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45. *The Fracture of a Semi-infinite Body Caused by an Inner Stress Origin and its Relation to the Earthquake Phenomena*

(*First paper*).

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1. Introduction.

It is believed by many geophysicists that earthqukes may be caused by brittle fracturing of the stressed earth's crust or the upper mantle. This mechanism of earthquake occurrence seems to be probable in shallow earthquakes. Therefore, it may be helpful for a better understanding of earthquake occurrence to investigate the fracturing phenomena of various materials from this point of view.

In this paper fracturing of a semi-infinite body caused by an inner stress origin is experimentally investigated. According to a previous study¹⁾, some important characteristics in fracture occurrence are remarkably affected by the following factors: the brittleness and the mechanical structure of the material and the spatial distribution of applied stresses. Therefore, these factors are also considered in this experiment. First, brittle materials are used for the experiment, because elastic waves of shock type occur in brittle fracturing. Then, the experiment is carried out on two different materials, of a completely homogeneous structure and a highly heterogeneous one.

The stress is applied by an inner stress source situated in a semi-infinite body. The stress distribution in a semi-infinite body including an inner stress source seems to be one of the most fundamental cases relating to certain seismological problems. A spherical or an ellipsoidal source situated in a semi-infinite body has been treated frequently as a simplified model of earthquake origin related to the problems of the generation of earthquake waves and the ground deformation²⁾. In recent systematic

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1) K. MOGI, *Bull. Earthq. Res. Inst.*, **40** (1962), 125-173.

2) For examples,
K. SEZAWA, *Bull. Earthq. Res. Inst.*, **7** (1929), 1-14.
N. YAMAKAWA, *Zisin* (*Journn. Seis. Soc. Japan*), [ii], **8** (1955), 84-98.

discussions on earthquake occurrence, T. Matsuzawa[3] proposed an elliptical stress source situated in a semi-infinite body as a cause of earthquakes. However, as the fracture phenomena are very complex even under such simple stress states, experimental approaches seem to be necessary for the investigation of the problem under consideration. The writer believes that experimental investigations on the brittle fracturing of a semi-infinite body caused by an inner stress source from the seismological view-point have been very few until now.

In this paper the successive occurrence of brittle fractures under the above-mentioned stress states are investigated by measurement of the elastic waves accompanying the fracture. Various fracture patterns in the medium are also discussed. Then, the fracturing phenomena observed in the laboratory experiment seem to be analogous in several respects to observed results in earthquake phenomena. It is hoped that this laboratory experiment may give an approach to the understanding of the mechanism of earthquake occurrence.

2. Experimental procedure.

The schematic view of the laboratory set is shown in Fig. 1. Pine resin was used as a homogeneous brittle material. The melted pine resin is gradually solidified in the cylindrical metal basin of diameter 30 cm and depth 13 cm. A small metallic body which acts as a stress source is buried at some depth from the surface in the pine resin medium. Since a fracturing caused by the inner stress source occurs only in the central part of the medium and the wall of the metal basin has no effect on the fracture occurrence, the medium may be regarded as a semi-infinite body.

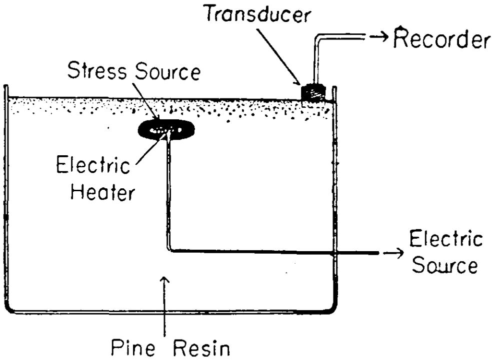

Fig. 1. Schematic view of experimental arrangement.

Stress was applied to the medium by thermal expansion of the

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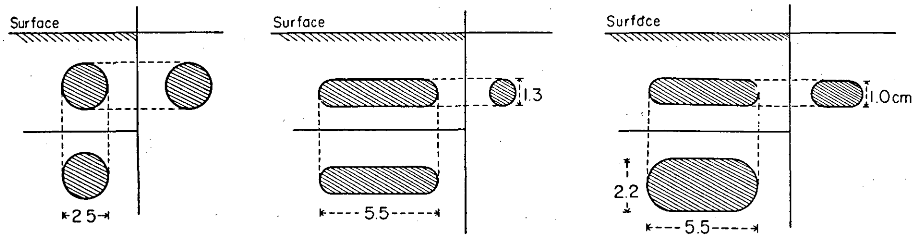
[3] T. MATSUZAWA, *Bull. Earthq. Res. Inst.*, **31** (1953), 179-201; **31** (1953), 249-253; **32** (1954), 341-347.
T. MATSUZAWA and H. HASEGAWA, *Bull. Earthq. Res. Inst.*, **32** (1954), 231-246.

metallic body which contains a small electric heat source inside. The shape of the stress source is spherical, cylindrical or ellipsoidal. Their dimensions are shown in Fig. 2. A stress source of an angular shape was also studied.



Sphere Cylinder Ellipsoid

Fig. 2. Dimensions of stress sources.

When the electric heater in the metallic body is switched on at the time $t=0$, the temperature of the metallic body increases gradually with time, that is,

$$T=T_0(1+\alpha t)\ , \tag{1}$$

where T_0 and α are constants. As the heat conductivity of the pine resin*) is very small as compared with that of the metal, the heat transmission from the metallic body to the surrounding pine resin medium is not appreciable during the period of measurement. The volume of the metallic body is approximately represented by

$$V=V_0(1+\beta t)\ , \tag{2}$$

where V_0 and β are constants. The stress around the source is then accumulated gradually until a remarkable fracture occurs to release the stress. If a boundary condition on the surface of the source is given, the stress distribution in the medium may be calculated on the basis of an elasticity theory. However, the boundary condition on the surface of the source in the present experiment is very complicated; the quantitative calculation of stress distribution was not undertaken.

