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2. *The Directivity of Tsunami due to the Bottom Irregularities [I].*

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Introduction.

The author treated the tsunami caused by the deformation of a portion of the sea bottom, the shape of which is elliptical.[1),2)] The anisotropy of the wave origin produces the directivity of tsumami. On the other hand, it is also expected that the bottom irregularities in the vicinity of the wave origin give rise to the directivity of the tsunami. In this and subsequent papers the author will consider these problems. In the first place, the two dimensional cases, where the submerged vertical cliff is placed in water and near it a portion of the bottom or cliff periodically vibrates, are considerred in this purview. Studies are separated into three Parts, i. e.,

Part I: the case where the vertical cliff submerged in water vibrates horizontally (Fig. I, 1).

Part II: the case where a portion of the bottom in deep water vibrates vertically (Fig. II, 1).

Part III: the case where a portion of the bottom in shallow water vibrates vertically (Fig. III, 1).

Part I.

(I, 1) *Theory.*

With x and z denoting Cartesian co-ordinates, x being measured at the undisturbed free surface of water and z vertically upward, and D_j ($j=1$ and 2) denoting the domains ($0<x$, $0>z>-H$) and ($x<0$, $0>z>-H+h$) respectively, the velocity potentials ϕ_j ($j=1$, 2) in the domains D_j ($j=1$, 2) satisfy the equations of continuity:

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\phi_j=0\,. \tag{1}$$

1) T. MOMOI, *Bull. Earthq. Res. Inst.*, **40** (1962), 297-307.
2) T. MOMOI, *Bull. Earthq. Res. Inst.*, **40** (1962), 288-296.

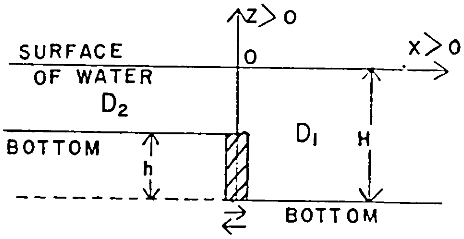

Fig. I, 1.

The surface condition ($z=0$) are

$$\left.\begin{aligned}\frac{\partial\phi_j}{\partial t}&=-g\zeta_j\ ,\\ \frac{\partial\zeta_j}{\partial t}&=\frac{\partial\phi_j}{\partial z}\ ,\end{aligned}\right\}\ (j=1,\ 2) \tag{2}$$

or $$\frac{\partial^2\phi_j}{\partial t^2}+g\frac{\partial\phi_j}{\partial z}=0\ (j=1,\ 2), \tag{3}$$

ζ_j ($j=1$, 2) being the elevation of water from the undisturbed free surface of water in the domain D_j ($j=1$, 2) respectively, g the acceleration of gravity, and t a variable of time.

The bottom conditions are

$$\left.\begin{aligned}&\frac{\partial\phi_1}{\partial z}=0\quad (z=-H)\ ,\\ &\frac{\partial\phi_2}{\partial z}=0\quad (z=-H+z)\ ,\end{aligned}\right\} \tag{4}$$

where H is the depth of the deeper water, h the height of the cliff.

The boundary condition at the vertical cliff is that the horizontal velocity of water particle at the cliff sinusoidally vibrates

$$\frac{\partial\phi_1}{\partial x}=-i\omega D'_{\text{bot}}e^{-i\omega t}\ , \tag{5}$$

where D'_{bot} is the amplitude of vibration which is related to the horizontal displacement of the cliff by $D_{\text{bot}}=D'_{\text{bot}}e^{-i\omega t}$ (the only real part has a physical meaning).

For the case of vibration the equation (1) and the conditions (3)—(5) are reduced to

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\phi'_j=0\ , \tag{1'}$$

$$-\omega^2\phi'_j+g\frac{\partial\phi'_j}{\partial z}=0\quad (z=0)\ , \tag{3'}$$

$$\left.\begin{aligned}&\frac{\partial\phi'_1}{\partial z}=0\quad (z=-H)\ ,\\ &\frac{\partial\phi'_2}{\partial z}=0\quad (z=-H+h)\ ,\end{aligned}\right\} \tag{4'}$$

$$\frac{\partial \phi_1'}{\partial x} = -i\omega D'_{\text{bot}} \,, \tag{5'}$$

where $j=1,\ 2; \phi_j$ is the velocity potential eliminated time factor $e^{-i\omega t}$ (ω: angular frequency of vibration). Hereafter the primes ($'$) of ϕ_j' and D'_{bot} are omitted for simplicity.

In the domains D_j ($j=1,\ 2$) only the out-going progressive waves from the cliff and the waves being damped out as leaving the cliff remain, so that the expressions of ϕ_j ($j=1,\ 2$) must be of the following forms[3)-7)]

$$\phi_1 = A_0^{(1)} e^{+ia_0^{(1)}x} \cosh a_0^{(1)}(H+z) + \sum_{s=1}^{\infty} A_s^{(1)} e^{-a_s^{(1)}x} \cos a_s^{(1)}(H+z) \,, \tag{6}$$

$$\phi_2 = A_0^{(2)} e^{-ia_0^{(2)}x} \cosh a_0^{(2)}(H-h+z) + \sum_{s=1}^{\infty} A_s^{(2)} e^{+a_s^{(2)}x} \cos a_s^{(2)}(H-h+z) \,, \tag{7}$$

where $A_0^{(1)}$, $A_s^{(1)}$ ($s=1,\ 2,\ 3,\cdots$), $A_0^{(2)}$, $A_s^{(2)}$ ($s=1,\ 2,\ 3,\cdots$) are arbitrary constants to be determined by the conditions at the origin ($x=0$); $a_0^{(1)}$, $a_s^{(1)}$ ($s=1,\ 2,\ 3,\cdots$) and $a_0^{(2)}$, $a_s^{(2)}$ ($s=1,\ 2,\ 3,\cdots$) are *eigen* values of the equation (1′) due to the conditions (3′) and (4′), i. e., the solutions of $\omega^2 = a_0^{(1)} g \tanh a_0^{(1)} H = -a_s^{(1)} g \tan a_s^{(1)} H$ ($s=1,2,3,\cdots$) and $\omega^2 = a_0^{(2)} g \tanh a_0^{(2)}(H-h) = -a_s^{(2)} g \tan a_s^{(2)}(H-h)$ respectively.

Substituting (6) and (7) into the former of the equations (2), allowing for the abbreviation of the vibrating time factor $\exp(-i\omega t)$, we have the wave heights ζ_j ($j=1,2$) in the domains D_j ($j=1,\ 2$) as follows:

$$\zeta_1 = \frac{i\omega}{g} \cdot A_0^{(1)} e^{-i(\omega t - a_0^{(1)}x)} \cdot \cosh a_0^{(1)} H + \frac{i\omega}{g} \cdot e^{-i\omega t} \cdot \left\{ \sum_{s=1}^{\infty} A_s^{(1)} e^{-a_s^{(1)}x} \cos a_s^{(1)} H \right\} \,, \tag{8}$$

$$\zeta_2 = \frac{i\omega}{g} \cdot A_0^{(2)} e^{-i(\omega t + a_0^{(2)}x)} \cdot \cosh a_0^{(2)}(H-h) + \frac{i\omega}{g} \cdot e^{-i\omega t} \cdot \left\{ \sum_{s=1}^{\infty} A_s^{(2)} e^{+a_s^{(2)}x} \cos a_s^{(2)}(H-h) \right\} \,. \tag{9}$$

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3) T. H. HAVELOCK, "Forced Surface-Waves on Water," *Phil. Mag.*, **8** (1929), 569-576.

4) T. MOMOI, "The Effect of Coastlines on the Tsunami (1)," *Bull. Earthq. Res. Inst.*, **40** (1962), 719.

5) T. MOMOI, "The Effect of Coastlines on the Tsunami (2)," *ditto*, **40** (1962), 733.

6) T. MOMOI, "On Water Waves Generated by a Vibrating Bottom (Two-dimensional Case)," *Zisin* [**ii**], **15** (1962), 52, (in Japanese).

7) K. TAKANO, "Effects d'un obstacle parallélépipédique sur la propagation de la houle," *La Houille Blanche* (Mai 1960), 247.

In order to determine the arbitrary constants $A_0^{(1)}$, $A_s^{(1)}$ $(s=1,2,3,\cdots)$, $A_0^{(2)}$, $A_s^{(2)}$ $(s=1, 2, 3,\cdots)$ we have two available conditions at $x=0$:

$$\phi_1=\phi_2 \quad (0>z>-H+h)\ , \tag{10}$$

(from the continuity of pressure),

$$\frac{\partial\phi_1}{\partial x}=\begin{cases}\dfrac{\partial\phi_2}{\partial x} & (0>z>-H+h)\ ,\\ -i\omega D_{\text{bot}} & (-H+h>z>-H)\ ,\end{cases} \tag{11}$$

(the first being the continuity of velocity of water particles and the second the horizontal velocity of water particles at the cliff.)

Putting (6) and (7) into (10) and (11), the conditions (10) and (11) become

$$A_0^{(1)}\cosh a_0^{(1)}(H+z)+\sum_{s=1}^{\infty}A_s^{(1)}\cos a_s^{(1)}(H+z)$$

$$=A_0^{(2)}\cosh a_0^{(2)}(H-h+z)+\sum_{s=1}^{\infty}A_s^{(2)}\cos a_s^{(2)}(H-h+z)\quad(0>z>-H+h)\ , \tag{12}$$

$$+ia_0^{(1)}A_0^{(1)}\cosh a_0^{(1)}(H+z)+\sum_{s=1}^{\infty}(-a_s^{(1)})A_s^{(1)}\cos a_s^{(1)}(H+z)$$

$$=\begin{cases}-ia_0^{(2)}A_0^{(2)}\cosh a_0^{(2)}(H-h+z)\\ \quad+\displaystyle\sum_{s=1}^{\infty}(+a_s^{(2)})A_s^{(2)}\cos a_s^{(2)}(H-h+z)\quad(0>z>-H+h)\ ,\\ -i\omega D_{\text{bot}}\quad(-H+h>z>-H)\ .\end{cases} \tag{13}$$

