BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 39 (1961), pp. 359-366

12. *A New Problem Concerning Surface Waves.*

(Preliminary Notes)

By Kiyoshi KANAI,

Earthquake Research Institute.

(Read Mar. 28 and May 23, 1961.—Received June 30, 1961.)

The features of seismic waves are usually influenced to a large extent by the discontinuity surfaces in the earth, and the mathematical treatment of them is carried out on the assumption that boundary conditions, as adjacent media, are undoubtedly fixed in relation to each other,—that is, displacements as well as stresses at the boundary surface of two media have perfect continuity. As a matter of fact, in general, such treatment has been adopted in order to simplify the problem, and pays slight consideration to the actual conditions of the earth. Nevertheless the results of mathematical studies on seismic waves by means of the treatment mentioned above agree with the results of analytical studies of actual seismograms. It is, however, uncertain whether or not the conditions at the discontinuity surfaces in the earth are correctly determined fixed. In order to clear the uncertainties just mentioned, the problem of waves, under the conditions of surfaces of discontinuity in which the normal components of stress and displacement are continuous, on the other hand, the transversal components of stress and displacement are not, were solved.[1] Furthermore, it is assumed that, at the same surface, there is a tangential resistance that is proportional to the relative tangential velocity.[2] In the previous paper, it has been ascertained that if a discontinuity surface in the earth's crust has a frictional resistance of a dissipative nature, the input energy of the waves is partly absorbed by the discontinuity surface in question, even were there no damping resistance in the media through which the waves are transmitted.

1) G. NISHIMURA and K. KANAI, "On the Effects of Discontinuity Surfaces upon the Propagation of Elastic Waves," *Bull. Earthq. Res. Inst.*, **11** (1933), 123-186, 595-631: **12** (1934), 277-316, 331-367, (in Japanese),

S. NATSUME, *Monthly Meeting of the Earthquake Research Institute*, (Oct. 1953).

2) K. SEZAWA and K. KANAI, "A Fault Surface or a Block Absorbs Seismic Wave Energy," *Bull. Earthq. Res. Inst.*, **18** (1940), 465-482.

In this paper, the case will be discussed in which Love-type waves are transmitted in a singly stratified visco-elastic layer residing on the semi-infinite visco-elastic body under the same above-mentioned conditions at the surface of discontinuity.

Let ρ, μ, ν, u and H be the density, elastic constant, coefficient of solid viscosity, displacement and thickness of layer. Suffix 1 and 2 represent the layer and the subjacent medium, respectively. Then the equation of the SH-type wave motion of each medium may be written as follows:

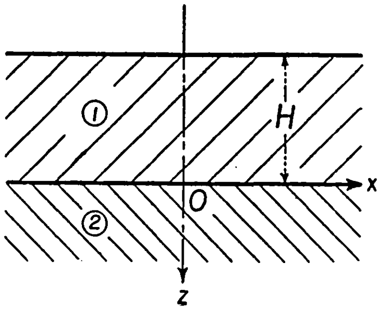

Fig. 1.

$$\left.\begin{aligned}\left(\mu_1+\nu_1\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u_1}{\partial z^2}+\frac{\partial^2 u_1}{\partial x^2}\right)=\rho_1\frac{\partial^2 u_1}{\partial t^2}\ ,\\ \left(\mu_2+\nu_2\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 u_2}{\partial z^2}+\frac{\partial^2 u_2}{\partial x^2}\right)=\rho_2\frac{\partial^2 u_2}{\partial t^2}\ .\end{aligned}\right\} \tag{1}$$

The resulting displacements of the layer and the subjacent medium are expressed by

$$\left.\begin{aligned}u_1&=(A\cos s_1 z+B\sin s_1 z)\exp\left[i(p_0 t-fx)\right]\ ,\\ u_2&=C\exp\left[-s_2 z+i(p_0 t-fx)\right]\ ,\end{aligned}\right\} \tag{2}$$

where A, B and C are arbitrary constants determined by boundary conditions and

$$s_1^2=\frac{\rho_1 p_0^2}{\mu_1+ip_0\nu_1}-f^2\ , \qquad s_2^2=f^2-\frac{\rho_2 p_0^2}{\mu_2+ip_0\nu_2}\ . \tag{3}$$

