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1. *Some Remarks on Generation of Waves from Elliptical Wave Origin.*

By Takao MOMOI,

Earthquake Research Institute.

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1. Introduction

The author has already considered the directivity of waves produced by the "uniform" elevation of elliptical wave origin[1]. Lately Takahasi and Hatori of this Institute have made an experiment of the directivity of waves caused by the "nonuniform" elevation of elliptical wave origin[2]. Using the form of the wave origin in the above-mentioned authors' experiment, the conformity of the experiment and theory is examined in this paper.

2. Theory and Discussions

When a portion of the bottom elevates instantaneously, the formulae of wave height at a point distant from the wave origin are easily obtained, with a little change from the preceding paper[3], i. e.,

$$\zeta_r = \zeta' \Big/ \left\{ \frac{1}{g'} \cdot \frac{1}{\sqrt{2\pi}} \cdot c^{1/2} \cdot \sqrt{\frac{1}{r'}} \right\} \cdot \frac{r_0}{D}\,, \tag{1}$$

(I) when $t < \dfrac{r'-1}{c}$,

$$\zeta_r \equiv 0\,, \tag{2}$$

(II) when $\dfrac{r'-1}{c} < t < \dfrac{r'+1}{c}$,

1) T. MOMOI, *Bull. Earthq. Res. Inst.*, **40** (1962), 297.
2) R. TAKAHASI and T. HATORI, *Bull. Earthq. Res. Inst.*, **40** (1962), 873.
3) T. MOMOI, *loc. cit.*, 1).

$$\zeta_r=\frac{r_0}{D_c}\sum_{m=0}^{\infty}\sum_{u=1}^{\infty}G_{2m,u}^{A}\cdot\lambda_{2m,u}\cdot J_{2m}'(\lambda_{2m,u})\cdot\cos 2m\theta$$

$$\cdot\Big[\pi\cdot N_{2m}(\lambda_{2m,u})\cdot\frac{1}{\sqrt{\lambda_{2m,u}}}\cdot\sin\left\{\frac{1}{2}\lambda_{2m,u}(ct-r'-1)+\frac{4m+1}{4}\pi\right\}\cdot$$

$$\cdot\sin\left\{\frac{1}{2}\lambda_{2m,u}(ct-r'+1)\right\}$$

$$+\frac{1}{\pi}\{L_{2m,u}(-1)-L_{2m,u}(ct-r')\}\Big]\,. \qquad (3)$$

(III) when $\dfrac{r'+1}{c}<t$,

$$\zeta_r=\frac{r_0}{D_c}\cdot\sum_{m=0}^{\infty}\sum_{u=1}^{\infty}G_{2m,u}^{A}\cdot\lambda_{2m,u}\cdot J_{2m}'(\lambda_{2m,u})\cdot\cos 2m\theta$$

$$\cdot\Big[-M_{2m,u}(-1)+M_{2m,u}\{-(ct-r')\}$$

$$+\frac{1}{\pi}\{L_{2m,u}(-1)-L_{2m,u}(+1)\}$$

$$+\pi\cdot N_{2m}(\lambda_{2m,u})\cdot\frac{1}{\sqrt{\lambda_{2m,u}}}\cdot\sin\frac{4m+1}{4}\pi\cdot\sin\lambda_{2m,u}\Big]\,, \qquad (4)$$

where

$$\left.\begin{aligned} L_{2m,u}(x)&=\int_0^{\infty}df_0\,\frac{\sqrt{f_0}}{f_0^2+\lambda_{2m,u}^2}\cdot K_{2m}(f_0)e^{f_0x}\,,\\ M_{2m,u}(x)&=\int_0^{\infty}df_0\,\frac{\sqrt{f_0}}{f_0^2+\lambda_{2m,u}^2}\cdot J_{2m}(f_0)e^{f_0x}\,,\end{aligned}\right\}; \qquad (5)$$

$$G_{2m,u}^{A}=\frac{4\varepsilon_{2m}}{\pi[J_{2m+1}(\lambda_{2m,u})]^2}\int_0^{\pi/2}\cdot\int_0^1 D_0'(r',\theta)\cos 2m\theta\cdot\cdot J_{2m}(\lambda_{2m,u}r')\cdot d\theta\cdot r'dr'\;; \qquad (6)$$

D_c stands for the displacement of the center of the wave origin; other notations and definitions are the same as those used before[3].