The systems of functions $\{\cosh a_0^{(1)}(H+z),\ \cos a_s^{(1)}(H+z)\ (s=1,2,3,\cdots)\}$ and $\{\cosh a_0^{(2)}(H-h+z),\ \cos a_s^{(2)}(H-h+z)\ (s=1, 2, 3,\cdots)\}$ have orthogonalities in the range $0>z>-H$ and $0>z>-H+h$ respectively, i.e.,[8),9),10)]

$$\int_{-H}^{0}\cosh^2 a_0^{(1)}(H+z)dz=I_0^{(H)}\quad\text{(non-zero)}\ ;$$

$$\int_{-H}^{0}\cos^2 a_s^{(1)}(H+z)dz=I_s^{(H)}\quad\text{(non-zero)}\quad(s=1,\ 2,\ 3,\cdots)\ ;$$

$$\int_{-H}^{0}\cosh a_0^{(1)}(H+z)\cos a_s^{(1)}(H+z)dz=0\quad(s=1,\ 2,\ 3,\cdots)\ ;$$

8) T. H. HAVELOCK, *loc. cit.*, 3).
9) T. MOMOI, *loc. cit.*, 6).
10) K. TAKANO, *loc. cit.*, 7).

$$\int_{-H}^{0} \cos a_s^{(1)}(H+z) \cos a_r^{(1)}(H+z)dz=0 \quad \left(s\neq r: \begin{array}{l} s=1,\ 2,\ 3,\cdots \\ r=1,\ 2,\ 3,\cdots \end{array}\right);$$

and

$$\int_{-H+h}^{0} \cosh^2 a_0^{(2)}(H-h+z)dz=I_0^{(H-h)} \quad \text{(non-zero)};$$

$$\int_{-H+h}^{0} \cos^2 a_s^{(2)}(H-h+z)dz=I_s^{(H-h)} \quad \text{(non-zero)} \quad (s=1,\ 2,\ 3,\cdots);$$

$$\int_{-H+h}^{0} \cos a_0^{(2)}(H-h+z) \cos a_s^{(2)}(H-h+z)dz=0 \quad (s=1,\ 2,\ 3,\cdots);$$

$$\int_{-H+h}^{0} \cos a_s^{(2)}(H-h+z) \cos a_r^{(2)}(H-h+z)dz=0 \quad \left(s\neq r: \begin{array}{l} s=1, 2, 3,\cdots \\ r=1, 2, 3,\cdots \end{array}\right);$$

Hence multiplying (12) by $\cosh a_0^{(2)}(H-h+z)$, $\cos a_s^{(2)}(H-h+z)$ $(s=1,2,3,\cdots)$ and integrating $-H+h$ to 0 with respect to z, the relation (12) is reduced to the following equations:

$$A_0^{(1)}\cdot I_{0,0}[(^{0}_{-H+h}), H, H-h]+\sum_{s=1}^{\infty} A_s^{(1)}\cdot I_{s,0}[(^{0}_{-H+h}), H, H-h]=A_0^{(2)}\cdot I_0^{(H-h)} \quad (14)$$

$$A_0^{(1)}\cdot I_{0,s}[(^{0}_{-H+h}), H, H-h]+\sum_{s'=1}^{\infty} A_{s'}^{(1)}\cdot I_{s',s}[(^{0}_{-H+h}), H, H-h]=A_s^{(2)}\cdot I_s^{(H-h)} \quad (15)$$

$$(s=1,\ 2,\ 3,\cdots,\ \infty),$$

where

$$I_{0,0}[(^{0}_{-H+h}), H, H-h]=\int_{-H+h}^{0} \cosh a_0^{(1)}(H+z) \cosh a_0^{(2)}(H-h+z)dz$$
$$=\frac{1}{(a_0^{(2)})^2-(a_0^{(1)})^2}[a_0^{(2)} \sinh a_0^{(2)}(H-h) \cosh a_0^{(1)}H$$
$$-a_0^{(1)}\{\cosh a_0^{(2)}(H-h) \sinh a_0^{(1)}H-\sinh a_0^{(1)}h\}],$$

$$I_{s,0}[(^{0}_{-H+h}), H, H-h]=\int_{-H+h}^{0} \cos a_s^{(1)}(H+z) \cosh a_0^{(2)}(H-h+z)dz$$
$$=\frac{1}{(a_s^{(1)})^2+(a_0^{(2)})^2}[a_s^{(1)}\{\sin a_s^{(1)}H \cosh a_0^{(2)}(H-h)-\sin a_s^{(1)}h\}$$
$$+a_0^{(2)} \cos a_s^{(1)}H \sinh a_0^{(2)}(H-h)] \quad (s=1,\ 2,\ 3,\cdots,\ \infty),$$

$$I_0^{(H-h)}=\int_{-H+h}^{0} \cosh^2 a_0^{(2)}(H-h+z)dz$$
$$=\frac{1}{2}\left[\frac{1}{2a_0^{(2)}} \sinh 2a_0^{(2)}(H-h)+(H-h)\right],$$

$$I_{0,s}[(^{0}_{-H+h}), H, H-h]=\int_{-H+h}^{0} \cosh a_0^{(1)}(H+z) \cos a_s^{(2)}(H-h+z)dz$$

$$=\frac{1}{(a_s^{(2)})^2+(a_0^{(1)})^2}[a_s^{(2)} \sin a_s^{(2)}(H-h) \cosh a_0^{(1)}H$$

$$+a_0^{(1)}\{\cos a_s^{(2)}(H-h) \sinh a_0^{(1)}H-\sinh a_0^{(1)}h\}]$$

$$(s=1,\ 2,\ 3,\cdots,\ \infty)\ ,$$

$$I_{s',s}[(^{0}_{-H+h}), H, H-h]=\int_{-H+h}^{0} \cos a_{s'}^{(1)}(H+z) \cos a_s^{(2)}(H-h+z)dz$$

$$=\frac{1}{(a_{s'}^{(1)})^2-(a_s^{(2)})^2}[a_{s'}^{(1)}\{\sin a_{s'}^{(1)}H \cos a_s^{(2)}(H-h)-\sin a_{s'}^{(1)}h\}$$

$$-a_s^{(2)} \cos a_{s'}^{(1)}H \sin a_s^{(2)}(H-h)] \quad (s,\ s'=1,\ 2,\ 3,\cdots,\ \infty)\ ,$$

$$I_s^{(H-h)}=\int_{-H+h}^{0} \cos^2 a_s^{(2)}(H-h+z)dz$$

$$=\frac{1}{2}\left\{\frac{1}{2a_s^{(2)}} \sin 2a_s^{(2)}(H-h)+(H-h)\right\} \quad (s=1,2,3,\cdots,\infty)\ .$$

In like manner, applying the operators $\int_{-H}^{0} \cosh a_0^{(1)}(H+z)dz$ and $\int_{-H}^{0} \cos a_s^{(1)}(H+z)dz$ to (13), we have

$$\frac{-i\omega D_{\rm bot}}{a_0^{(1)}} \sinh a_0^{(1)}h-ia_0^{(2)}A_0^{(2)}I_{0,0}[(^{0}_{-H+h}), H-h, H]$$

$$+\sum_{s=1}^{\infty} a_s^{(2)}A_s^{(2)}I_{s,0}[(^{0}_{-H+h}), H-h, H]=+ia_0^{(1)}A_0^{(1)}I_0^{(H)}\ , \tag{16}$$

$$\frac{-i\omega D_{\rm bot}}{a_s^{(1)}} \sin a_s^{(1)}h-ia_0^{(2)}A_0^{(2)}\cdot I_{0,s}[(^{0}_{-H+h}), H-h, H]$$

$$+\sum_{s'=1}^{\infty} a_{s'}^{(2)}A_{s'}^{(2)}I_{s',s}[(^{0}_{-H+h}), H-h, H]=-a_s^{(1)}A_s^{(1)}I_s^{(H)} \quad (s=1,\ 2,\ 3,\cdots,\ \infty)\ , \tag{17}$$

where

$$I_{0,0}[(^{0}_{-H+h}), H-h, H]=\int_{-H+h}^{0} \cosh a_0^{(2)}(H-h+z) \cosh a_0^{(1)}(H+z)dz$$

$$=I_{0,0}[(^{0}_{-H+h}), H, H-h] \quad \text{(already given)}\ ,$$

$$I_{s,0}[(^{0}_{-H+h}), H-h, H]=\int_{-H+h}^{0} \cos a_s^{(2)}(H-h+z) \cosh a_0^{(1)}(H+z)dz$$

$$=I_{0,s}[(^{0}_{-H+h}), H, H-h] \quad (s=1,\ 2,\ 3,\cdots,\ \infty)$$

$$\text{(already given)}\ ,$$

$$I_{0,s}[(^{0}_{-H+h}), H-h, H]=\int_{-H+h}^{0}\cosh a_0^{(2)}(H-h+z)\cos a_s^{(1)}(H+z)dz$$
$$=I_{s,0}[(^{0}_{-H+h}), H, H-h] \quad (s=1,\ 2,\ 3,\cdots,\ \infty)$$
(already given) ,

$$I_{s',s}[(^{0}_{-H+h}), H-h, H]=\int_{-H+h}^{0}\cos a_{s'}^{(2)}(H-h+z)\cos a_s^{(1)}(H+z)dz$$
$$=I_{s,s'}[(^{0}_{-H+h}), H, H-h] \quad (s,\ s'=1,\ 2,\ 3,\cdots,\ \infty)$$
(already given) ,

$$I_0^{(H)}=\int_{-H}^{0}\cosh^2 a_0^{(1)}(H+z)dz=\frac{1}{2}\left[\frac{1}{2a_0^{(1)}}\sinh 2a_0^{(1)}H+H\right],$$

$$I_s^{(H)}=\int_{-H}^{0}\cos^2 a_s^{(1)}(H+z)dz=\frac{1}{2}\left[\frac{1}{2a_s^{(1)}}\sin 2a_s^{(1)}H+H\right]$$
$$(s=1,\ 2,\ 3,\cdots,\ \infty)\ .$$

The arbitrary constants $A_0^{(1)}$, $A_s^{(1)}$ $(s=1, 2, 3,\cdots)$, $A_0^{(2)}$ and $A_s^{(2)}$ $(s=1, 2, 3,\cdots)$ can be obtained as solutions of the *infinite* simultaneous equations (14), (15), (16) and (17).

(I, 2) *Used Values.*

We used the following values for determining the arbitrary constants $A_0^{(1)}$, $A_s^{(1)}$, $A_0^{(2)}$and $A_s^{(2)}$ $(s=1, 2, 3,\cdots)$:

depth in the deep side of water $(H)=10$ cm,
depth in the shallow side of water $(H-h)=5$ cm
or the height of the submerged cliff $(h)=5$ cm,
period of vibration of the cliff $(T)=1$ sec,
acceleration of gravity $(g)=980$ cm/sec^2,
amplitude of vibration of the cliff (D_{bot}; in form omitted prime) $=1$ cm.

