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39. *The Effects of Coastlines on the Tsunami (1) and some Remarks on the Chilean Tsunami.*

By Takao Momoi,

Earthquake Research Institute.

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1. Introduction.

Few theories on the effects of coastlines on the tsunami are available, although we have considerable empirical and experimental knowledges of them. Hence, in this paper and subsequent papers, we develop theories on the effects of coastlines on the tsunami. In the first place, in this paper, we treated tsunamis plunging into a winding canal or a long winding bay, and also compared the theoretically obtained results with actual phenomena accompanied by the Chilean Tsunami.

2. Theory.

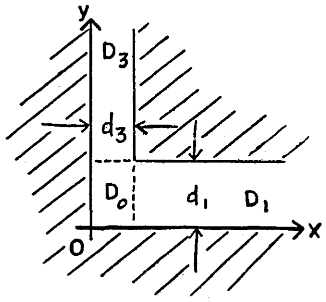

Fig. 1.

In order to analyse waves plunging into a perpendicularly bent canal (refer to Fig. 1), we used Cartesian co-ordinates, x- and y-axis being taken horizontally on the undisturbed free surface of water along the brims of the canal and z-axis vertically upwards.

Assuming that (refer to Fig. 1)

D_1: the domain in the range, $x>d_3$, $d_1>y>0$, $0>z>-H$, where H is the depth of water.

D_3: the domain in the range, $d_3>x>0$, $y>d_1$, $0>z>-H$,

D_0: the domain in the range, $d_3>x>0$, $d_1>y>0$, $0>z>-H$,

Φ_1 : the velocity potential in the domain D_1,

Φ_3 : the velocity potential in the domain D_3,

Φ_0 : the velocity potential in the domain D_0,

g : the acceleration of gravity,

t : the variable of time,

then we have, as basic equations,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi_j=0 \quad (j=1,\ 3,\ 0)\ , \tag{1}$$

as the surface conditions,

$$\frac{\partial^2\Phi_j}{\partial t^2}+g\frac{\partial\Phi_j}{\partial z}=0 \quad (z=0\,;\ j=1,\ 3,\ 0)\ , \tag{2}$$

as the bottom conditions,

$$\frac{\partial\Phi_j}{\partial z}=0 \quad (z=-H;\ j=1,\ 3,\ 0)\ , \tag{3}$$

and also, as the boundary conditions,

$$\frac{\partial\Phi_1}{\partial y}=0 \quad (y=0 \text{ and } d_1,\ x>d_3)\ , \tag{4}$$

$$\frac{\partial\Phi_3}{\partial x}=0 \quad (x=0 \text{ and } d_3,\ y>d_1)\ , \tag{5}$$

$$\left.\begin{aligned}\frac{\partial\Phi_0}{\partial y}=0 \quad (y=0,\ d_3>x>0)\\ \frac{\partial\Phi_0}{\partial x}=0 \quad (x=0,\ d_1>y>0)\end{aligned}\right\}. \tag{6}$$

Provided that the problem is confined to the vibrating case, the basic equations (1), the surface conditions (2), the bottom conditions (3), and the boundary conditions (4), (5), (6) are reduced to

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi_j'=0 \quad (j=1,\ 3,\ 0)\ , \tag{1'}$$

$$-\omega^2\Phi_j'+g\frac{\partial\Phi_j'}{\partial z}=0 \quad (z=0\,;\ j=1,\ 3,\ 0)\ , \tag{2'}$$

$$\frac{\partial\Phi_j'}{\partial z}=0 \quad (z=-H;\ j=1,\ 3,\ 0)\ , \tag{3'}$$

$$\frac{\partial\Phi_1'}{\partial y}=0 \quad (y=0 \text{ and } d_1,\ x>d_3)\ , \tag{4'}$$

$$\frac{\partial\Phi_3'}{\partial x}=0 \quad (y=0 \text{ and } d_3,\ y>d_1)\ , \tag{5'}$$

$$\left.\begin{aligned}\frac{\partial \Phi_0'}{\partial y}&=0 \quad (y=0,\ d_3>x>0)\\ \frac{\partial \Phi_0'}{\partial x}&=0 \quad (x=0,\ d_1>y>0)\end{aligned}\right\}, \tag{6'}$$

where ω is the angular frequency of the vibration, and Φ_j' ($j=1,\ 3,\ 0$) stand for the velocity potentials from which time factor $\exp(-i\omega t)$ is excluded. In the subsequent discussions, we omit the primes of the velocity potentials Φ_j' for simplicity.

Suppose that the plane wave being propagated along the canal from $x=+\infty$ is given by

$$A_1^{(1)}e^{-ia_0x}\cosh a_0(H+z)\ , \tag{7}$$

where a_0 is determined by Airy's relation $\omega^2=a_0 g\tanh a_0H$ and $A_1^{(1)}$ stands for the amplitude of the plunging wave, we can easily see that the wave mentioned above satisfies the basic equation (1′) and the boundary conditions (2′), (3′), (4′).

