BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 40, (1962), pp. 733-746

40. *The Effects of Coastlines on the Tsunami (2) and some Remarks on the Chilean Tsunami.*

By Takao MOMOI,

Earthquake Research Institute.

(Read Mar. 17, 1962.—Received Sept. 29, 1962.)

1. Introduction.

In the preceding paper[1] we studied the tsunami running into the right-angled long "canal". In this paper we study the tsunami plunging into a long crooked "bay".

2. General Theory.

In order to analyse the waves running into a right-angled bay (Fig. 1), we used Cartesian co-ordinates, x and y-axis being taken horizontally on the undisturbed free surface of water along the rims of the bay and z-axis vertically upwards.

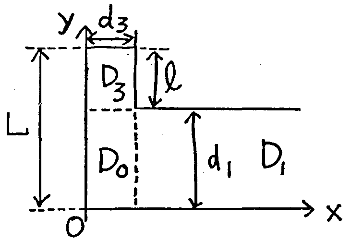

Fig. 1.

Assuming that (Fig. 1)

D_1; the domain in the range $x>d_3$, $d_1>y>0$ and $0>z>-H$, where H is the depth of water,

D_3; the domain in the range $d_3>x>0$, $d_1<y<L$ and $0>z>-H$,

D_0; the domain in the range $d_3>x>0$, $d_1>y>0$ and $0>z>-H$,

Φ_1, Φ_3 and Φ_0; the velocity potentials in the domain D_1, D_3 and D_0,

g ; the acceleration of gravity,

t ; the variable of time,

then we have, as basic equations,

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi_j=0 \quad (j=1,\ 3,\ 0)\,, \tag{1}$$

as the surface conditions,

1) T. MOMOI, *Bull. Earthq. Res. Inst.*, **40** (1962), 723. This paper is referred to as paper A in the subsequent discussion.

$$\frac{\partial^2 \Phi_j}{\partial t^2} + g\frac{\partial \Phi_j}{\partial z} = 0 \quad (z=0\,;\ j=1,\ 3,\ 0)\ , \tag{2}$$

as the bottom conditions,

$$\frac{\partial \Phi_j}{\partial z} = 0 \quad (z=-H;\ j=1,\ 3,\ 0)\ , \tag{3}$$

and also, as the boundary conditions,

$$\frac{\partial \Phi_1}{\partial y} = 0 \quad (y=0 \text{ and } d_1,\ x>d_3)\ , \tag{4}$$

$$\frac{\partial \Phi_3}{\partial x} = 0 \quad (x=0 \text{ and } d_3,\ L>y>d_1)\ , \tag{5}$$

$$\frac{\partial \Phi_3}{\partial y} = 0 \quad (y=L,\ d_3>x>0)\ , \tag{6}$$

$$\left.\begin{aligned} \frac{\partial \Phi_0}{\partial y} &= 0 \quad (y=0,\ d_3>x>0)\ , \\ \frac{\partial \Phi_0}{\partial x} &= 0 \quad (x=0,\ d_1>y>0)\ . \end{aligned}\right\} \tag{7}$$

Except for condition (6), other conditions and the equation are identical in form with those mentioned in paper A. Hence using the same definitions and notations as those in paper A, we have the velocity potential Φ_1 and Φ_0 as follows (refer to paper A):-

$$\begin{aligned} \Phi_1 = {} & A_1^{(1)} e^{-i a_0 x} \cdot \cosh a_0(H+z) \\ & + \sum_{m=0}^{\infty} A_1^{(2)m} \cdot \cos \frac{m\pi}{d_1} y \cdot e^{+ik_x^{(1)m} x} \cdot \cosh a_0(H+z)\ , \end{aligned} \tag{8}$$

$$\Phi_0 = \sum_{f_0} A_0(f_0) \cdot \cos k_x^{(0)} x \cdot \cos k_y^{(0)} y \cdot \cosh a_0(H+z)\ , \tag{9}$$

Next, let us consider the velocity potential Φ_3 in the domain D_3. The separation of the variables of the velocity potential Φ_3 in the equation (1) and the conditions (2), (3), (5), (6) leads to

$$\frac{d^2 \Phi_x^{(3)}}{dx^2} = -(k_x^{(3)})^2 \Phi_x^{(3)}\ , \tag{10}$$

$$\frac{d^2 \Phi_y^{(3)}}{dy^2} = -(k_y^{(3)})^2 \Phi_y^{(3)}\ , \tag{11}$$

$$\frac{d^2\Phi_z^{(3)}}{dz^2}=a_0^2\Phi_z^{(3)} , \tag{12}$$

where

$$\Phi_3=\Phi_x^{(3)}(x)\cdot\Phi_y^{(3)}(y)\cdot\Phi_z^{(3)}(z)\cdot\exp(-i\omega t)$$