The stress application by thermal expansion is favourable for highly sensitive measurement of the elastic waves of shock type (the elastic shocks) accompanying the brittle fractures of materials, as the method

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*) 1.6×10^{-4} cal/cm sec °C. This was measured by K. HÔRAI from the present pine resin specimen.

for stress application is completely noiseless. The successive occurrence of elastic shocks was observed by the same acoustic method as in the previous experiment[4]. The elastic shocks were picked up by accelerometers which were set on the surface of the medium. The signals were then recorded by a tape-recorder and were re-recorded by an electromagnetic oscillograph for measurements.

The deformation of the surface of the semi-infinite body caused by the inner stress source was measured by an electric resistance strain meter. The changes of strain in the stressed region were continuously recorded and were compared with the frequency curves of the elastic shocks.

As mentioned before, the heterogeneous structure of the material has an important influence on the occurrence of the fractures. In this experiment, pine resin mixed with fine pumice particles was used as a heterogeneous brittle material.

3. Experimental results.

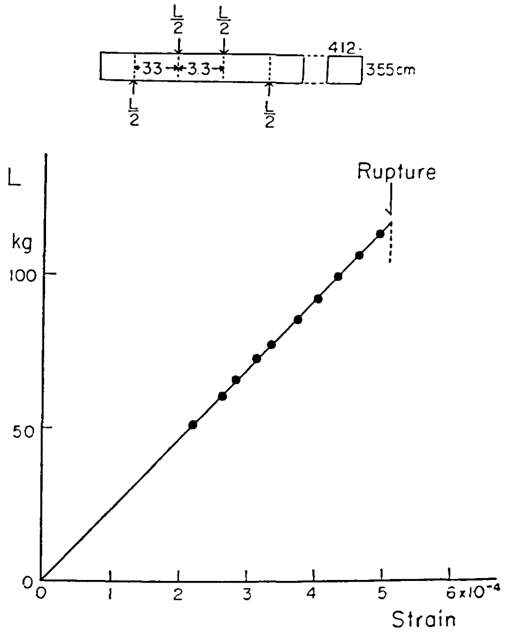

Fig. 3. Relation between the load and the strain of a pine resin specimen in a uniform bending test. (The strain is tension on the tensile side).

First, mechanical properties of the pine resin were investigated by bending it. The test pieces were made in a prism shape of the dimension 3.55 cm × 4.12 cm × 15 cm, and then a uniform bending moment which increased with a constant rate (0.47 kg/cm^2 sec) was applied to the central part of the specimen. As shown in Fig. 3, the stress-strain relation is quite linear to a sudden rupture but, preceding the rupture, local minor fractures do not occur. This fact shows that the pine resin is a homogeneous brittle material at room temperature and under the increasing stress with the rate (0.47 kg/cm^{2}sec). According

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[4] K. Mogi, *loc. cit.*, 1).

to the result, the flexural strength of the pine resin is 20.4 kg/cm^2 and its Young's modulus is 4.1×10^4 kg/cm^2.

When the volume of the inner source increased at a constant rate, the fracturing of the semi-infinite body was measured by the abovementioned methods. The results are described below.

(1) *The mode of the successive occurrence of fractures.*

When the volume of the inner source in the homogeneous brittle medium increases monotonically, the stress and the strain in the medium increase gradually and the deformation is nearly elastic until a main rupture takes place. Minor fractures do not occur before the main rupture. Finally the main rupture accompanied by a predominantly large elastic shock breaks out suddenly and the fracturing region appears in the central part around the inner source. A large number of elastic shocks accompanying local fractures which follow the main rupture are then observed. The frequency curves of the elastic shocks in this process are shown in Fig. 5. The changes of the strain at the surface of the central region are represented in Fig. 6, that is, strain increases monotonically to the main rupture and increases abruptly at the time of the rupture. Thereafter the deformation increases remarkably with the successive occurrence of local fractures. The relation among various quantities is schematically shown in Fig. 7. It is noteworthy that small elastic shocks do not occur before the main rupture in the homogeneous medium under the near uniform stress.

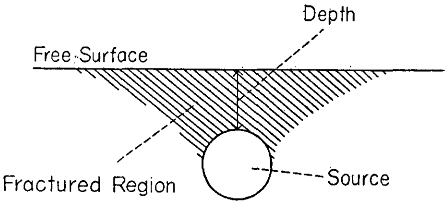

Fig. 4. Fractured region which appears in a homogeneous medium by main fracturing.

The depth of the inner source has a close relation to the generation of the remarkable fracturing region at the time of main rupture and also to the successive occurrence of elastic shocks after the main shock. As seen in Fig. 5, the number of elastic shocks increases with the depth of the stress source. When the stress source is deeper, the volume of the stressed region between the surface of the medium and the stress source is larger, and so the fracturing region is larger and the number of elastic shocks is also larger. However, when the depth of the source is too large as compared with the dimension of the source, a dense crack pattern does not appear at the time of main fracturing and only a few radial cracks occur. The number of small elastic shocks which follow the main shock is also small. This is explained by the fact that a highly stressed region between the surface of medium and the stress source does not appear in the case of deep source. The experimental result therefore suggests that the free surface of medium has an important influence on the generation of the dense crack pattern at the

