BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 40 (1962), pp. 685-692

37. *A Possible Cause of Earth-Current Anisotropy.*

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(Read Sept. 18, 1962.—Received Sept. 19, 1962.)

Summary

Discussion is made on the influence of the anomalous concentration of electric currents as found beneath Japan at times of geomagnetic disturbance on earth-currents observed at the surface. An electric field amounting to as much as 20 per cent of the one induced directly in the earth by a geomagnetic change may well be expected to be set up as the result of the underground circuit which has been supposed to exist from analyses of anomalies of geomagnetic variation in Japan. Earth-current anisotropy can be raised not only by near-surface geology and topography but also by non-uniformity in the mantle.

1. Introduction

It has long been noticed that direction of earth-currents observed at an observatory is usually restricted to a certain azimuth. At Kakioka Magnetic Observatory, for instance, short-poriod changes in earth-potential predominate in direction $N73°E$ or its reverse[1]. According to the theory of electromagnetic induction within an anisotropic earth as advanced by T. Rikitake and M. Sawada[2], the ratio of the maximum of electrical conductivity to its minimum is estimated as 4 or thereabouts at Kakioka.

Earth-current anisotropy has been customarily ascribed to non-uniform distribution of earth resistivity around the observation point. Although the extent of such a geological or topographical control of earth-currents does not appear to have been worked out, no other theory has been effected so far. It seems to the writer that the earth-current anisotropy found at some places in Japan is so large that it would be difficult to account for all the anisotropy by the near-surface geology and topography only.

1) T. YOSHIMATSU, *Mem. Kakioka Mag. Obs. Suppl.*, **1** (1957).
2) T. RIKITAKE and M. SAWADA, *Bull. Earthq. Res. Inst.*, **40** (1962), 657.

It has been made clear by the writer and his colleagues[3)-11)] that an enormous concentration of electric current develops somewhere underneath Japan whenever a magnetic disturbance, bay, polar magnetic storm or such like, is observed. Only geomagnetic results have been analyzed for arriving at the conclusion. As has been emphasized by G. D. Garland[12)], however, earth-currents over the anomalous area might reflect the unusual underground structure. It is therefore of interest and importance to analyze earth-current results in connection with the underground condition which has been brought out by analyses of geomagnetic data. It might be possible to account for the extermely large earth-current anisotropy by taking into consideration the complicated distribution of electrical conductivity in the upper mantle.

The aim of this paper is to estimate the possible influence of the current-flow concentrated in the upper mantle on earth-currents at the earth's surface. The pattern and magnitude of the underground current-flow have been roughly obtained by analyzing geomagnetic data.

2. Model of the underground circuit

In Fig. 1 is reproduced the distribution of changes in the geomagnetic vertical component (ΔZ) over Japan at the time of a bay that occurred on April 18, 1958 when we could have 12 observatories, permanent and temporary, in Japan. The elliptical distribution strongly suggests that electric currents are flowing along a circuit which lies underneath Japan. Although there is no guarantee that such an interpretation of the anomaly of ΔZ is unique, the concetration of induced currents would certainly be the most straightforward way of explaining the anomaly. It is not possible to determine the depth of the circuit accurately. But

3) T. RIKITAKE, I. YOKOYAMA and Y. HISHIYAMA, *Bull. Earthq. Res. Inst.*, **30** (1952), 207, **31** (1953), 19, 89, 101 and 119.

4) T. RIKITAKE and I. YOKOYAMA, *Bull. Earthq. Res. Inst.*, **33** (1955), 297.

5) T. RIKITAKE, I. YOKOYAMA, S. UYEDA, T. YUKUTAKE and E. NAKAGAWA, *Bull. Earthq. Res. Inst.*, **36** (1958), 1.

6) T. RIKITAKE, S. UYEDA, T. YUKUTAKE, I. TANAOKA and E. NAKAGAWA, *Bull. Earthq. Res. Inst.*, **37** (1959), 1.

7) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **37** (1959), 545.

8) T. RIKITAKE, *Geophys. Journ.*, **2** (1959), 276.

9) T. RIKITAKE and I. YOKOYAMA, *Journ. Geomagn. Geoelectr.*, **5** (1953), 59.

10) T. RIKITAKE and I. YOKOYAMA, *Naturwissenschaften*, **41** (1954), 420.

11) T. RIKITAKE, T. YABU and K. YAMAKAWA, *Bull. Earthq. Res. Inst.*, **40** (1962), 693.