and

$$(k_x^{(3)})^2+(k_y^{(3)})^2=a_0^2 ; \tag{13}$$

$$-\omega^2\Phi_z^{(3)}+g\frac{d\Phi_z^{(3)}}{dz}=0 \quad (z=0) ; \tag{14}$$

$$\frac{d\Phi_z^{(3)}}{dz}=0 \quad (z=-H) ; \tag{15}$$

$$\frac{d\Phi_x^{(3)}}{dx}=0 \quad (x=0 \text{ and } d_3,\ L>y>d_1) ; \tag{16}$$

$$\frac{d\Phi_y^{(3)}}{dy}=0 \quad (y=L,\ d_3>x>0) . \tag{17}$$

From (10), (12), (14), (15) and (16), we have (after paper A)

$$\left.\begin{aligned}\Phi_z^{(3)}&\sim\cosh a_0(H+z) ,\\ \Phi_x^{(3)}&\sim\cos k_x^{(3)}x ,\end{aligned}\right\} \tag{18}$$

where

$$k_x^{(3)}=\frac{m\pi}{d_3} \quad (m=0,\ 1,\ 2,\ 3,\cdots) . \tag{19}$$

Now since we have another condition (17) for the velocity potential $\Phi_y^{(3)}$ which denotes that the canal is closed, the following solution, different from the progresive wave in the case of the open canal in paper A, is obtained for $\Phi_y^{(3)}$

$$\Phi_y^{(3)}\sim\cos k_y^{(3)m}(L-y) , \tag{20}$$

where from (13) and (19), $k_y^{(3)m}$ $(m=0,\ 1,\ 2,\ 3,\cdots)$ are given by

$$k_y^{(3)m}=\sqrt{a_0^2-\left(\frac{m\pi}{d_3}\right)^2} . \tag{21}$$

Finally we obtain from (18) and (20)

$$\Phi_3=\sum_{m=0}^{\infty}A_3^m\cos\frac{m\pi}{d_3}x\cdot\cos\{k_y^{(3)m}(L-y)\}\cdot\cosh a_0(H+z) , \tag{22}$$

where A_3^m ($m=0, 1, 2, 3, \cdots$) are arbitrary constants.

In order to determine the arbitrary constants $A_1^{(2)m}$ ($m=0, 1, 2, 3, \cdots$), A_3^m ($m=0, 1, 2, 3, \cdots$) and $A_0(f_0)$, we have the next conditions;

$$\left.\begin{array}{ll} \Phi_1=\Phi_0 & \text{(continuity of pressure)}, \\ \left.\begin{array}{l} \dfrac{\partial \Phi_1}{\partial x}=\dfrac{\partial \Phi_0}{\partial x} \\ \dfrac{\partial \Phi_1}{\partial y}=\dfrac{\partial \Phi_0}{\partial y} \end{array}\right\} & \left(\begin{array}{l}\text{continuity of velocity} \\ \text{of water particles}\end{array}\right), \end{array}\right\} \tag{23}$$

at $x=d_3$,

$$\left.\begin{array}{ll} \Phi_3=\Phi_0 & \text{(continuity of pressure)}, \\ \left.\begin{array}{l} \dfrac{\partial \Phi_3}{\partial x}=\dfrac{\partial \Phi_0}{\partial x} \\ \dfrac{\partial \Phi_3}{\partial y}=\dfrac{\partial \Phi_0}{\partial y} \end{array}\right\} & \left(\begin{array}{l}\text{continuity of velocity} \\ \text{of water particles}\end{array}\right), \end{array}\right\} \tag{24}$$

at $y=d_1$.

In the same manner as in paper A, putting (8) and (9) into (23), applying the operators

$$\int_0^{d_1} dy \cos \frac{m\pi}{d_1} y \quad (m=0, 1, 2, 3, \cdots),$$

$$\int_0^{d_1} dy \sin \frac{m\pi}{d_1} y \quad (m=1, 2, 3, \cdots),$$

to them and then making use of the long wave approximation such as

$$\cos k_x^{(0)} d_3 \simeq 1, \quad \sin k_y^{(0)} d_1 \simeq k_y^{(0)} d_1, \quad \sin k_x^{(0)} d_3 \simeq k_x^{(0)} d_3, \quad |k_y^{(0)}| d_1 \ll m\pi$$
$$(m=1, 2, 3, \cdots)$$

or

$$(k_y^{(0)} d_1)^2 - (m\pi)^2 \simeq -(m\pi)^2,$$

we get (refer to paper A):

$$A_1^{(1)} e^{-ia_0 d_3} + A_1^{(2)0} \cdot e^{+ia_0 d_3} = \sum_{f_0} A_0(f_0), \tag{25}$$

$$\frac{1}{2} A_1^{(2)m} \cdot e^{+ik_x^{(1)m} \cdot d_3} = \sum_{f_0} A_0(f_0) \left\{ -\frac{\cos m\pi}{(m\pi)^2} \right\} (k_y^{(0)} d_1)^2 \quad (m=1, 2, 3, \cdots), \tag{26}$$