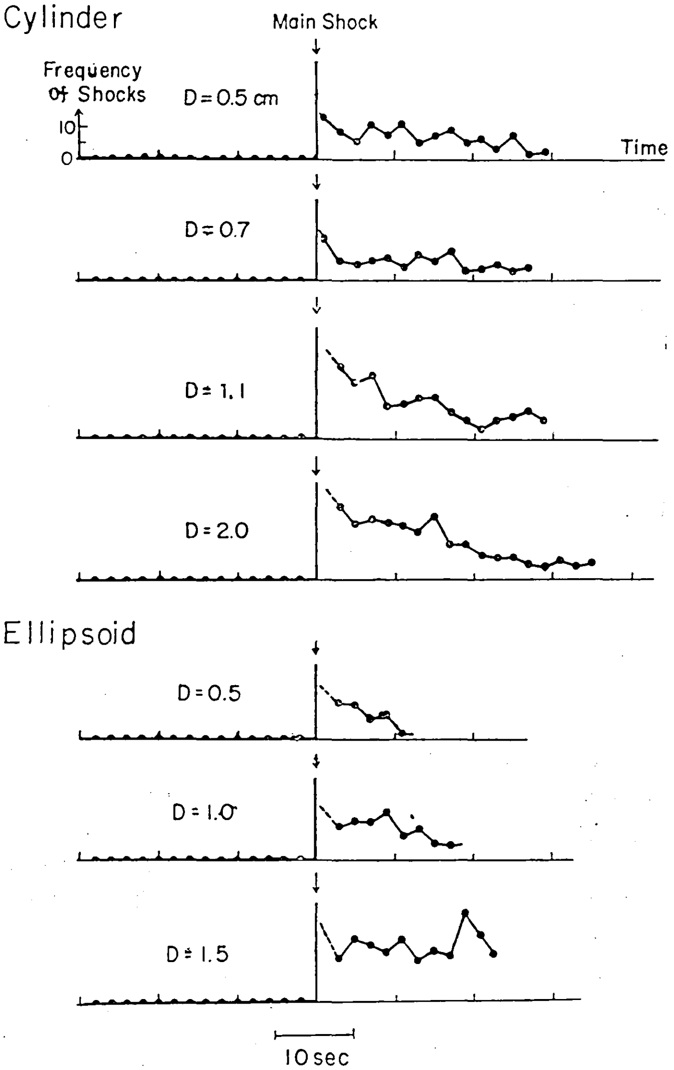

Fig. 5. Changes in frequency of elastic shocks accompanying fractures of a homogeneous material caused by an inner stress source of round shape. *D*: depth of source.

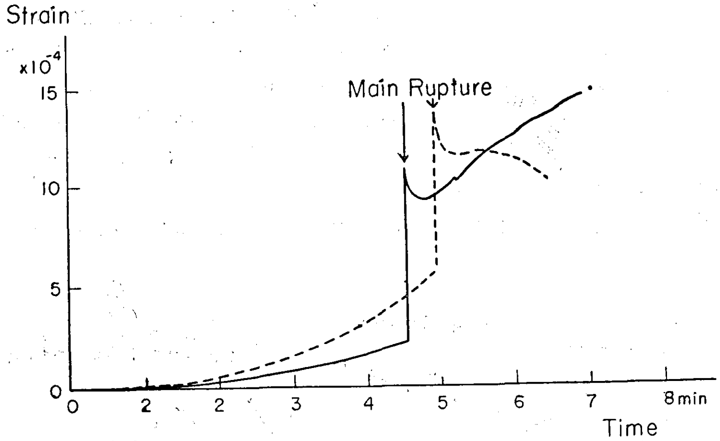

Fig. 6. Changes in the strain of the stressed region (the surface of medium just above the source).

time of the main fracturing and the remarkable occurrence of small elastic shocks after the main shock.

The above-mentioned mode of the shock occurrence is analogous to that of aftershocks in earthquakes.

On the other hand, the mode of the successive occurrence of the fracturing in heterogeneous materials is very different from that in the homogeneous one. Fig. 8 shows the frequency curves of the elastic shocks caused by the fracturing of the heterogeneous medium including an inner stress source. The changes in the elastic energy dispersed as elastic shocks are also shown. The fractures begin to occur at the initial stage of stress application. Their frequency increases gradually with time and decreases after some duration. Any single predominant principal shock is not observed, but a rather continuous series of elastic shocks. A similar mode of successive occurrence of elastic shocks also occurs, caused by

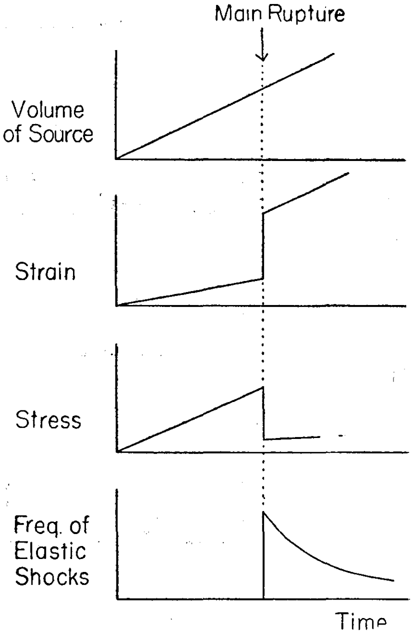

Fig. 7. Relation between the strain stress and shock occurrence in the case of a homogeneous medium.

the application of concentrated stress, for example, the frequency curves of the elastic shocks caused by an inner stress source of an angular shape in the homogeneous medium are similar. Here, it is noted that the stress around the source distributes concentratively in both cases. Therefore, it is deduced that this mode of shock occurrence is caused by concentrated stresses.

