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4. *Accuracy of the Rayleigh Wave Method for Studying the Earthquake Mechanism.*

By Keiiti AKI,

Earthquake Research Institute.

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1. Introduction

Analyses on the long period surface waves have been attempted by various authors for the purpose of obtaining useful information on earthquake sources. These analyses aim at removing the effects of the inhomogeneous wave media, the sphericity of the earth and the polar phase shift from the observed seismograms, so that they can be interpreted by means of available theories on wave generation, which are mostly concerned with the point source or finite source problem in the homogeneous half space.

In the present paper, we shall confine ourselves to Rayleigh waves, for which the theory of generation has been most extensively studied. Also, we shall be concerned with the phase characteristics (of the Fourier components) of the waves, the measuring of which is at present done more accurately than the measuring of the amplitude spectrum.

Recently, a question was raised as to whether a quick analysis of Rayleigh waves could be useful in Tsunami warning work. The Tsunami may possibly be more effectively generated from a dip slip earthquake than from a strike slip earthquake of the same magnitude. As described in the next section, the difference in the phase angle of (a Fourier component of) Rayleigh waves is expected to be $\pi/2$ between these two types of earthquakes. Therefore, accuracy of at least $\pi/4$ is necessary in order to distinguish them. We shall discuss later the possibility of attaining this accuracy at present.

2. The theory of Rayleigh wave generation

Lamb (1904) solved the problem of Rayleigh wave generation from a concentrated force applied at a point on the surface of a homogeneous isotropic half space. If the force is of the form R exp $i\omega t$, directed downward to the half space at $t=0$, $r=0$, the vertical displacement of Rayleigh waves at a long distance r, is expressed by his equation (160)

$$w=\frac{\kappa R}{2\mu}K\sqrt{\frac{2}{\pi\kappa r}}\exp i\left(\omega t-\kappa r+\frac{3}{4}\pi\right) \tag{1}$$

In the above equation, π is added to the phase factor in Lamb's original equation, because the upward displacement is taken as positive in the present paper. In this equation, the term $\frac{\kappa R}{2\mu}K\sqrt{\frac{2}{\pi\kappa r}}$ is real positive and does not produce any phase shift. Since $\kappa=\omega/c$ (c being the phase velocity of Rayleigh waves) it is proportional to ω in the case of the homogeneous half space, and the term κr in the phase factor will disappear if we choose the time variable $t'=t-\frac{r}{c}$. The only important phase term in this case, therefore, is $+3/4\cdot\pi$. We shall call this term the phase angle of the source function and designate this by ϕ. In the case of progressive Rayleigh waves, the corresponding phase angle for the horizontal displacement is related to that for the vertical, and it is sufficient to define the phase angle only for the vertical displacement.

If the force at the source is directed upward, the phase angle of the source function will be $-\pi/4$, which is obtained by the addition of $-\pi$ to that for the downward force.

The case of the tangential force to the surface is also studied by Lamb. If the force is directed toward the station, ϕ is $-3/4\cdot\pi$, and if it is directed away from the station, ϕ is $+\pi/4$. These results may be easily obtained by an application of reciprocal theorem to the result of the vertical source. According to Knopoff and Gangi (1959), under the assumption of homogeneous boundary conditions, the displacement u_x at P due to the unit force f_y at Q is the same as the displacement u_y at Q due to the unit force f_x at P. In our case, P and Q are both on the surface. Taking the cylindrical coordinates (r, z) with the origin at the point where the force is applied, we may express the reciprocity relation as follows,

$$u_r \text{ due to } f_z=-u_z \text{ due to } f_r$$

Here, z is taken to be upward positive and r is taken to be outward positive. Since u_r is advanced in phase by $\pi/2$ from u_z in progressive Rayleigh waves from any sources, u_r due to f_z is advanced in phase by $\pi/2$ from u_z due to f_z. The phase angle of u_z due to f_z is $-\pi/4$ as shown before. Therefore, the phase angle of u_z due to f_r is

$-\frac{\pi}{4}+\frac{\pi}{2}-\pi=-\frac{3}{4}\pi$. Thus, ϕ is $-\frac{3}{4}\pi$ for force directed toward the station, and is $+\frac{3}{4}\pi$ for the force directed away from the station.