12) G. D. GARLAND, *Methods and Techniques in Geophysics*, **1** (1960), 304.

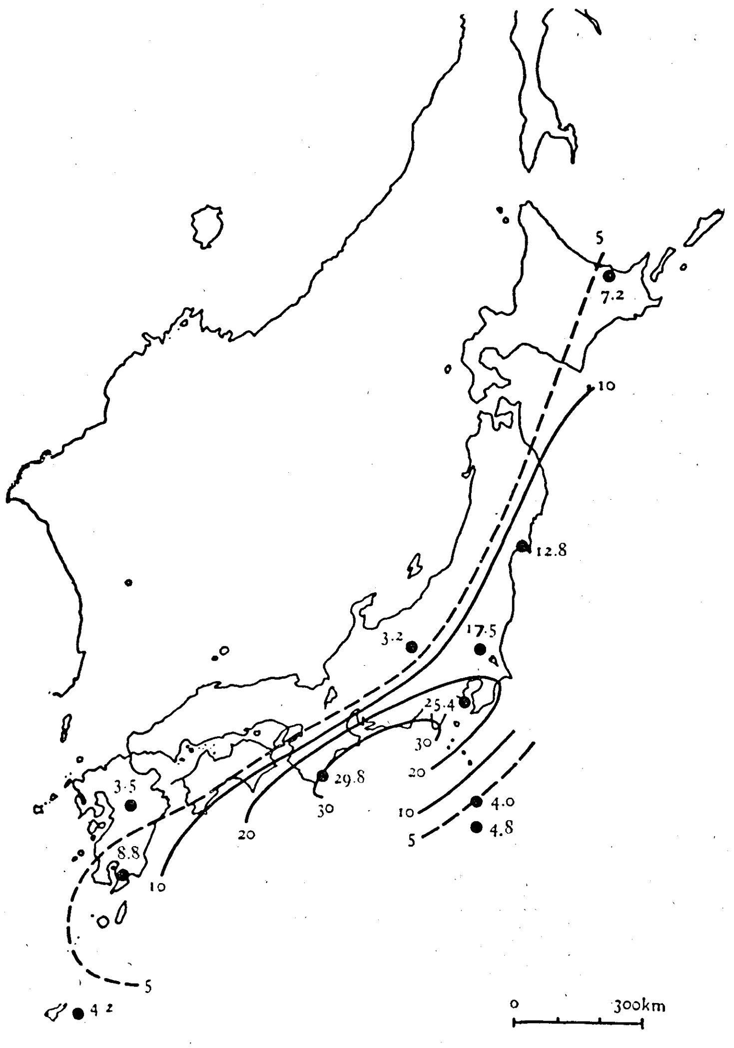

Fig. 1. The distribution of ΔZ in units of gamma for the maximum epoch of a geomagnetic bay on Apr. 18, 1958.

for various reasons the depth has been supposed to be somewhere between 100 *km* and 200 *km*.

The amount of current-flow has been estimated as 3×10^4 *amperes* for an inducing field of 100 *gammas* on the assumption that the circuit is circular with a radius of 200 *km*. If we assume a line current, 5×10^4 *amperes* are required for the explanation of the anomaly. Since the shape of the circuit is not determined accurately, a hypothetical model of the circuit, a circular one with a radius of 200 *km*, is assumed in the following. It is also assumed that the circuit plane is parallel to the ground surface. The reason why such an intense current flows along the circuit is not discussed here though it is likely that the current comes up from the deeper part of the mantle as has been surmised in the previous papers.

3. Electromagnetic induction by a time-dependent circular current within a conducting sheet

Let us assume a circular current (radius: a) parallel to a plane sheet, the distance between the circuit and the sheet being taken as z_0. It is assumed here that only the near-surface layer of the earth is conducting ignoring the conductivity deep in the mantle.

Taking a cylindrical coordinate system (r, ϕ, z) of which the origin is taken at the centre of the circuit and the z axis is taken positive upwards, the magnetic potential of the circular current at any point is expressed as

$$W=2\pi ja\int_0^\infty e^{-\lambda z}J_1(\lambda a)J_0(\lambda r)d\lambda \tag{1}$$

for $z>0$, where j is the intensity of the electric current flowing in the circuit. J_1 and J_0 are Bessel functions.

