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13. *Reflection and Refraction of Elastic Waves at a Corrugated Boundary Surface. Part II.*[1]

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Chapter 1. Introduction

The pioneering seismological studies of Lord Rayleigh in 1885 and C. G. Knott in 1888 on the theory of elastic surface waves and reflection and refraction of the same body waves respectively, which in themselves were directed along the way laid by Navier, Poisson, Cauchy, Green and Stokes with a view to optical application, gave profound stimulus to many theoretical investigators with the results that in-

1) The study of the case of a SH wave, which is appropriately included as a chapter in this paper, has already been published as Part I in this Bulletin (Vol. 38 (1960), pp. 177). Therefore in the present Part II, a general review is made from the broader point of view of all the subjects including those in Part I.

numerable papers on the theory of elastic waves were written and great progress has now been made thereby towards an understanding of earthquake phenomena.

However, there still remains many points to be clarified for a full understanding of the complicated nature of seismic waves. The problems, as it seems to the writer, are rooted in part in the complexities of the mechanism of an earthquake and in part in the structure of the earth in which earthquake waves are propagated. In fact, if the structure of the earth were very simple, for example uniform and homogeneous, we would be able to grasp the nature of the earthquake motion much more easily because the seismic wave propagated from the origin to the surface would have no chance to be modified by reflection and refraction at the boundaries of different media. We may therefore say that the voluminous existent seismograms of natural earthquakes and artificial seismic shootings pile up vainly barren of important further information which could be drawn if there were more data or some theoretical clues could be found. Thus theoretical studies of the propagation of elastic waves are deemed to occupy a very important position in seismology.

One of the most important problems on elastic waves is the study of the reflection and refraction of elastic waves at a boundary surface or surfaces. With regard to cases of a plane boundary surface or surfaces, there have been many studies by senior authorities, but an important and interesting problem in this field which has not yet been studied in detail, is the effect on the propagation of seismic waves of corrugation or roughness of a boundary surface.

It is well-known in geophysics and geology that there are many strata which are not bounded by planes but by corrugated interfaces especially in mountainous regions. Many of the geophysical exploration methods such as seismic, gravity and other prospecting and also geodetic measurement on a larger scale have revealed such evidences in various parts of the world.

We may also count many other phenomena supporting the above inference, amongst which those cited below seem to the writer to deserve notice.

(1) The distribution of earthquake intensity in north-eastern Japan through Hokkaido in the case of an earthquake originating under the sea off the Pacific coast of Hokkaido. In these earthquakes the seismic intensity attenuates more rapidly in a direction perpendicular to the

island arc from Hokkaido to the Kwanto District than in a direction parallel to the same arc. It is to be remarked in most of the above mentioned regions that the former direction is across the central mountain range while the latter is parallel to it[2].

There are many other regions in Japan which display abnormally high or low seismic intensities in comparison to those in the neighbourhood of like epicentral distances.

(2) A remarkably similar situation, found by H. E. Tatel and others in the Andes, South America, although it cannot yet be claimed to be fully established. In the course of seismic explorations by these investigators in the area, they found the fact that the seismic waves across the mountain range could not be observed clearly at a distance while on a similar spread along the Andes they succeeded in their work, which fact was ascribed by them to the difference in dissipation of the energy in the two directions[3].

(3) The smallness of the reflected waves from the Mohorovičić discontinuity (' Moho ') in western California as reported by Tatel and others[4]. Needless to say, the observation of reflected waves from the ' Moho ' is one of the powerful methods of elucidating the crustal structure, and the difficulty in one region and no difficulty in the other in the above inference is a very interesting fact in need of explanation, although the above fact mentioned by Tatel and others was not found during another exploration made by G. G. Shor in the vicinity[5]. According to the Japanese experiences, though meagre, the success or failure in getting good reflected waves from the ' Moho ' seems to depend on the crustal conditions under the observing point[6].

In view of these circumstances, it is desirable to obtain some clue

2) For example, H. KAWASUMI, *Publ. Bureau Central Séism. Intern., Série A, Trav. Sci., Fasc.* No. **19** (1956), 99.

K. WADATI and T. HIRONO, *Publ. Bureau Central Séism. Intern., Série A, Trav. Sci., Fasc.* No. **20** (1959), 221.

3) H. E. TATEL, M. A. TUVE and COLLEAGUES, "Abstract of paper presented at 39th Annual Meeting of American Geophysical Union held in 1958", *Trans. Amer. Geophys. Un.*, **39** (1958), 533.

4) H. E. TATEL, L. H. ADAMS and M. A. TUVE, *Proc. Amer. Phil. Soc.*, **97** (1953), 658.

5) G. G. SHOR, *Trans. Amer. Geophys. Un.*, **36** (1955), 133.