On substituting these values into the Airy's relations $\omega^2=a_0^{(1)}g\cdot\tanh a_0^{(1)}H=-a_s^{(1)}g\cdot\tan a_s^{(1)}H$ $(s=1, 2, 3,\cdots)$ and $\omega^2=a_0^{(2)}g\cdot\tanh a_0^{(2)}(H-h)=-a_s^{(2)}g\cdot\tan a_s^{(2)}(H-h)$ $(s=1, 2, 3,\cdots)$, we obtain, by method of trial and error,

$$\left.\begin{array}{lll}
a_0^{(1)}=0.068060\ , & a_1^{(1)}=0.30090\ , & a_2^{(1)}=0.62185\ ,\\
a_3^{(1)}=0.93821\ , & a_4^{(1)}=1.2534\ , & a_5^{(1)}=1.5682\ ,\\
a_6^{(1)}=1.8828\ , & a_7^{(1)}=2.1973\ , & a_8^{(1)}=2.5117\ ;\\
a_0^{(2)}=0.028500\ , & a_1^{(2)}=0.61526\ , & a_2^{(2)}=1.2502\ ,\\
a_3^{(2)}=1.8807\ , & a_4^{(2)}=2.5100\ , & a_5^{(2)}=3.1390\ ,\\
a_6^{(2)}=3.7678\ , & a_7^{(2)}=4.3964\ , & a_8^{(2)}=5.0249\ ,
\end{array}\right\} \qquad (18)$$

where the calculation has been made up to $s=8$.

In the following section we shall be concerned with the method of calculation of the infinite simultaneous equations (14), (15), (16) and (17).

(I, 3) *Outlines of Numerical Analysis.*

The simultaneous equations (14), (15), (16) and (17) possess an infinite number of unknowns $A_0^{(1)}$, $A_s^{(1)}$, $A_0^{(2)}$ and $A_s^{(2)}$ $(s=1, 2, 3, \cdots)$. For calculation, these equations, in general, are approximated by a large number (N) of terms such that the contribution of the omitted terms is considered as negligibly small. On the assumption that; $A_s^{(1)}$, $A_s^{(2)}$ $(s>N)\equiv 0$; $\sum_{s=1}^{\infty}\equiv\sum_{s=1}^{N}$; the equations beyond $s=N$ are able to be omitted, the equations (14), (15), (16) and (17) become $2(N+1)$ simultaneous equations with $2(N+1)$ unknowns $A_0^{(1)}$, $A_s^{(1)}$, $A_0^{(2)}$ and $A_s^{(2)}$ $(s=1, 2, 3, \cdots, N)$. Here the validity of the approximation by a finite number of terms must be examined. From a purely mathematical point of view, necessary and sufficient conditions for the existence of the solutions of the infinite simultaneous equations have not been known so far.

Hence, if the variations become smaller as a number of unknowns successively increasing, then it may be interpreted that the solutions of these simultaneous equations converge. Such an interpretation for convergence is sometimes permitted in the world of applied mathematics. Thus, this convention is also followed in this purview.

Since the coefficients of the equations (14)—(17) are complex, the solutions $A_0^{(1)}$, $A_s^{(1)}$, $A_0^{(2)}$ and $A_s^{(2)}$ $(s=1, 2, 3, \cdots, N)$ are generally of forms, i. e.

$$\left.\begin{aligned} A_0^{(1)} &= x_0^{(1)} + iy_0^{(1)}\ , \\ A_s^{(1)} &= x_s^{(1)} + iy_s^{(1)} \quad (s=1,\ 2,\ 3, \cdots,\ N)\ , \\ A_0^{(2)} &= x_0^{(2)} + iy_0^{(2)}\ , \\ A_s^{(2)} &= x_s^{(2)} + iy_s^{(2)} \quad (s=1,\ 2,\ 3, \cdots,\ N)\ , \end{aligned}\right\} \tag{19}$$

where

$$\begin{aligned} &x_0^{(1)},\ y_0^{(1)},\ x_s^{(1)},\ y_s^{(1)} \quad (s=1,\ 2,\ 3, \cdots, N)\ , \\ &x_0^{(2)},\ y_0^{(2)},\ x_s^{(2)},\ y_s^{(2)} \quad (s=1,\ 2,\ 3, \cdots, N) \end{aligned}$$

are all real.

On substitution for (14)—(17) from (19) and taking the real and imaginary parts respectively, the number of the equations (14)—(17) changes from $2(N+1)$ to $4(N+1)$. By solving these $4(N+1)$ equations, we can obtain the solutions $A_0^{(1)}$, $A_s^{(1)}$, $A_0^{(2)}$ and $A_s^{(2)}$ $(s=1, 2, 3, \cdots, N)$ by virtue of (19).

(I, 4) *The Result of Computation and the Convergence.*

Using the values described in section (I, 2) and $a_0^{(1)}$, $a_s^{(1)}$, $a_0^{(2)}$ and $a_s^{(2)}$ ($s=1, 2, 3, \cdots, 8$) in (18), which were prescribed by Airy's relations, the equations (14)—(17) are respectively solved for the successive increase of N from 3 to 8.

For easy evaluation of the computed values we took new expressions as given below, instead of $A_0^{(1)}$, $A_s^{(1)}$, $A_0^{(2)}$ and $A_s^{(2)}$ ($s=1, 2, 3, \cdots, 8$),

$$\left.\begin{aligned} \zeta_0^{(1)} &= \frac{\omega}{g} A_0^{(1)} \cosh a_0^{(1)} H\,, \\ \zeta_s^{(1)} &= \frac{\omega}{g} A_s^{(1)} \cos a_s^{(1)} H \quad (s=1, 2, 3, \cdots, 8)\,, \\ \zeta_0^{(2)} &= \frac{\omega}{g} A_0^{(2)} \cosh a_0^{(2)} (H-h)\,, \\ \zeta_s^{(2)} &= \frac{\omega}{g} A_s^{(2)} \cos a_s^{(2)} (H-h) \quad (s=1, 2, 3, \cdots, 8)\,. \end{aligned}\right\} \tag{20}$$

Then the expressions (8) and (9) of the wave heights become

$$\left.\begin{aligned} \zeta_1 &= i\cdot\zeta_0^{(1)} e^{-i(\omega t - a_0^{(1)} x)} + \sum_{s=1}^{8} i\cdot\zeta_s^{(1)} e^{-i\omega t}\cdot e^{-a_s^{(1)} x}\,, \\ \zeta_2 &= i\cdot\zeta_0^{(2)} e^{-i(\omega t + a_0^{(2)} x)} + \sum_{s=1}^{8} i\cdot\zeta_s^{(2)} e^{-i\omega t}\cdot e^{+a_s^{(2)} x}\,, \end{aligned}\right\} \tag{21}$$

where the only real parts must be retained in consideration of physical meaning.

From (21), we can see that:

$\zeta_0^{(1)}$, $\zeta_0^{(2)}$ are the amplitudes of the out-going waves in the x-positive and -negative directions respectively;

$\zeta_s^{(1)}$, $\zeta_s^{(2)}$ the amplitudes for the s-th mode of the disturbances on the x-positive and -negative sides of the origin, which are exponentially decreased with respect to position.

Actual calculations were made by use of the OKITAC-5090 at the Computation Centre of Tokyo University.

The results of computations are given below:

when $N=3$,

$$\left.\begin{aligned}
\zeta_0^{(1)} &= -0.26211\cdot\exp(-i\cdot 0.077321)\ ,\\
\zeta_1^{(1)} &= -0.093632\cdot\exp(+i\cdot 1.5103)\ ,\\
\zeta_2^{(1)} &= +0.0092889\cdot\exp(+i\cdot 1.4213)\ ,\\
\zeta_3^{(1)} &= -0.0088734\cdot\exp(+i\cdot 1.4523)\ ;
\end{aligned}\right\} \qquad (22)$$

$$\left.\begin{aligned}
\zeta_0^{(2)} &= -0.24382\cdot\exp(+i\cdot 0.26128)\ ,\\
\zeta_1^{(2)} &= -0.019312\cdot\exp(+i\cdot 1.4501)\ ,\\
\zeta_2^{(2)} &= +0.012315\cdot\exp(+i\cdot 1.4621)\ ,\\
\zeta_3^{(2)} &= -0.0052134\cdot\exp(+i\cdot 1.4531)\ ;
\end{aligned}\right\} \qquad (22')$$

when $N=4$,

$$\left.\begin{aligned}
\zeta_0^{(1)} &= -0.26191\cdot\exp(-i\cdot 0.056969)\ ,\\
\zeta_1^{(1)} &= -0.093527\cdot\exp(+i\cdot 1.5188)\ ,\\
\zeta_2^{(1)} &= +0.0091280\cdot\exp(+i\cdot 1.4514)\ ,\\
\zeta_3^{(1)} &= -0.0087136\cdot\exp(+i\cdot 1.5008)\ ,\\
\zeta_4^{(1)} &= +0.0052814\cdot\exp(+i\cdot 1.4989)\ ;
\end{aligned}\right\} \qquad (23)$$

$$\left.\begin{aligned}
\zeta_0^{(2)} &= -0.24339\cdot\exp(+i\cdot 0.21937)\ ,\\
\zeta_1^{(2)} &= -0.019735\cdot\exp(+i\cdot 1.4591)\ ,\\
\zeta_2^{(2)} &= +0.010435\cdot\exp(+i\cdot 1.4987)\ ,\\
\zeta_2^{(2)} &= -0.0059309\cdot\exp(+i\cdot 1.4839)\ ,\\
\zeta_4^{(2)} &= +0.0029704\cdot\exp(+i\cdot 1.4722)\ ;
\end{aligned}\right\} \qquad (23')$$

when $N=5$,

$$\left.\begin{aligned}
\zeta_0^{(1)} &= -0.26189\cdot\exp(-i\cdot 0.045966)\ ,\\
\zeta_1^{(1)} &= -0.093432\cdot\exp(+i\cdot 1.5288)\ ,\\
\zeta_2^{(1)} &= +0.0089681\cdot\exp(+i\cdot 1.4832)\ ,\\
\zeta_3^{(1)} &= -0.0086214\cdot\exp(+i\cdot 1.5258)\ ,\\
\zeta_4^{(1)} &= +0.0042848\cdot\exp(+i\cdot 1.5050)\ ,\\
\zeta_5^{(1)} &= -0.0041853\cdot\exp(+i\cdot 1.5608)\ ;
\end{aligned}\right\} \qquad (24)$$