This shock occurrence is similar to that of earthquake swarms and

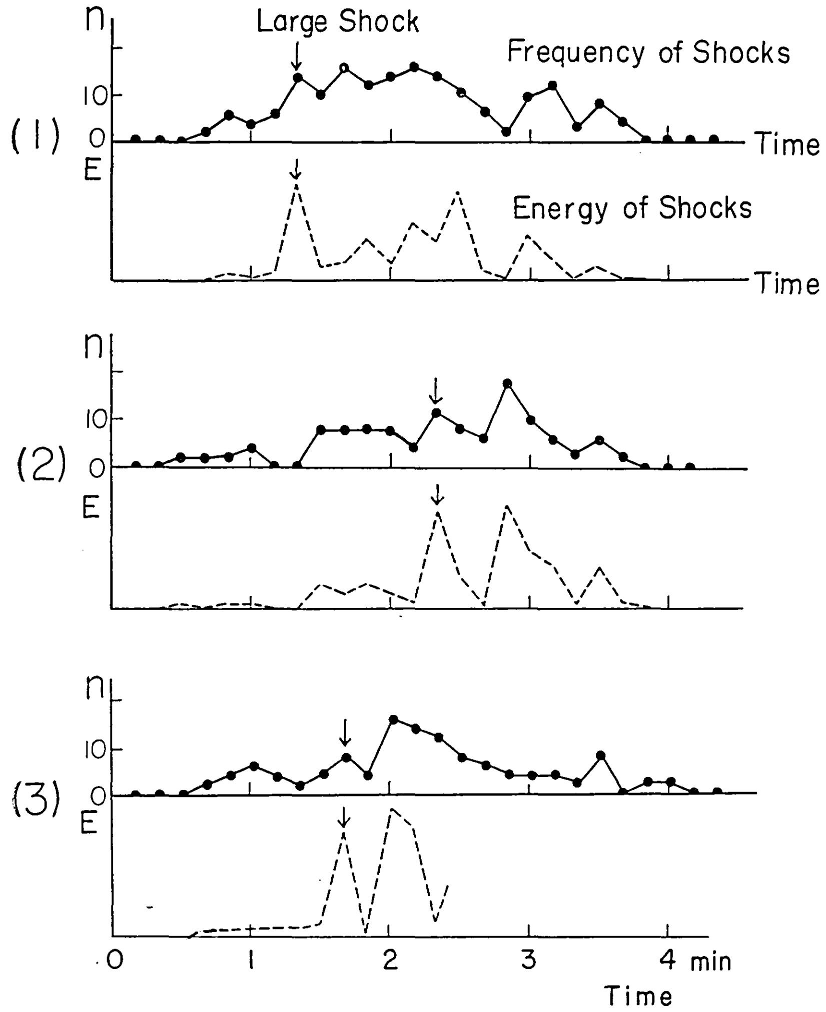

Fig. 8. Changes in the frequency of elastic shocks and their energy in a heterogeneous medium. The scale of energy is conventional.

of some volcanic earthquakes[5].

(2) *Crack patterns in the fracturing region.*

As mentioned above, at the time of the main fracture of the homogeneous medium a dense crack pattern appears in the region between the surface of medium and the inner stress source. Fig. 13 shows photographs of these crack patterns on the surface of the medium. In many cases, a large number of cracks start from a point or a line just above the inner stress source. The cracks grow radially to the outer region and stop at a finite length. At the same time, some tangential cracks cross the radial cracks. It is clear from the patterns that the former cracks occur after the latter (Fig. 9). The crack patterns distribute symmetrically around the center of the stressed region in many cases, but in some cases they are sometimes very irregular.

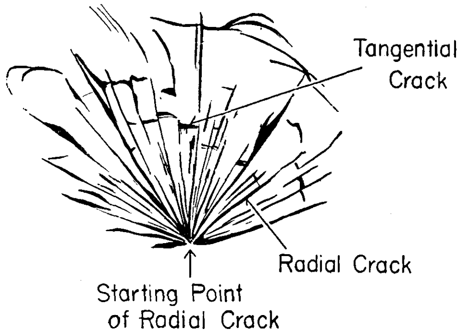

Fig. 9. Example of crack pattern.

Besides the dense crack patterns, a few large cracks develop also radially from the same origin and they arrive to the wall of the basin. They occur in the homogeneous medium only, and do not appear in the heterogeneous one.

The various types of crack patterns observed are shown in Figs. 10 and 11, and some interesting features are described below:

In the case of the spherical source, many cracks develop radially from a point just above the center of the source, and the crack pattern is always symmetrical around the point. Thus, the shape of the fracturing region is circular.

When the source is cylindrical or ellipsoidal in shape, the cracks develop radially from a point or a line just above the center of the stress source in many cases and the patterns are symmetrical, with the fracturing region elliptical in shape. Sometimes the starting point of the cracks are located off the center of the stressed region and the crack patterns are not symmetrical but irregular. In other cases, the dense crack pattern distributes only on one side of a large fault while the other side of the fault is free from fracturing (Fig. 10 (5), (6)).

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[5] K. Mogi, *Bull. Volcanologique*, (1962), (in press).

Thus, although the medium is nearly homogeneous and the stress distribution is symmetrical, the crack patterns are frequently not symmetrical and are very irregular. This fact seems to indicate that the fracture phenomenon is highly structure-sensitive, or the occurrence of fracture is remarkably affected by the micro-irregularities in the medium. Such irregular distributions of the crack pattern should be regarded as an inherent characteristic of fracture phenomena, as like the fluctuation