When j is time-dependent, some electric currents are induced in the sheet. The potential of magnetic field produced by the currents is denoted by W_i, while the current function is written as Ψ. According to the theory of electromagnetic induction within a plane sheet[13], we have

$$\Psi=(2\pi)^{-1}W_i \tag{2}$$

and

$$\rho\left(\frac{\partial^2 W_i}{\partial z^2}\right)=2\pi D\left(\frac{\partial W}{\partial z}+\frac{\partial W_i}{\partial z}\right) \tag{3}$$

at the positive side of the sheet, where ρ is the resistivity of the sheet and D is time-operator $\partial/\partial t$. It is also obvious that W_i satisfies

$$\nabla^2 W_i=0\ . \tag{4}$$

13) A. T. PRICE, *Quart. Journ. Mech. Applied Math.*, **2** (1949), 283.

Solving (4) with proper account of boundary condition (3), Ψ is obtained as

$$\Psi = -2\pi D j a \int_0^\infty \frac{e^{-\lambda z_0} J_1(\lambda a)}{\rho\lambda + 2\pi D} J_0(\lambda r) d\lambda \,, \tag{5}$$

so that we see that the current-flow in the sheet is circular.

In the case of a periodic change having a period T, we put

$$D = i\alpha \qquad (\alpha = 2\pi/T) \,. \tag{6}$$

The current intensity is then given as

$$i_\phi = \frac{jK}{a}\Big[K\int_0^\infty \frac{\xi}{\xi^2+K^2} e^{-(z_0/a)\xi} J_1(\xi) J_1\Big(\frac{r}{a}\xi\Big) d\xi + i\int_0^\infty \frac{\xi^2}{\xi^2+K^2} e^{-(z_0/a)\xi} J_1(\xi) J_1\Big(\frac{r}{a}\xi\Big) d\xi\Big] \,, \tag{7}$$

where

$$K = 2\pi\alpha a \rho^{-1} \,. \tag{8}$$

(7) enables us to estimate current intensity or electric field (E_ϕ) provided the integrals involved are performed numerically. Assuming $\rho = 10^6$ *e. m. u.*, $T = 1$ *hour*, $a = 200$ *km* and $z_0 = 100$ *km*, $|E_\phi|$ is calculated

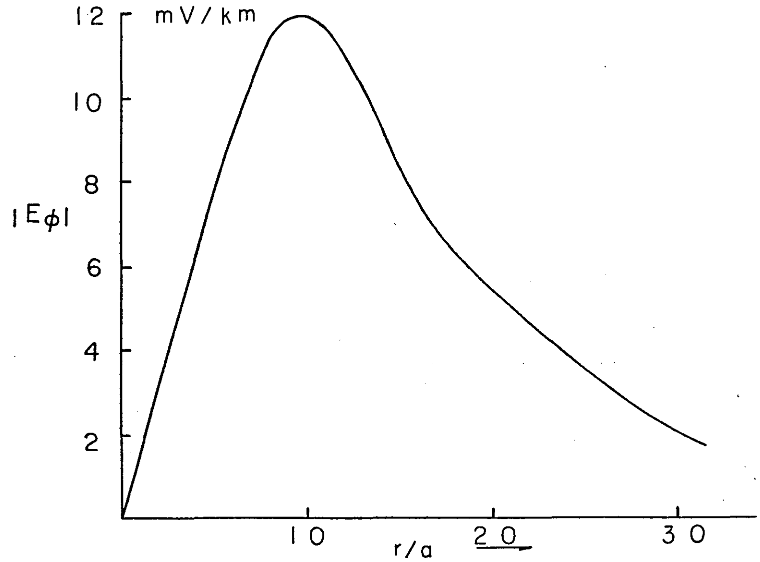

Fig. 2. The intensity of electric field at the earth's surface.

for various values of r/a as shown in Fig. 2. In the calculation, j is assumed as 4×10^4 *amperes*, that is, the mean value of the current intensities in the case of circular and line circuits when we have a 100 *gamma* inducing field over Japan.

In Fig. 2, it is seen that the induced electric field is the largest approximately at $r=a$, while the decrease with the increase in r is gradual. From (7), we also see that E_ϕ is affected very little by the choice of ρ. Even if we take $\rho=10^5\,e.m.u.$ instead of $10^6\,e.m.u.$, E_ϕ changes very slightly.