6) THE RESEARCH GROUP FOR EXPLOSION SEISMOLOGY, *Bull. Earthq. Res. Inst.*, **33** (1955), 699; **37** (1959), 89; **37** (1959), 495.

T. MATUZAWA, *Bull. Earthq. Res. Inst.*, **37** (1959), 123.

T. MATUZAWA, T. MATUMOTO and S. ASANO, *Bull. Earthq. Res. Inst.*, **37** (1959), 509.

from theoretical knowledge of the effects of corrugation on the propagation of elastic waves. This is one of the geophysical significances of the writer's attempt in this paper.

It was Lord Rayleigh who opened the gate of study on the present problem, and introduced an ingeneous method towards this purpose although his problem was confined to sound and light waves[7]. Recently considerations concerning the reflection of a sound wave at the sea surface have been examined by many investigators and various methods have been attempted in these studies[8]. With respect to seismic waves, S. Homma considered the problem of their reflection at a corrugated free surface for the first time[9], and R. Sato solved the same problem by Rayleigh's method[10].

The present writer has been studying the case of the reflection of elastic waves at a corrugated boundary surface between two different media, using the same method as Lord Rayleigh, and part of the results obtained will be reported in this paper.

Chapter 2. Fundamental Equations

§ 1. Equation of Boundary Surface

The coordinate axes are taken as shown in Fig. 1. That is, the x- and y-axis are on the mean horizontal surface of the boundary of the two media and z-axis is taken vertically downwards.

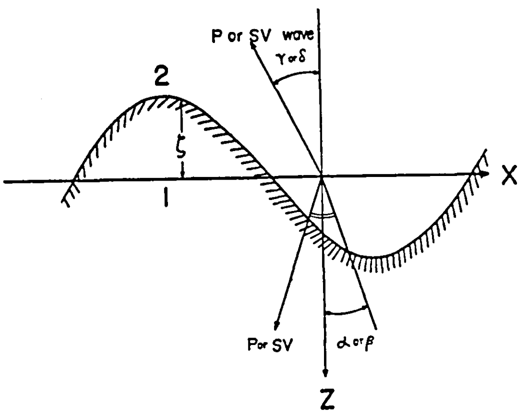

Fig. 1.

The equation of the boundary surface is given as $z=\zeta$, where ζ is assumed to be a periodic function of x but independent of y, the mean value of which being zero by assumption. Taking the origin of coordinates suit-

7) LORD RAYLEIGH, *Proc. Roy. Soc. Lond.*, A **79** (1907), 399.
8) For example, CARL ECKART, *Journ. Acoust. Soc. Amer.*, **25** (1953), 566.
E. O. LACASCE and P. TAMARKIN, *Journ. Appl. Phys.*, **27** (1956), 138.
9) S. HOMMA, *Quart. Journ. Seism.*, **11** (1940-41), 349; **12** (1942), 17 (in Japanese).
10) R. SATO, *Zisin*, [ii], **8** (1955), 8 (in Japanese).

ably, we can represent ζ by Fourier's series without a loss of generality as follows:

$$\zeta=\sum_{n=1}^{\infty}(\zeta_n e^{inpx}+\zeta_{-n}e^{-inpx})$$
$$=c_1\cos px+c_2\cos 2px+s_2\sin 2px+\cdots+c_n\cos npx+s_n\sin npx+\cdots, \tag{1}$$

where

$$\zeta_1=\zeta_{-1}=c_1/2\ , \qquad \zeta_{\pm n}=(c_n\mp is_n)/2\ .$$

When the boundary surface is expressible by one cosine term, i.e. $\zeta=c\cos px$, the wave length of the corrugation is $2\pi/p$.

§ 2. Equations of Motion and Boundary Conditions

We will consider the case, in the following, where a plane wave with a period of $2\pi/\omega$ is incident with an angle of incidence α in the case of a dilatational wave or β in the case of a distorsional wave to the boundary from the lower medium to the upper one. The quantities concerning the lower medium will be shown by suffix 1 and those of the upper medium, by suffix 2.

Equations of motion in the case of the incidence of P or SV waves are given by using displacement potentials ϕ and ψ as follows:

$$(\nabla^2+h_i^2)\phi_i=0\ , \qquad (\nabla^2+\sigma_i h_i^2)\psi_i=0\ , \tag{2}$$

where

∇^2=Laplacian, λ_i, μ_i=Lamé's constants,
$h_i^2=\rho_i\omega^2/(\lambda_i+2\mu_i)$, ρ_i =density,
$\sigma_i=(\lambda_i+2\mu_i)/\mu_i$, i =1 or 2.