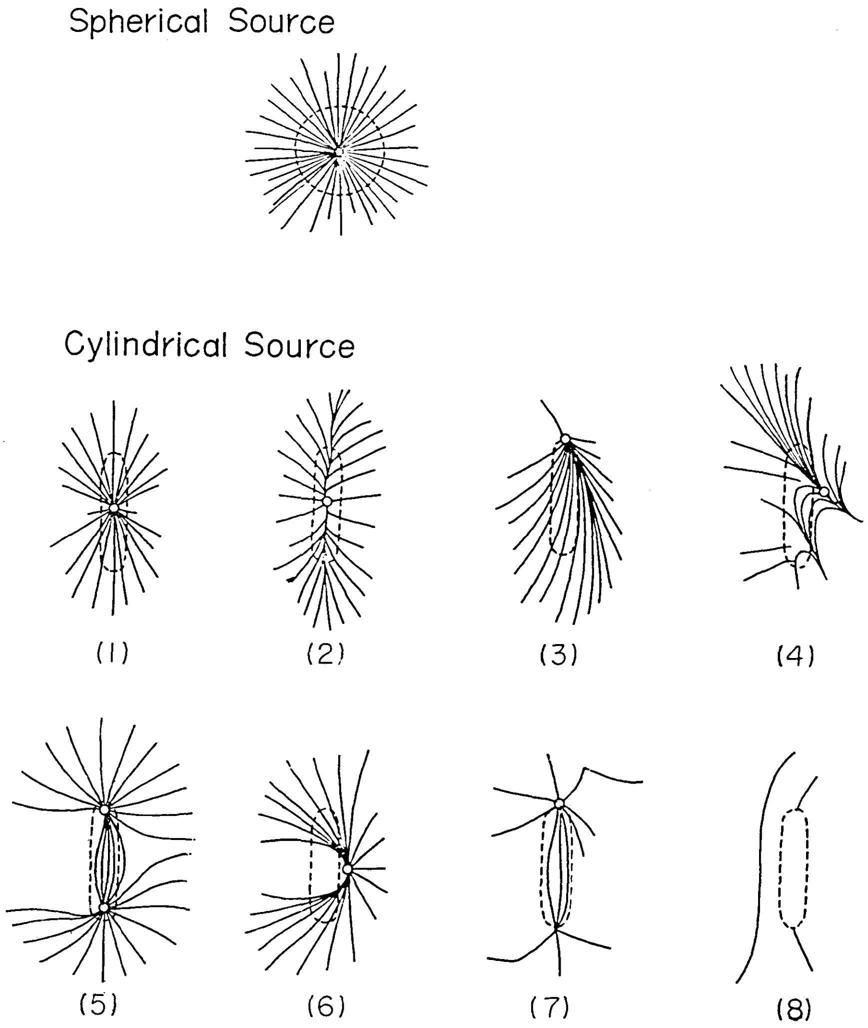

Fig. 10. Crack patterns on the surface of medium in the cases of a spherical source and a cylindrical one. (7), (8): deep source.

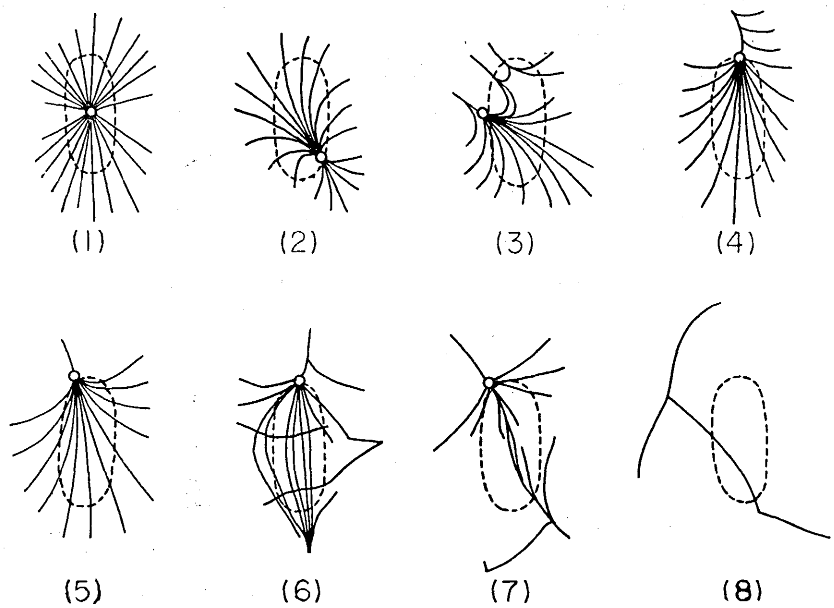

Fig. 11. Crack patterns on the surface of medium in the case of an ellipsoidal source. (7), (8): deep source.

of fracturing time⁽⁶⁾.

In the case of a deep source, the above-mentioned dense crack pattern is not remarkable, but a few large cracks develop only.

4. Discussion—Comparison of the results of experiment with actual earthquake phenomena.

The above-mentioned results of experiment are analogous to earthquake phenomena in several ways. In this section, the main results of the experiment are summarized and they are compared with the related problems in earthquakes.

(1) A main fracture (or a main shock) in a semi-infinite homogeneous brittle medium including an inner stress source is followed by many elastic shocks. The mechanism of occurrence of many elastic shocks following the main shock is deduced as follows: the stress around the inner source increases gradually and, after some duration, the stressed region breaks out suddenly. Then a large part of the accumulated

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6) K. Mogi, *loc. cit.*, 1).

strain energy is liberated, partly as an elastic energy accompanying the principal shock and partly as a fracturing energy expended for the crack formation. Since the fracturing takes place in the region between the surface of medium and the inner stress source resulting in the high stress concentrations in the region, many local fractures occur successively at these stress concentrated points following the main fracture.

This mode of shock occurrence is analogous to that of the aftershocks in earthquakes. Therefore, the above-mentioned mechanism may also be possible in earthquakes which are believed to be caused by fracturing of the earth's crust. This implies that the aftershock phenomena in earthquakes may be attributable to the fracturing characteristics of the earth's crust(7).

(2) When the depth of the stress source in a homogeneous medium is very large as compared with the dimension of the stress source, the remarkable fracturing region does not appear.

On the other hand, T. Matsuzawa(8) pointed out that the aftershocks in deep focus earthquakes are not outstanding in many cases. Furthermore, he discussed the fact that this is attributable to the effect of the free surface of the earth's crust. The present results of experiment seem to be analogous to the result in earthquakes.

(3) In the cases where the shape of stress source is cylindrical or ellipsoidal, the fracturing area on the surface of the medium is elliptical in shape and the starting point of the crack pattern is frequently located near the center of the fracturing region. However, this point is sometimes located at a corner of the region and, in other cases, the fracturing pattern distributes only on one side of a large crack. This fact seems to be analogous to the following phenomena(9) in earthquakes. That is, the aftershock region is frequently elliptical in shape and an epicenter of principal shock is located on the boundary of the aftershock region. Then, the aftershock region frequently distributes only on one side of an earthquake fault. These phenomena seem to be attributable to the structure-sensitive property of fracturing.

(4) If the medium is heterogeneous or the stress is applied concentratively, the mode of the shock occurrence is very different from that in the homogenous medium under uniformly applied stress. When the inner stress source increases gradually, the elastic shocks also increase gradually and, after some duration, they decrease. There is no

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7) K. MOGI, *Bull. Earthq. Res. Inst.*, **40** (1962), 105-120.