$$\left.\begin{aligned}
\zeta_0^{(2)} &= -0.24367\cdot\exp(+i\cdot 0.19739)\ ,\\
\zeta_1^{(2)} &= -0.020878\cdot\exp(+i\cdot 1.4613)\ ,\\
\zeta_2^{(2)} &= +0.0084581\cdot\exp(+i\cdot 1.5050)\ ,\\
\zeta_3^{(2)} &= -0.0073669\cdot\exp(+i\cdot 1.5099)\ ,\\
\zeta_4^{(2)} &= +0.0036473\cdot\exp(+i\cdot 1.4889)\ ,\\
\zeta_5^{(2)} &= -0.0019965\cdot\exp(+i\cdot 1.4949)\ ;
\end{aligned}\right\} \qquad (24')$$

when $N=6$,

$$\left.\begin{aligned}
\zeta_0^{(1)}&=-0.26188\cdot\exp(-i\cdot 0.029923)\,,\\
\zeta_1^{(1)}&=-0.093389\cdot\exp(+i\cdot 1.5367)\,,\\
\zeta_2^{(1)}&=+0.0089521\cdot\exp(+i\cdot 1.5015)\,,\\
\zeta_3^{(1)}&=-0.0086206\cdot\exp(+i\cdot 1.5132)\,,\\
\zeta_4^{(1)}&=+0.0042315\cdot\exp(+i\cdot 1.5139)\,,\\
\zeta_5^{(1)}&=-0.0041153\cdot\exp(+i\cdot 1.5021)\,,\\
\zeta_6^{(1)}&=+0.0031234\cdot\exp(+i\cdot 1.5141)\,;
\end{aligned}\right\}\tag{25}$$

$$\left.\begin{aligned}
\zeta_0^{(2)}&=-0.24373\cdot\exp(+i\cdot 0.12361)\,,\\
\zeta_1^{(2)}&=-0.021103\cdot\exp(+i\cdot 1.5002)\,,\\
\zeta_2^{(2)}&=+0.0080142\cdot\exp(+i\cdot 1.5132)\,,\\
\zeta_3^{(2)}&=-0.0075016\cdot\exp(+i\cdot 1.5236)\,,\\
\zeta_4^{(2)}&=+0.0038297\cdot\exp(+i\cdot 1.5013)\,,\\
\zeta_5^{(2)}&=-0.0015360\cdot\exp(+i\cdot 1.5036)\,,\\
\zeta_6^{(2)}&=+0.00083216\cdot\exp(+i\cdot 1.4921)\,;
\end{aligned}\right\}\tag{25'}$$

when $N=7$,

$$\left.\begin{aligned}
\zeta_0^{(1)}&=-0.26187\cdot\exp(-i\cdot 0.012361)\,,\\
\zeta_1^{(1)}&=-0.093351\cdot\exp(+i\cdot 1.5421)\,,\\
\zeta_2^{(1)}&=+0.0089411\cdot\exp(+i\cdot 1.5362)\,,\\
\zeta_3^{(1)}&=-0.0086200\cdot\exp(+i\cdot 1.5243)\,,\\
\zeta_4^{(1)}&=+0.0041132\cdot\exp(+i\cdot 1.5264)\,,\\
\zeta_5^{(1)}&=-0.0041024\cdot\exp(+i\cdot 1.5161)\,,\\
\zeta_6^{(1)}&=+0.0030024\cdot\exp(+i\cdot 1.5362)\,,\\
\zeta_7^{(1)}&=-0.0029926\cdot\exp(+i\cdot 1.5039)\,;
\end{aligned}\right\}\tag{26}$$

$$\left.\begin{aligned}
\zeta_0^{(2)}&=-0.24401\cdot\exp(+i\cdot 0.093245)\,,\\
\zeta_1^{(2)}&=-0.021300\cdot\exp(+i\cdot 1.5216)\,,\\
\zeta_2^{(2)}&=+0.0079345\cdot\exp(+i\cdot 1.5261)\,,\\
\zeta_3^{(2)}&=-0.0076187\cdot\exp(+i\cdot 1.5391)\,,\\
\zeta_4^{(2)}&=+0.0040032\cdot\exp(+i\cdot 1.5241)\,,\\
\zeta_5^{(2)}&=-0.0014321\cdot\exp(+i\cdot 1.5249)\,,\\
\zeta_6^{(2)}&=+0.00074259\cdot\exp(+i\cdot 1.5213)\,,\\
\zeta_7^{(2)}&=-0.00021817\cdot\exp(+i\cdot 1.5162)\,;
\end{aligned}\right\}\tag{26'}$$

when $N=8$,

$$\left.\begin{aligned}
\zeta_0^{(1)} &= -0.26187\cdot\exp(-i\cdot 0.0093921)\,,\\
\zeta_1^{(1)} &= -0.093311\cdot\exp(+i\cdot 1.5520)\,,\\
\zeta_2^{(1)} &= +0.0089401\cdot\exp(+i\cdot 1.5721)\,,\\
\zeta_3^{(1)} &= -0.0086105\cdot\exp(+i\cdot 1.5537)\,,\\
\zeta_4^{(1)} &= +0.0040932\cdot\exp(+i\cdot 1.5319)\,,\\
\zeta_5^{(1)} &= -0.0039996\cdot\exp(+i\cdot 1.5466)\,,\\
\zeta_6^{(1)} &= +0.0029998\cdot\exp(+i\cdot 1.5495)\,,\\
\zeta_7^{(1)} &= -0.0028327\cdot\exp(+i\cdot 1.5418)\,,\\
\zeta_8^{(1)} &= +0.0019346\cdot\exp(+i\cdot 1.5532)\,;
\end{aligned}\right\} \qquad (27)$$

$$\left.\begin{aligned}
\zeta_0^{(2)} &= -0.24421\cdot\exp(+i\cdot 0.042361)\,,\\
\zeta_1^{(2)} &= -0.021521\cdot\exp(+i\cdot 1.5528)\,,\\
\zeta_2^{(2)} &= +0.0078134\cdot\exp(+i\cdot 1.54326)\,,\\
\zeta_3^{(2)} &= -0.0076234\cdot\exp(+i\cdot 1.5536)\,,\\
\zeta_4^{(2)} &= +0.0048211\cdot\exp(+i\cdot 1.5462)\,,\\
\zeta_5^{(2)} &= -0.0013291\cdot\exp(+i\cdot 1.5416)\,,\\
\zeta_6^{(2)} &= +0.00070039\cdot\exp(+i\cdot 1.5361)\,,\\
\zeta_7^{(2)} &= -0.00011391\cdot\exp(+i\cdot 1.5199)\,,\\
\zeta_8^{(2)} &= +0.000053211\cdot\exp(+i\cdot 1.5236)\,.
\end{aligned}\right\} \qquad (27')$$

From the results of calculations, the values of $\zeta_0^{(1)}$ and $\zeta_0^{(2)}$ respectively vary $-0.26211\cdot\exp(-i\cdot 0.077321)$(from(22)) to $-0.26187\cdot\exp(-i\cdot 0.0093921)$ (from (27)) and $-0.24382\cdot\exp(+i\cdot 0.26128)$ to $-0.24421\cdot\exp(+i\cdot 0.042361)$ (refer to (22′) and (27′)). It seems very probable that

$$\left.\begin{aligned}
\zeta_0^{(1)} &\longrightarrow -0.261\cdots\\
\zeta_0^{(2)} &\longrightarrow -0.244\cdots
\end{aligned}\right\} \quad (N\to\infty)\,,$$

taking into account that the exponential parts of $\zeta_0^{(1)}$, $\zeta_0^{(2)}$ are tending to a unit. As far as $\zeta_1^{(1)}$ to $\zeta_3^{(1)}$ and $\zeta_1^{(2)}$ to $\zeta_3^{(2)}$ are concerned, the exponential parts tend to $\exp(+i\cdot\pi/2)$ and $\exp(+i\cdot\pi/2)$ respectivly and the orders of the amplitudes remain on the whole unchanged. It can, therefore, be concluded that the infinite simultaneous equations (14)—(17) have solutions of convergence. They may be considered to be approximately given by putting the exponential parts of $\zeta_0^{(1)}$, $\zeta_0^{(2)}$ to 1 and those of $\zeta_1^{(1)}$—$\zeta_8^{(1)}$ and $\zeta_1^{(2)}$—$\zeta_8^{(2)}$ to $\exp(+i\cdot\pi/2)$ in (27) and (27′), that is to say

$$\left.\begin{array}{lll}\zeta_0^{(1)}=-0.26187\,, & \zeta_1^{(1)}=-0.093311\cdot i\,, & \zeta_2^{(1)}=+0.0089401\cdot i\,,\\ \zeta_3^{(1)}=-0.0086105\cdot i\,, & \zeta_4^{(1)}=+0.0040932\cdot i\,, & \zeta_5^{(1)}=-0.0039996\cdot i\,,\\ \zeta_6^{(1)}=+0.0029998\cdot i\,, & \zeta_7^{(1)}=-0.0028327\cdot i\,, & \zeta_8^{(1)}=+0.0019346\cdot i\,;\end{array}\right\} \quad (28)$$

$$\left.\begin{array}{lll}\zeta_0^{(2)}=-0.24421\,, & \zeta_1^{(2)}=-0.021521\cdot i\,, & \zeta_2^{(2)}=+0.0078134\cdot i\,,\\ \zeta_3^{(2)}=-0.0076234\cdot i\,, & \zeta_4^{(2)}=+0.0048211\cdot i\,, & \zeta_5^{(2)}=-0.0013291\cdot i\,,\\ \zeta_6^{(2)}=+0.00070039\cdot i\,, & \zeta_7^{(2)}=-0.00011391\cdot i\,, & \zeta_8^{(2)}=+0.000053211\cdot i\,.\end{array}\right\} \quad (28')$$

In a problem of tsunami, of much interest are the amplitudes of the out-going waves leaving the cliff, or $\zeta_0^{(1)}$ and $\zeta_0^{(2)}$.

When the values described in section (I, 2) were used, the amplitudes of the out-going waves are $|\zeta_0^{(1)}|=0.26187$ cm (the first of (28)) and $|\zeta_0^{(2)}|=0.24421$ cm (the first of (28′)). The wave in water of deep depth is in height larger than that in water of shallow depth by $0.26187-0.24421=0.01766$ (cm). The ratio of $|\zeta_0^{(1)}|$ to $|\zeta_0^{(2)}|$ is $0.26187/0.24421=1.0723$, i. e., the former is larger than the latter by 0.0723%.

(I, 5) *Supplement.*

In the preceding sections, when the appropriate physical values as in section (I, 2) are given, the procedures for obtaining the solutions and their convergence are described.