4. Dicussion and conclusion

We are now in a position to compare the effect of the underground circuit with the electric field induced directly in the earth. Assuming that the earth can be treated as a plane sheet over a non-conducting layer of thickness H and underlain by a conductor (conductivity: σ), the relation between the electric field and the magnetic field becomes

$$\frac{E_x}{H_y}=\frac{i\alpha\rho M}{\dfrac{2\pi}{L\rho}+4\pi i\alpha M}, \tag{9}$$

for a periodic change having a wave-length L. M is defined by

$$M=1-\frac{\sqrt{1+s^2}-1}{\sqrt{1+s^2}+1}e^{-(4\pi/L)H}, \tag{10}$$

where

$$s^2=\frac{4\pi\sigma i\alpha}{\left(\dfrac{2\pi}{L}\right)^2}. \tag{11}$$

Table 1. The intensities of electric field induced by a 100 *gamma* field in layered earth.

ρ	Electric field
$10^5\,e.m.u.$	$7.9\,mV/km$
10^6	65
10^7	120

mod. (E_x/H_y) can be calculated from (9) with the aid of (10) and (11). Taking the same period as before, intensities of electric field induced by a 100 *gamma* field are calculated as given in Table 1 for three values of ρ, while $H=400\,km$ and $\sigma=10^{-12}\,e.m.u.$ are assumed.

If induction by a uniform inducing field in a uniform semi-infinite earth (conductivity: σ_0) is assumed, the relation between E_x and H_y at the surface becomes

$$\frac{E_x}{H_y}=\sqrt{\frac{i\alpha}{4\pi\sigma_0}} \tag{12}$$

by which intensities of electric field induced by a 100 *gamma* field are easily calculated as given in Table 2.

Table 2. The intensities of electric field induced by a 100 *gamma* field in uniform earth.

σ_0	Electric field
10^{-14} *e. m. u.*	118 *mV/km*
10^{-13}	37
10^{-12}	12
10^{-11}	4

It is not known what the distribution of electrical conductivity is beneath Japan. Yoshimatsu[1] has estimated on the basis of an analysis of S_q, that the conductivity amounts to 5.3×10^{-15} *e. m. u.* at Kakioka on condition that the uniform model is assumed. He has also proposed a possible model of layered earth. According to that model, the upper 6 *km* is composed of material having a conductivity 1.3×10^{-13} *e. m. u.* while the conductivity becomes 5.0×10^{-15} *e. m. u.* below that depth. An analysis of *s.s.c.*, *s.i.* and such like made by Rikitake and Sawada[2] suggests that ρ at Kakioka is of the order of 10^6 *e. m. u.* which does not differ much from Yoshimatsu's second model. Although these estimates are by no means complete because they are based on simple models of the earth, ρ may be taken as 10^6 *e. m. u.* at Kakioka provided the layered earth model is taken for granted. In that case, it is seen that the electric field due to the induction by the currents flowing in the underground circuit amounts to some 20 per cent of the one induced directly by the magnetic change in the earth.

The position of the circuit is not known. Judging from the distribution of anomalous ΔZ, however, the circuit must run from east to west somewhere beneath Kakioka or a little south. Whenever the magnetic field changes to the north, we observe an increase in the downward component of geomagnetic field. That means that the current flows clockwise in the circuit seen from above. The induced currents at the earth's surface should then be anti-clockwise. We see therefore that the induced electric field is enhanced by the effect of the deep-seated current circuit in the case of a northerly change in geomagnetic field. On the contrary nothing anomalous happens when we have an east-west change in geomagnetic field because very small currents are induced in the circuit as has been made clear by an analysis of *s.f.e.*[11] The above feature of earth-current anisotropy seems likely to fit in qualitatively with the observational results at Kakioka.

Although the anisotropy of earth-currents examined in the above is not large enough for the explanation of all the anisotropy observed, there still remains a possibility of choosing favourable values for various quantities involved. If a shallower depth of the circuit is taken, for example, the influence of the circuit on the electric field at the surface becomes greater. At the present stage of investigation, however, it dose not seem possible to discuss these points in more detail. Until the nature of concentration of electric currents beneath Japan is made a little clearer, nothing definite can be said. The most important point of this discussion is, however, that earth-current anisotropy is caused not only by near-surface geology and topography but also by some agency seated in the mantle.

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37. 地電流異方性の原因について

地震研究所　力　武　常　次

地磁気変動のさい，日本の地下に巨大な電流が集中して流れることが明らかにされている．この電流変化が地表付近に誘導作用をおよぼし，地電流が特定の方向に卓越することがわかつた．地電流の異方性は地表付近の不均一ばかりでなく，このようにマントル内に起因する原因によつても発生するものと考えられる．