BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 40 (1962) pp. 653-655

35. *A Note on the Relation between the Initial Motion and the Azimuthal Characteristic of a Focus from the View-point of the Phase Shift near the Origin.*

By Yasuo SATÔ,

Earthquake Research Institute.

(Read July 24, 1962.—Received Aug. 29, 1962.)

1. The discovery of the polar phase shift associated with the propagation of waves on a spherical surface produced new knowledge of the phase relation of surface waves.[1] Because of the nature of the associated Legendre function, waves on a spherical surface have somewhat longer wave lengths in the region of the pole and antipode as compared with other areas of the sphere. Similar results had been found previously for the source region in the case of waves propagating on a plane surface.[2] These two phenomena, however, are not quite independent, but are connected by the formula[3]

$$n^{-m}P_n^m(\cos\theta)\sim J_m(\xi)\ ,\qquad \text{when } n\to\infty\ , \tag{1}$$

where $\theta=\xi/n$ and θ is not very small.

The left side of the above expression represents the wave form on a spherical surface, while the right side that on a plane surface. When n and ξ are large, but the ratio ξ/n is moderate, both sides of the expression (1) have the asymptotic form[4]

$$\sqrt{\frac{2}{\pi\zeta}}\cos\left[\zeta-\frac{m\pi}{2}-\frac{\pi}{4}\right], \tag{2}$$

where ζ stands for either θ or ξ in the case of the spherical and plane boundary problems respectively.

1) J. N. BRUNE, J. E. NAFE and L. E. ALSOP, "The Polar Phase Shift of Surface Waves on a Sphere," *B. S. S. A.*, **51** (1961), 247.

2) Y. SATÔ, "Synthesis of Dispersed Surface Waves by Means of Fourier Transform," *B. S. S. A.*, **50** (1960), 417.

3) WHITTAKER and WATSON, *Modern Analysis*, 4th ed. (1935), p. 367.

4) With regard to the asymptotic expansion of the associated Legendre function care must be taken because there are various definitions, most of which use the identical notations.

Since the argument of the cosine function is not ζ only, but has two additional terms, the effect of these terms must be considered, especially when the motion at the origin is to be deduced using data observed at large distances from the source. The last term, $\pi/4$, is connected with the polar phase shift and is independent of the order of the functions. The second term, however, contains the factor m; consequently the phase angle at the origin is gravely affected by an erroneous estimation of m. For example, if $m=2$ and is assumed to be zero by mistake, the sense of the motion at the origin proves to be reversed. If m is an odd number, G waves G_n and G_{n+1} (or Rayleigh waves R_n and R_{n+1}) have the additional phase difference π caused by the second term in (2).

2. The theoretical seismogram, obtained as the result of numerical calculation of the propagation of torsional oscillations on a homogeneous elastic mantle[5], will serve as an example of the application of the foregoing theory.

Suppose that the force applied at the origin is unknown, and that the value of m which determines the azimuthal characteristic of the focus is to be found. The ordinary method of Fourier analysis can be used to calculate the phase velocity by the comparision of two observations, say G_1 and G_3. If the same method is applied to the data G_1 and G_2, the correct value of phase velocity cannot be determined unless m is known to be either even or odd. If m is an odd number, a correction must be added to compensate for the phase shift caused by the passage over the antipode.

Independent evidence concerning the nature of the source and the value of the factor m is available from body waves. The study of ScS waves indicates a wave form similar to that in Fig. 1a, from which the type of the force applied at the origin may be inferred. The motion near the origin deduced from Fourier synthesis[2] has a phase relation which agrees with the above wave form if m is correctly assumed to be zero. The assumption of values

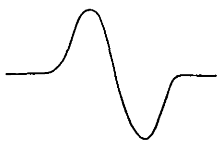

Fig. 1a

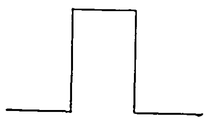

Fig. 1b

Theoretical ScS waves on the surface of a homogeneous elastic mantle (Fig. 1a), and the torsional force applied around the pole. (Fig. 1b) Axial symmetry ($m=0$) is assumed for the calculation.

5) Y. SATÔ, T. USAMI, M. LANDISMAN and M. EWING, "Propagation of Torsional Disturbances on a Radially Heterogeneous Sphere, Case of Homogeneous Mantle," (in press).

for m which are different from zero will result in an incorrect time function at the origin.

In this way the value of m is connected both with the phase relation near the origin and the azimuthal characteristic, which permits the determination of one from the other.

---

35. 震央付近での位相変化より震源での動きと方位特性の関係を求めること

地震研究所 佐藤泰夫

1. 球面上の波の伝播に関しては，いはゆる Polar phase shift の発見によつて[1]，一進歩がなされたが，これは Legendre 函数の性質により，球面上を伝はる波にあつては，極附近で他の部分より波長が長くなることにもとづくものである．同様のことは，点源より発せられた波が平面上を伝はる場合について，さきに見出されてゐるが[2]，この二つの現象は独立なものではなく，

$$n\to\infty\ ,\quad \theta=\xi/n\ ,\ (\theta \text{ は非常に小さくはない})$$

のときに知られてゐる関係式

$$n^{-m}P_n^m(\cos\theta)\sim J_m(\xi)$$

によつて結びつけられてゐる．この式の左辺は球面上，右辺は平面上を伝はる波を表現してゐるが，両辺は共に

$$\sqrt{\frac{2}{\pi\zeta}}\cos\left[\zeta-\frac{m\pi}{2}-\frac{\pi}{4}\right]$$

の形の漸近展開を持つ．(ζ は θ もしくは ξ を表はす)

上式 cos 内の変数は ζ のみでなく，二つの余分の項をもつ．そのうち後の項 $\pi/4$ は Polar phase shift を与へるものであるが，G_1 と G_2 の位相差から速度を求めるとき，又遠くの観測から震源での動き，位相などを求めようとするとき等には考慮しなくてはならない．

2. 例へば，理論的に求めた捩れ振動の，一様な球の上の伝播を例にとる[5]．正しい m の値を知ることなしには，震源で最初どちら向きの力が加へられたかは不明である．しかし，震源での力の向きが何らかの方法によつて推定できるならば，(たとへば Fig. 1 に示した *ScS* 波の動きを参考にするなど) m を求めることも可能となる．また，G_1，G_2 の差から位相速度を求めるときにも，m が奇数か偶数かによつて，異なつた補正を行はなくてはいけない．

このやうに m は震源の方位特性，および位相の両方に密接に関連してゐるので，一方を知れば m を通じて他方を知り得るといふ関係がある．