To illustrate the variation of the ratios of the amplitudes $|\zeta_0^{(1)}|/|\zeta_0^{(2)}|$ of the out-going waves versus the ratios of the depths $H/H-h$ of both sides of the cliffs, let us supplement two more numerical results here. Since the solutions are given without proofs of convergence, we have given these results in this section as only supplementary remarks of the fore-going ones.

When the height of the cliff $h=6$ cm (other used values are the same as those given in section (I, 2)), that is, the ratio of the depths $H/H-h=10/4$, the results are, to the approximation of nine terms ($N=8$), as follows:

$$\left.\begin{array}{lll}\zeta_0^{(1)}=-0.26721\,, & \zeta_1^{(1)}=-0.093412\cdot i\,, & \zeta_2^{(1)}=+0.0089601\cdot i\,,\\ \zeta_3^{(1)}=-0.0086209\cdot i\,, & \zeta_4^{(1)}=+0.0041523\cdot i\,, & \zeta_5^{(1)}=-0.0041021\cdot i\,,\\ \zeta_6^{(1)}=+0.0030021\cdot i\,, & \zeta_7^{(1)}=-0.0029451\cdot i\,, & \zeta_8^{(1)}=+0.0022459\cdot i\,;\end{array}\right\} \quad (29)$$

$$\left.\begin{array}{lll}\zeta_0^{(2)}=-0.23936\,, & \zeta_1^{(2)}=-0.022542\cdot i\,, & \zeta_2^{(2)}=+0.0079461\cdot i\,,\\ \zeta_3^{(2)}=-0.0077912\cdot i\,, & \zeta_4^{(2)}=+0.0048916\cdot i\,, & \zeta_5^{(2)}=-0.0015432\cdot i\,,\\ \zeta_6^{(2)}=+0.00075132\cdot i\,, & \zeta_7^{(2)}=-0.00013401\cdot i\,, & \zeta_8^{(2)}=+0.000065319\cdot i\,.\end{array}\right\} \quad (29')$$

When the height of the cliff $h=4$ cm (other values being referred to the section (I, 2)), i.e., the ratio of the depths $H/H-h=10/6$, the amplitudes become, to the nine terms approximation,

$$\left.\begin{array}{lll} \zeta_0^{(1)}=-0.26001\,, & \zeta_1^{(1)}=-0.093105\cdot i\,, & \zeta_2^{(1)}=+0.0087901\cdot i\,, \\ \zeta_3^{(1)}=-0.0082016\cdot i\,, & \zeta_4^{(1)}=+0.0040213\cdot i\,, & \zeta_5^{(1)}=-0.0032921\cdot i\,, \\ \zeta_6^{(1)}=+0.0024321\cdot i\,, & \zeta_7^{(1)}=-0.0024613\cdot i\,, & \zeta_8^{(1)}=+0.0015001\cdot i\,; \end{array}\right\} \quad (30)$$

$$\left.\begin{array}{lll} \zeta_0^{(2)}=-0.24932\,, & \zeta_1^{(2)}=-0.020032\cdot i\,, & \zeta_2^{(2)}=+0.0074301\cdot i\,, \\ \zeta_3^{(2)}=-0.007539\cdot i\,, & \zeta_4^{(2)}=+0.0042091\cdot i\,, & \zeta_5^{(2)}=-0.0010931\cdot i\,, \\ \zeta_6^{(2)}=+0.00065238\cdot i\,, & \zeta_7^{(2)}=-0.00010002\cdot i\,, & \zeta_8^{(2)}=+0.000050113\cdot i\,. \end{array}\right\} \quad (30')$$

Then the ratios of the amplitudes of the out-going waves, $|\zeta_0^{(1)}|/|\zeta_0^{(2)}|$, are; when $H/H-h=10/4$, from the first of (29) and (29'),

$$|\zeta_0^{(1)}/\zeta_0^{(2)}|=0.26721/0.23936 = 1.1164\,; \quad (31)$$

when $H/H-h=10/6$, from the first of (30) and (30'),

$$|\zeta_0^{(1)}/\zeta_0^{(2)}|=0.26001/0.24932 = 1.0429\,. \quad (32)$$

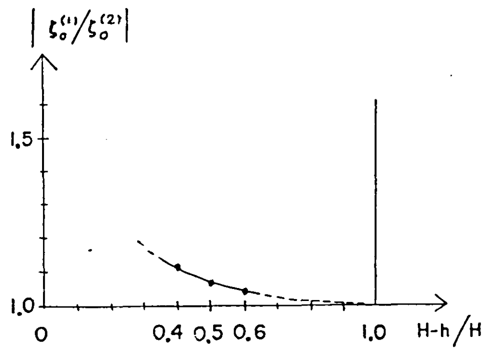

Fig. I, 2

The ratios (31), (32) and that obtained in section (I, 4) (the last is the case where $H/H-h=10/5$) are plotted in Fig. I, 2 for the variation of the depth ratios.

Part II.

(II, 1) *Theory.*

In a manner similar to Part I, the co-ordinates (x, z) are centered at the mean surface of water above the cliff, the x-axis being taken horizontally and the z-axis vertically upwards. In the present Part, the case is considered where a portion of the bottom on the lower side of the submerged step is vibrated (Fig. II, 1). Let the length of the vibrating region be l; the domains in the range $(0>z>-H,\ l>x>0)$, $(0>z>-H,\ x>l)$ and $(0>z>-H+h,\ 0>x)$ D_0, D_1 and D_2 respectively (refer to Fig. II, 1); the velocity potentials in the domains D_j $(j=0, 1, 2)$

ϕ_j $(j=0,\ 1,\ 2)$ respectively. Then the basic equations in the domains D_j $(j=0,\ 1,\ 2)$ are, in incompressible fluid,

$$\Delta\phi_j=0 \quad (j=0,\ 1,\ 2)\ , \qquad (1)$$

where

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\ .$$

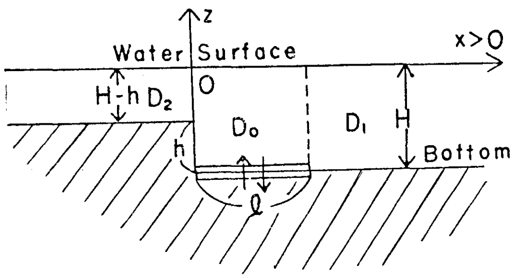

Fig. II, 1.

The surface and bottom conditions for the case of vibration are:

$$\text{in the domain } D_0,\quad \left.\begin{aligned} -\omega^2\phi_0+g\frac{\partial\phi_0}{\partial z}&=0 \quad (z=0)\ , \\ \frac{\partial\phi_0}{\partial z}&=-i\omega D_{\text{bot}} \quad (z=-H)\ , \end{aligned}\right\} \qquad (2)$$

where ω is an angular frequency of vibration, D_{bot} the amplitude of vibration of a portion of the bottom and ϕ_0, D_{bot} have been expressed in simplified form omitted primes which must be added in full form when the time factor $\exp(-i\omega t)$ is excluded (this point should be referred to Part I and this convention will be followed in subsequent discussions and in Part III);

$$\text{in the domain } D_1,\quad \left.\begin{aligned} -\omega^2\phi_1+g\frac{\partial\phi_1}{\partial z}&=0 \quad (z=0)\ , \\ \frac{\partial\phi_1}{\partial z}&=0 \quad (z=-H)\ ; \end{aligned}\right\} \qquad (3)$$

$$\text{in the domain } D_2,\quad \left.\begin{aligned} -\omega^2\phi_2+g\frac{\partial\phi_2}{\partial z}&=0 \quad (z=0)\ , \\ \frac{\partial\phi_2}{\partial z}&=0 \quad (z=-H+h)\ . \end{aligned}\right\} \qquad (4)$$

The velocity potentials ϕ_j $(j=1,\ 2)$ in the domains D_j $(j=1,\ 2)$, satisfying the conditions (3) and (4), are given as follows (refer to Part I):

$$\phi_1=B_0^{(1)}e^{+ia_0^{(1)}x}\cosh a_0^{(1)}(H+z)+\sum_{s=1}^{\infty}B_s^{(1)}e^{-a_s^{(1)}x}\cos a_s^{(1)}(H+z)\ , \qquad (5)$$

$$\phi_2=A_0^{(2)}e^{-ia_0^{(2)}x}\cosh a_0^{(2)}(H-h+z)+\sum_{s=1}^{\infty}A_s^{(2)}e^{+a_s^{(2)}x}\cos a_s^{(2)}(H-h+z)\ , \qquad (6)$$

where $A_0^{(2)}$, $A_s^{(2)}$ $(s=1, 2, 3, \cdots)$, $B_0^{(1)}$, $B_s^{(1)}$ $(s=1, 2, 3, \cdots)$ stand for the arbitrary constants.

On the other hand, the solution of the equation (1) in the domain D_0 that satisfies the condition (2) has already been given by the author[11], i. e.

$$\phi_0=(A_0^{(0)}e^{-ia_0^{(0)}x}+B_0^{(0)}e^{+ia_0^{(0)}x})\cosh a_0^{(0)}(H+z)$$
$$+\sum_{s=1}^{\infty}(A_s^{(0)}e^{+a_s^{(0)}x}+B_s^{(0)}e^{-a_s^{(0)}x})\cos a_s^{(0)}(H+z)-i\omega D_{\mathrm{bot}}(z+g/\omega^2)\,, \qquad (7)$$

where $A_0^{(0)}$, $B_0^{(0)}$, $A_s^{(0)}$ and $B_s^{(0)}$ $(s=1, 2, 3, \cdots)$ are the arbitrary constants.

The expression (7) was given in the paper written in Japanese, so let us outline the derivation of (7) here.

