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13. *General Method of Treatment of Tsunami Caused by the Displacement of a Portion of the Bottom with an Arbitrary Form.*

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1. Introduction

We studied the general method of treatment of water waves produced by a vibrating bottom with an arbitrary form[1] and its application to a square vibrating bottom[2] before.

In this paper, we intend to develop the above-mentioned method more generally to treat Tsunami caused by the displacement—instead of by vibration—of a portion of the bottom with an arbitrary form.

2. Basic Equation and Boundary Conditions

We use polar coordinates (r, θ, z), taken (r, θ) at the undisturbed free surface of water and z vertically upwards. Then we have, as the surface conditions,

$$\left.\begin{aligned} \zeta&=\frac{1}{-g}\left(\frac{\partial\Phi}{\partial t}\right) \\ \frac{\partial\Phi}{\partial z}&=\frac{\partial\zeta}{\partial t} \end{aligned}\right\} (z=0)\,, \tag{1}$$

or

$$\frac{\partial^2\Phi}{\partial t^2}+g\frac{\partial\Phi}{\partial z}=0 \quad (z=0)\,, \tag{2}$$

and, as the bottom condition,

$$\frac{\partial\Phi}{\partial z}=\eta \quad (z=-H)\,, \tag{3}$$

1) T. MOMOI, *Bull. Earthq. Ses. Inst.* **40** (1962) 261-271. In the subsequent discussions, this paper is referred to as paper (X).

2) T. MOMOI, *Bull. Earthq. Res. Inst.* **40** (1962) 273-285.

where

ζ ; elevation of free surface of water,
H ; depth of water,
g ; acceleration of gravity,
t ; variable of time,
η ; velocity of the bottom displacement,
Φ ; velocity potential which satisfies the equation

$$\left(\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}+\frac{\partial^2\Phi}{\partial z^2}\right)=0\,. \tag{4}$$

3. Expression of the Equation and Boundary Conditions in Dimensionless Form with Rergard to Length

Supposing that

$$r_1=\frac{r}{r_0}\,,\qquad \Phi_1=\frac{\Phi}{r_0^{\,2}}\,,$$

$$z_1=\frac{r}{r_0}\,,\qquad \eta_1=\frac{\eta}{r_0}\,,$$

$$g_1=\frac{g}{r_0}\,,\qquad \zeta_1=\frac{\zeta}{r_0}\,,$$

$$\text{and}\qquad H_1=\frac{H}{r_0}\,,$$

where r_0 is the radius of a circumscribed circle of a portion of the water bottom to be moved, we have from (2), (3) and (4)

$$\frac{\partial\Phi_1}{\partial t^2}+g_1\frac{\partial\Phi_1}{\partial z_1}=0\quad(z_1=0)\,, \tag{5}$$

$$\frac{\partial\Phi_1}{\partial z_1}=\eta_1\quad(z_1=-H_1)\,, \tag{6}$$

$$\left(\frac{\partial^2}{\partial r_1^{\,2}}+\frac{1}{r_1}\frac{\partial}{\partial r_1}+\frac{1}{r_1^{\,2}}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial z_1^{\,2}}\right)\Phi_1=0\,. \tag{7}$$

4. Fourier Trannsform of the Basic Equation and Boundary Conditions

Let Φ' and η' be transformed functions of Φ_1 and η_1 as follows ;

$$\left.\begin{aligned}\Phi_1&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\Phi' e^{-\iota\omega t}d\omega\\ \eta_1&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\eta' e^{-\iota\omega t}d\omega\end{aligned}\right\}, \tag{8}$$

whereupon the condition (5), (6) and the equation (7) become

$$-\omega^2\Phi'+g_1\frac{\partial\Phi'}{\partial z_1}=0 \quad (z_1=0), \tag{9}$$

$$\frac{\partial\Phi'}{\partial z_1}=\eta' \quad (z_1=-H_1), \tag{10}$$

$$\left(\frac{\partial^2}{\partial r_1^2}+\frac{1}{r_1}\frac{\partial}{\partial r_1}+\frac{1}{r_1^2}\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial z_1^2}\right)\Phi'=0\,. \tag{11}$$

5. Fourier Bessel Expression of the Bottom Condition

Let D or D_1 $(=D/r_0)$ in dimensionless form be the bottom displacement.

Then $$\eta=\frac{\partial D}{\partial t} \quad \text{or} \quad \eta_1=\frac{\partial D_1}{\partial t} \tag{12}$$

Here we expand the displacement (D_1) of the bottom by Fourier Bessel series as follows;

$$D_1=\sum_{n=0}^{\infty}\sum_{u=1}^{\infty}(A_{n,u}(t)\cos n\theta+B_{n,u}(t)\sin n\theta)\cdot J_n(\lambda_{n,u}r_1)\,, \tag{13}$$

where $A_{n,u}(t)$ and $B_{n,u}(t)$ are Fourier Bessel coefficients and functions of time, and $\lambda_{n,u}$ $(u=1, 2,\cdots)$ are the positive roots of $J_n(\lambda_{n,u})=0$ (refer to paper (X)).

Consequently, the combination of (12) and (13) gives

$$\eta_1=\sum_{n=0}^{\infty}\sum_{u=1}^{\infty}\left(\frac{dA_{n,u}}{dt}\cdot\cos n\theta+\frac{dB_{n,u}}{dt}\cdot\sin n\theta\right)\cdot J_n(\lambda_{n,u}r_1)\,. \tag{14}$$

Next, supposing that $A'_{n,u}(\omega)$ and $B'_{n,u}(\omega)$ are Fourier transforms of $A_{n,u}(t)$ and $B_{n,u}(t)$ respectively, i. e.,

$$\left.\begin{matrix}A_{n,u}(t)\\ B_{n,u}(t)\end{matrix}\right\}=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\left\{\begin{matrix}A'_{n,u}(\omega)\\ B'_{n,u}(\omega)\end{matrix}\right\}e^{-\iota\omega t}d\omega\,, \tag{15}$$

we obtain from (8), (10), (14) and (15)

$$\frac{\partial \Phi'}{\partial z_1}=\eta'=-i\omega\sum_{n=0}^{\infty}\sum_{n=1}^{\infty}(A'_{n,u}(\omega)\cos n\theta+B'_{n,u}(\omega)\sin n\theta)\cdot J_n(\lambda_{n,u}r_1)\ . \qquad (16)$$

6. Solutions of Equation (11)

The equation (11) and the boundary conditons (9) and (16) are identical in form with those discussed in the paper (X)[1] in the case of the vibrating bottom. In paper (X)[1] ω is always positive, while in this paper ω has no such restriction.