8) T. MATSUZAWA, *Bull. Earthq. Res. Inst.*, **32** (1954), 341-347.

9) T. MATSUZAWA, *Zisin-gaku (Seismology)*, (1950).

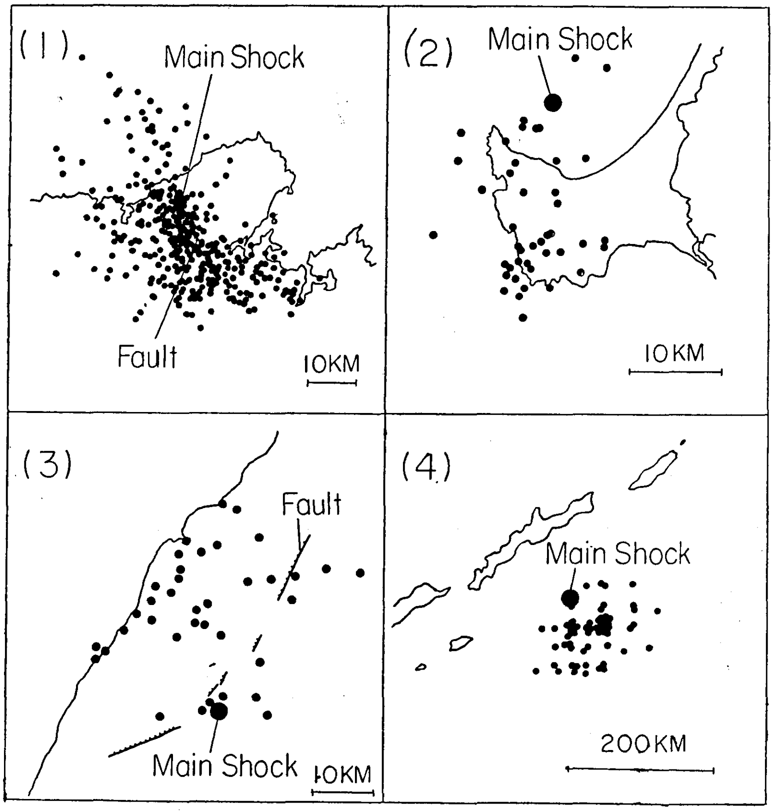

Fig. 12. Epicentral distributions of aftershocks.

(1) Tango Earthquake of March 7, 1927. Epicenter: 135.1°E; 35.6°N (After N. Nasu[10]).

(2) Ogasima Earthquake of May 1, 1939. Epicenter: 139.8°E; 39.9°N (After T. Hagiwara[11]).

(3) Earthquake of April 21, 1935 (Formosa). Epicenter: 120.8°E; 24.4°N (After T. Ishikawa[12]).

(4) Off Yetorup Earthquake of Nov. 7, 1958.[13] Epicenter: 148.5°E; 44.3°N.

predominantly large shock. These facts imply that the earthquake swarm may occur in the heterogeneous medium and/or be caused by the application of the concentrated stress. On the other hand, the mode of aftershock occurrence may be caused by uniformly applied stress to a homogeneous medium.

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[10] N. Nasu, *Bull. Earthq. Res. Inst.*, **6** (1929), 245-331.

[11] T. Hagiwara, *Bull. Earthq. Res. Inst.*, **18** (1940), 252-264.

[12] T. Ishikawa, *Quarterly Journal of Seismology*, **9** (1935-37), 29-36.

[13] *Seis. Bull. Japan Meteor. Agency*, (1958).

5. Summary and acknowledgment.

In this paper the fracturing of a semi-infinite body caused by an inner stress source was experimentally investigated. The results are analogous to earthquake phenomena in several aspects. Therefore, the writer thinks that the experiment may be considered as a simple model one for earthquake occurrence. This attempt is a first step in this direction, and a quantitative study will subsequently be carried out. In the previous paper[14], it was indicated that the magnitude-frequency relation of earthquakes and their time distribution seem to be attributable to the fundamental characteristics of the fracturing of the earth's materials. These results support the idea that earthquakes are caused by the fracturing of the earth's crust, which is a heterogeneous brittle medium, under some simple stress states.

The writer wishes to express his sincere thanks to Professor T. Minakami for his valuable advice and encouragement.

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45. 半無限体の内部力源による破壊およびそれに関連した地震発生の二，三の問題 (第1報)

地震研究所 茂 木 清 夫

地震は地殼の脆性破壊に伴う衝擊性波動であるとの立場から，種々の状態での破壊の発生特性を研究することによつて地震の起こり方を明らかにする手掛りが得られると考えられる．このような立場から，半無限体の内部力源による破壊の発生過程および破壊群の空間的分布状態を実験的に調べて地震の場合と比較考察した．実験における半無限媒体が，地震の場合の地殼に相当するとすれば，大きい地震の発生に関する一つの簡単な模型実験と見ることができる．

半無限脆性体として，松脂を大型円形容器に熔融して後，徐々に凝固させたものをとり，内部力源として，小型電熱器を内蔵した球形あるいは楕円体形などの金属塊をとつて適当の深さに埋設し，金属塊の熱膨脹によつて力を作用させた．

破壊は主として力源と媒質表面との間の部分に発生するが，それに伴う衝擊性弾性波群を振動計で連続記録した．実験の結果を要約すると次のとおりである．

(1) 媒質が均質一様な場合は，応力の増加とともに主破壊が突然発生し，それにひきつづいて多数の小破壊が続発する．これは地震の場合の余震発生によく似ている．さらに力源の深さが力源の大きさに比較してある程度大きい場合は，このような小破壊群の発生が比較的顕著でない．これも深発地震に余震が少ないことに相当するように見える．

(2) 楕円体型または円筒型力源の場合には，主破壊の際にあらわれる破砕域の分布は楕円形となる．ときとして，主破壊の出発点が破砕域の縁にくることが少なくない．また，大きい割れ目の一方だけが破砕して，他方はほとんど非破壊の状態で残る場合がある．これらの諸性質は破壊の基本的な