The point differing in this paper from the preceding work[4] is that the factor $D_0'(r',\theta)$ included in the expression (6) depends on the position

4) T. MOMOI, *loc. cit.*, 1).

(r', θ), whereas $D_0'(r', \theta)$ in the preceding paper[4] is constant within the wave origin by virtue of the "uniform" elevation.

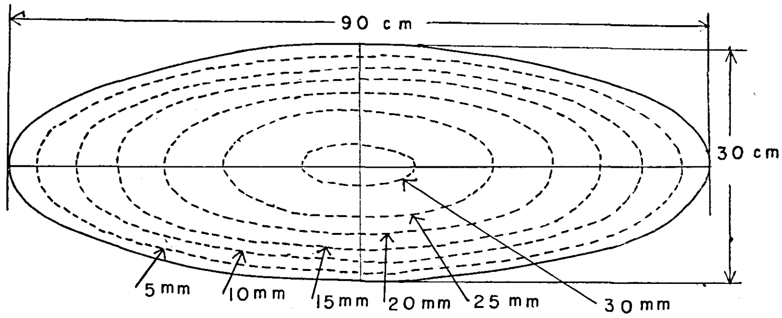

Fig. 1. The broken lines marked 5 mm to 30 mm denote the elevation of the bottom in Takahasi and Hatori's experiment.

In order to calculate the expression (6) by use of the form of the bottom elevation in Takahasi and Hatori's experiment[5] which is shown in Fig. 1, the integration (6) is transformed into a summation of small portions, i. e.,

$$G_{2m,u}^A = \frac{4\varepsilon_{2m}}{\pi[J_{2m+1}(\lambda_{2m,u})]^2} \cdot \sum_{i,j} D_0'(r_{ij}', \theta_{ij}) \cos 2m\theta_{ij} \cdot J_{2m}(\lambda_{2m,u} r_{ij}') \Delta x' \Delta y', \qquad (7)$$

where $r_{i,j}' = \sqrt{x_i^2 + y_j^2}$; $\theta_{ij} = \tan^{-1} \frac{y_j}{x_i}$;

$\Delta x'$ and $\Delta y'$ being small intervals of divided meshes.

For convenience of calculation, Fig. 1 is rewritten such that the displacement (D_c) of the center and a half length of the major axis become a unit, which is shown in Fig. 2. The displacement $\{D_0'(r_{ij}', \theta_{ij})\}$ at the point (x_i, y_j) is uniform inside the small mesh $(\Delta x' \Delta y')$ (Fig. 3). The small intervals $\Delta x'$ and $\Delta y'$ of the mesh are taken as $\Delta x' = 0.02$ and $\Delta y' = 0.005$. The reason why the latter is smaller in size than the former is that the profile of the displacement in the y-direction has more rapid variation than that in the x-direction. The values of the displacement $\{D_0'(r_{ij}', \theta_{ij})\}$ at the point (r_{ij}', θ_{ij}) can easily be read by drawing the lines on Fig. 2 at every interval mentioned above. Using the values of

5) R. TAKAHASI and T. HATORI, *loc. cit.*, 2).

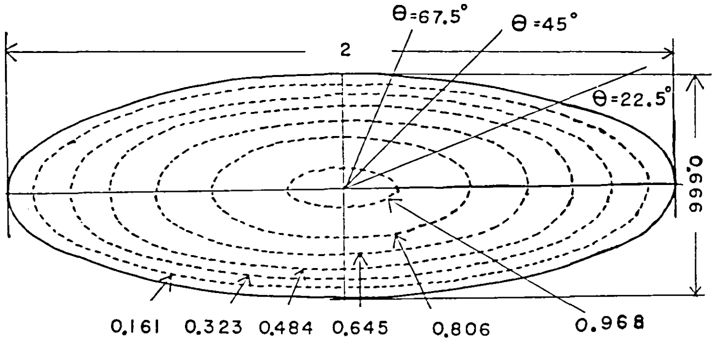

Fig. 2.