Evidence from the fault plane studies and other sources favor a couple or a double couple without moment as the force system at the earthquake source rather than a singlet which was assumed in Lamb's theory.

Let us take the x, y axes on the surface of the half space. If a singlet force, as shown in Fig. 1A, varies as exp $i\omega t$, the vertical displacement ξ (x, y) of Rayleigh waves at a distant point P will be proportional to

$$\cos\varphi \ \exp i\left(\omega t-\kappa r-\frac{3}{4}\pi\right) \tag{2}$$

The corresponding displacement due to a couple, as shown in Fig. 1B will be

$$\xi\left(x-\frac{\Delta x}{2},\ y\right)-\xi\left(x+\frac{\Delta x}{2},\ y\right)$$

$$=-\frac{\partial\xi}{\partial x}\Delta x \tag{3}$$

which is proportional to

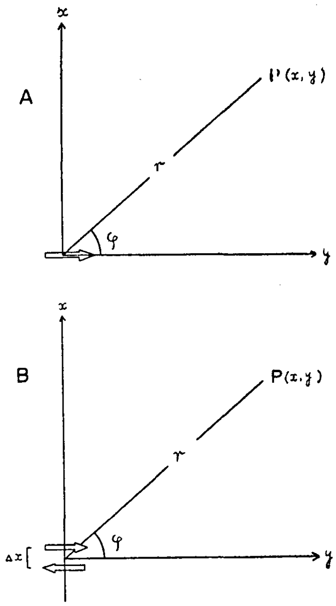

Fig. 1. A singlet force (A) and a couple (B). The x, y axes are taken on the surface of a half space.

$$i\kappa\sin\varphi\cos\varphi\cdot\exp i\left(\omega t-\kappa r-\frac{3}{4}\pi\right)$$

$$=\frac{1}{2}\ \kappa\sin 2\varphi\cdot\exp i\left(\omega t-\kappa r-\frac{\pi}{4}\right) \tag{4}$$

Thus the phase angle is greater by $\pi/2$ in the case of a couple than in the case of a singlet.

The double couple without momoment will show the same phase angle of Rayleigh waves as the single couple.

It has been emphasized by Honda (1954) that a double couple is equivalent, in the wave radiation pattern, to a radial force distributed on the surface, within a small circle around the origin, in the form of sin 2φ as the function of the azimuthal angle φ. The Rayleigh waves from such a source were studied by Nakano (1930). Nakano showed that the vertical displacement (we shall take the upward positive) at a long distance r, due to a radial force distribution of the form sin $n\varphi$ exp $i\omega t$ will be proportional to

$$\sin n\varphi \cdot \exp i \left\{\omega t - \kappa r + \frac{n}{2}\pi - \frac{5}{4}\pi\right\} \tag{5}$$

If $n=2$, the phase angle of the source function becomes $-\pi/4$, and is equal to that for the single couple (Eq. 4). Thus, a single couple, a double couple, and a radial force of four-lobed distribution give the same phase angle pattern, and we can not separate these three types from Rayleigh waves.

Lamb's source, which takes the form R exp $i\omega t$, represents the Fourier component of an impulse exerted at $t=0$. If an earthquake is a release of accumulated tectonic stress, a step function in time should be a better approximation than an impulse. Since the Fourier component of a step function is of the form $\frac{R}{i\omega}$ exp $i\omega t = \frac{R}{\omega}$ exp $i\left(\omega t - \frac{\pi}{2}\right)$, we must subtract $\pi/2$ from the phase angles for impulse source.

From the foregoing, we obtain the pattern of the phase angle of source functions as shown in Fig. 2A for a horizontal couple, which is a step function in time. We assume that this source represents a strike slip earthquake.