Let the super-suffix (1) and (2) designate the velocity potentials in the domain ($r_1<1$) where the bottom is to be moved and in the domain ($r_1>1$) where the bottom is at rest respectively.

Then in likely manner as in paper (X)[1], we obtain the velocity potentials $\Phi^{(1)\prime}$ and $\Phi^{(2)\prime}$ as follows;

$$\begin{Bmatrix}\Phi^{(1)\prime}(+|\omega|)\\ \Phi^{(1)\prime}(-|\omega|)\end{Bmatrix}=\sum_{n=0}^{\infty}\cosh a_0(H_1+z_1)\cdot\left[\begin{Bmatrix}C^{(1)+}_{n,0}\\ C^{(1)-}_{n,0}\end{Bmatrix}\cos n\theta+\begin{Bmatrix}S^{(1)+}_{n,0}\\ S^{(1)-}_{n,0}\end{Bmatrix}\sin n\theta\right]\cdot J_n(a_0r_1)$$
$$+\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\cos a_s(H_1+z_1)\cdot\left[\begin{Bmatrix}C^{(1)+}_{n,s}\\ C^{(1)-}_{n,s}\end{Bmatrix}\cos n\theta+\begin{Bmatrix}S^{(1)+}_{n,s}\\ S^{(1)-}_{n,s}\end{Bmatrix}\sin n\theta\right]\cdot I_n(a_sr_1)$$
$$-i\cdot\begin{Bmatrix}(+|\omega|)\\ (-|\omega|)\end{Bmatrix}\cdot\sum_{n=0}^{\infty}\sum_{n=1}^{\infty}\cdot\left[\begin{Bmatrix}A'_{n,u}(+|\omega|)\\ A'_{n,u}(-|\omega|)\end{Bmatrix}\cos n\theta\right.$$
$$\left.+\begin{Bmatrix}B'_{n,u}(+|\omega|)\\ B'_{n,u}(-|\omega|)\end{Bmatrix}\sin n\theta\right]\cdot J_n(\lambda_{n,u}r_1)\cdot(E_{n,u}e^{\lambda_{n,u}z_1}+F_{n,u}e^{-\lambda_{n,u}z_1})$$
$$\text{for }\begin{Bmatrix}\omega>0\\ \omega<0\end{Bmatrix}, \qquad (17)$$

where

$$E_{n,u}=\frac{1}{M_{n,u}}\cdot\frac{\omega^2+g_1\lambda_{n,u}}{\lambda_{n,u}},$$
$$F_{n,u}=\frac{1}{M_{n,u}}\cdot\frac{-\omega^2+g_1\lambda_{n,u}}{\lambda_{n,u}},$$
$$M_{n,u}=\begin{vmatrix}(-\omega^2+g_1\lambda_{n,u}), & -(\omega^2+g_1\lambda_{n,u})\\ e^{-\lambda_{n,u}H_1}, & -e^{+\lambda_{n,u}H_1}\end{vmatrix},$$

and

$$\begin{Bmatrix}\Phi^{(2)\prime}(+|\omega|)\\ \Phi^{(2)\prime}(-|\omega|)\end{Bmatrix}=\sum_{n=0}^{\infty}\cosh a_0(H_1+z_1)\cdot\left[\begin{Bmatrix}C^{(2)+}_{n,0}\\ C^{(2)-}_{n,0}\end{Bmatrix}\cos n\theta+\begin{Bmatrix}S^{(2)+}_{n,0}\\ S^{(2)-}_{n,0}\end{Bmatrix}\sin n\theta\right]\cdot\begin{Bmatrix}H^{(1)}_n(a_0r_1)\\ H^{(2)}_n(a_0r_1)\end{Bmatrix}$$
$$+\sum_{n=0}^{\infty}\sum_{s=1}^{\infty}\cos a_s(H_1+z_1)\cdot\left[\begin{Bmatrix}C^{(2)+}_{n,s}\\ C^{(2)-}_{n,s}\end{Bmatrix}\cos n\theta+\begin{Bmatrix}S^{(2)+}_{n,s}\\ S^{(2)-}_{n,s}\end{Bmatrix}\sin n\theta\right]\cdot K_n(a_sr_1)$$
$$\text{for }\begin{Bmatrix}\omega>0\\ \omega<0\end{Bmatrix}, \qquad (18)$$

where a_0 and ia_s $(s=1, 2, \cdots)$ are the real and imaginary roots of Airy's relation or $\omega^2 = a g_1 \tanh aH_1$; $C^{(1)\pm}_{n,u}$, $C^{(2)\pm}_{n,u}$, $S^{(1)\pm}_{n,u}$, $S^{(2)\pm}_{n,u}$ are arbitrary constants to be determined by the boundary conditions at $r_1=1$; the functions in the wavy brackets are taken in the same order.

7. Determination of the Arbitrary Constants

In order to cetermine the arbitrary constants $C^{(1)\pm}_{n,u}$, $C^{(2)\pm}_{n,u}$, $D^{(1)\pm}_{n,u}$ and $D^{(2)\pm}_{n,u}$ $(n=0, 1, 2, \cdots;\ u=0, 1, 2, \cdots)$, the next conditions are available, i. e.,

continuity of pressure:

$$\left(\frac{\partial \Phi_1^{(1)}}{\partial t}\right)_{r_1=1} = \left(\frac{\partial \Phi_1^{(2)}}{\partial t}\right)_{r_1=1}, \tag{19}$$

continuity of velocity components of water particle:

$$\left(\frac{\partial \Phi_1^{(1)}}{\partial r_1}\right)_{r_1=1} = \left(\frac{\partial \Phi_1^{(2)}}{\partial r_1}\right)_{r_1=1}, \tag{20}$$

$$\left(\frac{\partial \Phi_1^{(1)}}{\partial \theta}\right)_{r_1=1} = \left(\frac{\partial \Phi_1^{(2)}}{\partial \theta}\right)_{r_1=1}. \tag{21}$$

The substitution of Fourier expression (8) into (19), (20) and (21) yields

$$(\Phi^{(1)\prime})_{r_1=1} = (\Phi^{(1)\prime})_{r_1=1} \tag{22}$$

$$\left(\frac{\partial \Phi^{(1)\prime}}{\partial r_1}\right)_{r_1=1} = \left(\frac{\partial \Phi^{(2)\prime}}{\partial r_1}\right)_{r_1=1}, \tag{23}$$

$$\left(\frac{\partial \Phi^{(1)\prime}}{\partial \theta}\right)_{r_1=1} = \left(\frac{\partial \Phi^{(2)\prime}}{\partial \theta}\right)_{r_1=1}. \tag{24}$$

Here the condition (24) is involved in the condition (22). Hence the only conditions (22) and (23) are available.