Suppose that a particular solution of the equation (1), satisfying the condition (2), is ϕ_p and a general solution, satisfying a "homogeneous" condition:

$$-\omega^2\phi_g+g\frac{\partial\phi_g}{\partial z}=0 \quad (z=0)\,,$$

$$\frac{\partial\phi_g}{\partial z}=0 \quad (z=-H)\,,$$

the latter of which is an expression putting to zero in the second of (2), is denoted by ϕ_g, then the general solution of equation (1) under the condition (2) is in general given by

$$\phi_0=\phi_g+\phi_p\,. \qquad (8)$$

Such a formulation has been made for the case of three dimensions.[12]

The particular solution is $\phi_p=-i\omega D_{\mathrm{bot}}(z+g/\omega^2)$, from an intuitive consideration, whereas the general solution ϕ_g has the following form:

$$\phi_g=(A_0^{(0)}e^{-ia_0^{(0)}x}+B_0^{(0)}e^{+ia_0^{(0)}x})\cosh a_0^{(0)}(H+z)$$
$$+\sum_{s=1}^{\infty}(A_s^{(0)}e^{+a_s^{(0)}x}+B_s^{(0)}e^{-a_s^{(0)}x})\cos a_s^{(0)}(H+z)\,,$$

where $\exp(-ia_0^{(0)}x)$ and $\exp(+ia_0^{(0)}x)$ designate the progressive waves in the opposite directions: $\exp(-a_s^{(0)}x)$ and $\exp(+a_s^{(0)}x)$ the boundary corrections at either side of the vibrating region or $x=0$ and l; $a_0^{(0)}$, $a_s^{(0)}$

11) T. MOMOI, *Zisin* [ii], **15** (1962), 53, (in Japanese).
12) T. MOMOI, *Bull. Earthq. Res. Inst.*, **40** (1962), 265.

($s=1, 2, 3, \cdots$) the solutions of $\omega^2=a_0^{(0)}g \tanh a_0^{(0)}H=-a_s^{(0)} \tan a_s^{(0)}H$; $A_0^{(0)}$, $B_0^{(0)}$, $A_s^{(0)}$, $B_s^{(0)}$ ($s=1, 2, 3, \cdots$) the arbitrary constants.

Hence in consideration of (8), the general solution of the equation (1) under the condition (2) is of the form given in (7).

As the boundary conditions for determining the arbitrary constants contained in the expressions (5) to (7), we have:

$$\left.\begin{aligned} \phi_0&=\phi_1 \,, \\ \frac{\partial \phi_0}{\partial x}&=\frac{\partial \phi_1}{\partial x} \,, \end{aligned}\right\} \quad (x=l \,,\ 0>z>-H) \,, \tag{9}$$

the first being the continuity of pressure and the second the continuity of velocity of water particles;

$$\phi_2=\phi_0 \quad (x=0 \,,\ 0>z>-H+h) \,, \tag{10}$$

$$\left.\begin{aligned} \frac{\partial \phi_2}{\partial x}&=\frac{\partial \phi_0}{\partial x} \quad (x=0 \,,\ 0>z>-H+h) \,, \\ 0&=\frac{\partial \phi_0}{\partial x} \quad (x=0 \,,\ -H+h>z>-H) \,, \end{aligned}\right\} \tag{11}$$

where (10) stands for the continuity of pressure, the former of (11) the continuity of velocity of water particles and the latter the vanishing of the velocity of water particles at the cliff.

Substituting (5) and (6) for (9), and applying the operators (refer to Part I)

$$\int_{-H}^{0} \cosh a_0^{(1)}(H+z)dz \,, \quad \int_{-H}^{0} \cos a_s^{(1)}(H+z)dz \quad (s=1, 2, 3, \cdots) \,,$$

we have

$$\left.\begin{aligned} -M_0^{(0)}/I_0^{(H)}+(A_0^{(0)}e^{-ia_0^{(0)}l}+B_0^{(0)}e^{+ia_0^{(0)}l})&=B_0^{(1)}e^{+ia_0^{(1)}l} \,, \\ (-A_0^{(0)}e^{-ia_0^{(0)}l}+B_0^{(0)}e^{+ia_0^{(0)}l})&=B_0^{(1)}e^{+ia_0^{(1)}l} \,, \end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned} +M_s^{(0)}/I_s^{(H)}+(A_s^{(0)}e^{+a_s^{(0)}l}+B_s^{(0)}e^{-a_s^{(0)}l})&=B_s^{(1)}e^{-a_s^{(1)}l} \,, \\ (A_s^{(0)}e^{+a_s^{(0)}l}-B_s^{(0)}e^{-a_s^{(0)}l})&=-B_s^{(1)-a_s^{(1)}l} \,, \\ (s=1, 2, 3, \cdots) \,, \end{aligned}\right\} \tag{13}$$

where $M_0^{(0)}=+i\omega D_{\text{bot}}/a_0^{(0)2}$; $M_s^{(0)}=+i\omega D_{\text{bot}}/a_s^{(0)2}$; $I_0^{(H)}$ and $I_s^{(H)}$ have already been described in Part I.

Since the depths in the domains D_0 and D_1 are equal, so are the values of $a_0^{(0)}$, $a_s^{(0)}$ and $a_0^{(1)}$, $a_s^{(1)}$ $(s=1, 2, 3, \cdots)$ respectively. Hence these characters will appropriately be interchanged in the following.

In a manner similar to Part I, putting (6), (7) into (10), (11) and applying the operators,

$$\int_{-H+h}^{0}\begin{Bmatrix}\cosh a_0^{(2)}(H-h+z)\\ \cos a_s^{(2)}(H-h+z)\end{Bmatrix}dz \quad (s=1, 2, 3, \cdots),$$

to (10) and the oprators,

$$\int_{-H}^{0}\begin{Bmatrix}\cosh a_0^{(0)}(H+z)\\ \cos a_s^{(0)}(H+z)\end{Bmatrix}dz \quad (s=1, 2, 3, \cdots),$$

to (11); the following equations possessing an infinite number of unknowns are obtained:

$$\left.\begin{aligned}
A_0^{(2)}I_0^{(H-h)} &= -M_0^{(2)}+(A_0^{(0)}+B_0^{(0)})\cdot I_{0,0}[({}^{0}_{-H+h}), H, H-h]\\
&\quad +\sum_{s=1}^{\infty}(A_s^{(0)}+B_s^{(0)})\cdot I_{s,0}[({}^{0}_{-H+h}), H, H-h],\\
A_s^{(2)}I_s^{(H-h)} &= M_s^{(2)}+(A_0^{(0)}+B_0^{(0)})\cdot I_{0,s}[({}^{0}_{-H+h}), H, H-h]\\
&\quad +\sum_{s'=1}^{\infty}(A_{s'}^{(0)}+B_{s'}^{(0)})\cdot I_{s',s}[({}^{0}_{-H+h}), H, H-h] \quad (s=1, 2,3, \cdots),\\
-ia_0^{(2)}A_0^{(2)}\cdot I_{0,0}[({}^{0}_{-H+h}), H-h, H] &+ \sum_{s=1}^{\infty} a_s^{(2)}A_s^{(2)}\cdot I_{s,0}[({}^{0}_{-H+h}), H-h, H]\\
&= ia_0^{(0)}(-A_0^{(0)}+B_0^{(0)})\cdot I_0^{(H)},\\
-ia_0^{(2)}A_0^{(2)}\cdot I_{0,s}[({}^{0}_{-H+h}), H-h, H] &+ \sum_{s'=1}^{\infty} a_{s'}^{(2)}A_{s'}^{(2)}\cdot I_{s',s}[({}^{0}_{-H+h}), H-h, H]\\
&= a_s^{(0)}(A_s^{(0)}-B_s^{(0)})\cdot I_s^{(H)} \quad (s=1, 2, 3, \cdots),
\end{aligned}\right\} \tag{14}$$

where

$$M_0^{(2)} = +i\omega D_{\text{bot}}/a_0^{(2)2},$$
$$M_s^{(2)} = +i\omega D_{\text{bot}}/a_s^{(2)2},$$

the expressions $I_0^{(H-h)}$, $I_{0,0}[({}^{0}_{-H+h}), H, H-h]$, $I_{s,0}[({}^{0}_{-H+h}), H, H-h]$, $I_s^{(H-h)}$, $I_{0,s}[({}^{0}_{-H+h}), H, H-h]$, $I_{s',s}[({}^{0}_{-H+h}), H, H-h]$, $I_{0,0}[({}^{0}_{-H+h}), H-h, H]$, $I_{s,0}[({}^{0}_{-H+h}), H-h, H]$, $I_0^{(H)}$, $I_{0,s}[({}^{0}_{-H+h}), H-h, H]$, $I_{s',s}[({}^{0}_{-H+h}), H-h, H]$ and $I_s^{(H)}$ should be referred to Part I.

From (12) and (13), we have

$$A_0^{(0)} = \frac{M_0^{(0)}e^{+ia_0^{(0)}l}}{2I_0^{(H)}}, \tag{15}$$

$$B_0^{(0)}=B_0^{(1)}+\frac{M_0^{(0)}e^{-ia_0^{(0)}l}}{2I_0^{(H)}}\,, \tag{16}$$

$$A_s^{(0)}=-\frac{M_s^{(0)}\,e^{-a_s^{(0)}l}}{2I_s^{(H)}} \tag{17}$$

$$B_s^{(0)}=B_s^{(1)}-\frac{M_s^{(0)}\,e^{+a_s^{(0)}l}}{2I_s^{(H)}}\,. \tag{18}$$

Substituting for (14) from (15) and (17), the equations (14) become

$$\left.\begin{aligned}
&A_0^{(2)}I_0^{(H-h)}=-M_0^{(2)}+\left(\frac{M_0^{(0)}e^{+ia_0^{(0)}l}}{2I_0^{(H)}}+B_0^{(0)}\right)\cdot I_{0,0}[(^{0}_{-H+h}),\,H,\,H-h]\\
&\qquad+\sum_{s'=1}^{\infty}\left(-\frac{M_s^{(0)}\,e^{-a_s^{(0)}l}}{2I_s^{(H)}}+B_s^{(0)}\right)\cdot I_{s,0}[(^{0}_{-H+h}),\,H,\,H-h]\,,\\
&A_s^{(2)}I_s^{(H-h)}=M_s^{(2)}+\left(\frac{M_0^{(0)}e^{+ia_0^{(0)}l}}{2I_0^{(H)}}+B_0^{(0)}\right)\cdot I_{0,s}[(^{0}_{-H+h}),\,H,\,H-h]\\
&\qquad+\sum_{s'=1}^{\infty}\left(-\frac{M_{s'}^{(0)}\,e^{-a_{s'}^{(0)}l}}{2I_{s'}^{(H)}}+B_{s'}^{(0)}\right)\cdot I_{s',s}[(^{0}_{-H+h}),\,H,\,H-h]\\
&\qquad\qquad(s=1,\ 2,\ 3,\cdots)\,,\\
&-ia_0^{(2)}A_0^{(2)}\cdot I_{0,0}[(^{0}_{-H+h}),\,H-h,\,H]+\sum_{s=1}^{\infty}a_s^{(2)}A_s^{(2)}\cdot I_{s,0}[(^{0}_{-H+h}),\,H-h,\,H]\\
&\qquad=ia_0^{(0)}\left(-\frac{M_0^{(0)}\,e^{+ia_0^{(0)}l}}{2I_0^{(H)}}+B_0^{(0)}\right)\cdot I_0^{(H)}\,,\\
&-ia_0^{(2)}A_0^{(2)}\cdot I_{0,s}[(^{0}_{-H+h}),\,H-h,\,H]+\sum_{s'=1}^{\infty}a_{s'}^{(2)}A_{s'}^{(2)}\cdot I_{s',s}[(^{0}_{-H+h}),\,H-h,\,H]\\
&\qquad=a_s^{(0)}\left(-\frac{M_s^{(0)}\,e^{-a_s^{(0)}l}}{2I_s^{(H)}}-B_s^{(0)}\right)\cdot I_s^{(H)}\\
&\qquad\qquad(s=1,\ 2,\ 3,\cdots)\,.
\end{aligned}\right\} \tag{19}$$

Thus we have the infinite simultaneous equations (19), whose unkowns are $A_0^{(2)}$, $A_s^{(2)}$ $(s=1,\ 2,\ 3,\cdots)$, $B_0^{(0)}$, $B_s^{(0)}$ $(s=1,\ 2,\ 3,\cdots)$.