Let the tangential resistance at the discontinuity surface $z=0$ be

$$F\frac{\partial}{\partial t}\text{(relative displacement)}, \tag{4}$$

then the boundary conditions at the discontinuity, $z=0$, are

$$z=0\ ; \qquad \left(\mu_1+\nu_1\frac{\partial}{\partial t}\right)\frac{\partial u_1}{\partial z}=-F\frac{\partial}{\partial t}(u_1-u_2)\ , \tag{5}$$

$$\left(\mu_2+\nu_2\frac{\partial}{\partial t}\right)\frac{\partial u_2}{\partial z}=-F\frac{\partial}{\partial t}(u_1-u_2) \tag{6}$$

and the boundary condition at the free surface, $z=-H$, is

$$z=-H\ ;\qquad \left(\mu_1+\nu_1\frac{\partial}{\partial t}\right)\frac{\partial u_1}{\partial z}=0\ . \tag{7}$$

Substituting (2) in (5)-(7), we get

$$\cot s_1H-\frac{(\mu_1+ip_0\nu_1)s_1}{(\mu_2+ip_0\nu_2)s_2}+\frac{i(\mu_1+ip_0\nu_1)s_1}{p_0F}=0\ . \tag{8}$$

When $\nu_1=\nu_2=0$ and $F=\infty$, we get

$$\cot s_1H-\frac{\mu_1s_1}{\mu_2s_2}=0\ . \tag{8'}$$

This is of the same form as the usual velocity equation for Love-waves.

Now, we write

$$p_0=p+iq\ , \tag{9}$$

in which $2\pi/p$ and q are the period and damping coefficient of waves, respectively. (8) is transformed finally into the two expressions

$$\left.\begin{aligned} a\cos\alpha-b\cos\beta+c\cos\gamma=0\ ,\\ a\sin\alpha-b\sin\beta-c\sin\gamma=0\ ,\end{aligned}\right\} \tag{10}$$

where

$$a=(E_2^2+E_1^2)^{1/2}(F_1^2+F_2^2)^{1/4}\ ,$$

$$b=v\phi(G_1^2+G_2^2)^{1/2}(H_1^2+H_2^2)^{-1/4}(F_1^2+F_2^2)^{1/4}\ ,$$

$$c=v\phi(Y_1^2+Y_2^2)^{1/2}\ ,$$

$$\alpha=\tan^{-1}\left(\frac{E_1}{E_2}\right)+\frac{1}{2}\tan^{-1}\left(\frac{F_2}{F_1}\right)\ ,$$

$$\beta=\tan^{-1}\left(\frac{G_2}{G_1}\right)+\frac{1}{2}\tan^{-1}\left(\frac{F_2}{F_1}\right)-\frac{1}{2}\tan^{-1}\left(\frac{H_2}{H_1}\right)\ ,$$

$$\gamma=\tan^{-1}\left(\frac{Y_2}{Y_1}\right)\ ,$$

$$Y_1=\frac{\cos X_1\sin X_1}{\cos^2 X_1\,\mathrm{sh}^2 X_2+\sin^2 X_1\,\mathrm{ch}^2 X_2}\ ,$$

$$Y_2=\frac{\mathrm{ch}\,X_2\,\mathrm{sh}\,X_2}{\cos^2 X_1\,\mathrm{sh}^2 X_2+\sin^2 X_1\,\mathrm{ch}^2 X_2}\ ,$$

$$X_1=fH(F_1^2+F_2^2)^{1/4}\cos\left\{\frac{1}{2}\tan^{-1}\left(\frac{F_2}{F_1}\right)\right\}\ ,$$

$$X_2 = fH(F_1^2 + F_2^2)^{1/4} \sin\left\{\frac{1}{2}\tan^{-1}\left(\frac{F_2}{F_1}\right)\right\},$$