In the first place, let us consider the waves reflected at the corner of the bent canal.

Separation of the variables of the velocity potential Φ_1 in the equation (1′), and the conditions (2′), (3′), (4′), leads to:

from (1′), $$\frac{d^2\Phi_x^{(1)}}{dx^2}=-(k_x^{(1)})^2\Phi_x^{(1)}\ , \tag{8}$$

$$\frac{d^2\Phi_y^{(1)}}{dy^2}=-(k_y^{(1)})^2\Phi_y^{(1)}\ , \tag{9}$$

$$\frac{d^2\Phi_z^{(1)}}{dz^2}=a_0^2\Phi_z^{(1)}\ , \tag{10}$$

where $$\Phi_1=\Phi_x^{(1)}(x)\cdot\Phi_y^{(1)}(y)\cdot\Phi_z^{(1)}(z)$$

and $$(k_x^{(1)})^2+(k_y^{(1)})^2=a_0^2\ ; \tag{11}$$

from (2′), $$-\omega^2\Phi_z^{(1)}+g\frac{d\Phi_z^{(1)}}{dz}=0 \quad (z=0)\ ; \tag{12}$$

from (3′), $$\frac{d\Phi_z^{(1)}}{dz}=0 \quad (z=-H)\ ; \tag{13}$$

from (4′), $$\frac{d\Phi_y^{(1)}}{dy}=0 \quad (y=0 \text{ and } d_1,\ x>d_3)\ . \tag{14}$$

A solution of (10) under the conditions (12) and (13) is

$$\Phi_z^{(1)} \sim \cosh a_0(H+z) \, , \tag{15}$$

where a_0 is a real and positive root of Airy's relation

$$\omega^2 = a_0 g \tanh a_0 H \, .$$

And also solving (9) under the condition (14), we have

$$\Phi_y^{(1)} \sim \cos \frac{m\pi}{d_1} y \quad (m=0, 1, 2, 3, \cdots) \, , \tag{16}$$

where $\qquad k_y^{(1)} = m\pi/d_1 \, .$

Accordingly, (11) becomes

$$k_x^{(1)} = \pm\sqrt{a_0^2 - \left(\frac{m\pi}{d_1}\right)^2} \quad (\equiv k_x^{(1)m}) \, .$$

Here since we consider the reflected waves, this expression reduces to

$$k_x^{(1)m} = +\sqrt{a_0^2 - \left(\frac{m\pi}{d_1}\right)^2} \quad (m=0, 1, 2, \cdots) \, , \tag{17}$$

where $k_1^{(1)m}$ is imaginary, when $a_0 < m\pi/d_1$.

Then the solution of (8) becomes

$$\Phi_x^{(1)} \sim e^{+ik_x^{(1)m} \cdot x} \, . \tag{18}$$

From (15), (16) and (18), we have the velocity potential of the reflected wave as series, i.e.,

$$\sum_{m=0}^{\infty} A_1^{(2)m} \cdot \cos \frac{m\pi}{d_1} y \cdot e^{+ik_x^{(1)m} \cdot x} \cdot \cosh a_0(H+z) \, , \tag{19}$$

where $A_1^{(2)m}$ $(m=0, 1, 2, \cdots)$ are arbitrary constants.

Thus from (7) and (19) we finally obtain the velocity potential in the domain D_1 as follows;

$$\Phi_1 = A_1^{(1)} e^{-ia_0 x} \cdot \cosh a_0(H+z) + \sum_{m=0}^{\infty} A_1^{(2)m} \cos \frac{m\pi}{d_1} y \cdot e^{+ik_x^{(1)m} \cdot x} \cdot \cosh a_0(H+z) \, . \tag{20}$$

In the same manner, we obtain the velocity potential Φ_3 in the domain D_3 from (1′), (2′), (3′) and (5′), that is to say,

$$\Phi_3 = \sum_{n=0}^{\infty} A_3^n \cos \frac{n\pi}{d_3} x \cdot e^{+ik_y^{(3)n} \cdot y} \cdot \cosh a_0(H+z) \, , \tag{21}$$

where A_3^n $(n=0, 1, 2, 3, \cdots)$ are arbitrary constants and

$$k_y^{(2)n} = +\sqrt{a_0^2 - \left(\frac{n\pi}{d_3}\right)^2} \quad (n=0, 1, 2, \cdots) .$$

Next, let us consider the velocity potential Φ_0 in the domain D_0.