$$(-ia_0)A_1^{(1)}e^{-ia_0d_3}\cdot d_1+ia_0A_1^{(2)0}\cdot d_1\cdot e^{+ia_0d_3}=\sum_{f_0}A_0(f_0)(-1)(k_x^{(0)})^2d_1d_3\ , \tag{27}$$

$$\frac{1}{2}\cdot ik_x^{(1)m}A_1^{(2)m}\cdot d_1\cdot e^{+ik_x^{(1)m}\cdot d_3}$$
$$=\sum_{f_0}A_0(f_0)(-1)(k_x^{(0)})^2\cdot d_3\cdot\left\{-\frac{\cos m\pi}{(m\pi)^2}\right\}\cdot(k_y^{(0)}d_1)^2\cdot d_1 \tag{28}$$
$$(m=1,\ 2,\ 3,\cdots)\ ;$$

And also putting (9) and (22) into (24) and applying the operators

$$\int_0^{d_3}dx\cos\frac{m\pi}{d_3}x\quad(m=0,\ 1,\ 2,\ 3,\cdots)\ ,$$
$$\int_0^{d_3}dx\sin\frac{m\pi}{d_3}x\quad(m=1,\ 2,\ 3,\cdots)\ ,$$

to them, we have:

$$\left.\begin{aligned}
&A_3^0\cdot d_3\cdot\cos\{a_0(L-d_1)\}\\
&\qquad=\sum_{f_0}A_0(f_0)\cdot\cos k_y^{(0)}d_1\cdot\frac{1}{k_x^{(0)}}\cdot\sin k_x^{(0)}d_3\ ,\\
&A_3^m\cdot\frac{d_3}{2}\cdot\cos\{k_y^{(3)m}(L-d_1)\}\\
&\qquad=\sum_{f_0}A_0(f_0)\cdot\cos k_y^{(0)}d_1\cdot\frac{(k_x^{(0)}d_3)d_3}{(k_x^{(0)}d_3)^2-(m\pi)^2}\cdot\sin k_x^{(0)}d_3\cdot\cos m\pi\\
&\qquad\qquad(m=1,\ 2,\ 3,\cdots)\ ,\\
&A_3^0\cdot d_3\cdot a_0\cdot\sin\{a_0(L-d_1)\}\\
&\qquad=\sum_{f_0}A_0(f_0)\cdot k_y^{(0)}\cdot(-1)\cdot\sin k_y^{(0)}d_1\cdot\frac{1}{k_x^{(0)}}\cdot\sin k_x^{(0)}d_3\ ,\\
&A_3^m\cdot\frac{d_3}{2}\cdot k_y^{(3)m}\cdot\sin\{k_y^{(3)m}(L-d_1)\}\\
&\qquad=\sum_{f_0}A_0(f_0)\cdot k_y^{(0)}\cdot(-1)\cdot\sin k_y^{(0)}d_1\cdot\frac{(k_x^{(0)}d_3)d_3}{(k_x^{(0)}d_3)^2-(m\pi)^2}\cdot\cos m\pi\\
&\qquad\cdot\sin k_x^{(0)}d_3\quad(m=1,\ 2,\ 3,\cdots)\ .
\end{aligned}\right\} \tag{29}$$

By virtue of the long wave approximation, (29) becomes

$$A_3^0\cdot\cos\{a_0(L-d_1)\}=\sum_{f_0}A_0(f_0)\ , \tag{30}$$

$$\frac{1}{2}A_3^m\cdot\cos\{k_y^{(3)m}(L-d_1)\}=\sum_{f_0}A_0(f_0)\left\{-\frac{\cos m\pi}{(m\pi)^2}\right\}\cdot(k_x^{(0)}d_3)^2 \tag{31}$$

$$(m=1,\ 2,\ 3,\cdots)\ ,$$

$$A_3^0\cdot a_0d_3\cdot\sin\{a_0(L-d_1)\}=\sum_{f_0}A_0(f_0)(-1)(k_y^{(0)})^2\cdot d_1d_3\ , \tag{32}$$

$$A_3^m\cdot\frac{d_3}{2}\cdot k_y^{(3)m}\cdot\sin\{k_y^{(3)m}(L-d_1)\}$$

$$=\sum_{f_0}A_0(f_0)(-1)(k_y^{(0)})^2\cdot d_1\cdot\left\{-\frac{\cos m\pi}{(m\pi)^2}\right\}(k_x^{(0)}d_3)^2\cdot d_3 \tag{33}$$

$$(m=1,\ 2,\ 3,\cdots)\ .$$

Now from (25) and (30), we have

$$A_1^{(1)}e^{-ia_0d_3}+A_1^{(2)0}e^{+ia_0d_3}=A_3^0\cdot\cos\{a_0(L-d_1)\}\ . \tag{34}$$

The combination of (27) and (32) yields

$$(-ia_0)A_1^{(1)}e^{-ia_0d_3}\cdot d_1+ia_0A_1^{(2)0}\cdot d_1\cdot e^{+ia_0d_3}+A_3^0\cdot a_0d_3\cdot\sin\{a_0(L-d_1)\}$$
$$=\sum_{f_0}A_0(f_0)(-1)a_0^2d_1d_3\ ,$$

where $(k_x^{(0)})^2+(k_y^{(0)})^2=a_0^2$ is used.