In the same manner, we obtain the pattern of phase angles for a dip slip earthquake as shown in Fig. 2B.

It is implicit in these patterns that the faults are vertical. Further, we must take into account the effect of focal depth as well as of the finiteness of the source, if necessary.

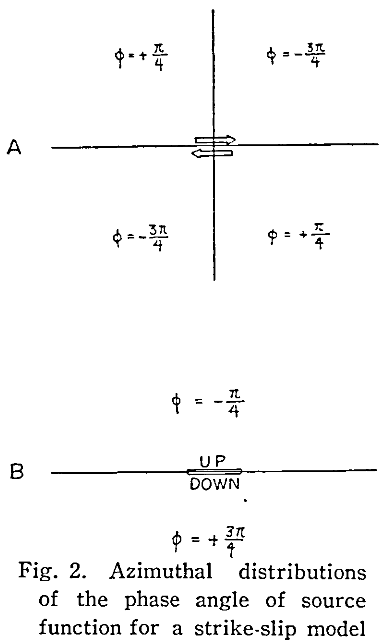

Fig. 2. Azimuthal distributions of the phase angle of source function for a strike-slip model (A) and for a dip-slip model (B).

3. Two methods of the phase equalization

The vertical displacement of Rayleigh waves from an earthquake may be expressed as

$$x(t)=\frac{1}{\pi}\int_0^\infty |X(\omega)| \cos\left(\omega t-\frac{\omega r}{c(\omega)}-\phi_{in}+\frac{m}{2}\pi+\phi\right) d\omega, \tag{6}$$

where,

$|X(\omega)|$: the absolute value of the Fourier transform of $x(t)$, $X(\omega)=\int_{-\infty}^{\infty} x(t)e^{-i\omega t}\, dt$,

$c(\omega)$: the phase velocity,

r : the epicentral distance,

ω : the angular frequency, $2\pi/\mathrm{T}$,

$\frac{\omega r}{c(\omega)}$: the phase delay due to propagation,

ϕ_{in} : the instrumental phase delay

$\frac{m}{2}\pi$: the polar phase advances introduced by Brune, Nafe and Alsop (1961), m being the number of polar or antipodal passages which the waves made,

ϕ : the phase angle of the source function.

Brune, Nafe and Oliver (1960) and Brune (1961) use the term "initial phase of Rayleigh waves." Since they reduce the problem of the wave generation in three to two dimensions, a phase advance of $\pi/4$ which occurs when the waves leave the source is taken into account. This initial phase ϕ_0 is related to our phase angle of source function ϕ by the following formula,

$$\phi_0=\phi-\frac{\pi}{4} \tag{7}$$

Two methods have been proposed for the phase equalization of the observed record $x(t)$ to obtain ϕ or ϕ_0. One is the Fourier analysis method, in which the Fourier analysis or the stationary phase analysis is used to obtain the phase angle φ of Fourier components of

$$x(t)=\frac{1}{\pi}\int_0^\infty |X(\omega)| \cos(\omega t+\varphi)\, d\omega \tag{8}$$

The comparison of this with Eq. 6 gives

$$\varphi\pm 2n\pi=-\frac{\omega r}{c(\omega)}-\phi_{in}+\frac{m}{2}\pi+\phi \tag{9}$$

This equation gives us ϕ directly, if the phase velocity, the epicentral distance and the instrumental phase delay are known. This method has been used by Sato (1955, 1956), Brune, Nafe and Oliver (1960), Brune (1961), Brune, Benioff and Ewing (1961) and others.