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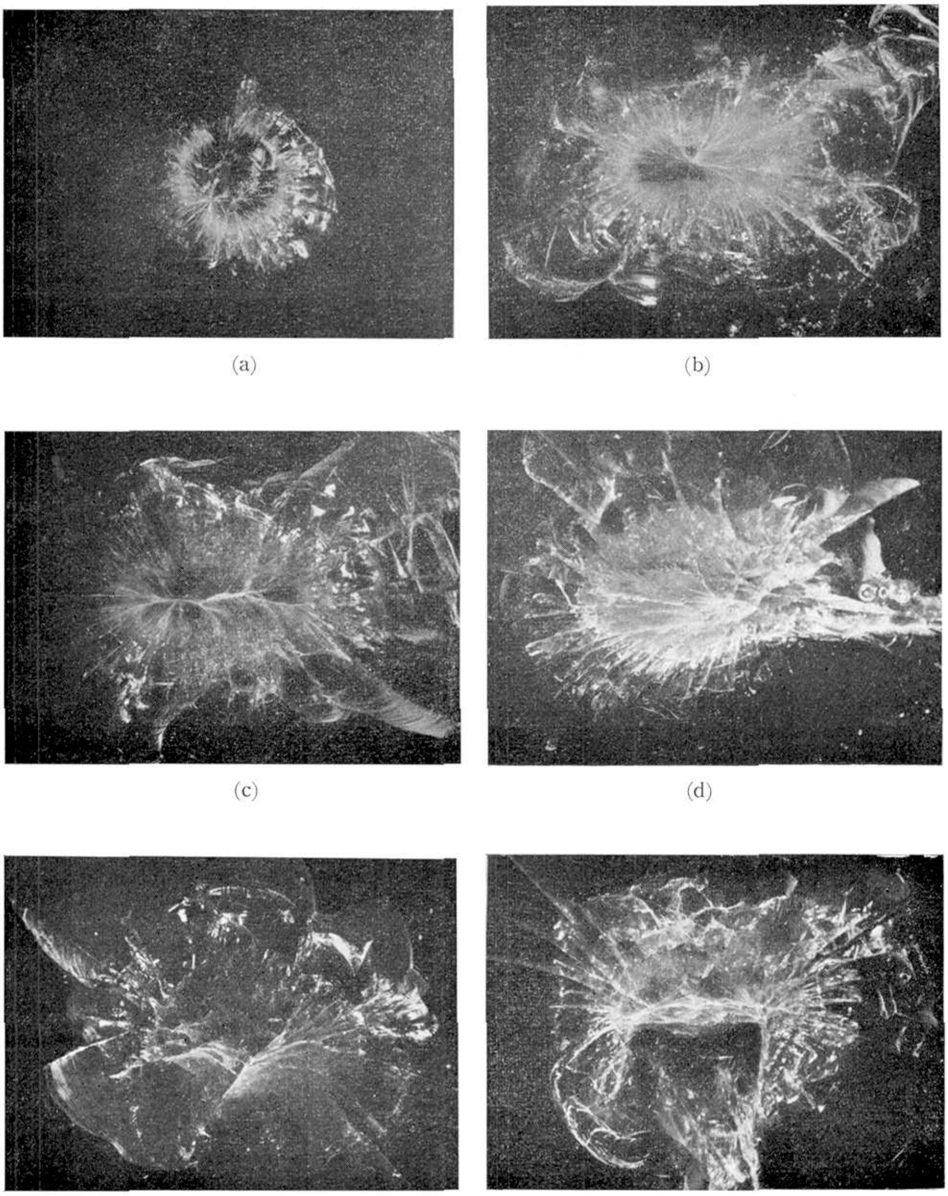
14) K. MOGI, *loc. cit.*, 1).



(a) (b)

(c) (d)

(e) (f)

（震研彙報 第四十号 図版 茂木）

Fig. 13. Crack patterns on the surface of medium.
(a): spherical stress source.
(b), (c), (d), (e), (f): cylindrical stress source.