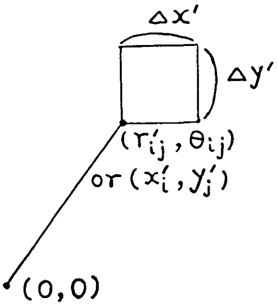

Fig. 3.

$D'_0(r'_{i.j}, \theta_{ij})$ read in such a way, we can evaluate the coefficients (7) of the Fourier Bessel expansion of the bottom displacement of Takahasi and Hatori's experiment. These results are shown in Table I.

The conformity of the profile of the experiment with that expanded by the Fourier Bessel coefficients $G^A_{2m,u}$ in Table I is fairly good, as shown in Fig. 4.

Since the theory in the preceding paper[6] was developed within the scope of long wave approximation, the number of terms of the Fourier Bessel coefficients is limited by the condition

$$\lambda_{2m,u} \ll \frac{r_0}{H} . \tag{8}$$

Table I. The Fourier Bessel coefficients $G^A_{2m,u}$ of the bottom displacement of Takahasi and Hatori's experiment

m \ u	1	2	3	4	5
0	+0.312	+0.358	+0.221	+0.124	+0.023
1	+0.302	+0.621	+0.341	+0.221	
2	+0.521	+0.223	+0.193		
3	+0.413	+0.260			
4	+0.211				

6) T. MOMOI, *loc. cit.*, 1).

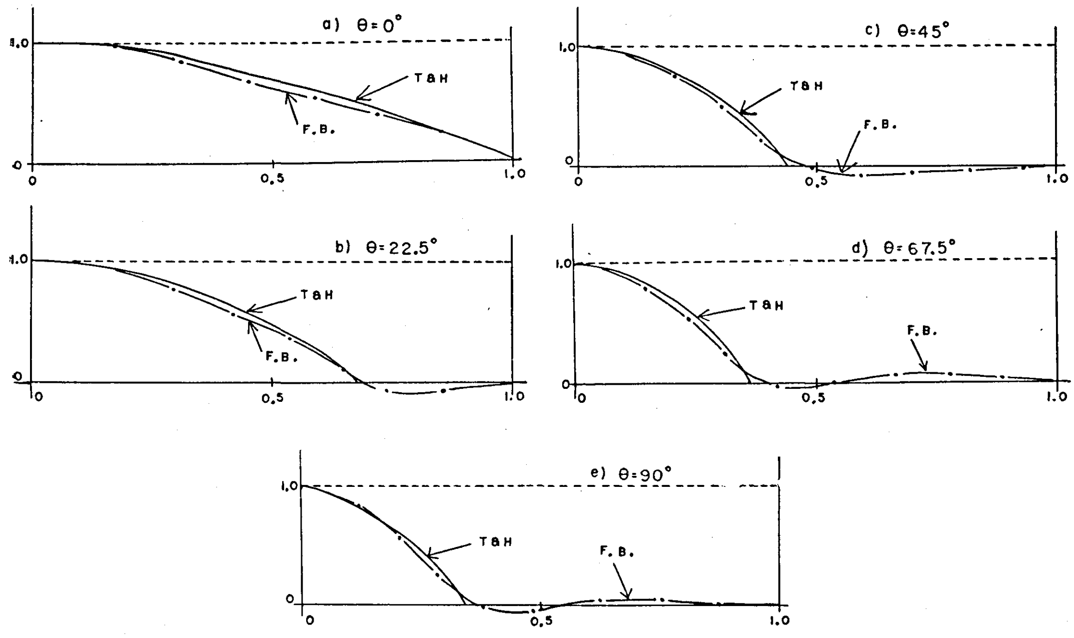

Fig. 4.

"T & H" curves denote the profiles of the bottom displacement in Takahasi and Hatori's experiment and "F. B." curves the profiles due to the Fourier Bessel coefficients $G^{A}_{2m,u}$ tabulated in Table I.

