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25. *Surface Waves and Layered Structures. Part 1. Influence of a low velocity layer and some study on Lg and Rg waves.*

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1. Introduction

The nature of surface waves has been investigated by many seismologists from various points of view.[1)-2)] Amongst others, the dispersion of these waves has become one of the most interesting problems in these days.[3)] The reason for this current trend of study lies in the fact that the clarification of regional variation in the crustal structure of the earth has become a central problem of seismology and the usual approach to this problem through the study of travel-time curves of bodily seismic waves in respective parts of the world is hindered by the paucity of adequate seismographic stations, while the study of only a few relevant seismograms of surface waves, transmitted through the region in question, enables us to gain a clue to the problem, because the dispersion of these waves is strongly influenced by the crustal structures concerned. The possibility of the latter approach has hitherto been proved by theoretical calculations as well as actual observations. But there is no method of obtaining a unique solution of the crustal structure from observations of dispersion curves of Lr and Lq waves, since much difficulty has been felt by each investigator in finding even an approximate inference of the actual structure to comply with the observations. Although some investigators have gone so far as to calculate dispersion curves in the cases of complicated crustal structures composed of tens of different layers in accordance

1) Y. SATÔ, "On Elastic Surface Waves," *Zisin*. [ii], **6** (1953), 13.

2) M. EWING, W. JARDETZKY, and F. PRESS, *Elastic Waves in Layered Media*, (New York: McGraw-Hill, 1957).

3) e.g. J. DORMAN, M. EWING, and J. OLIVER, "Study of the Shear Velocity Distribution in the Upper Mantle by Mantle Rayleigh Waves," *Bull. Seism. Soc. Amer.*, **50** (1960), 87.

with the known earth's structure to explain average dispersion curves. But accurately speaking, there are no true average curves of dispersion for the whole earth, and in addition, the regional variation of them is so remarkable that we have first to derive regional dispersion curves and then find corresponding crustal structures of the first approximation before going into further details.

For such a purpose, we must find the general characteristics of dispersion curves of respective waves of corresponding to particular crustal structures, and then find the main factors whose variation can cover wide variation in velocity, and the other particular features of dispersion curves and the spectral characteristics of the amplitudes, etc.

Recently, long period seismographs of high sensitivity have been accomplished, and long surface waves with periods of hundreds or a thousand seconds are being clearly observed, while remarkable progress of with high speed electronic computers encourage us to proceed into detailed studies involving complicated calculations which would be almost impossible otherwise. In view of these circumstances, the writer ventured to make some contribution to the above mentioned problem by some study on the influence of low velocity layers in the earth's crust on surface waves, and by presenting a systematic calculation of dispersion curves of surface waves covering wide variations of conceivable parameters of the physical constants of the layers along which these waves are propagated.

This paper contains parts of the writer's study touching on such line, and is composed of two parts, the one dedicated to the study of the effect of a low velocity layer on surface waves and some study on Lg and Rg waves, and the other pertaining to the systematic calculation of the dispersion curves of Lr and Lq waves in various layered structures below the ocean bottom.

2. Love waves propagated along the surface of a stratified medium with a low velocity layer.

Our knowledge on the internal structure of the earth has recently advanced from the hypothesis of continuous monotonous increase in velocity v or v/r (r being the radius vector from the earth's centre) up to the core boundary which information was known in the early years of this century. It has been revealed that the crustal structure is almost certainly subjected to regional variation while the more generally prevalent structure of the mantle also includes some peculiarities other than there hitherto recognized. Some seismologists proposed the hypothesis of the existence of the first order discontinuity while others suggested a minimum in the velocity distribution (a low velocity layer) in the mantle. The existence of two other low velocity layers in the crust has also been pointed out. But crucial tests have yet to be made from points of view other than the time distance curves P and S waves. Verification of the latter hypothesis has recently been made by a comparison of the dispersion curve of the mantle Rayleigh wave as calculated on the suggested earth's structure and that actually observed,[3] and the result seems to be very encouraging.

We have already obtained the velocity equation of Love waves propagated along the surface of a multi-layered media.[4] We shall now examine the effect of a low velocity layer on the dispersion of Love waves and their amplitude distribution within the layers for a reference to the above mentioned verification.

Here we shall study the case where three layers overlie on a semi-infinite medium, and for simplicity, we will assume that the thicknesses of these layers are equal $(=H)$ to each other, while the rigidities in the layers and the subjacent medium are in the ratios 1 : 3 : 2 : 4 respectively and their densities are equal; that is, the third layer is a low velocity layer.

The relevant velocity equation in this case is at once derived from our general formula as follows:

$$\begin{aligned}&\{c_1c_2-\chi_{12}(\tilde\beta_1/\tilde\beta_2)s_1s_2\}\{c_3-\chi_{34}(\tilde\beta_3/\beta_4)s_3\}\\&-\{\chi_{23}(\tilde\beta_2/\tilde\beta_3)c_1s_2+\chi_{13}(\tilde\beta_1/\tilde\beta_3)s_1c_2\}\{s_3+\chi_{34}(\tilde\beta_3/\beta_4)c_3\}\\&=0\end{aligned}$$

where $c_k=\cos\tilde\beta_k H_k\ ,\qquad s_k=\sin\tilde\beta_k H_k\ ,$

$\beta_k=\sqrt{f^2-p^2/V_k^2}=i\tilde\beta_k\ ,\qquad \chi_{jk}=\mu_j/\mu_k\ ,$

H_k = thickness of k th layer,

V_k = shear volocity of k th layer,

μ_k = rigidity of k th layer,

f = wave number,

p = frequency.

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4) Y. SATÔ and R. YAMAGUCHI, "Velocity Equation of Love Waves Propagated in Multi-layered Media," *Zisin*, [ii], **12** (1959), 61.