The method for obtaining the solutions of infinite simultaneous equations were outlined in Part I.

Before proceeding to numerical analysis, let us consider the expressions of the wave height.

By use of the relation $\partial\phi/\partial t=-g\zeta$ $(z=0)$, the wave heights ζ_j $(j=0,\ 1,\ 2)$ in the domains D_j $(j=0,\ 1,\ 2)$ become:

from (5),

$$\zeta_1 = i\cdot\zeta_0^{(1)} e^{-i\omega t + i a_0^{(1)} x} + i\cdot\sum_{s=1}^{\infty} \zeta_s^{(1)} e^{-i\omega t}\cdot e^{-a_s^{(1)} x} ; \tag{20}$$

from (6),

$$\zeta_2 = i\cdot\zeta_0^{(2)} e^{-i\omega t - i a_0^{(1)} x} + i\cdot\sum_{s=1}^{\infty} \zeta_s^{(2)} e^{-i\omega t}\cdot e^{+a_s^{(2)} x} ; \tag{21}$$

from (7),

$$\zeta_0 = i\cdot e^{-i\omega t}(\zeta_0^{(0)} e^{-i a_0^{(0)} x} + \bar{\zeta}_0^{(0)} e^{+i a_0^{(0)} x}) + i\cdot\sum_{s=1}^{\infty} e^{-i\omega t}(\zeta_s^{(0)} e^{+a_s^{(0)} x} + \bar{\zeta}_s^{(0)} e^{-a_s^{(0)} x}) + e^{-i\omega t} D'_{\mathrm{bot}} ; \tag{22}$$

where

$$\left.\begin{aligned}
\zeta_0^{(1)} &= \frac{\omega}{g}\cdot B_0^{(1)}\cdot\cosh a_0^{(1)} H , \\
\zeta_s^{(1)} &= \frac{\omega}{g}\cdot B_s^{(1)}\cdot\cos a_s^{(1)} H \quad (s=1,\ 2,\ 3,\cdots) , \\
\zeta_0^{(2)} &= \frac{\omega}{g}\cdot A_0^{(2)}\cdot\cosh a_0^{(2)} (H-h) , \\
\zeta_s^{(2)} &= \frac{\omega}{g}\cdot A_s^{(2)}\cdot\cos a_s^{(2)} (H-h) \quad (s=1,\ 2,\ 3,\cdots) , \\
\zeta_0^{(0)} &= \frac{\omega}{g}\cdot A_0^{(0)}\cdot\cosh a_0^{(0)} H , \\
\bar{\zeta}_0^{(0)} &= \frac{\omega}{g}\cdot B_0^{(0)}\cdot\cosh a_0^{(0)} H , \\
&\left.\begin{aligned}
\zeta_s^{(0)} &= \frac{\omega}{g}\cdot A_s^{(0)}\cdot\cos a_s^{(0)} H , \\
\bar{\zeta}_s^{(0)} &= \frac{\omega}{g}\cdot B_s^{(0)}\cdot\cos a_s^{(0)} H ,
\end{aligned}\right\} \quad (s=1,\ 2,\ 3,\cdots) ,
\end{aligned}\right\} \tag{23}$$

and D'_{bot} contained in (18) is expressed in full form adding a dash (′).

(II, 2) *Used Values.*

Used values to solve the equations (19) are the same as those given in section (I, 2) of Part I except for the length of the wave origin (l) which is taken as 50 cm.

When these values are used, "eigin" values $a_0^{(0)}$ (or $a_0^{(1)}$), $a_s^{(0)}$ (or $a_s^{(1)}$) ($s=1,\ 2,\ 3,\cdots$), $a_0^{(2)}$ and $a_s^{(2)}$ ($s=1,\ 2,\ 3,\cdots$) are determined as given in

(18) of Part I.

(II, 3) *The Results of Computation.*

Following the procedures used in Part I, we have the ratio of the amplitudes of the waves leaving the cliff, i. e. $|\zeta_0^{(2)}/\zeta_0^{(1)}|$ from (16) and (19). We are less interested in the case of vibration of the bottom, so that the ratio of the amplitudes, when $N=4$ (N stands for a number up to which the unknowns $A_0^{(2)}$, $B_0^{(0)}$, $A_s^{(2)}$ and $B_s^{(0)}$ ($s=1, 2, \cdots, N$) are only retained to solve the infinite simultaneous equations (19)), has been computed alone, i. e., $|\zeta_0^{(2)}/\zeta_0^{(1)}| \fallingdotseq 1.48$ ($|\zeta_0^{(2)}|=0.773$ cm, $|\zeta_0^{(1)}|=0.522$ cm).

Part. III.

(III, 1) *Theory.*

In this Part, the case where a portion of the bottom in shallow water neighbouring the cliff vibrates vertically, is considered (Fig. III, 1).

The definitions and notations to be used in this Part are identical with those used in Parts I and II.

Referring to Fig. III, 1, we have:

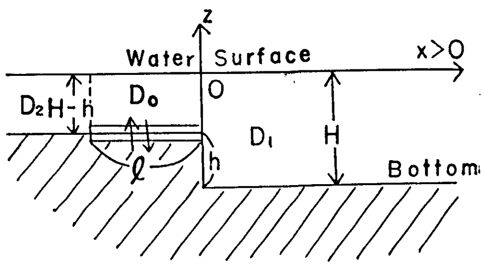

Fig. III, 1.

in the domain D_0,

$$\left.\begin{aligned} &\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\phi_0=0\ , \\ &-\omega^2\phi_0+g\frac{\partial\phi_0}{\partial z}=0 \quad (z=0)\ , \\ &\frac{\partial\phi_0}{\partial z}=-i\omega D_{\text{bot}} \quad (z=-H+h)\ ; \end{aligned}\right\} \tag{1}$$

in the domain D_1,

$$\left.\begin{aligned} &\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\phi_1=0\ , \\ &-\omega^2\phi_1+g\frac{\partial^2\phi_1}{\partial z}=0 \quad (z=0)\ , \\ &\frac{\partial\phi_1}{\partial z}=0 \quad (z=-H)\ ; \end{aligned}\right\} \tag{2}$$

in the domain D_2,

$$\left.\begin{aligned} &\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial z^2}\right)\phi_2=0 , \\ &-\omega^2\phi_2+g\frac{\partial\phi_2}{\partial z}=0 \quad (z=0) , \\ &\frac{\partial\phi_2}{\partial z}=0 \quad (z=-H+h) , \end{aligned}\right\} \tag{3}$$

where D_0, D_1, D_2, stand for the domains $-l<x<0$, $0<x$, $x<-l$ respectively, instead of $0<x<l$, $l<x$, $x<0$ in the case of Part II.

The solutions of (1), (2) and (3) are as follows:

$$\left.\begin{aligned} \phi_1&=B_0^{(1)}e^{+ia_0^{(1)}x}\cosh a_0^{(1)}(H+z)+\sum_{s=1}^{\infty}B_s^{(1)}e^{-a_s^{(1)}x}\cos a_s^{(1)}(H+z) , \\ \phi_2&=A_0^{(2)}e^{-ia_0^{(2)}x}\cosh a_0^{(2)}(H-h+z)+\sum_{s=1}^{\infty}A_s^{(2)}e^{+a_s^{(2)}x}\cos a_s^{(2)}(H-h+z) , \\ \phi_0&=(A_0^{(0)}e^{-ia_0^{(0)}x}+B_0^{(0)}e^{+ia_0^{(0)}x})\cosh a_0^{(0)}(H-h+z) \\ &\quad+\sum_{s=1}^{\infty}(A_0^{(0)}e^{+a_s^{(0)}x}+B_s^{(0)}e^{-a_s^{(0)}x})\cos a_s^{(0)}(H-h+z)-i\omega D_{\text{bot}}(z+g/\omega^2) , \end{aligned}\right\} \tag{4}$$

where, in this case, $a_0^{(2)}=a_0^{(0)}$ and $a_s^{(2)}=a_s^{(0)}$, in place of $a_0^{(0)}=a_0^{(1)}$ and $a_s^{(0)}=a_s^{(1)}$ in the case of Part II.