Substituting (30) into the above equation, we get

$$(-ia_0d_1)A_1^{(1)}e^{-ia_0d_3}+(ia_0d_1)\cdot A_1^{(2)0}\cdot e^{+ia_0d_3}$$
$$=A_3^0[\cos\{a_0(L-d_1)\}\cdot(-1)\cdot a_0^2d_1d_3-a_0d_3\sin\{a_0(L-d_1)\}]\ . \tag{35}$$

Now solving the equations (34) and (35), we obtain

$$\left.\begin{aligned}A_3^0&=\frac{2e^{-ia_0d_3}}{\cos a_0l-i\left(\cos a_0l\cdot a_0d_3+\dfrac{d_3}{d_1}\cdot\sin a_0l\right)}A_1^{(1)}\ ,\\ A_1^{(2)0}&=\frac{\cos a_0l+i\left(\cos a_0l\cdot a_0d_3+\dfrac{d_3}{d_1}\cdot\sin a_0l\right)}{\cos a_0l-i\left(\cos a_0l\cdot a_0d_3+\dfrac{d_3}{d_1}\cdot\sin a_0l\right)}\cdot e^{-i\cdot 2a_0d_3}\cdot A_1^{(1)}\ ,\end{aligned}\right\} \tag{36}$$

where $l=L-d_1$.

And also we have from $\{(26)/d_1^2\}+\{(31)/d_3^2\}$

$$\frac{1}{2}A_1^{(2)m}\cdot e^{+ik_x^{(1)m}\cdot d_3}\cdot\frac{1}{d_1^2}+\frac{1}{2}A_3^m\cos\{k_y^{(3)m}(L-d_1)\}\cdot\frac{1}{d_3^2}=\sum_{f_0}A_0(f_0)\cdot\left\{-\frac{\cos m\pi}{(m\pi)^2}\right\}\cdot a_0^2,$$

where $(k_x^{(0)})^2+(k_y^{(0)})^2=a_0^2$ is used.

Substituting (30) into the above equation, we get

$$\frac{1}{2}A_1^{(2)m}\cdot e^{+ik_x^{(1)m}\cdot d_3}\cdot\frac{1}{d_1^2}+\frac{1}{2}\cdot A_3^m\cos\{k_y^{(3)m}(L-d_1)\}\cdot\frac{1}{d_3^2}$$

$$=A_3^0\cos\{a_0(L-d_1)\}\cdot\left\{-\frac{\cos m\pi}{(m\pi)^2}\right\}\cdot a_0^2\,. \tag{37}$$

And the reduction $\{(28)/d_1^2\}=\{(33)/d_3^2\}$ leads to

$$\frac{1}{d_3}A_3^m\cdot k_y^{(3)m}\cdot\sin\{k_y^{(3)m}(L-d_1)\}=\frac{1}{d_1}\cdot ik_x^{(1)m}A_1^{(2)m}\cdot e^{+ik_x^{(2)m}\cdot d_3}\,. \tag{38}$$

Now solving the equations (36), (37) and (38), we obtain

$$\left.\begin{aligned}
A_3^m&=\frac{4d_3\cos a_0l\cdot\left\{-\dfrac{\cos m\pi}{(m\pi)^2}\right\}\cdot a_0^2\cdot e^{-ia_0d_3}A_1^{(1)}}{\left[\dfrac{1}{d_3}\cdot\cos k_y^{(3)m}l+\dfrac{1}{d_1}\cdot\dfrac{k_y^{(3)m}}{ik_x^{(1)m}}\sin k_y^{(3)m}l\right]\cdot\left[\cos a_0l-i\left(\cos a_0l\cdot a_0d_3+\dfrac{d_3}{d_1}\sin a_0l\right)\right]}\,,\\
A_1^{(2)m}&=\frac{d_1k_y^{(3)m}\cdot\sin k_y^{(3)m}l}{d_3\cdot ik_x^{(1)m}\cdot e^{+ik_x^{(1)m}\cdot d_3}}\cdot A_3^m\,,\\
&(m=1,\ 2,\ 3,\cdots)\,,
\end{aligned}\right\} \tag{39}$$

where $l=L-d_1$.