In the same way as in the domain D_1, separation of variables in (1′), (2′), (3′) and (6′) yields

$$\left.\begin{aligned} \frac{d^2\Phi_x^{(0)}}{dx^2} &= -(k_x^{(0)})^2\Phi_x^{(0)} , \\ \frac{d^2\Phi_y^{(0)}}{dy^2} &= -(k_y^{(0)})^2\Phi_y^{(0)} , \\ \frac{d^2\Phi_z}{dz^2} &= a_0^2\Phi_z^{(0)} , \end{aligned}\right\} \tag{22}$$

where $$\Phi_0 = \Phi_x^{(0)}(x)\cdot\Phi_y^{(0)}(y)\cdot\Phi_z^{(0)}(z)$$

and $$(k_x^{(0)})^2 + (k_y^{(0)})^2 = a_0^2 ; \tag{23}$$

$$\left.\begin{aligned} -\omega^2\Phi_z^{(0)} + g\frac{d\Phi_z^{(0)}}{dz} &= 0 \quad (z=0) ; \\ \frac{d\Phi_z^{(0)}}{dz} &= 0 \quad (z=-H) ; \\ \left.\begin{aligned} \frac{d\Phi_y^{(0)}}{dy} &= 0 \quad (y=0,\ d_3>x>0) \\ \frac{d\Phi_x^{(0)}}{dx} &= 0 \quad (x=0,\ d_1>y>0) \end{aligned}\right\} ; \end{aligned}\right\} . \tag{24}$$

From (22) and (24), we can easily obtain a particular solution in the domain D_0 as

$$\Phi_0 \sim \cos k_x^{(0)}x \cdot \cos k_y^{(0)}y \cdot \cosh a_0(H+z) .$$

Thus, superposing the above expression throughout the range which is permitted in the relation (23) (it goes without saying that complex $k_x^{(0)}$ and $k_y^{(0)}$ are permitted), we finally obtain the general solution in the domain D_0 as

$$\Phi_0 = \sum_{f_0} A_0(f_0)\cdot\cos k_x^{(0)}x \cdot \cos k_y^{(0)}y \cdot \cosh a_0(H+z) , \tag{25}$$

where $A_0(f_0)$ is a function of f_0, and f_0 denotes a pair of $k_x^{(0)}$ and $k_y^{(0)}$

restricted by the relation (23), i. e.,

$$f_0=(k_x^{(0)},\ k_y^{(0)})\ ,$$

and hence $\sum_{f_0}$ stands for the integration covering all pairs of $f_0=(k_x^{(0)},\ k_y^{(0)})$ that are permissible in the relation

$$(k_x^{(0)})^2+(k_y^{(0)})^2=a_0^2\ .$$

In order to determine the arbitrary constants $A_1^{(2)m}$ $(m=0,\ 1,\ 2,\cdots)$, A_3^n $(n=0,\ 1,\ 2,\ 3,\cdots)$ and $A_0(f_0)$, we have the next conditions:

$$\left.\begin{array}{ll} \Phi_1=\Phi_0 & \text{(continuity of pressure)}\ , \\ \left.\begin{array}{l} \dfrac{\partial\Phi_1}{\partial x}=\dfrac{\partial\Phi_0}{\partial x} \\ \dfrac{\partial\Phi_1}{\partial y}=\dfrac{\partial\Phi_0}{\partial y} \end{array}\right\} & \left(\begin{array}{l}\text{continuity of velocity} \\ \text{of water particles}\end{array}\right)\ , \end{array}\right\}\ \text{at } x=d_3\ ; \tag{26}$$

and

$$\left.\begin{array}{ll} \Phi_3=\Phi_0 & \text{(contiunity of pressure)}\ , \\ \left.\begin{array}{l} \dfrac{\partial\Phi_3}{\partial x}=\dfrac{\partial\Phi_0}{\partial x} \\ \dfrac{\partial\Phi_3}{\partial y}=\dfrac{\partial\Phi_0}{\partial y} \end{array}\right\} & \left(\begin{array}{l}\text{continuity of velocity} \\ \text{of water particles}\end{array}\right)\ , \end{array}\right\}\ \text{at } y=d_1\ . \tag{27}$$

From (20), (25) and (26), we get

$$A_1^{(1)}e^{-ia_0d_3}+\sum_{m=0}^{\infty}A_1^{(2)m}\cos\frac{m\pi}{d_1}y\cdot e^{+ik_x^{(1)m}\cdot d_3}$$
$$=\sum_{f_0}A_0(f_0)\cos k_x^{(0)}d_3\cdot\cos k_y^{(0)}y\ , \tag{28}$$

$$(-ia_0)A_1^{(1)}e^{-ia_0d_3}+\sum_{m=0}^{\infty}(ik_x^{(1)m})A_1^{(2)m}\cos\frac{m\pi}{d_1}y\cdot e^{+ik_x^{(1)m}\cdot d_3}$$
$$=\sum_{f_0}A_0(f_0)\cdot k_x^{(0)}(-1)\sin k_x^{(0)}d_3\cdot\cos k_y^{(0)}y\ , \tag{29}$$

$$\sum_{m=1}^{\infty}A_1^{(2)m}\cdot\frac{m\pi}{d_1}\cdot(-1)\cdot\sin\frac{m\pi}{d_1}y\cdot e^{+ik_x^{(1)m}\cdot d_3}$$
$$=\sum_{f_0}A_0(f_0)\cdot\cos k_x^{(0)}d_3\cdot k_y^{(0)}(-1)\cdot\sin k_y^{(0)}y \tag{30}$$