The results of the calculation are represented in Figs. 1 through to 9. Dispersion curves in Fig. 1 show the existence of a very remarkable new branch C other than those of the higher modes (II and III) to the fundamental one (I). The phase velocity of this C branch approaches the shear wave velocity in the low velocity layer as the period decreases to zero. It is also remarkable that the group velocities of the second mode (II) and the branch C, as indicated by the broken lines in the same figure have almost stationary values nearly equal to the

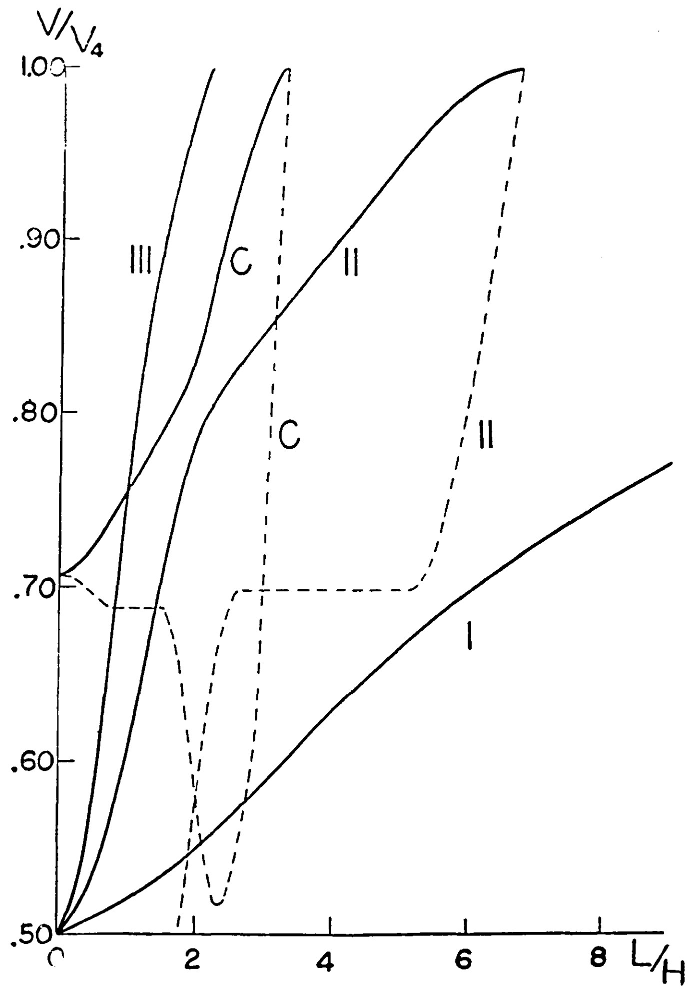

Fig. 1. Phase (solid line) and group (broken line) velocity dispersion curves of Love waves for the case $\mu_1 : \mu_2 : \mu_3 : \mu_4 = 1 : 3 : 2 : 4$, $\rho_1=\rho_2=\rho_3=\rho_4$, $H_1=H_2=H_3=H$.

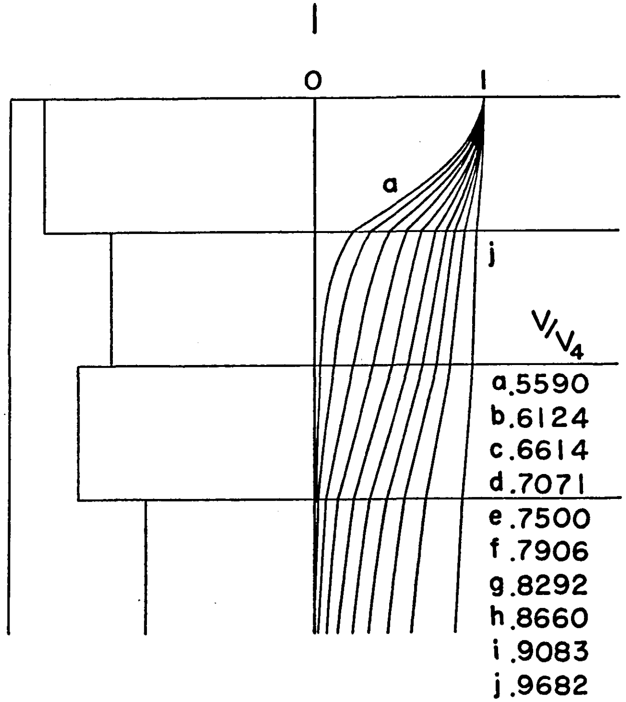

Fig. 2. Amplitude distributions of branch I for various phase velocities.

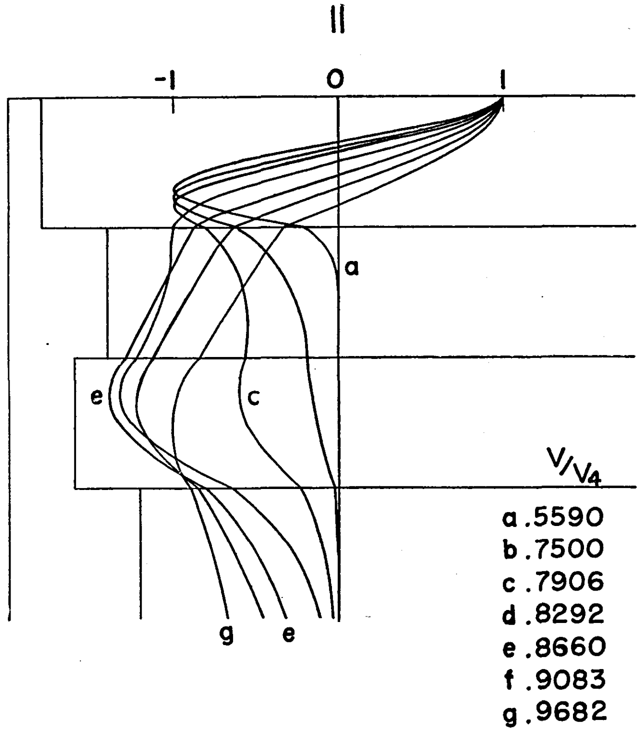

Fig. 3. Amplitude distributions of branch II for various phase velocities.

