, each term of the expressions (II.15′) and (II.16′) represents the multi-reflection of the waves between the two steps of the canal.

If necessary, the higher modes of the waves can be computed in the same manner as in Part I.

Concluding Remarks

As far as the dimension of the buffer domain is concerned, some ambiguous points are present, which are accounted for by the ambiguity of the long wave approximation, that is to say, kd is very small compared with unit.

When d_4 (the dimension of the buffer domain) tends to zero, the expression (I.21) (the higher modes of waves in the larger canal) also tends to zero. Then no correction for the irregularity of the bonudary becomes applicable. For sufficient correction, d_4 must be taken as large as possible, but there exists the other limitation that

$$kd\ll 1\,,$$

for application of the first reduction of Momoi's method. Accordingly, as far as the higher modes of waves are concerned, the arguments should be limited to qualitative ones alone.

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25. 巾の変わる水路における津波

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本報告において，筆者は桃井の方法を巾の変る水路に適用し，次の結論を導いた.

巾の広い方から津波が押し寄せてくる場合，巾の広い方への反射波および巾の狭い方への進行波の第零次モードはそれぞれ，(I. 12) の前式および後式によつて与えられる．この零次モードは kd (但し，k は進入波の波数，d は水路の巾とす) の第一次近似の範囲で出された式であり，水路の高次のモード (零次モードを除いた) の計算を kd の一次の範囲に限れば，巾の広い水路へ反射する高次モードの波は (I. 21) で与えられる．更に進んで巾の狭い水路への高次モード波の位数評価をおこなうと，それは kd の二次の位数であることがわかる.

上に述べた高次のモードに対する考察は，長波近似で，kd が 1 より十分に小さいという曖昧な仮定に基いているために，単に order を評価し，定性的な議論をおこなうにとどまつている.