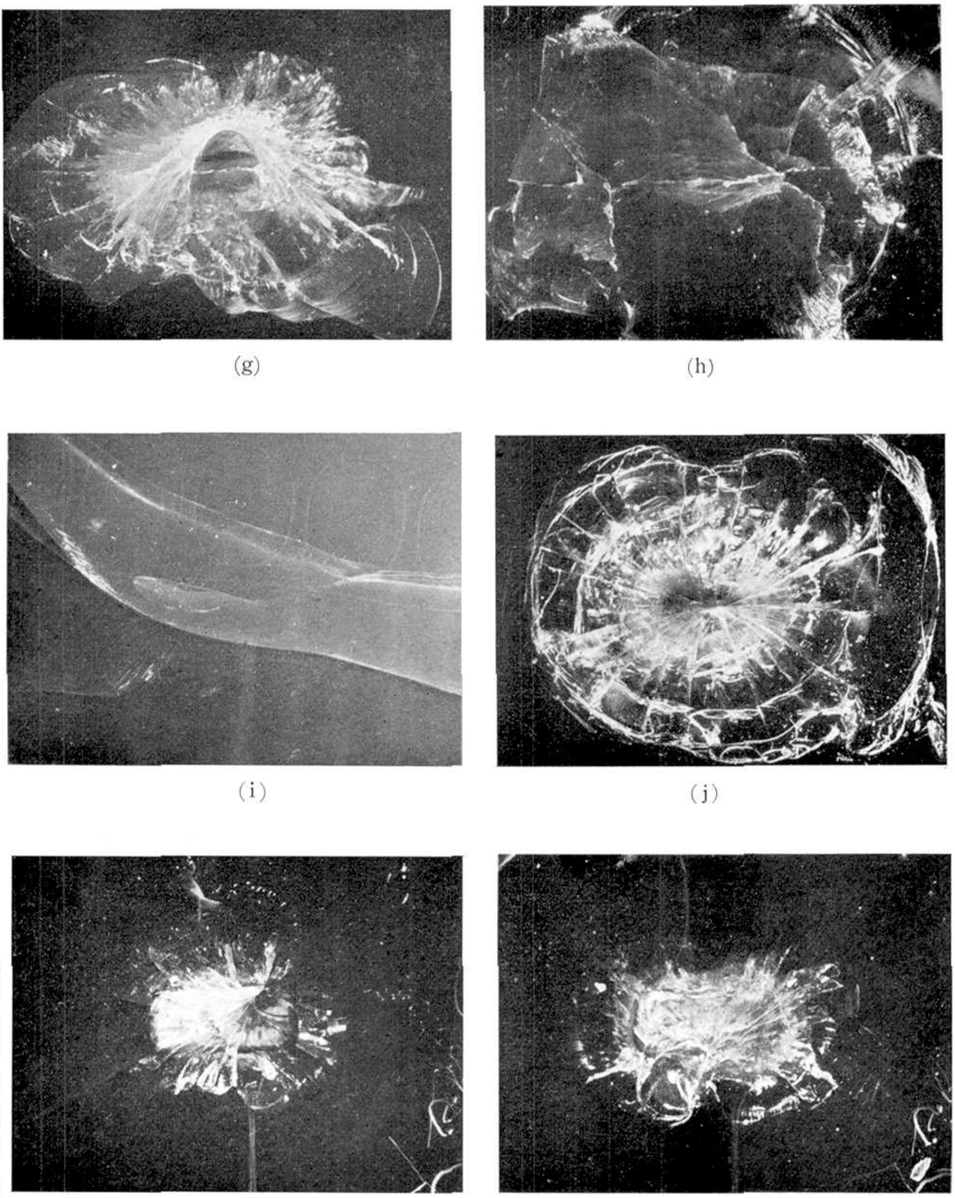


Fig. 13. Crack patterns on the surface of medium.
(g), (h), (i): cylindrical stress source ((i): deep source).
(j), (k), (l): ellipsoidal stress source.

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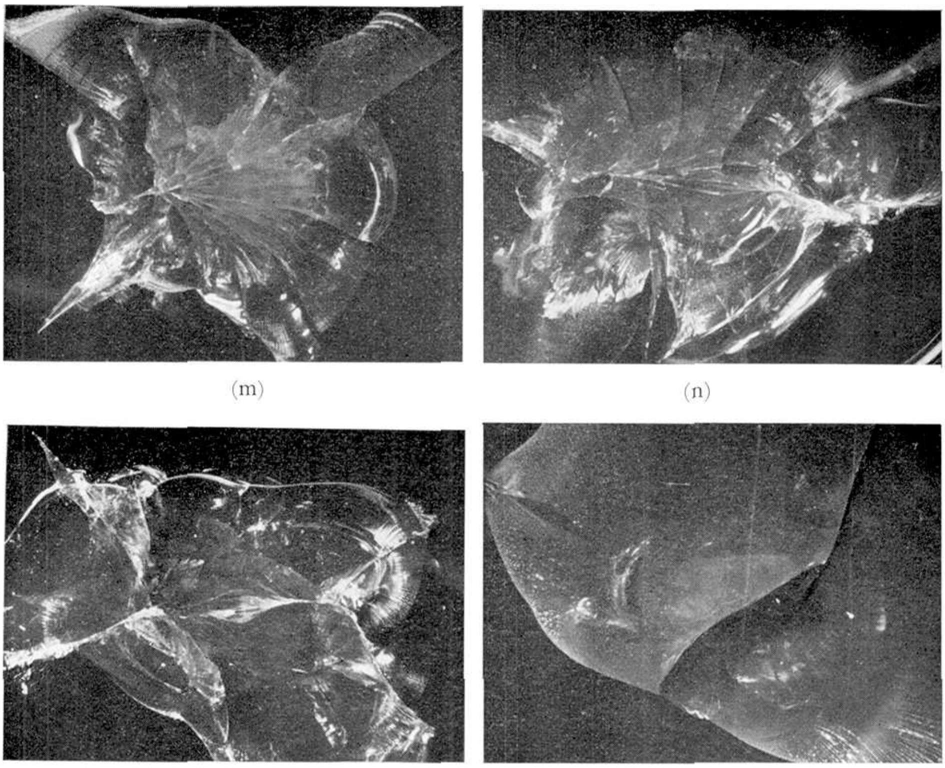
(m) (n)
(o) (p)

Fig. 13. Crack patterns on the surface of medium.
(m), (n), (o), (p): ellipsoidal stress source ((p): deep source).

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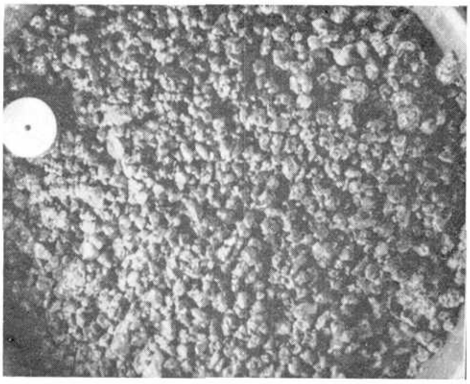

Fig. 14. Mixture of pine resin and pumice particles.

特性に起因するものであるが，地震の余震域の分布と本震の震央あるいは地震断層との関係について知られている特徴と類似している点が少なくない．

(3) 媒質の構造が不均一な場合，あるいは力源の形が著しく不規則な場合は，破壊の頻度および大きさは，応力の増加と共に次第に増大する．これは，媒質内の応力が集中的に分布するために局部破壊が頻発することによると考えられる．この場合の破壊群の発生過程は群発地震などに類似している．

以上，半無限体の内部力源による破壊の発生過程について調べたが，このような最も簡単な応力分布による破壊の発生が地震の起こり方といくつかの重要な点で類似していることは注目される．このような立場から，より定量的に研究を進めることによつて，地震の起こり方と媒質の構造および応力状態との関係を明らかにする手掛りが得られると考えられる．