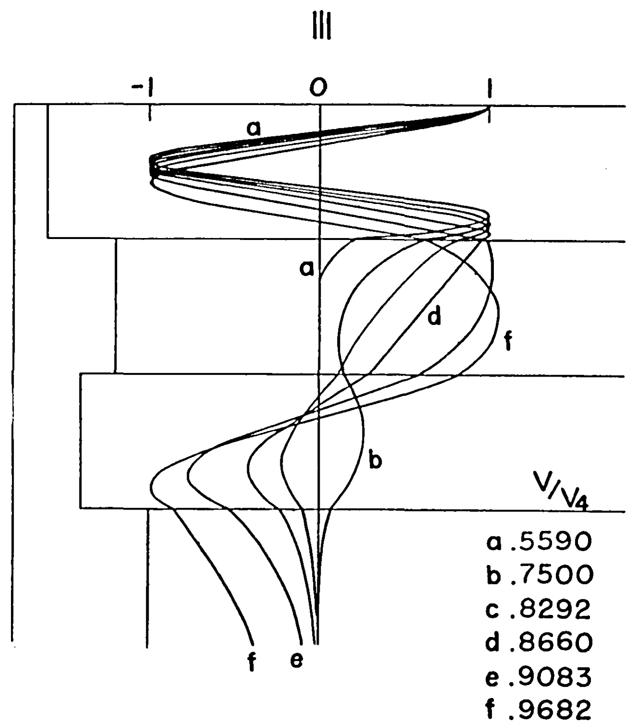

Fig. 4. Amplitude distributions of branch III for various phase velocities.

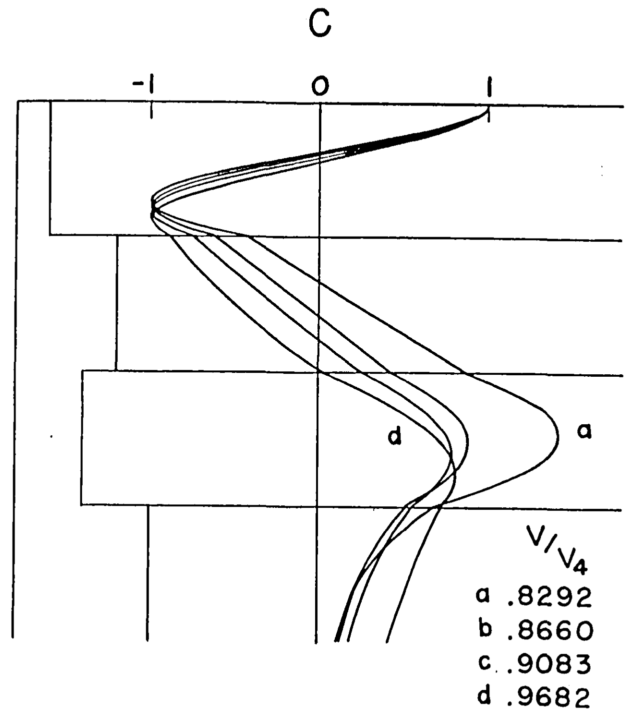

Fig. 5. Amplitude distributions of branch C for various phase velocities.

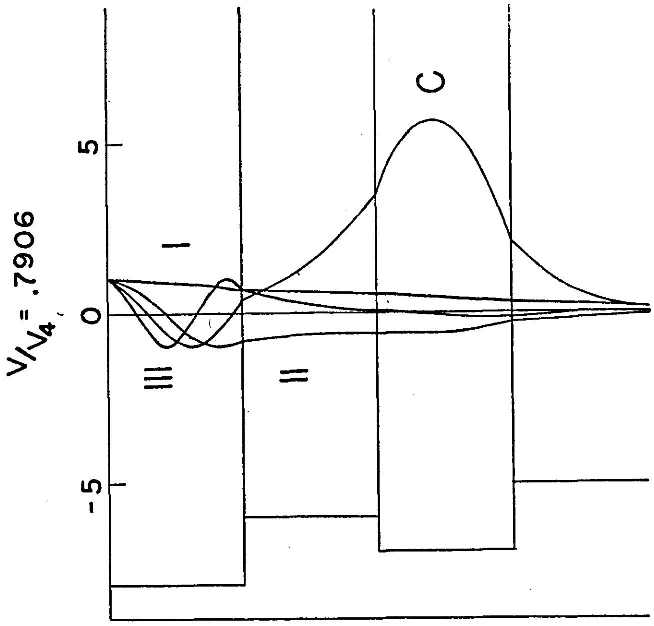

Fig. 7. Comparisons of amplitude distribution of each branch at $V/V_4=0.7906$.