Applying the operators $O_p^c(m)=\int_0^{d_1}\cos\frac{m\pi}{d_1}y\cdot dy$ $(m=0,\ 1,\ 2,\cdots)$ to

(28) and (29), and taking account of the orthogonality of the function series $\left\{\cos\frac{m\pi}{d_1}y,\ (m=0,\ 1,\ 2,\ 3,\cdots)\right\}$ in the range $d_1>y>0$, the relations (28) and (29) are reduced to:

from $O_p^c(0)\cdot(28)$,

$$A_1^{(1)}e^{-ia_0d_3}\cdot d_1+A_1^{(2)0}\cdot d_1\cdot e^{+ia_0d_3}=\sum_{f_0}A_0(f_0)\cdot\cos k_x^{(0)}d_3\cdot\frac{1}{k_y^{(0)}}\cdot\sin k_y^{(0)}d_1\,; \tag{31}$$

from $O_p^c(m)\cdot(28)$ $(m=1,\ 2,\ 3,\cdots)$,

$$A_1^{(2)m}\cdot\frac{d_1}{2}\cdot e^{+ik_x^{(1)m}\cdot d_3}$$
$$=\sum_{f_0}A_0(f_0)\cdot\cos k_x^{(0)}d_3\cdot\frac{k_y^{(0)}d_1\cdot d_1}{(k_y^{(0)}d_1)^2-(m\pi)^2}\cdot\sin k_y^{(0)}d_1\cdot\cos m\pi\,; \tag{32}$$

from $O_p^c(0)\cdot(29)$,

$$(-ia_0)\cdot A_1^{(1)}e^{-ia_0d_3}\cdot d_1+(+ia_0)\cdot A_1^{(2)0}\cdot d_1\cdot e^{+ia_0d_2}$$
$$=\sum_{f_0}A_0(f_0)\cdot k_x^{(0)}(-1)\cdot\sin k_x^{(0)}d_3\cdot\frac{1}{k_y^{(0)}}\cdot\sin k_y^{(0)}d_1\,; \tag{33}$$

from $O_p^c(m)\cdot(29)$ $(m=1,\ 2,\ 3,\cdots)$,

$$+ik_x^{(1)m}\cdot A_1^{(2)m}\cdot\frac{d_1}{2}\cdot e^{+ik_x^{(1)m}\cdot d_3}$$
$$=\sum_{f_0}A_0(f_0)\cdot k_x^{(0)}(-1)\cdot\sin k_x^{(0)}d_3\cdot\frac{(k_y^{(0)}d_1)^2d_1}{(k_y^{(0)}d_1)^2-(m\pi)^2}\cdot\cos m\pi\,. \tag{34}$$

Likewise, applying the operators $O_p^s(m)=\int_0^{d_1}\sin\frac{m\pi}{d_1}y\cdot dy$ $(m=1,\ 2,\ 3,\cdots)$ to (30) and taking account of the orthogonality of the functions $\left\{\sin\frac{m\pi}{d_1}y,\ (m=1,\ 2,\ 3,\cdots)\right\}$ in the range $d_1>y>0$, the relation (30) is reduced to

$$A_1^{(2)m}\cdot\frac{m\pi}{d_1}\cdot\frac{d_1}{2}\cdot e^{+ik_x^{(1)m}\cdot d_3}$$
$$=\sum_{f_0}A_0(f_0)\cdot\cos k_x^{(0)}d_3\cdot k_y^{(0)}\cdot\frac{m\pi\cdot d_1}{(k_y^{(0)}d_1)^2-(m\pi)^2}\cdot\sin k_y^{(0)}d_1\cdot\cos m\pi\,. \tag{35}$$

Now we find the above relation (35) to be equivalent to (32). Hence we consider only the relations (31), (32), (33) and (34).

Here we confine ourselves to the problem of waves long enough so that we can approximate $a_0 d_j \ll 1$ $(j=1, 3)$, $|k_x^{(0)}| d_3 \ll 1$, $|k_y^{(0)}| d_1 \ll 1$, that is to say, the case is considered when the width of the canal is very small compared with the wave length of a tsunami.

The above-mentioned approximation holds good for all canals and bays in Japan affected by the Chilean Tsunami which attacked the Coats of Japan.

Then the following reductions may be made, i. e.,

$$\cos k_x^{(0)} d_3 \simeq 1 \,, \quad \sin k_y^{(0)} d_1 \simeq k_y^{(0)} d_1 \,, \quad \sin k_x^{(0)} d_3 \simeq k_x^{(0)} d_3 \,,$$

$$|k_y^{(0)}| d_1 \ll m\pi \quad (m=1,\ 2,\ 3, \cdots) \ \text{or} \ (k_y^{(0)} d_1)^2 - (m\pi)^2 \simeq -(m\pi)^2 \,,$$

and hence the denominators on the right-hand sides of (32) and (34) are always non-zero.