$$E_1 = \frac{1}{1+\kappa_2}, \qquad E_2 = \frac{\kappa - \xi_1 - \xi_1\kappa^2}{1+\kappa^2},$$

$$F_1 = \frac{v^2\{(1-\kappa^2)(1-\xi_1\kappa) + 2\xi_1\kappa\}}{(1-\xi_1\kappa)^2 + \xi_1^2} - 1,$$

$$F_2 = \frac{v^2\{2\kappa(1-\xi_1\kappa) - \xi_1(1-\kappa^2)\}}{(1-\xi_1\kappa)^2 + \xi_1^2},$$

$$G_1 = \frac{m\{(1-\xi_1\kappa)(1-\xi_2\kappa) + \xi_1\xi_2\}}{(1-\xi_2\kappa)^2 + \xi_2^2},$$

$$G_2 = \frac{m\{\xi_1(1-\xi_2\kappa) - \xi_2(1-\xi_1\kappa)\}}{(1-\xi_2\kappa)^2 + \xi_2^2},$$

$$H_1 = 1 - \frac{mrv^2\{(1-\kappa^2)(1-\xi_2\kappa) + 2\xi_2\kappa\}}{(1-\xi_2\kappa)^2 + \xi_2^2},$$

$$H_2 = \frac{mrv^2\{2\kappa(1-\xi_2\kappa) - \xi_2(1-\kappa^2)\}}{(1-\xi_2\kappa)^2 + \xi_2^2}, \qquad (11)$$

and

$$\left.\begin{aligned} m &= \frac{\mu_1}{\mu_2}, \quad r = \frac{\rho_2}{\rho_1}, \quad v = \frac{p}{f}\sqrt{\frac{\rho_1}{\mu_1}}, \quad \kappa = \frac{q}{p}, \\ \xi_1 &= \frac{\nu_1 p}{\mu_1}, \quad \xi_2 = \frac{\nu_2 p}{\mu_2}, \quad \phi = \frac{F}{\sqrt{\mu_1\rho_1}}. \end{aligned}\right\} \qquad (12)$$

From (10) it is possible to get the velocity and damping coefficient of Love-type waves for any ratios of μ_2/μ_1, ρ_2/ρ_1, $\nu_1 p/\mu_1$, $\nu_2 p/\mu_2$, $F/\sqrt{\mu_1\rho_1}$ and fH.

In the present case, however, there are a number of quantities for determining the velocity and damping coefficient, the problem then being extremely complex. Whereupon, we tried various special methods of numerical calculation. For example, in some cases, the values of κ as well as ϕ were obtained, which satisfied the assumed values of v and fH, in other cases, higher orders of κ were neglected.

The results of calculating the various ratios of $F/\sqrt{\mu_1\rho_1}$ and fH in the case where $\rho_2/\rho_1=1$, $\mu_2/\mu_1=5$, as well as the conditions where $\nu_1 p/\mu_1 = \nu_2 p/\mu_2 = 0$, are shown in Table 1 and in Figs. 2, 3.

Fig. 3 tells us that the relation between the attenuation factor and the period of Love-type waves is not so simple, and neither the attenuation factor varies as the square of the frequency[3] nor the first power of it[4] then the attenuation factor takes a maximum values in the appropriate frequency of such waves.

Table 1.

$\frac{p}{f}\sqrt{\frac{\rho_1}{\mu_1}}$	$\frac{L}{H}$	$\frac{F}{\sqrt{\rho_1\mu_1}}$	$\frac{q}{p}$
2.15	10.47	1.1	0.2
	12.39	1.2	0.1
	15.09	1.4	0.05
	18.80	4.3	0.01
2.00	10.03	1.2	0.2
	10.90	1.8	0.1
	11.58	3.1	0.05
	11.70	6.9	0.02
	11.85	47.8	0.003
	12.70	0.74	0.866
1.732	7.62	47.8	0.005
	7.74	11.9	0.02
	7.79	2.5	0.1
	8.16	1.2	0.25
	8.61	0.94	0.4
	10.40	0.79	0.816
1.414	4.765	40.4	0.005
	4.775	8.1	0.025
	4.78	6.8	0.03
	5.02	3.2	0.07
	5.76	1.3	0.2
	6.34	0.93	0.3
	9.40	0.80	0.707
1.30	3.81	10.0	0.016
1.20	4.36	0.90	0.2
	8.32	0.75	0.6
1.10	1.97	10.0	0.005
	7.50	0.65	0.408