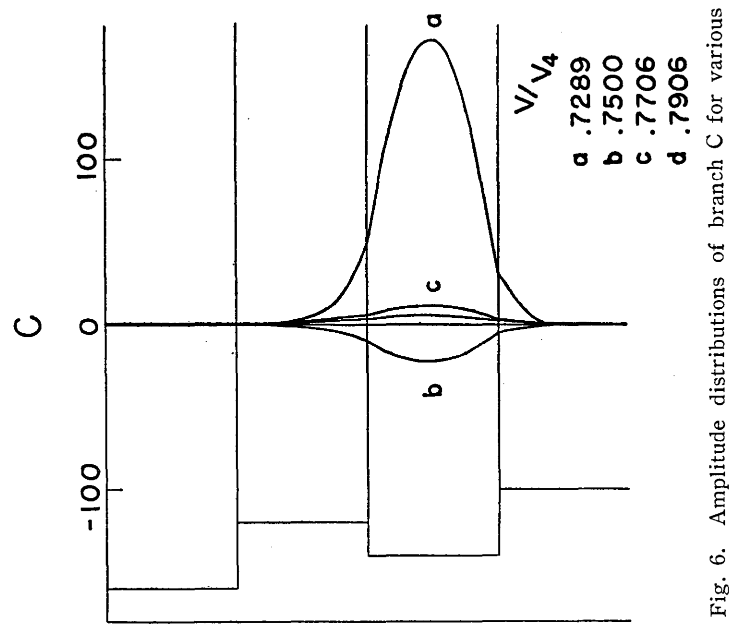

Fig. 6. Amplitude distributions of branch C for various phase velocities.

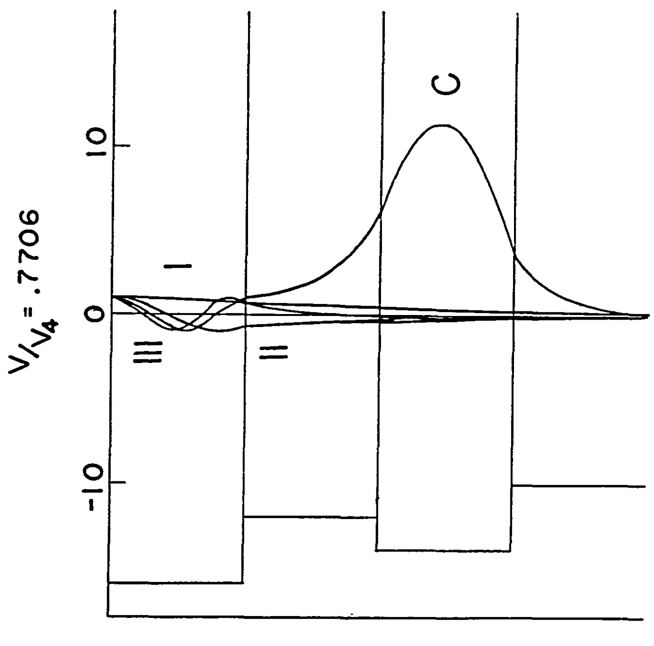

Fig. 8. Comparisons of amplitude distribution of each branch at $V/V_4 = .7706$.

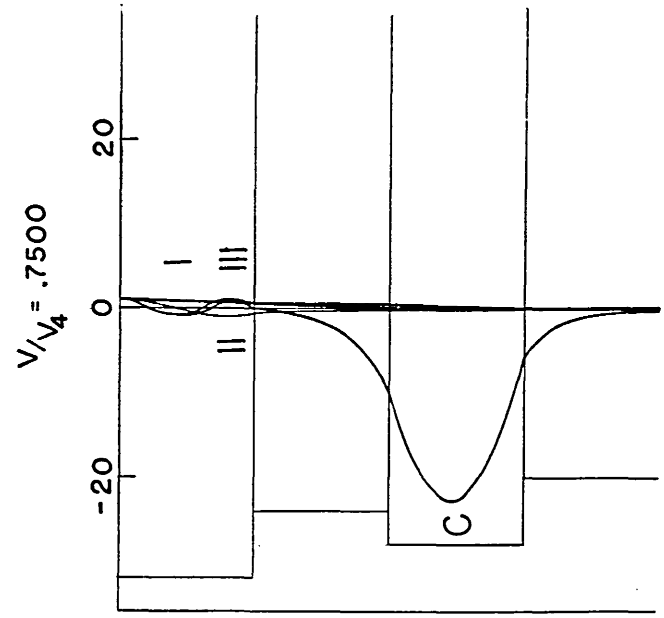

Fig. 9. Comparisons of amplitude distribution of each branch at $V/V_4 = .7500$.

low velocity in a considerable range of wave-lengths.

The next five figures from Fig. 2 to Fig. 6 indicate the amplitude distribution within the layers of the above mentioned branches, corresponding to the various phase velocities.

For branch II we see that the amplitude in the low velocity layer becomes maximum for the wave-length range at which the group velocity is stationary and nearly equal to the low velocity. The influence of the low velocity layer is most marked in the branch C as is seen from Fig. 7 to Fig. 9. In these figures, the distributions of the amplitudes of these branches, in the layers, are compared at certain levels of phase velocity. In the range of phase velocity near the low velocity, the amplitude of the C branch alone becomes very large in the low velocity layer. This branch is to be interpreted as a channel wave through the low velocity layer.

Although the present result is based on a simplified assumption and cannot, as it stands be applied to the actual problem, the general features revealed here will be of some value in the interpretation of actual observations, and will also be profitably referred to in more concrete studies, whenever a low velocity layer is taken into consideration at all.

As an example, some comments on the results we have already reported, will be made in the following.

3. Rayleigh waves propagated along the stratified medium with an internal low velocity layer.

The writer calculated in collaboration with T. Kizawa some examples of the present topic,[5)–6)] although the case treated is too specific for the present purpose because the calculation were made with a view to elucidating the characteristics of actually observed novel phases of volcanic earthquakes, as observed on the path across a shallow bay. It was thus assumed in the study, that the higher velocity surface layer is composed of water, and a sedimentary low velocity layer intervenes between the water layer and the usual granitic layer. The depth of the bay (100 m) is taken as the thickness of the water layer, and the thickness and the velocity of the low velocity layer were changed as parameters. And

5) T. KIZAWA, "Some New Phases Observed in a Study of Earthquake Swarms Relating to Volcanic Activity (I)," *Geophys. Mag.*, **29** (1960), 477.