By virtue of the long wave approximation such as

$$a_0d_j\ll 1\quad (i=1,\ 3)$$

or

$$\left.\begin{aligned}
k_x^{(1)m}&=\sqrt{a_0^2-\left(\frac{m\pi}{d_1}\right)^2}\simeq i\cdot\frac{m\pi}{d_1}\\
k_y^{(3)m}&=\sqrt{a_0^2-\left(\frac{m\pi}{d_3}\right)^2}\simeq i\cdot\frac{m\pi}{d_3}
\end{aligned}\right\}\quad (m=1,\ 2,\ 3,\cdots)\,, \tag{40}$$

the following reductions are valid,

$$\frac{k_y^{(3)m}}{k_x^{(1)m}}\simeq\frac{d_1}{d_3}\,,$$

$$\cos k_y^{(3)}l\simeq\cosh\frac{m\pi}{d_3}l\,,$$

$$\frac{1}{i}\sin k_y^{(3)m} l \simeq \sinh\frac{m\pi}{d_3} l\ .$$

By use of the above reductions, the expressions (39) become

$$\left.\begin{aligned} A_3^m &= \frac{4a_0^2 d_3^2 \cos a_0 l \cdot \left\{-\dfrac{\cos m\pi}{(m\pi)^2}\right\}\cdot e^{-ia_0 d_3} A_1^{(1)}}{\left[\cosh\dfrac{m\pi}{d_3} l + \sinh\dfrac{m\pi}{d_3} l\right]\cdot\left[\cos a_0 l - i\left(a_0 d_3\cdot\cos a_0 l + \dfrac{d_3}{d_1}\sin a_0 l\right)\right]} \\ A_1^{(2)m} &= \left(\frac{d_1}{d_3}\right)^2 \sinh\frac{m\pi}{d_3} l\cdot e^{(m\pi/d_1)\cdot d_3}\cdot A_3^m\ , \end{aligned}\right\} \quad (40)$$

where $l=L-d_1$.

Thus we have determined the arbitrary constants A_3^0, $A_1^{(2)0}$, A_3^m and $A_1^{(2)m}$ ($m=1, 2, 3,\cdots$) within the scope of the long wave approximation, which are given by (36) and (40).

By virtue of (40), the expressions of the velocity potentials Φ_1 and Φ_3 or (8) and (22) become as follows;

$$\begin{aligned} \Phi_1 = {} & A_1^{(1)} e^{-ia_0 x}\cdot\cosh a_0(H+z) + A_1^{(2)0}\cdot e^{+ia_0 x}\cdot\cosh a_0(H+z) \\ & + \sum_{m=1}^{\infty} A_1^{(2)m}\cdot\cos\frac{m\pi}{d_1} y\cdot e^{-(m\pi/d_1)\cdot x}\cdot\cosh a_0(H+z)\ , \end{aligned} \quad (41)$$

$$\begin{aligned} \Phi_3 = {} & A_3^0 \cos\{a_0(L-y)\}\cdot\cosh a_0(H+z) \\ & + \sum_{m=1}^{\infty} A_3^m \cos\frac{m\pi}{d_3} x\cdot\cosh\left\{\frac{m\pi}{d_3}(L-y)\right\}\cdot\cosh a_0(H+z)\ . \end{aligned} \quad (42)$$

From the expressions (41) and (42) we know that the reflected wave in the domain D_1 and the standing wave in the domain D_3 are expressed by $A_1^{(2)0}\cdot e^{+ia_0 x}\cdot\cosh a_0(H+z)$ and $A_3^0\cos\{a_0(L-y)\}\cdot\cosh a_0(H+z)$ respectively. Accordingly, the ratio of the amplitudes of the reflected and the surging wave in the domain D_1 is denoted by $|A_1^{(2)0}/A_1^{(1)}|$, and the ratio of the amplitudes of the standing wave in the domain D_3 and the surging wave by $|A_3^0\cos\{a_0(L-y)\}/A_1^{(1)}|$.

Thus we have from (36)

$$\left.\begin{aligned} & |A_1^{(2)0}/A_1^{(1)}| = 1 \\ \text{and}\quad & |A_3^0\cos\{a_0(L-y)\}/A_1^{(1)}| = \frac{2}{\sqrt{P}}|\cos\{a_0(L-y)\}|\ , \end{aligned}\right\} \quad (43)$$

where
$$P = \cos^2 a_0 l + \left(a_0 d_3\cdot\cos a_0 l + \frac{d_3}{d_1}\cdot\sin a_0 l\right)^2 . \quad (44)$$

Now the first expression of (43) means that the amplitude of the reflected wave in the domain D_1 is equal to that of the incoming wave.

The second expression of (43) means that the amplitude of the standing wave in the domain D_3 depends on the widths of the canals (or d_1, d_3), the wave number of the incoming wave (or a_0), the length of the crooked bay (or l), and the position (y) in the domain D_3.

At the head of the bay, the ratio of the standing wave in the domain D_3 and the incoming wave in the domain D_1 becomes, by substitution of $y=L$ into the second expression of (43),

$$|A_3^0/A_1^{(1)}|=\frac{2}{\sqrt{P}}\,, \tag{45}$$

where P is given by (44).

Now let us consider some particular cases.

i) when $d_3 \ll d_1$ (Fig. 2).

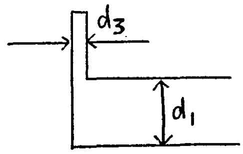

Fig. 2. The case of $d_3 \ll d_1$.