It is a noteworthy fact that the tendency of the relation between the attenuation factor to the period obtained in the present investigation agrees qualitatively with the results[5] of the observational studies

3) K. SEZAWA, "On the Decay of Waves in Visco-Elastic Solid Bodies," *Bull. Earthq. Res. Inst.*, **3** (1927), 43-53.

4) L. KNOPOFF, "The Seismic Pulse in Materials Possessing Solid Friction," *Bull. Seism. Soc. Amer.*, **46** (1956), 175-183, **49** (1959), 403-413.

5) M. EWING and F. PRESS, "An Investigation of Mantle Rayleigh Waves," *Bull. Seism. Soc. Amer.*, **44** (1954), 127-147.
ditto, "Mantle Rayleigh Waves from the Kamchatka Earthquake of November 4, 1952," *ditto*, 471-479.
Y. SATO, "Attenuation, Dispersion and the Wave Guide of the G Wave," *Bull. Seism. Soc. Amer.*, **48** (1958), 231-268.

of long period surface waves.

On the other hand, it should however be borne in mind that the

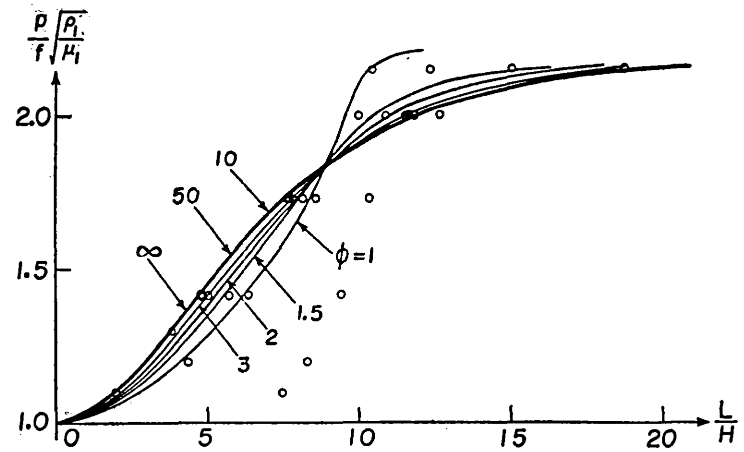

Fig. 2. Relation between the velocity and the period of Love-type waves. ($\rho_2/\rho_1=1$, $\mu_2/\mu_1=5$.)