Thus after some reductions, we obtain from (31), (32), (33) and (34)

$$A_1^{(1)} e^{-i a_0 d_3} + A_1^{(2)0} \cdot e^{+i a_0 d_3} = \sum_{f_0} A_0(f_0) \,, \tag{36}$$

$$\frac{1}{2} A_1^{(2)m} \cdot e^{+i k_x^{(1)m} \cdot d_3} = \sum_{f_0} A_0(f_0) \cdot \left\{ - \frac{\cos m\pi}{(m\pi)^2} \right\} \cdot (k_y^{(0)} d_1)^2 \tag{37}$$

$$(m=1,\ 2,\ 3, \cdots) \,,$$

$$(-i a_0) \cdot A_1^{(1)} e^{-i a_0 d_3} \cdot d_1 + i a_0 A_1^{(2)0} \cdot d_1 \cdot e^{+i a_0 d_3} = \sum_{f_0} A_0(f_0)(-1)(k_x^{(0)})^2 d_1 d_3 \,, \tag{38}$$

$$\frac{1}{2} \cdot i k_x^{(1)m} A_1^{(2)m} \cdot d_1 \cdot e^{+i k_x^{(1)m} \cdot d_3}$$

$$= \sum_{f_0} A_0(f_0)(-1)(k_x^{(0)})^2 \cdot d_1 d_3 \cdot \left\{ - \frac{\cos m\pi}{(m\pi)^2} \right\} \cdot (k_y^{(0)} d_1)^2 \tag{39}$$

$$(m=1,\ 2,\ 3, \cdots) \,,$$

where $k_y^{(0)}$ on the right-hand sides of (31) and (32) is tacitly assumed to be non-zero, but the relations (36) and (38) are also valid when $k_y^{(0)}=0$.

In like manner, we obtain the following relations from (20), (25) and (27);

$$A_3^0 \cdot e^{+i a_0 d_1} = \sum_{f_0} A_0(f_0) \,, \tag{40}$$

$$\frac{1}{2} A_3^m \cdot e^{+i k_y^{(3)m} \cdot d_1} = \sum_{f_0} A_0(f_0) \cdot \left\{ - \frac{\cos m\pi}{(m\pi)^2} \right\} \cdot (k_x^{(0)} d_3)^2 \ (m=1,\ 2,\ 3, \cdots) \,, \tag{41}$$

$$i a_0 A_3^0 \cdot d_3 \cdot e^{+i a_0 d_1} = \sum_{f_0} A_0(f_0)(-1)(k_y^{(0)})^2 d_1 d_3 \,, \tag{42}$$

$$\frac{1}{2} ik_y^{(3)m} A_3^m \cdot d_3 \cdot e^{+ik_y^{(3)m} \cdot d_1} = \sum_{f_0} A_0(f_0)(-1)(k_y^{(0)})^2 d_1 d_3 \cdot \left\{ -\frac{\cos m\pi}{(m\pi)^2} \right\} \cdot (k_x^{(0)} d_3)^2 \quad (43)$$

$$(m=1,\ 2,\ 3, \cdots).$$

Now from (36) and (40), we have

$$A_1^{(1)} e^{-ia_0 d_3} + A_1^{(2)0} \cdot e^{+ia_0 d_3} = A_3^0 e^{+ia_0 d_1} . \quad (44)$$

After some calculations and by use of $(k_x^{(0)})^2 + (k_y^{(0)})^2 = a_0^2$, we get from (40), (38), (42)

$$a_0 d_1 \cdot A_1^{(1)} \cdot e^{-ia_0 d_3} - a_0 d_1 \cdot A_1^{(2)0} \cdot e^{+ia_0 d_3} = A_3^0 e^{+ia_0 d_1} \cdot a_0 d_3 (1 - ia_0 d_1) . \quad (45)$$

From the above simultaneous equations (44) and (45), arbitrary constants $A_1^{(2)0}$ and A_3^0 are determined as follows;

$$A_1^{(2)0} = \frac{(d_1 - d_3) + ia \cdot d_1 d_3}{(d_1 + d_3) - ia_0 d_1 d_3} \cdot A_1^{(1)} \cdot e^{-i \cdot 2a_0 d_3} , \quad (46)$$

$$A_3^0 = \frac{2d_1}{(d_1 + d_3) - ia_0 d_1 d_3} \cdot A_1^{(1)} \cdot e^{-ia_0 (d_1 + d_3)} . \quad (47)$$

Next, the calculation $\{(37)/d_1^2\} + \{(41)/d_3^2\}$ yields

$$\left(\frac{1}{d_1^2}\right) \cdot \frac{1}{2} \cdot A_1^{(2)m} \cdot e^{+ik_x^{(1)m} \cdot d_3} + \left(\frac{1}{d_3^2}\right) \cdot \frac{1}{2} \cdot A_3^m \cdot e^{+ik_y^{(3)m} \cdot d_1}$$

$$= \left\{ \sum_{f_0} A_0(f_0) \right\} \cdot \left\{ -\frac{\cos m\pi}{(m\pi)^2} \right\} \cdot \{ (k_x^{(0)})^2 + (k_y^{(0)})^2 \} .$$