Making use of the conditions (22) and (23), we can determine the arbitrary constants $C^{(1)\pm}_{n,u}$, $C^{(2)\pm}_{n,u}$, $S^{(1)\pm}_{n,u}$ and $S^{(2)\pm}_{n,u}$ as follows in the same manner as in paper (X);

$$\left.\begin{aligned} C^{(1)\pm}_{n,0} &= \pm \frac{i\pi a_0}{2} \cdot \begin{Bmatrix} H_n^{(1)}(a_0) \\ H_n^{(2)}(a_0) \end{Bmatrix} \cdot R^{A_n}_{n,0}(\pm) \\ C^{(2)\pm}_{n,0} &= \pm \frac{i\pi a_0}{2} \cdot J_n(a_0) \cdot R^{A_n}_{n,0}(\pm) \end{aligned}\right\}$$

$$(n=0,\ 1,\ 2, \cdots),$$

$$
\left.
\begin{aligned}
&\left.\begin{aligned}
S_{n,0}^{(1)\pm} &= \pm\frac{i\pi a_0}{2}\cdot\begin{Bmatrix} H_n^{(1)}(a_0) \\ H_n^{(2)}(a_0)\end{Bmatrix}\cdot R_{n,0}^{B_n}(\pm) \\
S_{n,0}^{(2)\pm} &= \pm\frac{i\pi a_0}{2}\cdot J_n(a_0)\cdot R_{n,0}^{B_n}(\pm)
\end{aligned}\right\} \\
&\qquad (n=1,\ 2,\ 3,\cdots), \\
&\left.\begin{aligned}
C_{n,s}^{(1)\pm} &= a_s\cdot K_n(a_s)\cdot R_{n,s}^{A_n}(\pm) \\
C_{n,s}^{(2)\pm} &= a_s\cdot I_n(a_s)\cdot R_{n,s}^{A_n}(\pm)
\end{aligned}\right\} \\
&\qquad \begin{pmatrix} n=0,\ 1,\ 2,\cdots \\ s=1,\ 2,\ 3,\cdots \end{pmatrix}, \\
&\left.\begin{aligned}
S_{n,s}^{(1)\pm} &= a_s\cdot K_n(a_s) R_{n,s}^{B_n}(\pm) \\
S_{n,s}^{(2)\pm} &= a_s\cdot I_n(a_s) R_{n,s}^{B_n}(\pm)
\end{aligned}\right\} \\
&\qquad \begin{pmatrix} n=1,\ 2,\ 3,\cdots \\ s=1,\ 2,\ 3,\cdots \end{pmatrix},
\end{aligned}
\right\} \tag{25}
$$

where the double sign $(\pm)$ and the functions in the wavy brackets are taken in the same order (this convention is used in the following discussions); a_0 and ia_s $(s=1,\ 2,\cdots)$ are real and imaginary roots of Airy's relation;

$$
\left.
\begin{aligned}
R_{n,0}^{A_n}(\pm) &= \frac{1}{a_0}\cdot\sum_{u=1}^{\infty}\cdot i\cdot(\pm|\omega|)\cdot A'_{n,u}(\pm|\omega|)\cdot\lambda_{n,u}\cdot J'_n(\lambda_{n,u})\cdot\frac{1}{a_0^2-\lambda_{n,u}^2} \\
&\qquad (n=0,\ 1,\ 2,\cdots) \\
R_{n,0}^{B_n}(\pm) &= \frac{1}{a_0}\cdot\sum_{u=0}^{\infty}\cdot i\cdot(\pm|\omega|)\cdot B'_{n,u}(\pm|\omega|)\cdot\lambda_{n,u}\cdot J'_n(\lambda_{n,u})\cdot\frac{1}{a_0^2-\lambda_{n,u}^2} \\
&\qquad (n=1,\ 2,\ 3,\cdots) \\
R_{n,0}^{A_n}(\pm) &= \frac{1}{a_s}\cdot\sum_{u=1}^{\infty}\cdot i\cdot(\pm|\omega|)\cdot A'_{n,u}(\pm|\omega|)\cdot\lambda_{n,u}\cdot J'_n(\lambda_{n,u})\cdot\frac{-1}{a_s^2+\lambda_{n,u}^2} \\
&\qquad \begin{pmatrix} n=0,\ 1,\ 2,\cdots \\ s=1,\ 2,\ 3,\cdots \end{pmatrix}, \\
R_{n,s}^{B_n}(\pm) &= \frac{1}{a_s}\cdot\sum_{u=1}^{\infty}\cdot i\cdot(\pm|\omega|)\cdot B'_{n,u}(\pm|\omega|)\cdot\lambda_{n,u}\cdot J'_n(\lambda_{n,u})\cdot\frac{-1}{a_s^2+\lambda_{n,u}^2} \\
&\qquad \begin{pmatrix} n=1,\ 2,\ 3,\cdots \\ s=1,\ 2,\ 3,\cdots \end{pmatrix};
\end{aligned}
\right\} \tag{26}
$$

$\lambda_{n,u}$ $(u=1, 2, 3, \cdots)$ are the positive roots of the Bessel function $J_n(\lambda_{n,u})=0$ being taken in order from the smallest positive value.

8. Expression of Wave Height

Let $\zeta_1^{(1)}$ and $\zeta_1^{(2)}$ be the elevations of free surface of water from the undisturbed free surface of water in the domain $(r_1<1)$ and in the domain $(r_1>1)$ respectively.