6) T. KIZAWA and R. YAMAGUCHI, "Some New Phases Observed in a Study of Earthquake Swarms Relating to Volcanic Activity (II)," *Geophs. Mag.*, **30** (1960), 93.

the phase velocities and group velocities of Rayleigh waves in the media were calculated and compared with the observations. We could thus conclude that the observed novel third and fourth phases which accompanied the Usu volcanic earthquakes are to be explained by the fundamental and second modes of Rayleigh waves propagated along the medium of hypothetical structure, with a low velocity layer of velocity $V_p=1$ km/sec, $V_s=0.6$ km/sec and with a thickness of about 300–400 m.

The results of these calculations do not enable a direct comparison with the results mentioned in section 2 of the present paper, but we may almost certainly verify the last conclusion in section 2.

4. Lg and Rg waves

Of late years, the natures of the Lg and Rg phases have been actively discussed,[7)–18)] but no definite conclusion has yet been obtained. It seems to the writer that the segments of the stationary group velocity of the branches of II and *C* mentioned in section 2 may play some part in the observed Lg phase if a low velocity layer actually exists. There may also be a similar phenomenon in Rayleigh waves in similar circumstances, which is relevant to the interpretation of the Rg phase.

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7) F. PRESS, M. EWING, "Two Slow-Surface Waves across North America," *Bull. Seis. Soc. Amer.*, **42** (1952), 219.

8) I. LEHMANN, "On the short period surface wave 'Lg' and crustal structure," *Bull. d'Information U.G.G.I.*, **2**. annee, (1953), 248.

9) M. BÅTH, "The Elastic Waves Lg and Rg along Euro-Asiatic Paths," *Arkiv for Geofysik*, **2** (1954), 295.

10) B. GUTENBERG, "Channel Waves in the Earth's Crust," *Geophysics*, **20** (1955), 283.

11) J. OLIVER, M. EWING and F. PRESS, "Crustal Structure of the Arctic Regions from the Lg Phase," *Bull. Geol. Soc. Amer.*, **66** (1955), 1063.

12) F. PRESS, M. EWING and J. OLIVER, "Crustal Structure and Surface Wave Dispersion in Africa," *Bull. Seism. Soc. Amer.*, **46** (1956), 97.

13) M. BÅTH, "Some Consequences of the Existence of Low-Velocity Layers," *Estratto da Annali di Geofisica*, **9** (1956), 411.

14) M. BÅTH, "A Continental Channel Wave, guided by the Intermediate Layer in the Crust," *Geofisica Pura e Applicata*, **38** (1957), 19.

15) I. LEHMANN, "On Lg as Read in North America Records," *Annali di Geofisica*, **10** (1957), No. 1-2, 1.

16) J. OLIVER and M. EWING, "Higher Modes of Continental Rayleigh Waves," *Bull. Seism. Soc. Amer.*, **47** (1957), 187.

17) J. OLIVER and M. EWING, "Normal Mode of Continental Surface Waves," *Bull. Seism. Soc. Amer.*, **48** (1958), 33.

18) T. UTSU, "On the Lg Phase of Seismic Wave Observed in Japan (1)," *Quart. Jour. Seism. Japan Meteor. Agency*, **23** (1958), 61.

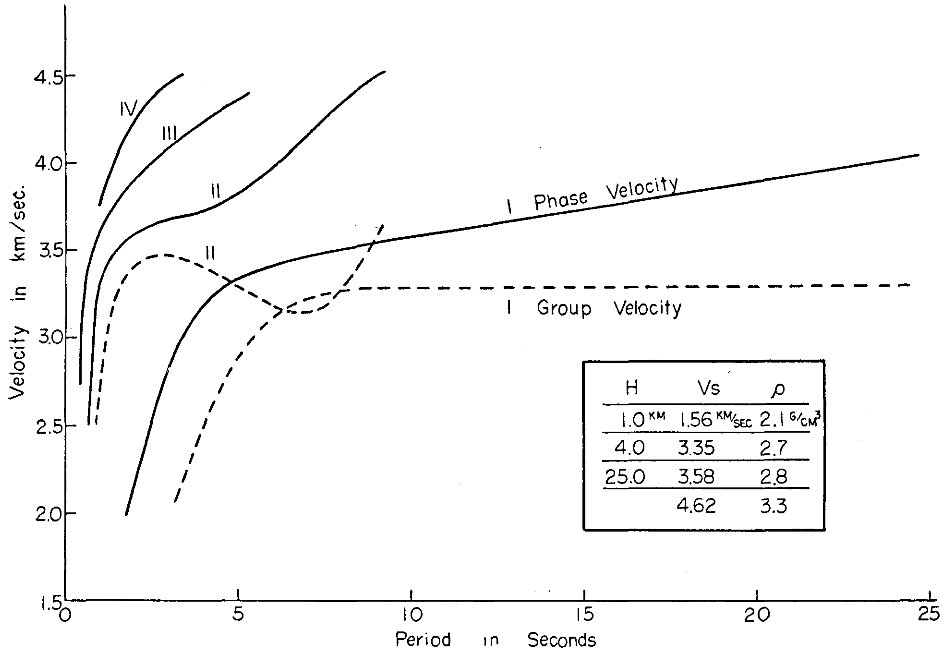

Fig. 10. Phase (solid line) and group (broken line) velocity dispersion curves of Love waves.