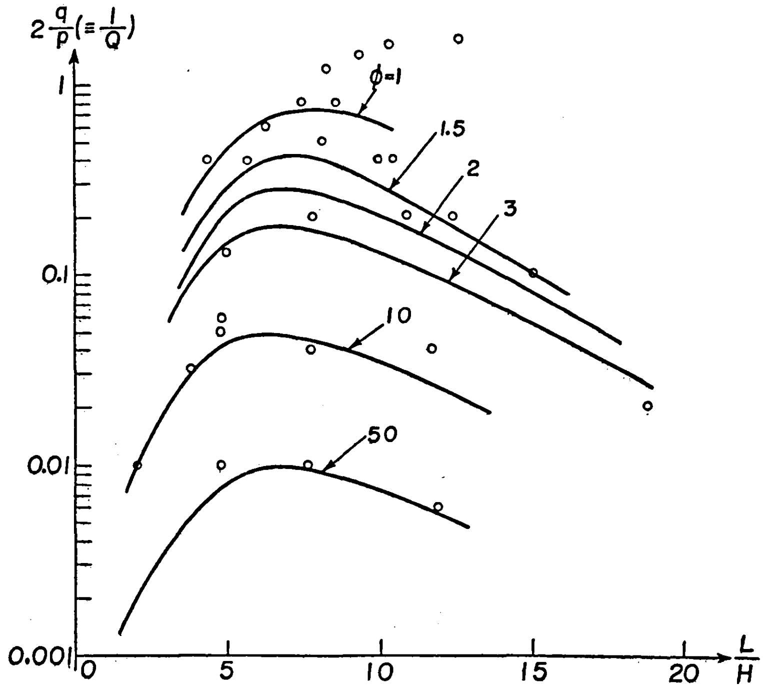

Fig. 3. Relation between the attenuation factor and the period of Love-type waves. ($\rho_2/\rho_1=1$, $\mu_2/\mu_1=5$.)

dispersion curve in the present case, except in the case of extremely small resistance at the discontinuity surface, coincides considerably with that of usual Love-waves.

In the case of Rayleigh-type waves, the boundary conditions at the discontinuity surface may be taken as follows:

$$\widehat{zz}_{1p}+\widehat{zz}_{1s}=\widehat{zz}_{2p}+\widehat{zz}_{2s}\ , \tag{13}$$

$$w_{1p}+w_{1s}=w_{2p}+w_{2s}\ , \tag{14}$$

$$\widehat{xz}_{1p}+\widehat{xz}_{1s}=-F\frac{\partial}{\partial t}\{(u_{1p}+u_{1s})-(u_{2p}+u_{2s})\}\ , \tag{15}$$

$$\widehat{xz}_{2p}+\widehat{xz}_{2s}=-F\frac{\partial}{\partial t}\{(u_{1p}+u_{1s})-(u_{2p}+u_{2s})\}\ , \tag{16}$$

in which $\widehat{zz}$, w, $\widehat{xz}$ and u represent normal stress, vertical displacement, tangential stress and horizontal displacement, respectively and suffixes 1, 2, p, and s represent the layer, the subjacent medium, P-waves and S-waves, respectively.

It is natural to consider that the influence of resistant phenomena at the discontinuity surface on Reyleigh-type waves is less than that on Love-type waves, because, in the former case, two of the four boundary conditions at the discontinuity surface have a bearing on the problem, in the latter case, nevertheless it is so with all the boundary conditions.

When the boundary conditions adopted here exist and also the above contentions are true, the amplitude relation between Rayleigh-type waves and Love-type waves is considered as follows:

$$\frac{L_c}{M_c}>\frac{L_m}{M_m}\ , \tag{17}$$

in which L, M, c and m represent Love-type waves, Rayleigh-type waves, earthquakes occurred in the crust and earthquakes occurred in the mantle.

In conclusion the author wishes to convey his thanks to Miss. S. Yoshizawa without whose help this work could not have been done.

12. 表面波に関する一つの問題（序報）

地震研究所　金　井　　清

地球内部には，いくつかの不連続面がある．その不連続面を地震波が通過するときの，境界条件としては，変位ならびに応力が連続という仮定が，一般に採用されてきた．しかし，ひるがえつて考えてみると，この仮定の是非は，よく吟味されたというものではなさそうである．

本研究は，上に述べた仮定を変えると，はたして，どんな結果が生れるかということを，しらべてみたものである．ここでは，不連続面で有限の大きさの，“すべり抵抗”のようなものを考えてみた．取扱つた地震波は SH 型表面波である．数値計算の結果，“すべり抵抗”の値が非常に小さい場合を別にすると，分散曲線は従来のものと大して変らないことがわかつた．しかし，減衰の性質としては，従来の理論的研究結果では見られないものが得られた．即ち，減衰は周期の 1 乗とか 2 乗とかに逆比例するというようなものではなく，ある周期に極大があることになつた．この結果は，最近，長周期の表面波に関する験震的研究結果と定性的には，よく合うものである．