Then we have from (1), (8), (17) and (18)

$$\zeta_1^{(1)}=\frac{1}{-g}\cdot\left(\frac{\partial\Phi_1^{(1)}}{\partial t}\right)_{z_1=0}=\frac{1}{-g}\cdot\frac{1}{\sqrt{2\pi}}\left[\int_0^{+\infty}\Phi^{(1)\prime}(+|\omega|)\cdot(-i\omega)e^{-i\omega t}d\omega\right.$$
$$\left.+\int_{-\infty}^0\Phi^{(1)\prime}(-|\omega|)\cdot(-i\omega)e^{-i\omega t}d\omega\right]_{z_1=0} \tag{27}$$

and

$$\zeta_1^{(2)}=\frac{1}{-g}\cdot\left(\frac{\partial\Phi_1^{(2)}}{\partial t}\right)_{z_1=0}=\frac{1}{-g}\cdot\frac{1}{\sqrt{2\pi}}\left[\int_0^{+\infty}\Phi^{(2)\prime}(+|\omega|)\cdot(-i\omega)e^{-i\omega t}d\omega\right.$$
$$\left.+\int_{-\infty}^0\Phi^{(2)\prime}(-|\omega|)\cdot(-i\omega)e^{-i\omega t}d\omega\right]_{z_1=0} \tag{28}$$

where $\Phi^{(1)\prime}(\pm|\omega|)$ and $\Phi^{(2)\prime}(\pm|\omega|)$ are given by (17) and (18).

Here (27) and (28) are formal expressions of wave height. In the next section, let us consider the asymptotic form of the out-going wave.

9. Asymptotic Form of the Out-going Wave

As in paper (X), the terms with factors $H_n^{(1)}(a_0r_1)$ and $H_n^{(2)}(a_0r_1)$ only remain at the distant point from the wave-generating origin.

Hence we have from (28) and (18)

$$\zeta_1^{(2)}\approx\frac{1}{-g_1}\cdot\frac{1}{\sqrt{2\pi}}\cdot\left[\int_0^{+\infty}d\omega\left\{(-i\omega)\cdot e^{-i\omega t}\cdot\sum_{n=0}^{\infty}\cosh a_0H_1\right.\right.$$
$$\left.\cdot(C_{n,0}^{(2)}\cos n\theta+S_{n,0}^{(2)+}\sin n\theta)\cdot H_n^{(1)}(a_0r_1)\right\}$$
$$+\int_{-\infty}^0 d\omega\left\{(-i\omega)\cdot e^{-i\omega t}\cdot\sum_{n=0}^{\infty}\cosh a_0H_1\right.$$
$$\left.\cdot(C_{n,0}^{(2)-}\cos n\theta+S_{n,0}^{(2)-}\sin n\theta)\cdot H_n^{(2)}(a_0r_1)\right\}, \tag{29}$$

where

$$\left.\begin{matrix} H_n^{(1)}(a_0 r_1) \\ H_n^{(2)}(a_0 r_1) \end{matrix}\right\} \doteqdot \sqrt{\frac{2}{\pi a_0 r_1}} \cdot e^{\pm i\left(a_0 r_1 - \frac{2n+1}{4}\pi\right)} \tag{30}$$

and ω is related with a_0 by virtue of Airy's relation or $\omega^2 = a_0 g_1 \tanh a_0 H_1$.

10. Changing the Variables

As ω is related with a_0 by the Airy's relation, the variable ω in (29) can be changed into the variable a_0 as follows;

$$\omega = \pm \sqrt{a_0 g_1 \tanh a_0 H_1}\,,$$

$$d\omega = \frac{1}{2}\sqrt{\frac{g_1}{a_0}} \cdot \frac{\pm 1}{\sqrt{\tanh a_0 H_1}} \cdot \left[\tanh a_0 H_1 + \frac{a_0 H_1}{\cosh^2 a_0 H_1}\right] da_0\,,$$

where the double sign $(\pm)$ is taken in the same order with $\omega \gtrless 0$, and the interval of ω $(-\infty,\ 0,\ +\infty)$ in integration corresponds with that of a_0 $(+\infty,\ 0,\ +\infty)$.

Consequently, the expression (29) becomes

$$\begin{aligned} \zeta_1^{(2)} \approx \frac{1}{-g_1} \cdot \frac{1}{\sqrt{2\pi}} \cdot \Bigg[\int_0^{+\infty} G\, da_0 \Big\{ & (-i|\omega|) \cdot e^{-i|\omega|t} \cdot \sum_{n=0}^{\infty} \cosh a_0 H_1 \\ & \cdot (C_{n,0}^{(2)+} \cos n\theta + S_{n,0}^{(2)+} \sin n\theta) \cdot H_n^{(1)}(a_0 r_1) \Big\} \\ + \int_0^{+\infty} G\, da_0 \Big\{ & (+i|\omega|) \cdot e^{+i|\omega|t} \cdot \sum_{n=0}^{\infty} \cosh a_0 H_1 \\ & \cdot (C_{n,0}^{(2)-} \cos n\theta + S_{n,0}^{(2)-} \sin n\theta) \cdot H_n^{(2)}(a_0 r_1) \Big\} \Bigg], \end{aligned} \tag{31}$$

where

$$|\omega| = \sqrt{a_0 g_1 \tanh a_0 H_1}\,, \tag{32}$$

$$G = \frac{1}{2}\sqrt{\frac{g_1}{a_0}} \cdot \frac{1}{\sqrt{\tanh a_0 H_1}} \cdot \left[\tanh a_0 H_1 + \frac{a_0 H_1}{\cosh^2 a_0 H_1}\right]. \tag{33}$$

11. Long Wave Approximation

In the foregoing section, the asymptotic formula was derived. In this section we apply long wave approximation such as $a_0 H_1 \ll 1$ to the asymptotic formula (31).

As a result of long wave approximation, the following reductions are valid,

$$\left.\begin{aligned} &\sinh(a_0H_1)\approx a_0H_1\ ,\\ &\cosh(a_0H_1)\approx 1\ ,\\ \text{or}\quad &\tanh(a_0H_1)\approx a_0H_1\ ,\end{aligned}\right\}\tag{34}$$

whence (32) and (33) are reduced to

$$|\omega|=a_0c\ ,\quad G=c\ ,\tag{35}$$

where $c=\sqrt{g_1H_1}$.