On the other hand, the Lg phase will be interpreted when the intermediate layer, with shear wave velocities of 3.5–3.7 km/sec has a thickness as great as five or more times of that of the upper layer.

As an example, we will introduce the model shown in Fig. 10. That model is one of the five models which were tried by the Research Group for Explosion Seismology in Japan, to decide the crustal structure in the northern Kwanto district. From Fig. 10 we can see that a maximum and a minimum group velocity of IInd mode appear in the shorter period range (2 to 7 sec.) and the group velocity of the fundamental mode has a stationary value (about 3.3 km/sec) at periods longer than about 7 sec.

It seems to the writer that the Lg_1 phase corresponds to the maximum and the Lg_2 phase is in agreement with the minimum and the stationary group velocity, since the amplitude at its phase, takes a large value. Perhaps the observed Lg phase in various places will be explained by the models conform to the example as mentioned above.

There may also be a similar phenomenon in the Rayleigh waves. However the fundamental mode brings Rg waves into actual existence in the following.

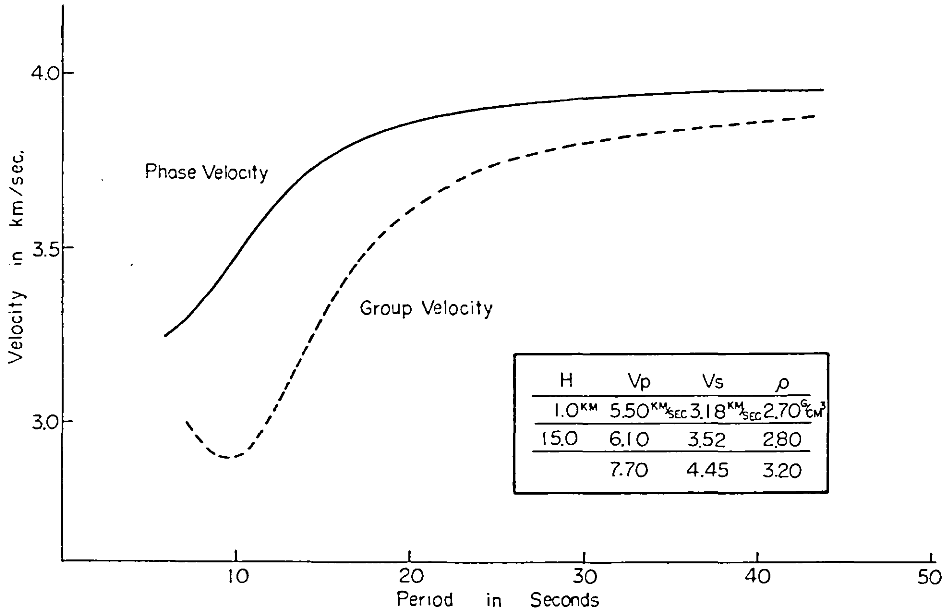

Fig. 11. Phase (solid line) and group (broken line) velocity dispersion curves of Rayleigh waves.

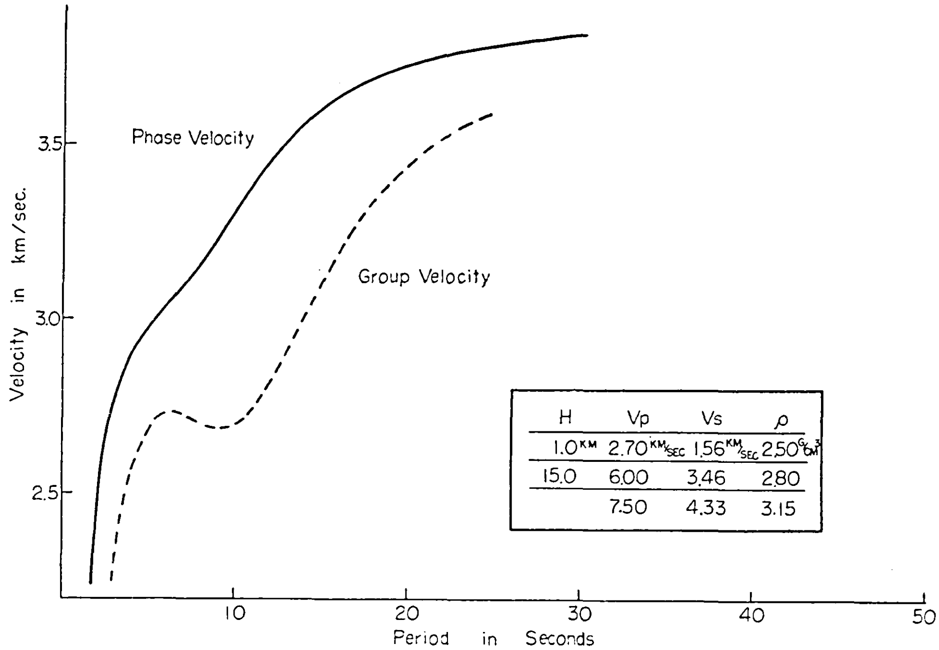

Fig. 12. Phase (solid line) and group (broken line) velocity dispersion curves of Rayleigh waves.

The crustal structures shown in Fig. 11 and Fig. 12 are the models tried also by the same group mentioned above in searching for the structure of the northern Kwanto area. We can see that the Rg phase will be not interpreted from those dispersion curves, even if the sedimentary layer is added as in Fig. 12. But the appearence of its phase may be possible by a change in the thickness and the velocities of the intermediate layer.

As mentioned above, it seems to the writer that the structure of the intermediate layer is decided to some extent from the observed Lg or Rg waves.