Suppose that a_0, d_1, d_3 are parameters and l or $a_0 l$ a variable, we have, on differentiation of P with respect to $a_0 l$,

$$\left.\begin{aligned}\frac{dP}{d(a_0 l)}&=\left\{-1+\left(\frac{d_3}{d_1}\right)^2\right\}\cdot\sin 2(a_0 l)+2a_0 d_3\left(\frac{d_3}{d_1}\right)\cdot\cos 2(a_0 l)\,,\\ \frac{d^2P}{d(a_0 l)^2}&=2\left[-\cos 2(a_0 l)\cdot\left\{1-\left(\frac{d_3}{d_1}\right)^2\right\}-2a_0 d_3\cdot\left(\frac{d_3}{d_1}\right)\cdot\sin 2(a_0 l)\right].\end{aligned}\right\} \tag{46}$$

Being taken account of $d_3 \ll d_1$, the above derivatives are reduced to

$$\left.\begin{aligned}\frac{dP}{d(a_0 l)}&\simeq-\sin 2(a_0 l)+2a_0 d_3\left(\frac{d_3}{d_1}\right)\cdot\cos 2(a_0 l)\,,\\ \frac{d^2P}{d(a_0 l)^2}&\simeq 2\left[-\cos 2(a_0 l)-2a_0 d_3\left(\frac{d_3}{d_1}\right)\cdot\sin 2(a_0 l)\right].\end{aligned}\right\} \tag{47}$$

Putting the first equation of (47) to zero, we have

$$\tan 2(a_0 l)=2a_0 d_3\cdot\left(\frac{d_3}{d_1}\right).$$

Since $a_0 d_3 \ll 1$ and $d_3 \ll d_1$, the above equation is nearly equal to zero, i.e., $\tan 2(a_0 l)\simeq 0$.

Then we have

$$\left.\begin{aligned}&2a_0 l\simeq m\pi \quad (m=0,\ 1,\ 2,\ 3,\cdots)\\ \text{or}\quad &\frac{l}{\lambda}=\frac{m}{4}\quad \text{from the above equation,}\end{aligned}\right\} \tag{48}$$

where λ is the wave length of the surging wave.

Substituting (48) into the second equation of (47), we have:
when $m=2n+1$ $(n=0,\ 1,\ 2,\cdots)$,

$$\frac{d^2P}{d(a_0l)^2}=+2>0\,;$$

when $m=2n$ $(n=0,\ 1,\ 2,\cdots)$,

$$\frac{d^2P}{d(a_0l)^2}=-2<0\,;$$

that is to say,
when $m=2n+1$ $(n=0,\ 1,\ 2,\cdots)$,

$a_0l\simeq\frac{m}{2}\pi$ gives for P the minimum value or $P=\left(\frac{d_3}{d_1}\right)^2$;

when $m=2n$ $(n=0,\ 1,\ 2,\cdots)$,

$a_0l\simeq\frac{m}{2}\pi$ gives for P the maximum value or $P=1$.

Now the ratio of the amplitues of the standing wave at the head of the bay and the surging wave in the straight canal, or (45), has the maximum value

$$|A_3^0/A_1^{(1)}|=2\cdot\frac{d_1}{d_3}\,,\ \text{when}\ l=\frac{2n+1}{4}\cdot\lambda\ (n=0,\ 1,\ 2,\cdots)$$

and the minimum value

$$|A_3^0/A_1^{(1)}|=2\,,\ \text{when}\ l=\frac{n}{2}\cdot\lambda\ (n=0,\ 1,\ 2,\cdots)\,. \tag{49}$$

Now we may conclude that

(1) the wave height at the head of an inlet in a large bay is always higher than that in the large bay itself or $|A_3^0|\geq 2|A_1^{(1)}|$, and the ratio between the former and the latter never exceeds the inverse ratio between the widths of the inlet and the large bay or $|A_3^0/2A_1^{(1)}|\leq d_1/d_3$;

(2) when the length of an inlet (l) is odd number times a quarter of the surging wave length (λ), the wave height at the head of the inlet takes the maximum value or $|A_3^0|=2\cdot d_1/d_3\cdot A_1^{(1)}$ as shown in (49);

(3) when the length of an inlet (l) is even number times a quarter of the surging wave length (λ), the wave height at the head of the

inlet takes the minimum value or $|A_3^0|=2|A_1^{(1)}|$ as shown in (49).

(4) when the length of an inlet (l) is very small compared with the surging wave length (λ), that is to say, $l/\lambda \simeq 0$, the wave height at the head of the inlet is almost equal to that in the large bay, as can be easily seen in the latter equation of (49).

ii) when $d_3=d_1$ (Fig. 3).

Substituting $d_3=d_1=d$ into (44), we get

$$P=1\ ,$$

where $|a_0 d \sin 2a_0 l| \leq a_0 d \ll 1$ is used.