And the substitution of (40) and $(k_x^{(0)})^2 + (k_y^{(0)})^2 = a_0^2$ into the above expression leads to

$$\left(\frac{1}{d_1^2}\right) \cdot \frac{1}{2} \cdot A_1^{(2)m} \cdot e^{+ik_x^{(1)m} \cdot d_3} + \left(\frac{1}{d_3^2}\right) \cdot \frac{1}{2} A_3^m \cdot e^{+ik_y^{(3)m} \cdot d_1}$$

$$= A_3^0 \cdot e^{+ia_0 d_1} \cdot \left\{ -\frac{\cos m\pi}{(m\pi)^2} \right\} \cdot a_0^2 .$$

Also after the operation $\{(39)/d_1^2\} = \{(43)/d_3^2\}$, we have

$$\frac{1}{d_1} \cdot k_x^{(1)m} \cdot A_1^{(2)m} \cdot e^{+ik_x^{(2)m} \cdot d_3} = \frac{1}{d_3} \cdot k_y^{(3)m} \cdot A_3^m \cdot e^{+ik_y^{(3)m} \cdot d_1} . \quad (49)$$

Now solving (48) and (49) by use of (47), we obtain $A_1^{(2)m}$ and A_3^m $(m=1,\ 2,\ 3, \cdots)$ as

$$A_3^m = A_1^{(1)} \cdot e^{-ia_0 d_3} \cdot e^{-ik_y^{(3)m} \cdot d_1} \cdot \left\{ -\frac{\cos m\pi}{(m\pi)^2} \right\} \cdot$$

$$\cdot \frac{2}{(d_1+d_3) - ia_0 d_1 d_3} \cdot \frac{a_0^2 d_1^2 d_3^2 \cdot k_x^{(1)m}}{(d_3 k_y^{(3)m} + d_1 k_x^{(1)m})} \quad (m=1, 2, 3, \cdots), \quad (50)$$

$$A_1^{(2)m} = A_1^{(1)} \cdot e^{-ia_0 d_3} \cdot e^{-ik_x^{(1)m} \cdot d_3} \cdot \left\{ -\frac{\cos m\pi}{(m\pi)^2} \right\} \cdot$$

$$\cdot \frac{2}{(d_1+d_3) - ia_0 d_1 d_3} \cdot \frac{a_0^2 d_1^3 d_3 \cdot k_y^{(3)m}}{(d_3 k_y^{(3)m} + d_1 k_x^{(1)m})} \quad (m=1, 2, 3, \cdots). \quad (51)$$

Thus we have determined the arbitrary constants $A_1^{(2)0}$, A_3^0, A_3^m and $A_1^{(2)m}$ $(m=1, 2, 3, \cdots)$ within the scope of the long wave approximation, which are given by (46), (47), (50) and (51).

The amplitudes of the reflected wave in the domain D_1 and the progressive wave in the domain D_3 are given by (46) and (47) respectively as follows;

$$|A_1^{(2)0}| = \sqrt{\frac{(d_1-d_3)^2 + (a_0 d_1 d_3)^2}{(d_1+d_3)^2 + (a_0 d_1 d_3)^2}} \cdot |A_1^{(1)}|, \quad (52)$$

$$|A_2^0| = \frac{2d_1}{\sqrt{(d_1+d_2)^2 + (a_0 d_1 d_3)^2}} \cdot |A_1^{(1)}|, \quad (53)$$

Here, let us consider a particular case where the crooked canal has uniform width or $d_1 = d_3 = d$.

Then the expressions (46), (47), (50), (51), (52) and (53) are reduced to the following forms;

$$A_1^{(2)0} = \frac{+ia_0 d}{2 - ia_0 d} \cdot A_1^{(1)} \cdot e^{-i \cdot 2a_0 d},$$

$$A_3^0 = \frac{2}{2 - ia_0 d} \cdot A_1^{(1)} \cdot e^{-i \cdot 2a_0 d}, \quad (55)$$

$$A_1^{(2)m} = A_2^m = A_1^{(1)} \cdot e^{-i \cdot a_0 d} \cdot e^{-ik_x^{(1)m} \cdot d} \cdot \frac{(a_0 d)^2}{2 - ia_0 d} \cdot \left\{ -\frac{\cos m\pi}{(m\pi)^2} \right\}, \quad (56)$$

where

$$k_x^{(1)m} = \sqrt{a_0^2 - \left(\frac{m\pi}{d} \right)^2} \quad (m=1, 2, 3, \cdots);$$

and

$$|A_1^{(2)0}| = \frac{a_0 d}{2} |A_1^{(1)}|, \quad (57)$$

$$|A_3^0| = \sqrt{1-\left(\frac{a_0 d}{2}\right)^2}\cdot|A_1^{(1)}|\,, \tag{58}$$

where $a_0 d \ll 1$ is used.