The alternative is the source function method, in which, first, the impulse response seismogram $g(t)$ is computed according to the equation

$$g(t)=\frac{1}{\pi}\int_{\omega_1}^{\omega_2}\cos\left(\omega t-\frac{\omega r}{c(\omega)}-\phi_{in}+\frac{m}{2}\pi\right)d\omega \tag{10}$$

The source function $y(\tau)$ is, then, computed by

$$\begin{aligned} y(\tau)&=\int_{-\infty}^{\infty}g(t)x(t+\tau)dt \\ &=\frac{1}{\pi}\int_{\omega_1}^{\omega_2}|X(\omega)|\cos(\omega\tau+\phi)d\omega \end{aligned} \tag{11)*$$

Since $|X(\omega)|$ is real positive, and usually uniform over the selected frequency range (ω_1, ω_2), it is possible to learn the value of ϕ with reasonable accuracy, by a visual inspection of the shape of $y(\tau)$. This method is a natural extension of Tukey's (1959) black box method, and has been used by Aki (1960a, 1960b, 1960c).

The Fourier analysis method is quantitative and straightforward in the determination of ϕ. On the other hand, the accuracy of visual determination of ϕ by the source function method, will not be better than $\pi/8$. However, the source function method has the great advantage that error in epicentral distance has very little effect on the determination of ϕ. Error in epicentral distance can be a very serious problem if the Fourier analysis method is used. A comparison of the errors liable in both methods is given in the next section.

4. Comparison of errors in the Fourier analysis method and the source function method.

The principal sources of errors in the determination of ϕ or ϕ_0 are the phase velocity and the epicentral distance. From Eq. 9, we have

$$\begin{aligned} \delta\phi&=\omega\delta\left(\frac{r}{c}\right) \\ &=\frac{\omega}{c}\delta r-\frac{\omega}{c^2}r\delta c \end{aligned} \tag{12}$$

This is the error formula in the determination of ϕ by the Fourier analysis method.

The source function will take the following form, when there are errors in r and c used in the computation of the impulse seismogram $g(t)$,

* The derivation of this equation is given in the Appendix.

$$y(\tau)=\frac{1}{\pi}\int_{\omega_1}^{\omega_2}|X(\omega)|\ \cos\left\{\omega\tau+\phi+\omega\delta\left(\frac{r}{c}\right)\right\}d\omega \tag{13}$$

The r/c in the above equation is the time required for travelling with the phase velocity, and may be called the phase delay time t_p of the wave medium. The t_p is related to the group delay time t_g (the time required for travelling with group velocity) by the following relation.

$$\begin{aligned} t_g &= r\frac{d(\omega/c)}{d\omega}, \\ &= t_p+\omega\frac{dt_p}{d\omega}, \\ &= t_p-T\frac{dt_p}{dT} \end{aligned} \tag{14}$$

If we expand $\delta\left(\frac{r}{c}\right)$ in Eq. 13 into a Taylor series of the variable T around $T_0\left(\frac{2\pi}{T_0}\right.$ being the centre frequency of the range $\left.(\omega_1,\ \omega_2)\right)$ we have $\delta\left(\frac{r}{c}\right)=(\delta t_p)_{T=To}+\frac{d(\delta t_p)}{dT}\ (T-T_0)+\cdots$.

Neglecting the terms higher than $T-T_0$, we have

$$\begin{aligned} \delta\left(\frac{r}{c}\right) &= (\delta t_p)_{T=To}-T_0\frac{d(\delta t_p)}{dT}+T\frac{d(\delta t_p)}{dT} \\ &= (\delta t_g)_{T=To}+T\frac{d(\delta t_p)}{dT} \end{aligned} \tag{15}$$

The first term on the right hand of this equation is the error in the group delay time at $T=T_0$. Since this term is constant, the effect of this error is simply to shift the position of $y(\tau)$ on the time axis without any change on its shape. Accordingly, this part of error does not affect the determination of ϕ by the source function method.