The existence or not of Lg waves with periods shorter than 2 sec. at a distance, enable us to gain a clue as to the existence of a low velocity layer.

We can see from Fig. 13 to Fig. 16 that a maximum and a minimum group velocity appear in the shorter period ranges when the thickness of the water layer is considerably shallower than that of the intermediate layer.

But in reality the structure which gives rise to the above mentioned

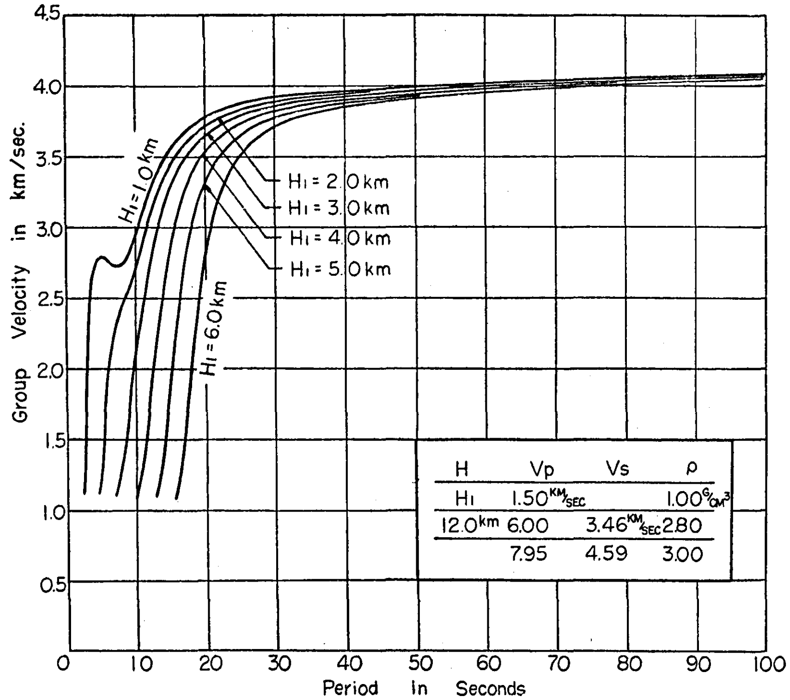

Fig. 13. Group velocity dispersion curves of Rayleigh waves.

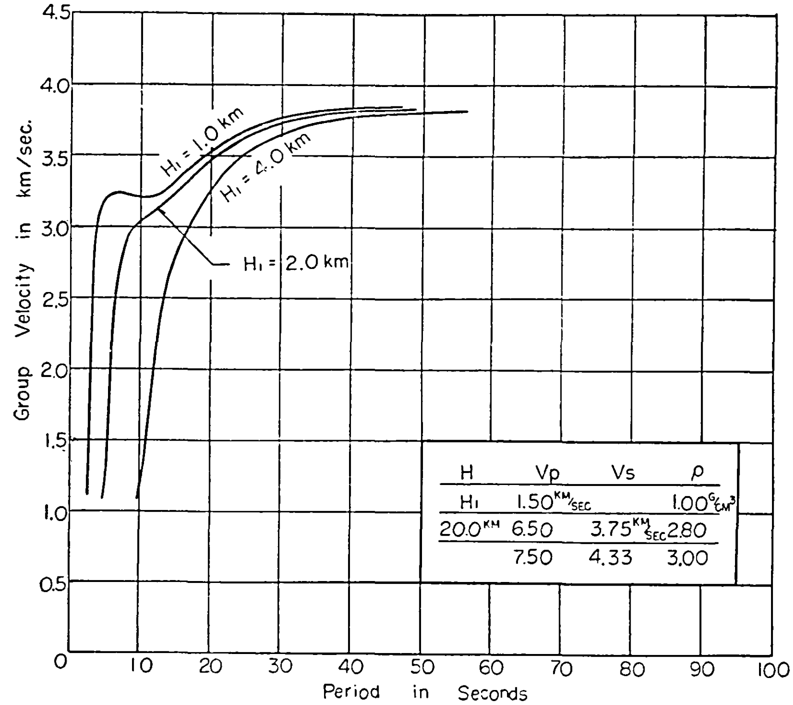

Fig. 14. Group velocity dispersion curves of Rayleigh waves.

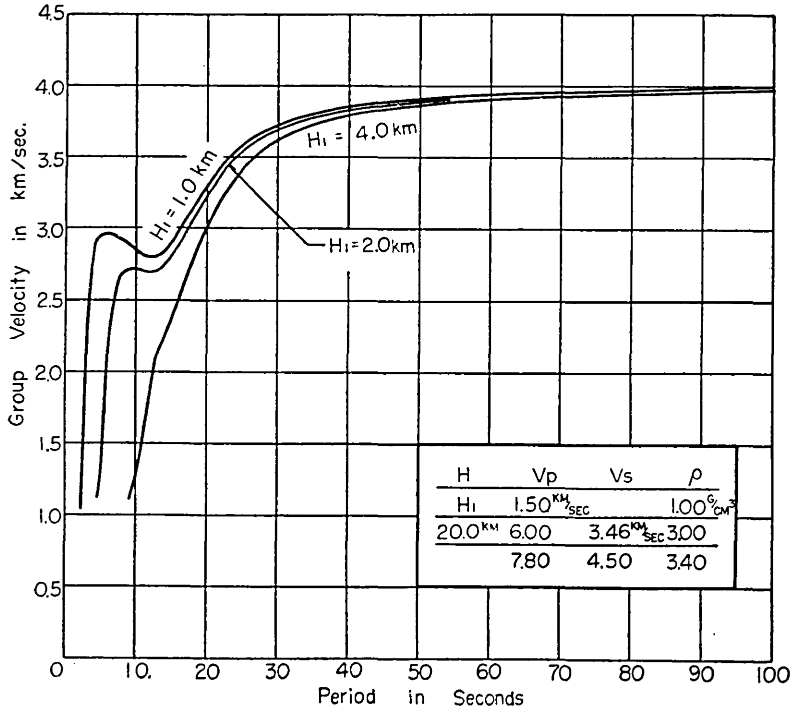

Fig. 15. Group velocity dispersion curves of Rayleigh waves.