Now from (56), (57) and (58), we can easily see that, for waves long enough to be put $a_0 d \ll 1$,

i) the order of the waves reflected at the crooked corner of the canal is lower than that of the plunging waves by $a_0 d$,

ii) the waves propagated through the canal are of the same order as the plunging waves (Fig. 2),

iii) the terms with the factors $A_1^{(2)m}$, A_3^m $(m=1, 2, 3, \cdots)$ in (20) and (21) denote exponentially damping waves, since $k_x^{(1)m}=k_y^{(3)m} =\sqrt{a_0^2-\left(\frac{m\pi}{d}\right)^2}$ $(m=1,2,3,\cdots)$ are imaginary by virtue of the long wave approximation or $a_0 d \ll m\pi/d$.

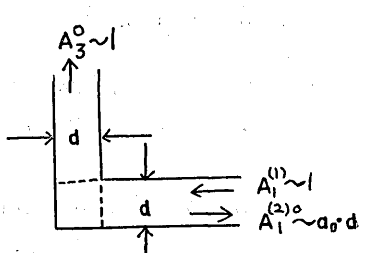

Fig. 2.

Now next, let us examine to what extent the two ratios $|A_3^0|/|A_1^{(1)}|$ between the progressive and the plunging wave, and $|A_1^{(2)0}|/|A_1^{(1)0}|$ between the reflected and the plunging wave depend on the ratio d_3/d_1 between the widths of the canal.

Assuming $d_3/d_1=D$, we have from (52) and (53)

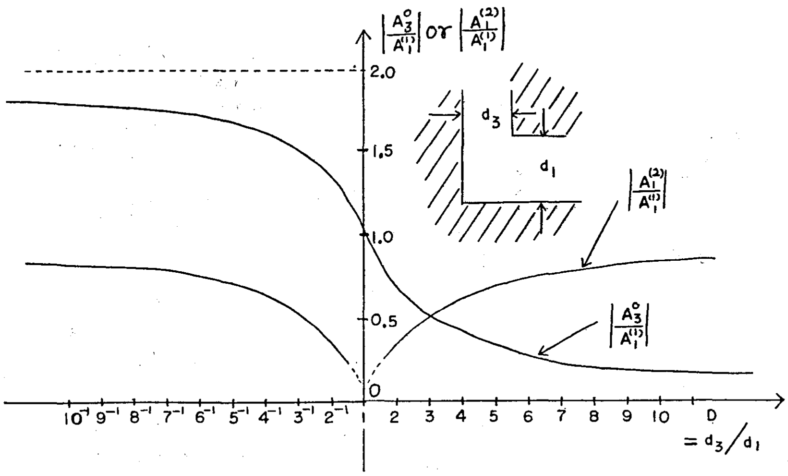

Fig. 3. The variations of $|A_3^0/A_1^{(1)}|$ and $|A_1^{(2)}/A_1^{(1)}|$ $(A_1^{(2)0}\equiv A_1^{(2)})$ versus D.

$$|A_1^{(2)0}| \fallingdotseq \left|\frac{D-1}{D+1}\right| \cdot |A_1^{(1)}| , \tag{59}$$

$$|A_3^0| \fallingdotseq \frac{2}{D+1} \cdot |A_1^{(1)}| , \tag{60}$$

where $a_0 d \ll 1$ is used, and the variation of $|A_1^{(2)0}|/A_1^{(1)}|$ and $|A_3^0|/|A_1^{(1)}|$ with respected to D is shown in Fig. 3.

From this figure we can see that the amplitude of the progressive wave is doubled in the case of $D \approx 0$.

In the next section we shall explain the extraordinary wave heights in the long crooked bays in the case of the Chilean Tsunami by use of the above-mentioned theory.

3. An Explanation of the Extraodinary Wave Heights in the case of the Chilean Tsunami.

As examples of the right-angled long bays we select Miyako Bay (Fig. 4), Hirota Bay (Fig. 5) Ōfunato Bay (Fig. 6), Okachi Bay (Fig. 7), Onagawa Bay (Fig. 8), which lie on the Pacific Ocean in the Tōhoku Districts of Japan, Shimoda Bay (Fig. 9) in Izu Peninsula, and Owase Bay (Fig. 10) in Kii Peninsula. In the case of the Chilean Tsunami, extraodinary wave heights were observed in these bays.

Now let us check in turn the applicability of the theory derived in the preceding section to the explanation of the extraordinary wave heights in each bay.

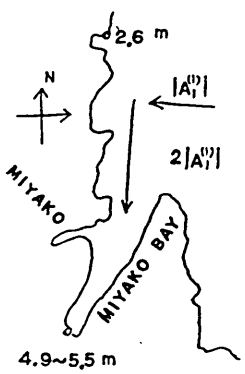

Fig. 4.

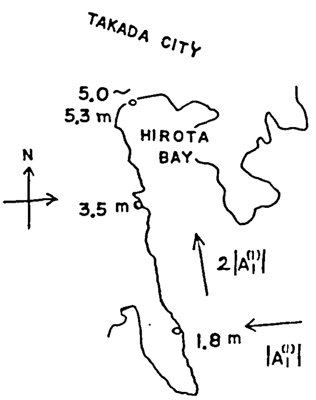

Fig. 5.