Putting τ' equal to $\tau+\delta t_g$, we have from Eqs. 13 and 15,

$$y(\tau')=\frac{1}{\pi}\int_{\omega_1}^{\omega_2}|X(\omega)|\ \cos\left\{\omega\tau'+\phi+2\pi\frac{d(\delta t_p)}{dT}\right\}d\omega \tag{16}$$

Thus, the error formula for the source function method corresponding to Eq. 12 for the Fourier analysis method is

$$\begin{aligned} \delta\phi &= 2\pi\frac{d(\delta t_p)}{dT} \\ &= 2\pi\delta\left(\frac{dt_p}{dT}\right) \end{aligned}$$

$$=2\pi\left\{-\frac{1}{c^2}\frac{dc}{dT}\delta r+\frac{2r}{c^3}\frac{dc}{dT}\delta c-\frac{r}{c^2}\delta\left(\frac{dc}{dT}\right)\right\} \tag{17}$$

By the use of these formulas, we shall compare both methods in the following four cases.

Case 1. Rayleigh waves which made two round trips of the earth,

$T=250$ sec $c=4.91$ km/sec $\frac{dc}{dT}=0.59\times10^{-2}$ km/sec^2 $r=80{,}000$ km

Case 2. Rayleigh waves which made one round trip of the earth,

$T=150$ sec $c=4.30$ km/sec $\frac{dc}{dT}=0.48\times10^{-2}$ km/sec^2 $r=40{,}000$ km

Case 3. Rayleigh waves which crossed the Pacific ocean,

$T=75$ sec $c=4.06$ km/sec $\frac{dc}{dT}=0.22\times10^{-2}$ km/sec^2 $r=10{,}000$ km

Case 4. Rayleigh waves which crossed North America,

$T=25$ sec $c=3.73$ cm/sec $\frac{dc}{dT}=2.4\times10^{-2}$ km/sec^2 $r=3{,}000$ km

Table 1 shows the deviations in the epicentral distance, in the phase velocity and in $\frac{dc}{dT}$ which independently produce the deviation of $+\pi/4$ in the value of the phase angle of the source function ϕ or of the initial phase ϕ_0. From this table, it is clear that a small error in the epicentral distance will affect the determination of ϕ by the Fourier analysis method. For instance, in case 2, in which the waves make a round trip of the earth, the ellipticity correction to the epicentral

Table 1 The deviations in epicentral distance r, the phase velocity c and $\frac{dc}{dT}$ which independently produce the deviation of $\pi/4$ in ϕ.

	Fourier analysis method		Source function method		
	δr in km	δc in km/sec	δr in km	δc in km/sec	$\delta\left(\frac{dc}{dT}\right)$ in km/sec^2
case 1	153	−0.0093	−500	0.0157	-0.38×10^{-4}
case 2	81	−0.0086	−482	0.0258	-0.58×10^{-4}
case 3	38	−0.015	−938	0.19	-2.1×10^{-4}
case 4	12	−0.015	−73	0.048	-5.8×10^{-4}

distance may become necessary. The difference between the meridional circumference and the equatorial one is 67.3 km, which is comparable to the value 81 km listed in Table I.

In cases 3 and 4, the accuracy of the epicentral location is required to be 38 km and 12 km respectively. This high accuracy seems difficult to obtain in most areas of the earth, except where a network of local stations is established.

On the other hand, when the source function method is used, the permissible error in the epicentral location is so large that the result of a rough preliminary determination can be safely used. This also permits us to use a small number of impulse response seismograms in the application of this method to many earthquakes.

As shown in Table I, the accuracy requirement in the phase velocity c is more severe in the Fourier analysis method than in the source function method. However, the accuracy requirement in $\frac{dc}{dT}$ is very strict in the source function method. Therefore, if the value of c is accurately known for a certain frequency, the application of the Fourier analysis method to that frequency component is recommended. On the other hand, if for some reason, the value of $\frac{dc}{dT}$ for a certain frequency range is dependable, the source function mothod is recommended. In any of the cases in Table I, the accuracy requirement to the phase velocity data is very strict.