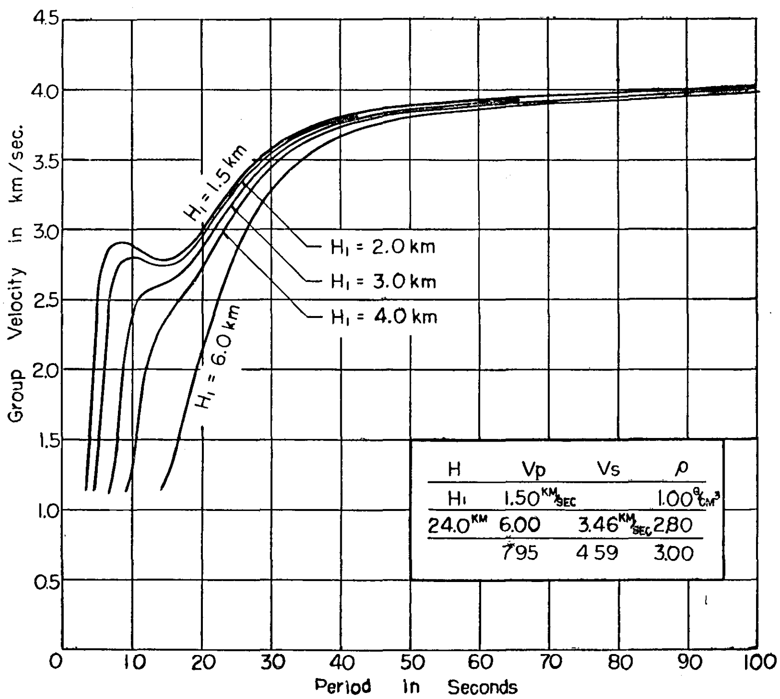

Fig. 16. Group velocity dispersion curves of Rayleigh waves.

peculiarity is to be interpreted as typically continental rather than oceanic, because the intermediate layer with a shear wave velocity of 3.5 km/sec has a thickness as great as 10 or more times that of the water layer. There may be some cognate reason for elucidating the observed Rg phase by the Rayleigh wave with the above mentioned maximum or minimum group velocity. It is therefore important to examine the crustal structure, where the Rg phase was observed as to whether it is in accordance with the theoretical requirement.

From the observations of Lg phases, it has been found that the crustal structure is typically continental without an exception in any large area where the water depth is less than about 2 km, whereas the contrary is the case along the path across the seas with depths of more than 3.5 km, where typically oceanic structures are expected to predominate. These facts are in harmony with the above mentioned condition for the appearance of Rg phase in our interpretation. Needless to say, the appearence of Lg waves postulates the existence of a sedimentary layer overlying the intermediate layer.

Acknowledgements

The author wishes to thank Prof. H. Kawasumi for valuable help and advice in carrying out this study. His thanks are also due to Prof. Y. Satô for his guidance and encouragement.

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25. 表面波と層構造 (1)

低速層の影響および Lg と Rg について

地震研究所　山　口　林　造

1. 最近における電子計算機の発達にともない，表面波と層構造の研究についても，詳しく議論されるようになつてきた．ここでは低速層の存在する場合，表面波の分散にどのような特徴が現われるかを検討した．簡単な構造に限つたのは手動計算のためである．また Lg および Rg 相について，具体的にどのような構造によつて説明できるかを示した．

2. 一般に低速層の存在する場合における Love 波の分散および振巾分布を簡単な構造を使つて計算した．すなわち，半無限層の上に3層をおき，剛性率の比を 1：3：2：4 にとり，層の厚さおよび密度は等しいとした．したがつて，第3層が低速層になる場合である．注目すべき結果は第1図に示されているごとく，新らしい分枝Cの存在することである．この分枝Cは周期が短かくなると，その位相速度は低速層の速度に近づく．そして振巾分布から分かるごとく，Channel wave の性格を持つているのでC分枝と名付けた．各分枝の振巾分布は第2図から第6図まで，また各分枝の振巾分布の比較を第7図から第9図までに示した．振巾分布から見て面白いことは，分枝IIで群速度のあまり変わらないところで，低速層に振巾が大きく出てくることである．

3. 液体-固体-固体からなる構造で，その中間層にあたる沈澱層の速度を水の層より遅くとつた場合の Rayleigh 波についてはすでに報告してあるが，上述の考えに似た場合を実際問題に適用したものである．

4. Lg および Rg を説明する場合に，もし低速層が地殻のなかに存在するとき，たとえその層がそれほど厚くなくても，その層に準拠して，上述の考察は有力なものとなるであろう．

また一方，中間層がかなり厚く，しかも，その横波の速度が Lg の速度に近いような場合，Lg 相の観測が可能であることを，実例をもつて第10図に示した．群速度の極大，極小およびあまり変わらない点に着目すればよい．

さらに Lg が海の深さ 3.5 km よりも深いような，いわゆる海洋構造では伝わらなくて，2 km より浅くなつて大陸構造になるときに，伝播していくことを，Rayleigh 波の計算において第13図から第16図にわたつて説明した．周期の短かいところで極大，極小が現われるが，それが Rg に対応するものと考えられる．Love 波においては海水層のかわりに沈澱層を考えれば良いわけである．