Substituting (34) and (35) into (31), we get

$$\begin{aligned}\zeta_1^{(2)}\approx\frac{1}{-g_1}\cdot\frac{1}{\sqrt{2\pi}}\Bigg[\int_0^{+\infty}c\,da_0\Big\{(-ia_0c)\cdot e^{-ia_0ct}\\ \cdot\sum_{n=0}^{\infty}(C_{n,0}^{(2)+}\cos n\theta+S_{n,0}^{(2)+}\sin n\theta)\cdot H_n^{(1)}(a_0r_1)\Big\}\\ +\int_0^{\infty}c\,da_0\Big\{(+ia_0c)\cdot e^{+ia_0ct}\\ \cdot(C_{n,0}^{(2)-}\cos n\theta+S_{n,0}^{(2)-}\sin n\theta)\cdot H_n^{(2)}(a_0r_1)\Big\}\Bigg]\ .\end{aligned}$$

Again substituting (25) and (26) into the above expression and after some reductions, we have

$$\zeta_1^{(2)}\approx\frac{1}{-g}\cdot\frac{1}{\sqrt{2\pi}}\cdot(I^{(+)}+I^{(-)})\ ,\tag{36}$$

where

$$I^{(\pm)}=+i\cdot\frac{\pi}{2}\cdot c^{3/2}\cdot\sum_{n=0}^{\infty}\sum_{u=1}^{\infty}(N_{n,u}^{(\pm)}\cos n\theta+P_{n,u}^{(\pm)}\sin n\theta)\cdot\lambda_{n,u}\cdot(\lambda_{n,u})\ ,\tag{37}$$

$$N_{n,u}^{(\pm)}=\pm\int_0^{+\infty}da_0\cdot J_n(a_0)\cdot\frac{a_0^2}{a_0^2-\lambda_{n,u}^2}\cdot A'_{n,u}(\pm a_0c)\cdot\begin{Bmatrix}H_n^{(1)}(a_0r_1)\\ H_n^{(2)}(a_0r_1)\end{Bmatrix}\cdot e^{\mp ia_0ct}\ ,\tag{38}$$

$$P_{n,u}^{(\pm)}=\pm\int_0^{+\infty}da_0\cdot J_n(a_0)\cdot\frac{a_0^2}{a_0^2-\lambda_{n,u}^2}\cdot B'_{n,u}(\pm a_0c)\cdot\begin{Bmatrix}H_n^{(1)}(a_0r_1)\\ H_n^{(2)}(a_0r_1)\end{Bmatrix}\cdot e^{\mp ia_0ct}\ ,\tag{39}$$

$$\left.\begin{aligned}A'_{n,u}(\pm a_0c)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}A_{n,u}(t)\cdot e^{\mp ia_0ct}dt\ ,\\ B'_{n,u}(\pm a_0c)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}B_{n,u}(t)\cdot e^{\mp ia_0ct}dt\ ,\end{aligned}\right\}\tag{40}$$

(from (15) and (35))

and $A_{n,u}(t)$, $B_{n,u}(t)$ are the Fourier Bessel coefficients in Fourier Bessel expansion of the movement of a portion of the bottom.

12. Integration of (38) and (39)

For the convenience of integration, the Bessel function $J_n(a_0)$ is converted into the Hankel function $\{H_n^{(1)}(a_0)+H_n^{(2)}(a_0)\}\cdot 1/2$.

Then the integrations (38) and (39) become

$$\left.\begin{aligned} N_{n,u}^{(\pm)} &= \pm\frac{1}{2}\left[\begin{Bmatrix} I_1^N \\ I_3^N \end{Bmatrix} + \begin{Bmatrix} I_2^N \\ I_4^N \end{Bmatrix}\right], \\ P_{n,u}^{(\pm)} &= \pm\frac{1}{2}\left[\begin{Bmatrix} I_1^P \\ I_3^P \end{Bmatrix} + \begin{Bmatrix} I_2^P \\ I_4^P \end{Bmatrix}\right], \end{aligned}\right\} \tag{41}$$

where

$$\left.\begin{aligned} \begin{matrix} I_1^N \\ I_1^P \end{matrix}\Big\} &= \int_0^{+\infty} da_0\cdot H_n^{(1)}(a_0)\cdot\frac{a_0^2}{a_0^2-\lambda_{n,u}^2}\cdot\begin{Bmatrix} A'_{n,u}(+a_0c) \\ B'_{n,u}(+a_0c) \end{Bmatrix}\cdot H_n^{(1)}(a_0r_1)\cdot e^{-ia_0ct}, \\ \begin{matrix} I_2^N \\ I_2^P \end{matrix}\Big\} &= \int_0^{+\infty} da_0\cdot H_n^{(2)}(a_0)\cdot\frac{a_0^2}{a_0^2-\lambda_{n,u}^2}\cdot\begin{Bmatrix} A'_{n,u}(+a_0c) \\ B'_{n,u}(+a_0c) \end{Bmatrix}\cdot H_n^{(1)}(a_0r_1)\cdot e^{-ia_0ct}, \\ \begin{matrix} I_3^N \\ I_3^P \end{matrix}\Big\} &= \int_0^{+\infty} da_0\cdot H_n^{(1)}(a_0)\cdot\frac{a_0^2}{a_0^2-\lambda_{n,u}^2}\cdot\begin{Bmatrix} A'_{n,u}(-a_0c) \\ B'_{n,u}(-a_0c) \end{Bmatrix}\cdot H_n^{(2)}(a_0r_1)\cdot e^{+ia_0ct}, \\ \begin{matrix} I_4^N \\ I_4^P \end{matrix}\Big\} &= \int_0^{+\infty} da_0\cdot H_n^{(2)}(a_0)\cdot\frac{a_0^2}{a_0^2-\lambda_{n,u}^2}\cdot\begin{Bmatrix} A'_{n,u}(-a_0c) \\ B'_{n,u}(-a_0c) \end{Bmatrix}\cdot H_n^{(2)}(a_0r_1)\cdot e^{+ia_0ct}. \end{aligned}\right\} \tag{42}$$

As one example of integration, we take the next form from (42), i. e.,

$$I_1^N=\int_0^{+\infty} da_0\cdot H_n^{(1)}(a_0)\cdot\frac{a_0^2}{a_0^2-\lambda_{n,u}^2}\cdot A'_{n,u}(+a_0c)\cdot H_n^{(1)}(a_0r_1)\cdot e^{-ia_0ct}, \tag{43}$$

and other integrations of (42) can be done in the same way.

Now substituting (40) into (43), we obtain

$$I_1^N=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} A_{n,u}(\alpha)Q_1 d\alpha, \tag{44}$$

where

$$Q_1=\int_0^{+\infty} da_0\cdot\frac{a_0^2}{a_0^2-\lambda_{n,u}^2}\cdot H_n^{(1)}(a_0)\cdot H_n^{(1)}(a_0r_1)\cdot e^{-ia_0c(t-\alpha)}. \tag{45}$$

In order to integrate (45), we take the path of integration on the complex plane as shown in Fig. 1.

Fig. 1.

(a) when $c(t-\alpha)-(r_1+1)<0$. (b) when $c(t-\alpha)-(r_1+1)>0$.