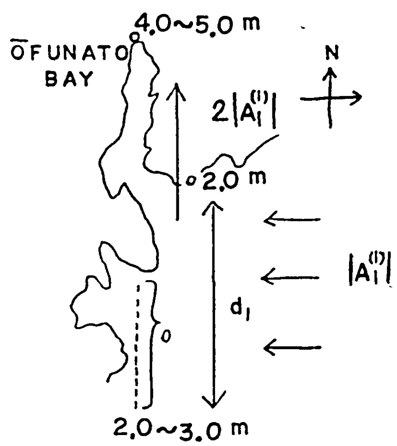

Fig. 6.

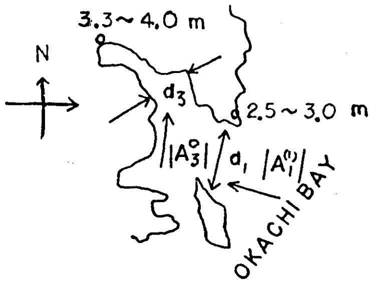

Fig. 7.

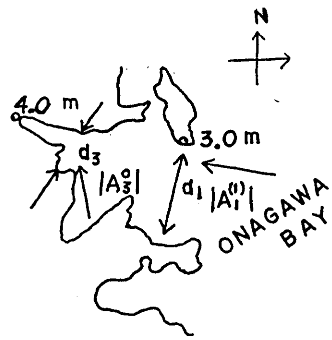

Fig. 8.

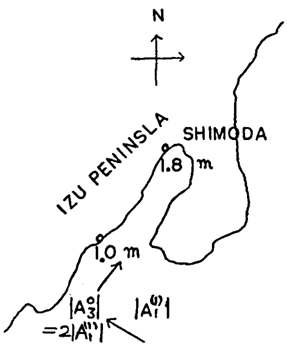

Fig. 9.

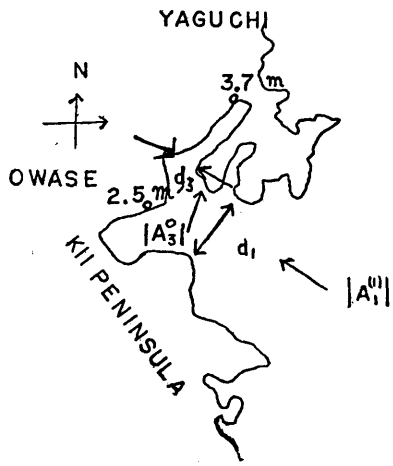

Fig. 10.

i) Miyako Bay (Fig. 4).

This bay is a marked example of our theory. The wave height inside the bay is twice as high as that outside it and then the ratio D is considered to be zero.

ii) Hirota Bay (Fig. 5).

The city of Takada in the inner part of Hirota Bay has a twofold wave height compared with the mouth of the bay, and here the ratio D may be regarded as nearly zero.

iii) Ōfunato Bay (Fig. 6).

This bay is an example of the doubled wave height where $D \simeq 0$.

iv) Okachi Bay (Fig. 7).

The ratio beween the observed wave heights inside and outside the bay is nearly 1.3, which equals the theoretically obtained value 1.3333 in the case of $D=1/2$.

v) Onagawa Bay (Fig. 8).

Likewise the ratio between the observed wave heights inside and outside the bay is nearly 1.3, which equals the theoretically obtained value 1.3333 in the case of $D=1/2$.

vi) Shimoda Bay (Fig. 9).

The ratio between the observed wave heights inside and outside the bay is 1.8, which is almost equal to the theoretically obtained ratio 2 in the case of $D=0$.

vii) Owase Bay (Fig. 10).

The ratio between the observed wave heights inside and outside the bay is approximately 1.5, which is equal to the theoretically obtained value 1.5.

Thus the conformity between the wave height ratios based on the observed wave heights inside and outside a bay and those theoretically deduced is fairly good.

Finally we should like particularly to mention that the extraodinary wave heights were mainly observed in lond right-angled canals on the occasion of the Chilean Tsunami.

4. Acknowledgement.

The author wishes to offer his thanks to Professor R. Takahasi, Assistant Professor K. Kajiura of this Institute and to Dr. K. Takano of the Geophysical Institute of Tokyo University for their kind discussions.

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39. 津波に対する海岸線の影響(1)とチリ津波についての二，三の注意

地震研究所　桃　井　高　夫

直角に曲つた巾の変わる水路に進入した波は角のところで一部は反射し，一部は回折して曲つた水路を進行して行く．このときの反射波および屈折波の波高の進入波の波高に対する比を，屈曲水路の巾の比をパラメーターとして理論的に算出した．そしてこれをチリ津波に適用して，かなりよくチリ津波の異常波高を説明できるのを知つた．