5. Discussions on some previous results

In a previous paper (Aki 1960b), the source functions of about 50 circum Pacific earthquakes were obtained from Rayleigh waves recorded at Pasadena. In the computation of the source function, the phase velocities for case 8099 of Dorman, Ewing and Oliver (1960) were used and the Pacific ocean is assumed as being uniformly covered by this model. The source functions are interpreted according to the simple generalization of Lamb's result as described in the present paper. We found that the result of the interpretation is consistent with (1) the abundance of strike slip earthquakes as revealed from the fault plane works, (2) the greater amplitude ratio of G waves to Rayleigh waves for strike slip earthquakes than for dip slip earthquakes, and (3) the so called "San Andreas hypothesis" of the circum Pacific tectionics.

We extended our method (Aki, 1960c) to the records of many IGY stations equipped with the Columbia type long period seismographs for three earthquakes in the circum Pacific belt, and we found that the azimuthal distributions of source functions are quadrant as expected. Additional data from the aftershocks of the Chilean earthquake of May 22, 1960 also supported the "San Andreas hypothesis."

Recently, it was recognized that the curvature of the earth affects appreciably the phase velocities of Rayleigh waves even in the short period range. Bolt and Dorman (1961) studied this effect in a period range from 300 sec to 25 sec for various models of a spherical gravitating earth. They give the following convenient formula for the curvature correction,

$$c = c_h(1 + 0.00016T) \tag{18}$$

where c is the phase velocity for the spherical gravitating earth model and c_h is that for the flat earth model with the identical layer parameters. According to Bolt and Dorman, this equation allows an estimation of c within 1% for $100 < T < 300$ sec from values computed for a flat earth model.

Case 8099 of Dorman, Ewing and Oliver, on which our previous study of the source function is based, is a flat earth model, and must be corrected for the curvature effect. Since our source functions were obtained for $35 < T < 150$ sec, a slightly different formula of curvature correction is used. The formula is

$$c = c_h(1.0034 + 0.00012T) \tag{19}$$

which was obtained to fit the values obtained by Bolt and Dorman for the above period range.

The correction to c_h according to Eq. 19 has negligible effect on the phase angle of the source function, but the correction to $\frac{dc_h}{dT}$ cannot be neglected. From Eq. 19, $\left(\frac{d(c-c_h)}{dT}\right)$ is deduced as being about 5×10^{-4} km/sec^2. This is about twice the value for case 3 of Table I which is the most representative of those studied in our paper (Aki, 1960b). Since Table I shows the deviation in $\frac{dc}{dT}$ which produces the deviation of $+\pi/4$ in the phase angle, the curvature correction to the phase angle of our source function amounts to $-\pi/2$. Old values of phase angles must be reduced by about $\pi/2$. This should lead to a serious modification of the conclusions given in our papers. We have reinterpreted all the source functions previously obtained, taking the curvature correction into account. We found that the new result is inconsistent with our generalization of Lamb's result, and the azimuthal distribution of the source functions does not show the quadrant pattern expected from a simple theory.

There is, however, one possibility for avoiding these grave modification in the conclusions. That is, the possibility that the phase velocities for case 8099 (flat earth model) may be closer to the true velocities than those for the curvature corrected 8099. Fig. 3 shows