(a) when $c(t-\alpha)-(r_1+1)<0$.

$$\left|\int_{c_1}\right| \leq \int_0^{\frac{\pi}{2}} R\cdot d\theta \cdot \frac{R^2}{R^2-\lambda_{n,u}^2}\cdot |H_n^{(1)}(a_0)\cdot H_n^{(1)}(a_0 r_1)|\cdot |e^{-ia_0 c(t-\alpha)}| \ , \tag{46}$$

where R is the radius of the circle c_1 and $a_0=Re^{i\theta}$ (Fig. 1(a)).

Now putting the asymptotic form of the Hankel function into (46) and after some computations, (46) becomes

$$\left|\int_{c_1}\right| \leq \int_0^{\frac{\pi}{2}} d\theta \cdot \frac{R^2}{R^2-\lambda_{n,u}^2}\cdot\frac{2}{\pi}\cdot\frac{1}{\sqrt{r}}\cdot e^{R\sin\theta\{c(t-\alpha)-(r_1+1)\}} \ .$$

Being $c(t-\alpha)-(r_1+1)<0$, this integration tends to zero when $R\rightarrow+\infty$.

Next, we evaluate the integration on the small semi-circle c_3.

Suppose that $a_0=\lambda_{n,u}+\delta r\cdot e^{i\varphi}$ on the circle c_3, i. e., δr is the radius of the circle c_3 and φ is the angle taken counter-clockwise at $a_0=\lambda_{n,u}$ from the positive side of the real axis. Then

$$\int_{c_3} = \int_{+\pi}^{0} i\cdot\delta r\cdot e^{i\varphi}\cdot d\varphi\cdot\frac{(\delta r e^{i\varphi}+\lambda_{n,u})^2}{\delta r\cdot e^{i\varphi}\cdot(\delta r\cdot e^{i\varphi}+2\lambda_{n,u})}$$

$$\cdot H_n^{(1)}(\delta r\cdot e^{i\varphi}+\lambda_{n,u})\cdot H_n^{(1)}((\delta r\cdot e^{i\varphi}+\lambda_{n,u})r_1)\cdot e^{-i\cdot(\delta r\cdot e^{i\varphi}+\lambda_{n,u})c(t-\alpha)}$$

$$=\int_{+\pi}^{0} i\cdot\delta\varphi\cdot\frac{(\delta r\cdot e^{i\varphi}+\lambda_{n,u})^2}{\delta r\cdot e^{i\varphi}+2\lambda_{n,u}} H_n^{(1)}(\delta r\cdot e^{i\varphi}+\lambda_{n,u})$$

$$\cdot H_n^{(1)}((\delta r\cdot e^{i\varphi}+\lambda_{n,u})r_1)\cdot e^{-i(\delta r\cdot e^{i\varphi}+\lambda_{n,u})c(t-\alpha)}$$

$$\longrightarrow \int_{+\pi}^{0} i\cdot d\varphi\cdot\frac{\lambda_{n,u}}{2}\cdot H_n^{(1)}(\lambda_{n,u})\cdot H_n^{(1)}(\lambda_{n,u}r_1)\cdot e^{-i\lambda_{n,u}c(t-\alpha)}$$

$$(\delta r\rightarrow 0)$$

$$=-i\cdot\pi\cdot\frac{\lambda_{n,u}}{2}\cdot H_n^{(1)}(\lambda_{n,u})\cdot H_n^{(1)}(\lambda_{n,u}r_1)\cdot e^{-i\lambda_{n,u}c(t-\alpha)} \ .$$

Next, we consider the integration on the path of the imaginary axis.

$$\int_{c_2}=\int_{+\infty\cdot i}^{0} d\alpha_0\cdot\frac{\alpha_0^2}{\alpha_0^2-\lambda_{n,u}^2}\cdot H_n^{(1)}(\alpha_0)\cdot H_n^{(1)}(\alpha_0 r_1)\cdot e^{-i\alpha_0 c(t-\alpha)}$$

$$=-i\int_0^{\infty} df_0\cdot\frac{f_0^2}{f_0^2+\lambda_{n,u}^2}\cdot H_n^{(1)}(+if_0)\cdot H_n^{(1)}(+if_0 r_1)\cdot e^{+f_0 c(t-\alpha)} ,$$

where the transformation of the variable, $\alpha_0=+if_0$, was made.

Since the integrant of (45) has no pole in the domain enclosed by the path shown in Fig. 1(a), the integration on this path can be put to zero,
or

$$Q_1+\int_{c_1}+\int_{c_2}+\int_{c_3}=0 \quad (R\to+\infty\ ,\quad \delta r\to 0)\ ,$$

where Q_1 is given by (45).

In consequence, Q_1 is expressed by use of the above-mentioned considerations as follows;

$$Q_1=+i\cdot\pi\cdot\frac{\lambda_{n,u}}{2}\cdot H_n^{(1)}(\lambda_{n,u})\cdot H_n^{(1)}(\lambda_{n,u}r_1)\cdot e^{-i\lambda_{n,u}c(t-\alpha)}$$

$$+i\int_0^{\infty} df_0\cdot\frac{f_0^2}{f_0^2+\lambda_{n,u}^2}\cdot H_n^{(1)}(+if_0)\cdot H_n^{(1)}(if_0 r_1)\cdot e^{+f_0 c(t-\alpha)} \tag{47}$$

$$\text{for}\quad c(t-\alpha)-(r_1+1)<0\ .$$

(b) when $c(t-\alpha)-(r_1+1)>0$.

In the same manner with the case of (a), we have

$$\int_{c_1'}\to 0 \quad (R\to+\infty)\ ,$$

$$\int_{c_3'}=+i\cdot\pi\cdot\frac{\lambda_{n,u}}{2}\cdot H_n^{(1)}(\lambda_{n,u})\cdot H_n^{(1)}(\lambda_{n,u}r_1)\cdot e^{-i\lambda_{n,u}c(t-\alpha)}\ ,$$

$$\int_{c_2'}=+i\int_0^{\infty} df_0\cdot\frac{f_0^2}{f_0^2+\lambda_{n,u}^2}\cdot H_n^{(1)}(-if_0)\cdot H_n^{(1)}(-if_0 r_1)\cdot e^{-f_0 c(t-\alpha)}\ ,$$

(refer to Fig. 1(b)) ,

Consequently, Q_1 becomes

$$Q_1=-i\cdot\pi\cdot\frac{\lambda_{n,u}}{2}\cdot H_n^{(1)}(\lambda_{n,u})\cdot H_n^{(1)}(\lambda_{n,u}r_1)\cdot e^{-i\lambda_{n,u}c(t-\alpha)}$$

$$-i\cdot\int_0^\infty df_0\cdot\frac{f_0^2}{f_0^2+\lambda_{n,u}^2}\cdot H_n^{(1)}(-if_0)\cdot H_n^{(1)}(-if_0r_1)\cdot e^{-f_0c(t-\alpha)} \tag{48}$$

$$\text{for}\quad c(t-\alpha)-(r_1+1)>0\ .$$

Thus, we obtained the expression of (45) as the numerically integrable form.