Hence (45) becomes

$$|A_3^0|=2|A_1^{(1)}|\ . \tag{50}$$

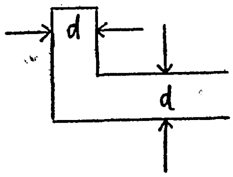

Fig. 3. The case of $d_3=d_1=d$.

In this case the amplitude of the standing wave in the domain D_3 is twice the amplitude of the incoming wave in the domain D_1.

iii) when $d_3 \gg d_1$ (Fig. 4).

Taking account of $(d_3/d_1) \gg 1$, we have from (46)

$$\left.\begin{aligned} \frac{dP}{d(a_0 l)} &\simeq \left(\frac{d_3}{d_1}\right)^2 \sin 2a_0 l + 2a_0 d_3 \cdot \left(\frac{d_3}{d_1}\right) \cos 2a_0 l\ , \\ \frac{d^2P}{d(a_0 l)^2} &\simeq \left(\frac{d_3}{d_1}\right)^2 \cdot 2 \cdot \cos 2a_0 l - 4a_0 d_3 \cdot \left(\frac{d_3}{d_1}\right) \cdot \sin 2a_0 l\ . \end{aligned}\right\} \tag{51}$$

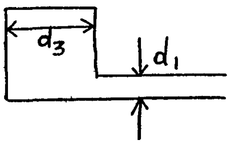

Fig. 4. The case of $d_3 \gg d_1$.

Putting the first equation of (51) to zero we have

$$\tan 2a_0 l + 2a_0 d_3 \cdot \left(\frac{d_1}{d_3}\right) = 0\ .$$

By virtue of the conditions $a_0 d_3 \ll 1$ and $(d_1/d_3) \ll 1$, the above equation is reduced to

$$\tan 2a_0 l \simeq 0\ .$$

Hence from the above expression, we have

$$2a_0 l = m\pi \quad (m=0,\ 1,\ 2,\ 3, \cdots) \text{ or } l/\lambda = \frac{m}{4}\ . \tag{52}$$

In the same manner as in section i) we find, by virtue of the second equation of (51), that

$$\left.\begin{array}{l} \text{when } m=2n+1 \ (n=0,\ 1,\ 2,\cdots) \text{ in (52) or } l=\dfrac{2n+1}{4}\lambda\,, \\ \qquad |A_3^0/2A_1^{(1)}|=d_1/d_3 \text{ as the minimum value;} \\ \text{when } m=2n \ (n=0,\ 1,\ 2,\cdots) \text{ in (52) or } l=\dfrac{n}{2}\lambda\,, \\ \qquad |A_3^0/2A_0^{(1)}|=1 \text{ as the maximum value.} \end{array}\right\} \tag{53}$$

(1) Now we may conclude that the wave height at the head of a large bay leading to a narrow canal is always lower than that in the narrow canal itself or $|A_3^0|\leq 2|A_1^{(1)}|$, and the ratio of the former and the latter never excceds the lower limit of the ratio of the widths of the narrow canal and the large bay or $|A_3^0/2A_0^{(1)}|\geq d_1/d_3$;

(2) when the length of a bay (l) is odd number times a quarter of the incoming wave length (λ), the wave height at the head of the bay has the minimum value or $|A_3^0|=2\cdot d_1/d_3\cdot|A_1^{(1)}|$ as shown in (53);

(3) when the length of a bay (l) is even number times a qutarer of the surging wave length (λ), the wave height at the head of the bay takes the maximum value or $|A_3^0|=2|A_1^{(1)}|$ as shown in (53).

3. Verification of General Theory by the Chilean Tsunami of 1960.

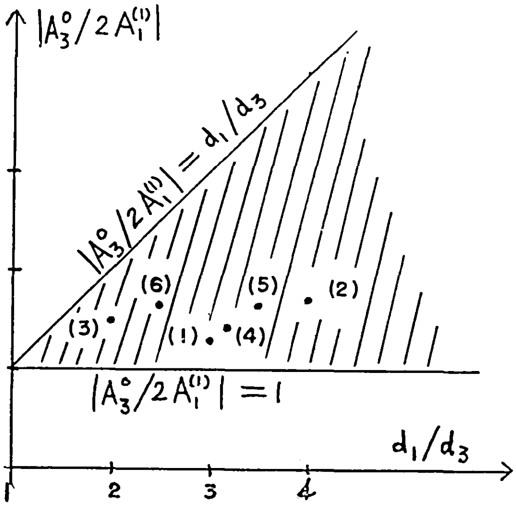

Fig. 5. The ratios of $|A_3{}^0/2A_1{}^{(1)}|$ obtained from the data of the Chilean Tsunami, where the inserted numbers correspond to the figures with the same number in Fig. 6.

In the following discussions, we assume that the depth of water is uniform, and hence the effects of the depth of water on the wave height of tsunami are disregarded.

Now by use of the data on the Chilean Tsunami we check the validity of the theory derived in the preceding section.

i) Examples of $d_3 \ll d_1$.