By the relation $\zeta=-(1/g)(\partial\phi/\partial t)$, the wave heights in the domains D_j $(=1, 2, 0)$ become, from (4),

$$\left.\begin{aligned} \zeta_1&=i\zeta_0^{(1)}e^{-i\omega t+ia_0^{(1)}x}+i\cdot\sum_{s=1}^{\infty}\zeta_s^{(1)}e^{-i\omega t}\cdot e^{-a_s^{(1)}x} , \\ \zeta_2&=i\zeta_0^{(2)}e^{-i\omega t-ia_0^{(2)}x}+i\cdot\sum_{s=1}^{\infty}\zeta_s^{(2)}e^{-i\omega t}\cdot e^{+a_s^{(1)}x} , \\ \zeta_0&=i\cdot e^{-i\omega t}(\zeta_0^{(0)}e^{-ia_0^{(0)}x}+\bar{\zeta}_0^{(0)}e^{+ia_0^{(0)}x}) \\ &\quad+i\cdot\sum_{s=1}^{\infty}e^{-i\omega t}(\zeta_s^{(0)}e^{+a_s^{(0)}x}+\bar{\zeta}_s^{(0)}e^{-a_s^{(0)}x})+e^{-i\omega t}D'_{\text{bot}} , \end{aligned}\right\} \tag{5}$$

where

$$\left.\begin{aligned} \zeta_0^{(1)}&=\frac{\omega}{g}B_0^{(1)}\cosh a_0^{(1)}H , \\ \zeta_s^{(1)}&=\frac{\omega}{g}B_s^{(1)}\cos a_s^{(1)}H \quad (s=1, 2, 3,\cdots) , \end{aligned}\right.$$

$$\left.\begin{aligned}
&\zeta_0^{(2)}=\frac{\omega}{g}A_0^{(2)}\cosh a_0^{(2)}(H-h)\ ,\\
&\zeta_s^{(2)}=\frac{\omega}{g}A_s^{(2)}\cos a_s^{(2)}(H-h)\quad (s=1,\ 2,\ 3,\cdots)\ ,\\
&\zeta_0^{(0)}=\frac{\omega}{g}A_0^{(0)}\cosh a_0^{(0)}(H-h)\ ,\\
&\bar{\zeta}_0^{(0)}=\frac{\omega}{g}B_0^{(0)}\cosh a_0^{(0)}(H-h)\ ,\\
&\left.\begin{aligned}
&\zeta_s^{(0)}=\frac{\omega}{g}A_s^{(0)}\cos a_s^{(0)}(H-h)\ ,\\
&\bar{\zeta}_s^{(0)}=\frac{\omega}{g}B_s^{(0)}\cos a_s^{(0)}(H-h)\ ,
\end{aligned}\right\}\ (s=1,\ 2,\ 3,\cdots)\ .
\end{aligned}\right\}\qquad (6)$$

Available conditions for determining the arbitrary constants are:

$$\left.\begin{aligned}
&\left.\begin{aligned}
&\phi_2=\phi_0\ ,\\
&\frac{\partial\phi_2}{\partial x}=\frac{\partial\phi_0}{\partial x}\ ,
\end{aligned}\right\}\ (x=-l\ ,\ 0>z>-H+h)\ ,\\
&\phi_0=\phi_1\quad (x=0\ ,\ \ 0>z>-H+h)\ ,\\
&\frac{\partial\phi_0}{\partial x}=\frac{\partial\phi_1}{\partial x}\quad (x=0\ ,\ \ 0>z>-H+h)\ ,\\
&0=\frac{\partial\phi_1}{\partial x}\quad (x=0\ ,\ \ -H+h>z>-H)\ .
\end{aligned}\right\}\qquad (7)$$

Substituting (4) for (7) and applying the operators:

$$\int_{-H+h}^{0}\begin{Bmatrix}\cosh a_0^{(2)}(H-h+z)\\ \cos a_s^{(2)}(H-h+z)\end{Bmatrix}dz\quad (s=1,\ 2,\ 3,\cdots)\ ,$$

$$\int_{-H}^{0}\begin{Bmatrix}\cosh a_0^{(1)}(H+z)\\ \cos a_s^{(1)}(H+z)\end{Bmatrix}dz\quad (s=1,\ 2,\ 3,\cdots)\ ,$$

we have (these procedures are the same as in Parts I and II),

$$\begin{aligned}
&A_0^{(2)}e^{+ia_0^{(2)}l}=-M_0^{(2)}/I_0^{(H-h)}+(A_0^{(0)}e^{+ia_0^{(0)}l}+B_0^{(0)}e^{-ia_0^{(0)}l})\ ,\\
&A_s^{(2)}e^{-a_s^{(2)}l}=+M_s^{(2)}/I_s^{(H-h)}+(A_s^{(0)}e^{-a_s^{(0)}l}+B_s^{(0)}e^{+a_s^{(0)}l})\quad (s=1,\ 2,\ 3,\cdots)\ ,
\end{aligned}$$

$$\left.\begin{aligned}
&-A_0^{(2)}e^{+ia_0^{(2)}l}=-A_0^{(0)}e^{+ia_0^{(0)}l}+B_0^{(0)}e^{-ia_0^{(0)}l}\ ,\\
&\quad A_s^{(2)}e^{-a_s^{(2)}l}=A_s^{(0)}e^{-a_s^{(0)}l}-B_s^{(0)}e^{+a_s^{(0)}l}\quad(s=1,\ 2,\ 3,\cdots)\ ,\\
&-M_0^{(2)}+(A_0^{(0)}+B_0^{(0)})\cdot I_0^{(H-h)}\\
&\qquad\qquad=B_0^{(1)}I_{0,0}[\tbinom{0}{-H+h}),H,H-h]+\sum_{s=1}^{\infty}B_s^{(1)}I_{s,0}[\tbinom{0}{-H+h}),H,H-h]\ ,\\
&+M_s^{(2)}+(A_s^{(0)}+B_s^{(0)})\cdot I_s^{(H-h)}\\
&\qquad\qquad=B_0^{(1)}I_{0,s}[\tbinom{0}{-H+h}),H,H-h]+\sum_{s'=1}^{\infty}B_{s'}^{(1)}I_{s',s}[\tbinom{0}{-H+h}),H,H-h]\ ,\\
&\qquad\qquad\qquad\qquad(s=1,\ 2,\ 3,\cdots)\ ,\\
&+ia_0^{(0)}(-A_0^{(0)}+B_0^{(0)})\cdot I_{0,0}[\tbinom{0}{-H+h}),H-h,H]\\
&\qquad+\sum_{s=1}^{\infty}a_s^{(0)}(A_s^{(0)}-B_s^{(0)})\cdot I_{s,0}[\tbinom{0}{-H+h}),H-h,H]=+ia_0^{(1)}B_0^{(1)}I_0^{(H)}\ ,\\
&+ia_0^{(0)}(-A_0^{(0)}+B_0^{(0)})\cdot I_{0,s}[\tbinom{0}{-H+h}),H-h,H]\\
&\qquad+\sum_{s'=1}^{\infty}a_{s'}^{(0)}(A_{s'}^{(0)}-B_{s'}^{(0)})\cdot I_{s',s}[\tbinom{0}{-H+h}),H-h,H]=-B_s^{(1)}a_s^{(1)}I_s^{(H)}\\
&\qquad\qquad\qquad\qquad(s=1,\ 2,\ 3,\cdots)\ ,
\end{aligned}\right\}\quad(8)$$

where the expressions $M_0^{(2)}$, $M_s^{(2)}$ $(s=1,\ 2,\ 3,\cdots)$, $I_0^{(H-h)}$, $I_s^{(H-h)}$ $(s=1,\ 2,\ 3,\cdots)$, $I_{0,0}[\tbinom{0}{-H+h}),H,H-h]$, $I_{s,0}[\tbinom{0}{-H+h}),H,H-h]$ $(s=1,\ 2,\ 3,\cdots)$, $I_{0,s}[\tbinom{0}{-H+h}),H,H-h]$ $(s=1,\ 2,\ 3,\cdots)$, $I_{s',s}[\tbinom{0}{-H+h}),H,H-h]$ $(s,s'=1,\ 2,\ 3,\cdots)$, $I_{0,0}[\tbinom{0}{-H+h}),H-h,H]$, $I_{s,0}[\tbinom{0}{-H+h}),H-h,H]$ $(s=1,\ 2,\ 3,\cdots)$, $I_0^{(H)}$, $I_{0,s}[\tbinom{0}{-H+h}),H-h,H]$ $(s=1,\ 2,\ 3,\cdots)$, $I_{s',s}[\tbinom{0}{-H+h}),H-h,H]$ $(s',s=1,\ 2,\ 3,\cdots)$ and $I_s^{(H)}$ $(s=1,\ 2,\ 3,\cdots)$ were given in Parts I and II.

(III, 2) *Used Values.*

The values used in this Part are the same as those in Part II, except that a vibrating portion of the bottom is placed on the upper side of the cliff in this case.

(III, 3) *The Results of Computation.*

Following the methods described in Parts I and II, the arbitrary constants are determined, by use of the expressions (6), as follows (actual computations were made by the OKITAC-5090 at the Computation Center of Tokyo University):

$$|\zeta_0^{(2)}/\zeta_0^{(1)}|\fallingdotseq 1.51\ \ (|\zeta_0^{(2)}|\fallingdotseq 0.924\,\mathrm{cm},\ \ |\zeta_0^{(1)}|\fallingdotseq 0.612\,\mathrm{cm})\ ,$$

where the only case of $N=4$ has been computed as in Part II, the case of vibration of the bottom is of less significance in the problem of generation of a tsunami. In due course let us consider the case of the instantaneously elevated bottom.

Here, the comparison of Parts II and III are given below:

(1) the amplitude ratios of both cases are nearly equal.

(2) the amplitudes in the case of Part II are smaller than those in Part III.

Firstly, this result seems to be inconsistent with the fact that the work done by a vibrating portion of the bottom in deep water should be greater than that in shallow water. However, further consideration shows this result to be self-consistent. Referring to the preceding paper[13] written in Japanese, in water of uniform depth, the wave height of the out-going wave from the "vibrating" wave origin varies sinusoidally as the length of a vibrating portion increasing. If the relation holds that (a half length of the wave origin)$\times$(a wave number)$=m\pi$ (m: integer), the out-going wave disappears irrespective of the length of the wave origin.

From this fact, we can find no difficulty in the interpretation of the result of this paper.

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2. 海底起伏にもとずく津波の指向性 [I]

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筆者は前報告で，津波波源域の非等方性にもとずく津波の指向性を論じた．

本論文および，以後の論文で，筆者は波源域の海底起伏にもとずく津波の指向性を論ずる．本論文は三部に分かれる．

まず，第一部では Fig. (I, 1) に示されるように，鉛直の崖が水平方向に振動する場合におこる水波の指向性を次の値に対して計算した．すなわち，

深い方の水深: 10粁,
浅い方の水深: 5粁,
振動の周期: 1秒,
振動振幅: 1粍.

このときおこる進行波の波高は

13) T. Momoi, *loc. cit.*, 11).

深い方へ 0.26....糎
浅い方へ 0.24....糎

である. 深い方への波は浅い方への波より約 10% 高い波高示す.

本報告の第 2 部においては, 段の "直下" の底の一部が, 鉛直方向に振動するときにおこる津波を論じ, 第 1 部と同じ数値に対して津波の指向性を計算した. (Fig. (II, 1).

第 3 部においては, 段の "直上" の底の一部が鉛直方向に振動するときの津波の指向性を, 第1, 2 部と同じ数値に対し計算した.

いずれにせよ, "振動した" 波源を用いて津波の指向性を論ずることは余り意味がないと思われるので, 各種の数値に対する, 発生津波の波高を計算することは敢えてせず, むしろ, 計算手順の紹介だけにとどめ, "振動していない" で, "瞬間的" に上げられた波源の問題に移ることにした.