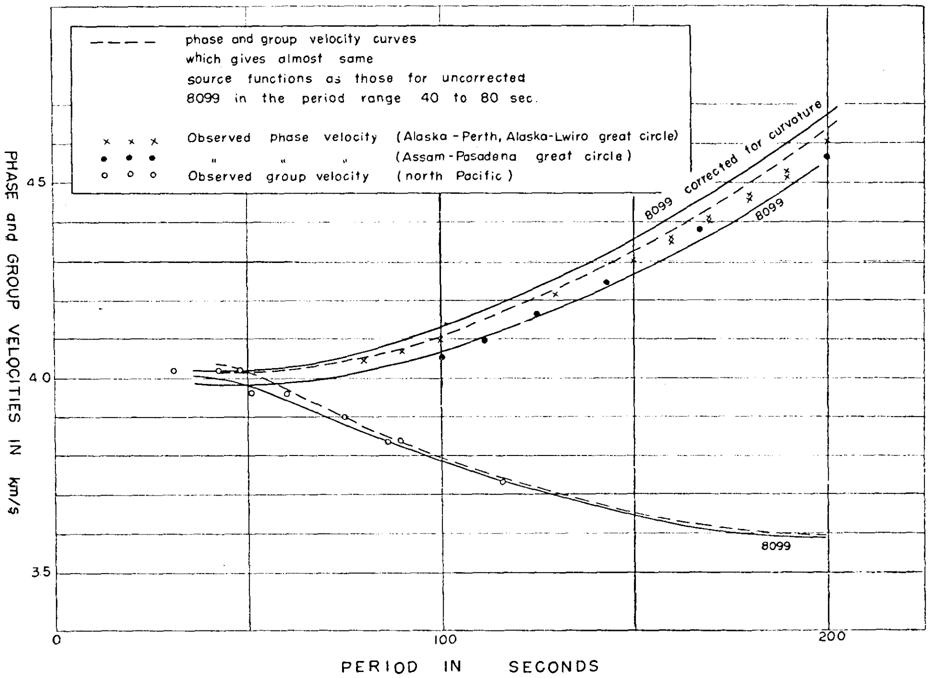

Fig. 3. Group and phase velocities of Rayleigh waves. Observed phase velocities are obtained by Brune (1961) and Brune, Nafe and Alsop (1961). Observed group velocities are obtained by Sutton, Major and Ewing (see Dorman etal (1960)). It seems that the observed phase velocities lie more parallel to, and also closer to, the curve for the uncorrected 8099 than for the curvature corrected.

the phase and group velocity curves (solid line) for both cases. Since the group velocities differ only slightly between them, only those for the flat earth model are shown. Observed phase and group velocities are also plotted in the figure. The observed group velocities are for the north Pacific, but the phase velocities are for several great circles. It can be seen that observed phase velocities lie more closely to the 8099 for the flat model than to the 8099 with the curvature correction. As stated in the preceding section, the only serious source of error in the source function method is $\frac{dc}{dT}$. The fact that the observed points (except for the Assam Pasadena great circle) lie nearly parallel to the phase velocity curve for the 8099 without curvature correction suggests that old values of the phase angle of the source functions may be nearly correct. This implies that the layer parameters in case 8099 should be modified so that the phase velocity with curvature correction agrees with the observed.

The dashed phase velocity curve in Fig. 3 is drawn parallel to the curve for the 8099 without curvature correction, in the period range 35 to 80 sec. Since the spectrum of our source function is mostly in this period range, this curve also gives the source function as almost the same as the 8099 without curvature correction. The corresponding group velocity curve is also shown by a dashed line, and agrees with the observed as well as that for 8099. If the true phase velocities lie parallel to and between the dashed curve and the curve for 8099 without curvature correction, we need not modify our conclusion given in the previous papers. At present, however, we cannot draw any firm conclusions on this problem, because we do not have a precise measurement of phase velocity for the purely Pacific path.

Brune's (1961) result on the radiation pattern of Rayleigh waves from the South East Alaska earthquake of July 10, 1958 do not agree with our generalization of Lamb's theory.* He suggests that the earthquake source may be an impulse rather than a step function in time. However, the accuracy in his determination of the initial phase seems not high enough to draw a definite conclusion on this problem. (His case corresponds roughly to cases 1 and 2 of Table 1)

The high accuracies required for the phase velocity data as shown in Table I may seem formidable, and one may suspect that it is impossible to gain an accurate determination of the earthquake mechanism from surface waves. However, these high accuracies have been already attained in the *P* travel times. We may, therefore, expect that a proper network of long period seismograph stations would give us the required accuracy in the phase velocity data.

* Recently Brune (personal communication) has found that a revision of the result on this earthquake is necessary, and the revised result agrees with our generalization of Lamb's theory.