Takahasi and Hatori's experiment was made in water depths of ($H=$) 5 cm and 17.3 cm by use of the elliptic wave origin with a half length ($r_0=$) 45 cm of the major axis. Since we are treating the problem of the directivity of tsunami (long wave), the case of $H=17.3$ cm is not considered in this paper. Hence condition (8) becomes

$$\lambda_{2m,u} \ll \frac{45}{5}(=9.00) . \qquad (9)$$

In the fore-going paper[7] condition (8) was interpreted as $\lambda_{2m,u} < r_0/H$ instead of $\lambda_{2m,u} \ll r_0/H$. Therefore the terms of the Fourier Bessel coefficients permitted by the condition $\lambda_{2m,u} < r_0/H$ were summed up. In this paper the number of terms is gradually increased until the last term permitted by the condition $\lambda_{2m,u} < r_0/H$ is added. Following this procedure, the conformity of the experiment and the theory is checked.

In the same manner as in the fore-going paper[8], $\lambda_{2m,u}$ permissible

7) T. Momoi, *loc. cit.*, 1).
8) T. Momoi, *loc. cit.*, 1).

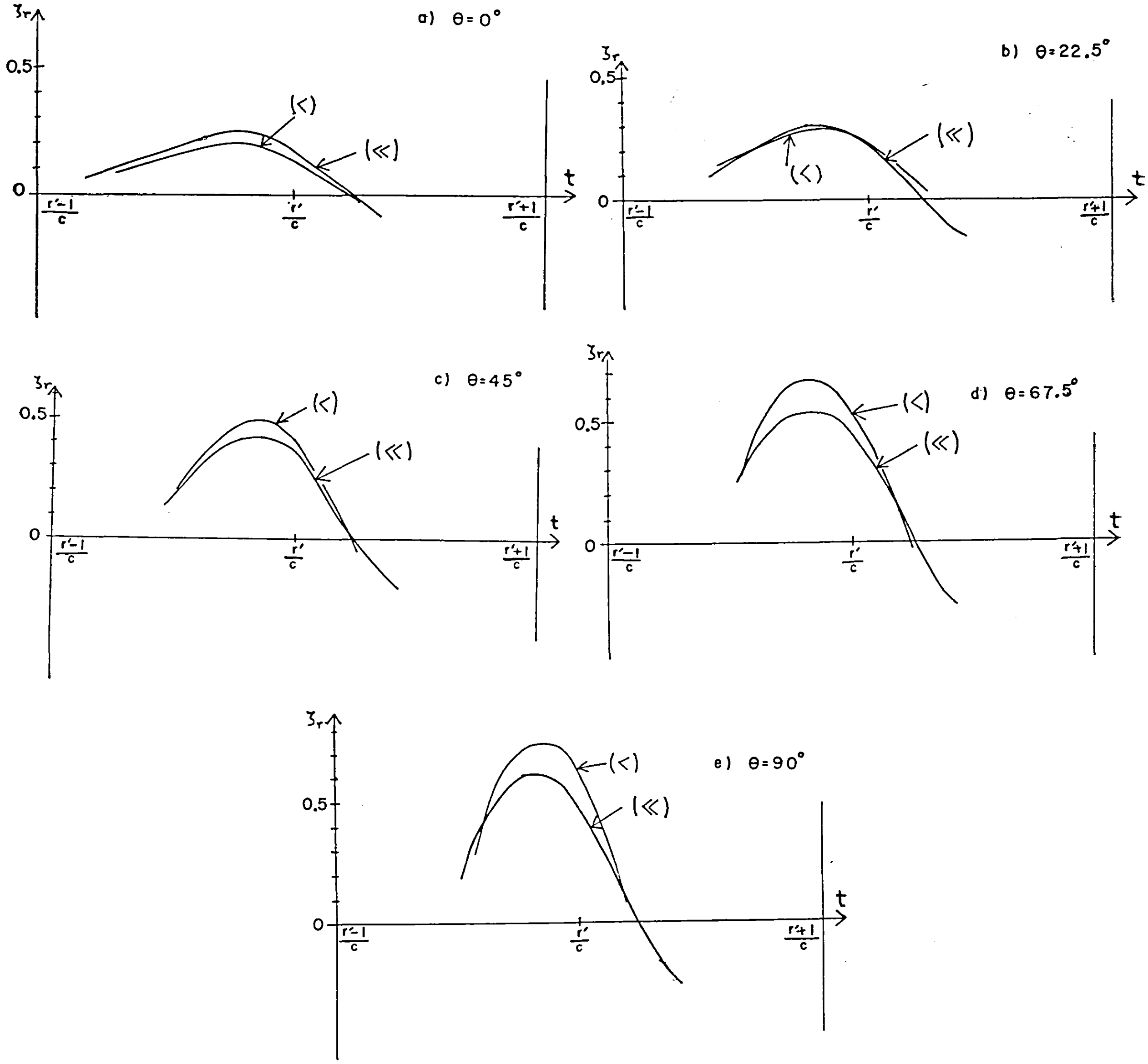

Fig. 5. The values of θ correspond to the directions θ in Fig. 2.