In the same manner we can obtain numerically integrable forms for the other integrations of (42), that is to say,

$$\left.\begin{aligned}
I_1^P&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}B_{n,u}(\alpha)\cdot Q_1\cdot d\alpha\ ,\\
\begin{Bmatrix}I_2^N\\ I_2^P\end{Bmatrix}&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\begin{Bmatrix}A_{n,u}(\alpha)\\ B_{n,u}(\alpha)\end{Bmatrix}\cdot Q_2\cdot d\alpha\ ,\\
\begin{Bmatrix}I_3^N\\ I_3^P\end{Bmatrix}&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\begin{Bmatrix}A_{n,u}(\alpha)\\ B_{n,u}(\alpha)\end{Bmatrix}\cdot Q_3\cdot d\alpha\ ,\\
\begin{Bmatrix}I_4^N\\ I_4^P\end{Bmatrix}&=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\begin{Bmatrix}A_{n,u}(\alpha)\\ B_{n,u}(\alpha)\end{Bmatrix}\cdot Q_4\cdot d\alpha\ ,
\end{aligned}\right\} \tag{49}$$

where Q_1 has already given in (47) and (48),

$$\left.\begin{aligned}
Q_2=&\pm i\cdot\pi\cdot\frac{\lambda_{n,u}}{2}\cdot H_n^{(2)}(\lambda_{n,u})\cdot H_n^{(1)}(\lambda_{n,u}r_1)\cdot e^{-i\lambda_{n,u}c(t-\alpha)}\\
&\pm i\cdot\int_0^\infty df_0\cdot\frac{\lambda_0^2}{f_0^2+\lambda_{n,u}^2}\cdot H_n^{(2)}(\pm if_0)\cdot H_n^{(1)}(\pm if_0r_1)\cdot e^{\pm f_0c(t-\alpha)}\\
&\qquad\text{for}\quad c(t-\alpha)-(r_1-1)\lesseqgtr 0\ ,\\
Q_3=&\mp i\cdot\pi\cdot\frac{\lambda_{n,u}}{2}\cdot H_n^{(1)}(\lambda_{n,u})\cdot H_n^{(2)}(\lambda_{n,u}r_1)\cdot e^{+i\lambda_{n,u}c(t-\alpha)}\\
&\mp i\cdot\int_0^\infty df_0\cdot\frac{\lambda_0^2}{f_0^2+\lambda_{n,u}^2}\cdot H_n^{(1)}(\mp if_0)\cdot H_n^{(2)}(\mp if_0r_1)\cdot e^{\pm f_0c(t-\alpha)}\\
&\qquad\text{for}\quad c(t-\alpha)-(r_1-1)\lesseqgtr 0\ ,\\
Q_4=&\mp i\cdot\pi\cdot\frac{\lambda_{n,u}}{2}\cdot H_n^{(2)}(\lambda_{n,u})\cdot H_n^{(2)}(\lambda_{n,u}r_1)\cdot e^{+i\lambda_{n,u}c(t-\alpha)}\\
&\mp i\cdot\int_0^\infty df_0\cdot\frac{f_0^2}{f_0^2+\lambda_{n,u}^2}\cdot H_n^{(2)}(\mp if_0)\cdot H_n^{(2)}(\mp if_0r_1)\cdot e^{\pm f_0c(t-\alpha)}\\
&\qquad\text{for}\quad c(t-\alpha)-(r_1+1)\lesseqgtr 0\ .
\end{aligned}\right\} \tag{50}$$

Now if the time variation of the bottom displacement is given, the integrations (38) and (39) are numerically possible and the elevation $\zeta_1^{(2)}$ is determined from (36).

13. Final Results

Substituting (37) into (36), we have

$$\zeta_1^{(2)} \approx \frac{1}{-g} \frac{1}{\sqrt{2\pi}} \cdot i \cdot \frac{\pi}{2} \cdot C^{3/2} \cdot \sum_{n=0}^{\infty} \sum_{u=1}^{\infty} [(N_{n,u}^{+} + N_{n,u}^{-}) \cos n\theta + (P_{n,u}^{+} + P_{n,u}^{-}) \sin n\theta] \cdot \lambda_{n,u} \cdot J_n'(\lambda_{n,u}) \tag{51}$$

where

$$\left.\begin{matrix} N^{+}+N^{-} \\ P^{+}+P^{-} \end{matrix}\right\} = \frac{1}{2} \cdot \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left\{\begin{matrix} A_{n,u}(\alpha) \\ B_{n,u}(\alpha) \end{matrix}\right\} \cdot (Q_1+Q_2-Q_3-Q_4) d\alpha \tag{52}$$

(from (41), (44) and (49) .

Here, $(Q_1+Q_2-Q_3-Q_4)$ is given from (47), (48) and (50) after some computations as follows ;

(i) when $t - \frac{r_1 - 1}{c} < \alpha$,

$$(Q_1+Q_2-Q_3-Q_4) = 2 \cdot i \cdot \int_0^{\infty} d f_0 \cdot \frac{f_0^2}{f_0^2 + \lambda_{n,u}^2} \cdot e^{+f_0 c(t-\alpha)} \cdot \{J_n(+if_0) H_n^{(1)}(+if_0 r_1) + J_n(-if_0) \cdot H_n^{(2)}(-if_0 r_1)\} . \tag{53}$$