We assume that $|A_3^0|$ and $|2A_1^{(1)}|$ are given by the wave heights observed at the head of a small inlet and

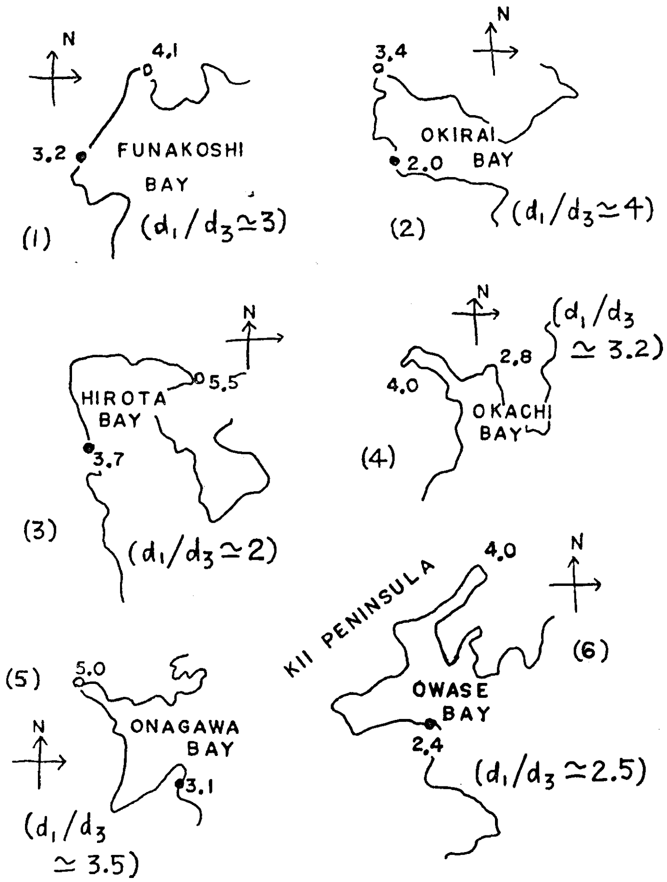

Fig. 6. Examples of the bays in the case $d_3 \ll d_1$.
○ Observation point of $|A_3^0|$,
● Observation point of $2|A_1^{(1)}|$,
Inserted values denote the observed wave heights (units: m).

near the mouth of a large bay respectively. Then we plotted the ratio $|A_3^0/2A_1^{(1)}|$ obtained from the data of the Chilean Tsunami in Fig. 5. Now Fig. 5 shows that the ratios $|A_3^0/2A_1^{(1)}|$ obtained from the actual data fall in the range $d_1/d_3 \geq |A_3^0/2A_1^{(1)}| \geq 1$, that is to say, Fig. 5 verifies the validity of the theory derived in the preceding section. The inserted numbers in Fig. 5 correspond to the figures with the same numbers in Fig. 6.

ii) Example of $d_3 = d_1$.

In this case we have only one example, viz., the Bay of Ise (Fig. 7),

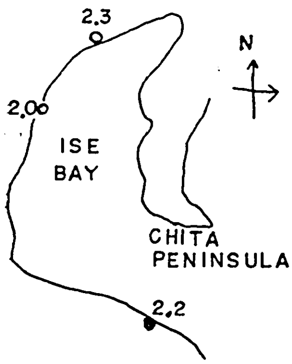

Fig. 7.

throughout which the wave heights are almost uniform.

iii) Examples of $d_3 \gg d_1$.

In this case we cannot find any suitable example from the data of the Chilean Tsunami.

4. Acknowledgment.

The author wishes to offer his thanks to Professor R. Takahasi and Assistant Professor K. Kajiura of this Institute and to Dr. K. Takano of the Geophysical Department of Tokyo University for their kind discussions.

---

40. 津波に対する海岸線の影響 (2) とチリ津波についての二，三の注意

地震研究所　桃　井　高　夫

著者は前報告において，直角に曲つた水路をとおる long wave について議論した．本論説において著者は，さらにこの canal の一端を閉じたときの水路，すなわち長い直角に曲つた湾について，そこに突入した津波をチリ津波の問題を加味して論じた．その結果は次のとおりである．Fig. 1 に示すごとく，湾の直線的な部分の巾を d_1，曲つた先の方の巾を d_3 とするとき，湾の一番奥の所での波高と直線的な水路のところでの波高の2倍との比 $|A_3^0|/2|A_1^{(1)}|$ は

i) $d_3 \ll d_1$ のとき，

$$d_1/d_3 \geq |A_3^0|/2|A_1^{(1)}| \geq 1\,,$$

ii) $d_3 = d_1$ のとき，

$$|A_3^0|/2|A_1^{(1)}| = 1\,,$$

iii) $d_3 \gg d_1$ のとき，

$$1 \geq |A_3^0|/2|A_1^{(1)}| \geq d_1/d_3\,,$$

の範囲を出ない．