by the condition $\lambda_{2m,u} < 45/5\,(=9.00)$ is given below;

$$\left.\begin{array}{l} \left.\begin{array}{l} \lambda_{0.1}(=2.4048) \\ \lambda_{0.2}(=5.5201) \\ \lambda_{0.3}(=8.6537) \end{array}\right\} \\ \left.\begin{array}{l} \lambda_{2.1}(=5.135) \\ \lambda_{2.2}(=8.417) \end{array}\right\} \\ \lambda_{4.1}(=7.586) \end{array}\right\} .$$

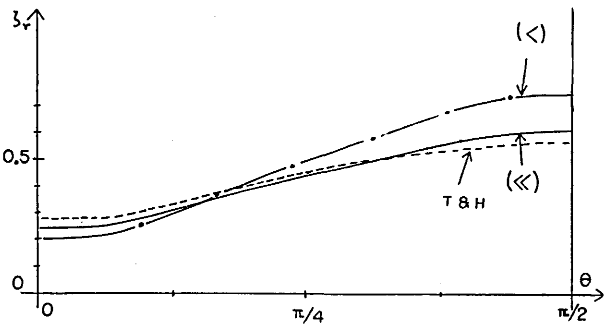

Fig. 6.

When a portion of the bottom is elevated instantaneously, it may be thought that the maximum of the wave height appears to be in the vicinity of $t=r'/c$ (refer to Figs. 1, (a), (b) and (c) in the preceding paper[9]). In order to avoid tedious calculations, the computations of wave heights are performed in this range, that is to say, only formula (3) is considered. The values of the Bessel and the Neumann functions in (3) are read from the Table of Bessel Function[10]. The integration of $L_{2m,u}(ct-r')$ is evaluated by *graphical* integration for the variation of time t. The calculated wave heights composed by modes permitted by $\lambda_{2m,u} < 6.00$ and 9.00 are shown in Figs. 5a, b, c, d, and e. In Fig. 5 the former is denoted by "(≪)" and the latter by "(<)". The maximum wave heights read from Fig. 5 are shown in Fig. 6 for the variation of direction θ. The "T & H" curve in Fig. 6 represents the result of Takahasi and Hatori's experiment. From Fig. 6, we find that the curve of "(≪)" is more coincident with the "T & H" curve than that of "(<)". It is interpreted that the better coincidence of (≪)'s and

9) T. MOMOI, *loc. cit.*, 1).

10) G. N. WATSON, *Theory of Bessel Functions* (Cambridge, 1922).

T & H's curves is the result of more rigorous application of long wave approximation.

The number of significant terms under the long wave approximation cannot be determined by theoretical consideration so far. It is necessary for experimental procedures.

Acknowledgment

The author is indebted to Professor R. Takahasi, Assistant Professor K. Kajiura and Mr. T. Hatori of this Institute for their helpful discussions.

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1. 楕円波源からの津波に関する二，三の注意

地震研究所　桃　井　高　夫

筆者は，先きに，"一様に"持ち上げられた，楕円波源よりの津波の指向性を論じた．最近，本研究所の高橋一羽鳥らによつて，楕円波源が"一様でなく"，持ち上げられたときに，起こされた津波の指向性に関する実験がおこなわれた．そこで本報告において，筆者は，この実験のときに，持ち上げられた楕円波源の断面を用いて数値計算をやり直し，津波の指向性を論じた．その結果，長波近似を，より厳格に適用するとき，理論と実験は，一応，一致するようである．併し，その厳格の程度の尺度は，未だ，理論的には厳密に与えられない．これは，新たに理論を改革するか，あるいは，実験により，長波近似の適用しうる尺度を，きめなければならない．