(ii) when $t - \frac{r_1 + 1}{c} < \alpha < t - \frac{r_1 - 1}{c}$,

$$\begin{aligned}(Q_1+Q_2-Q_3-Q_4) = & -\pi \cdot \lambda_{n,u} \cdot N_n(\lambda_{n,u}) \\ & \cdot \{H_n^{(1)}(\lambda_{n,u} r_1) \cdot e^{-i\lambda_{n,u} c(t-\alpha)} - H_n^{(2)}(\lambda_{n,u} r_1) \cdot e^{+i\lambda_{n,u} c(t-\alpha)}\} \\ & + i \cdot \int_0^{\infty} d f_0 \cdot \frac{f_0^2}{f_0^2 + \lambda_{n,u}^2} \\ & \cdot [H_n^{(1)}(+if_0) \cdot \{H_n^{(1)}(+if_0 r_1) \cdot e^{+f_0 c(t-\alpha)} - H_n^{(2)}(+if_0 r_1) \cdot e^{-f_0 c(t-\alpha)}\} \\ & - H_n^{(2)}(-if_0) \cdot \{H_n^{(1)}(-if_0 r_1) \cdot e^{-f_0 c(t-\alpha)} - H_n^{(2)}(-if_0 r_1) \cdot e^{+f_0 c(t-\alpha)}\}] .\end{aligned} \tag{54}$$

(iii) when $t - \frac{r_1 + 1}{c} > \alpha$,

$$(Q_1+Q_2-Q_3-Q_4) = -2 \cdot i \cdot \int_0^{\infty} d f_0 \cdot \frac{f_0^2}{f_0^2 + \lambda_{n,u}^2} \cdot e^{-f_0 c(t-\alpha)} \cdot \{J_n(-if_0) \cdot H_n^{(1)}(-if_0 r_1) + J_n(+if_0) \cdot H_n^{(2)}(+if_0 r_1)\} . \tag{55}$$

Now substituting the next forms into (53), (54) and (55), that is to say,

$$J_n(\pm i f_0)=e^{\pm(n/2)\cdot\pi i}J_n(f_0)\,,$$

$$\left.\begin{matrix}H_n^{(1)}(\pm i f_0 r_1)\\ H_n^{(2)}(\pm i f_0 r_1)\end{matrix}\right\}\doteqdot\sqrt{\frac{2}{\pm i f_0 r_1\pi}}\cdot\left\{\begin{matrix}e^{-(\pm f_0 r_1+(2n+1)/4\cdot\pi\cdot i)}\\ e^{+(\pm f_0 r_1+(2n+1)/4\cdot\pi\cdot i)}\end{matrix}\right\}\ \text{(asymptotic forms)}\,,$$

$$\left.\begin{matrix}H_n^{(1)}(+i f_0)\\ H_n^{(2)}(-i f_0)\end{matrix}\right\}=K_n(f_0)\cdot\left\{\begin{matrix}\left(\dfrac{+\pi i}{2}\cdot e^{+n\pi i/2}\right)^{-1}\\ \left(\dfrac{-\pi i}{2}\cdot e^{-n\pi i/2}\right)^{-1}\end{matrix}\right.,$$

and after some reductions, we obtain

(i) when $t-\dfrac{r_1-1}{c}<\alpha$,

$$(Q_1+Q_2-Q_3-Q_4)=0 \tag{56}$$

(ii) when $t-\dfrac{r_1+1}{c}<\alpha<t-\dfrac{r_1-1}{c}$,

$$\begin{aligned}(Q_1+Q_2-Q_3-Q_4)=&\pi\cdot\lambda_{n,u}\cdot N_n(\lambda_{n,u})\cdot\\ &\cdot\sqrt{\frac{2}{\pi\lambda_{n,u}r_1}}\cdot(2i)\cdot\sin\left[\lambda_{n,u}\{c(t-\alpha)-r_1\}+\frac{2n+1}{4}\pi\right]\\ &+i\cdot\int_0^\infty d f_0\cdot\frac{f_0^2}{f_0^2+\lambda_{n,u}^2}\cdot(-4)\cdot\frac{K_n(f_0)}{\pi}\cdot\sqrt{\frac{2}{f_0 r_1\pi}}\cdot\\ &\cdot e^{+f_0\{c(t-\alpha)-r_1\}}\cdot\cos n\pi\end{aligned} \tag{57}$$

(iii) when $t-\dfrac{r_1+1}{c}>\alpha$,

$$\begin{aligned}(Q_1+Q_2-Q_3-Q_4)=-4\cdot i\cdot\cos n\pi\cdot\sqrt{\frac{2}{r_1}}\cdot\frac{1}{\sqrt{\pi}}\int_0^\infty d f_0\cdot\frac{f_0^2}{f_0^2+\lambda_{n,u}^2}\cdot J_n(f_0)\cdot\frac{1}{\sqrt{f_0}}\\ \cdot e^{-f_0\{c(t-\alpha)-r_1\}}\,.\end{aligned} \tag{58}$$

Hence the integration (52) may be divided into two parts, i. e.,

$$\left.\begin{matrix}N^++N^-\\ P^++P^-\end{matrix}\right\}=\frac{1}{2}\cdot\frac{1}{\sqrt{2\pi}}(X_1+X_2)$$

$$X_1=\int_{-\infty}^{t-(r_1+1)/c}\left\{\begin{matrix}A_{n,u}(\alpha)\\ B_{n,u}(\alpha)\end{matrix}\right\}\cdot(Q_1+Q_2-Q_3-Q_4)d\alpha\,,$$

where $(Q_1+Q_2-Q_3-Q_4)$ is given by (58),

$$X_2=\int_{t-(r_1+1)/c}^{t-(r_1-1)/c}\begin{Bmatrix}A_{n,u}(\alpha)\\B_{n,u}(\alpha)\end{Bmatrix}\cdot(Q_1+Q_2-Q_3-Q_4)d\alpha\ ,\qquad (59)$$

where $(Q_1+Q_2-Q_3-Q_4)$ is given by (57), and the integration, $\int_{t-(r_1-1)/c}^{+\infty}$, is always zero by virtue of (56).

Thus, if we know the displacement of the bottom with regard to time or $A_{n,u}(t)$ and $B_{n,u}(t)$, we can evaluate the wave height at a point distant from the wave-generating origin by use of (51), (57), (58) and (59).

Now in the foregoing theory, the functions and values are treated in dimensionless from with regard to length, so the absolute wave height $\zeta^{(2)}$ is expressed as

$$\zeta^{(2)}=r_0\cdot\zeta_1^{(2)}\ .$$

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13. 任意の形をした海底の変位によつて おこされた津波の一般解法

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筆者は先に，任意の形をした“振動”水底によつておこされる水波を取扱う一般的方法を導入した.

筆者は本論文において，一般的な変位——必ずしも振動するとは限らない——によつておこされる津波の一般解法について，特に long wave approximation を用いて解く場合を論じた.

浪源域から遠い所での形式的な解は (51), (57), (58) と (59) で表わされている.