If the earth's surface is so far from uniform that the use of surface waves in the determination of the mechanism is impossible for those cases listed in Table I, we may still apply our method to the waves of shorter epicentral distances, because the errors in the phase angle of the source function are proportional to the distance. The writer believes that study on the long period Rayleigh pulse from near earthquakes, for which the fault plane solutions are known from P waves, will, give us another way to the solution of our problem.

Appendix

Since $g(t)$ and $x(t)$ are real and their Fourier transforms $X(\omega)$ and $G(\omega)$ satisfy the following relations,

$$X(-\omega)=X^*(\omega),$$
$$G(-\omega)=G^*(\omega),$$

where $G^*(\omega)$ and $X^*(\omega)$ are the complex conjugates of $G(\omega)$ and $X(\omega)$ respectively, we have the following formulas from the theorem on the Fourier transform,

$$g(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} G(\omega)e^{i\omega t}\,d\omega=\frac{1}{\pi}\int_0^{\infty}|G(\omega)|\cos(\omega t+\varphi_1(\omega))\,d\omega$$

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{i\omega t}\,d\omega=\frac{1}{\pi}\int_0^{\infty}|X(\omega)|\cos(\omega t+\varphi_2(\omega))\,d\omega$$

$$G(\omega)=|G(\omega)|e^{i\varphi_1(\omega)}=\int_{-\infty}^{\infty} g(t)e^{-i\omega t}dt=\int_{-\infty}^{\infty} g(t)\cos\omega t\,dt-i\int^{\infty} g(t)\sin\omega t\,dt$$

$$X(\omega)=|X(\omega)|e^{i\varphi_2(\omega)}=\int_{-\infty}^{\infty} x(t)e^{-i\omega t}dt=\int_{-\infty}^{\infty} x(t)\cos\omega t\,dt-i\int^{\infty} x(t)\sin\omega t\,dt$$

Eq. 11 can be obtained as follows,

$$\begin{aligned}
y(\tau)&=\int_{-\infty}^{\infty} g(t)\,x(t+\tau)\,dt\\
&=\frac{1}{\pi}\int_{-\infty}^{\infty} g(t)dt\int_0^{\infty}|X(\omega)|\cos\{\omega(t+\tau)+\varphi_2(\omega)\}\,d\omega\\
&=\frac{1}{\pi}\int_0^{\infty}|X(\omega)|\{|G(\omega)|\cos\varphi_1(\omega)\cdot\cos(\omega\tau+\varphi_2(\omega))+|G(\omega)|\sin\varphi_1(\omega)\cdot\\
&\qquad \sin(\omega\tau+\varphi_2(\omega)\}\,d\omega\\
&=\frac{1}{\pi}\int_0^{\infty}|X(\omega)|\cdot|G(\omega)|\cos(\omega\tau+\varphi_2(\omega)-\varphi_1(\omega))d\omega\\
&=\frac{1}{\pi}\int_0^{\infty}|r(\omega)|\cos(\omega\tau+\varphi_3(\omega))d\omega.
\end{aligned}$$

Thus, the absolute value of the Fourier transform of $y(\tau)$ is the product of those of $x(t)$ and $g(t)$, and the phase angle for $y(\tau)$ is the difference between those for $x(t)$ and $g(t)$.

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4. レーリー波を用いて発震機構を求める方法の精度について

地震研究所 安 芸 敬 一

長周期表面波を解析して，地震の震源に関する情報を求める試みはいくつか行われている．これらの解析は，結局，現存の表面波発生理論が半無限平面に関するものであるので，それと比べることができるように，実際の記録から，分散性媒質，地球の曲率，polar phase shift などの影響をとり除くことを目的としている．この論文では，現在行われている2つの解析法，フーリエ分析法と，source function 法とについて，現在どの程度の精度で，震源についての情報が得られ得るかを議論した．津波警報に関連して，レーリー波を用いて，地震が水平断層によるものか垂直断層によるものかを定めることの可能性についても論じた．現在われわれのもつている知識からは，この可能性は，やや無理である．しかし長周期地震観測の急速に発達しつつある現在，この可能性も近い将来実現されるであろう．