The displacement $(u, 0, w)$ is to be obtained from the potentials by the following formulae

$$u=\partial\phi/\partial x+\partial\psi/\partial z \quad \text{and} \quad w=\partial\phi/\partial z-\partial\psi/\partial y\ .$$

In the case of the incidence of SH waves, if the displacement v is parallel to the y-axis, the equations of motion are given as follows:

$$(\nabla^2+\sigma_i h_i^2)v_i=0\ . \tag{3}$$

Now let us deduce the conditions concerning the stress on a corrugated boundary surface in terms of displacement potentials ϕ and ψ or displacement v. Since the direction cosines of normal ν to the surface $z=\zeta(x)$ are $(1/\sqrt{1+\zeta'^2},\ 0,\ \zeta'/\sqrt{1+\zeta'^2})$ and those of tangent t, $(-\zeta'/\sqrt{1+\zeta'^2},\ 0,\ 1/\sqrt{1+\zeta'^2})$ where $\zeta'=d\zeta/dx$, normal and tangential stresses N_ν, T_ν, Y_ν in the νyt-system are connected to those X_x, Z_x, Z_z in the xyz-system by the following formulae,

$$
\begin{aligned}
N_\nu&=[(Z_z-X_x)\zeta'+Z_x(1-\zeta'^2)]/(1+\zeta'^2)\\
T_\nu&=[Z_z+X_x\cdot\zeta'^2-2Z_x\cdot\zeta']/(1+\zeta'^2)\\
Y_\nu&=[Y_z-Y_x\cdot\zeta']/\sqrt{1+\zeta'^2}
\end{aligned}
\qquad (4)
$$

Substituting the following well-known relations between displacement, stress component and displacement potentials into (4),

$$
\begin{aligned}
X_x&=\lambda\Delta+2\mu\frac{\partial u}{\partial x}=-\lambda h^2\phi+2\mu\frac{\partial^2\phi}{\partial x^2}+2\mu\frac{\partial^2\psi}{\partial x\partial z}\ ,\\
Z_x&=\mu\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)=\mu\left(2\frac{\partial^2\phi}{\partial x\partial z}-\sigma h^2\psi-2\frac{\partial^2\psi}{\partial x^2}\right),\\
Z_z&=\lambda\Delta+2\mu\frac{\partial w}{\partial z}=-\lambda h^2\phi+2\mu\frac{\partial^2\phi}{\partial z^2}-2\mu\frac{\partial^2\psi}{\partial x\partial z}\ ,\\
Y_z&=\mu\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=\mu\frac{\partial v}{\partial z}\ ,\qquad Y_x=\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)=\mu\frac{\partial v}{\partial x}\ .
\end{aligned}
$$

the representation of stress by displacement potentials or displacement is obtained as follows:

$$
\begin{aligned}
N_\nu&=\left[-2\mu\left(h^2\phi+2\frac{\partial^2\phi}{\partial x^2}+2\frac{\partial^2\psi}{\partial x\partial z}\right)\zeta'\right.\\
&\quad\left.+\mu\left(2\frac{\partial^2\phi}{\partial x\partial z}-\sigma h^2\psi-2\frac{\partial^2\psi}{\partial x^2}\right)(1-\zeta'^2)\right]\Big/(1+\zeta'^2)\ ,\\
T_\nu&=\left[-\lambda h^2\phi+2\mu\frac{\partial^2\phi}{\partial z^2}-2\mu\frac{\partial^2\psi}{\partial x\partial z}+\left(-\lambda h^2\phi+2\mu\frac{\partial^2\phi}{\partial x^2}+2\mu\frac{\partial^2\psi}{\partial x\partial z}\right)\zeta'^2\right.\\
&\quad\left.-2\mu\left(2\frac{\partial^2\phi}{\partial x\partial z}-\sigma h^2\psi-2\frac{\partial^2\psi}{\partial x^2}\right)\zeta'\right]\Big/(1+\zeta'^2)\ ,\\
Y_\nu&=\mu\left(\frac{\partial v}{\partial z}-\frac{\partial v}{\partial x}\zeta'\right)\Big/\sqrt{1+\zeta'^2}\ .
\end{aligned}
\qquad (5)
$$

At the gradient of ζ, i.e. ζ' is common to both media, the condition of continuity of N_ν is equivalent to that of $(1+\zeta'^2)N_\nu$ and so forth. Thus, in the case of incidence of P or SV waves, the conditions of the continuity of stresses are equivalent to those of the continuity of $(1+\zeta'^2)N_\nu$ and $(1+\zeta'^2)T_\nu$ at $z=\zeta$ and in the case of incidence of SH waves, the condition is equivalent to the continuity of $\sqrt{1+\zeta'^2}\,Y_\nu$ at $z=\zeta$.

Chapter 3. Cases of the Incidence of P and SV Waves

§ 1. Incident, Reflected and Refracted Waves

Next, the both cases of the incidence of P and SV waves are considered together, because the treatment for them is similar. The coordinate system used is shown in Fig. 1.

In regard to the incident, regularly reflected and regularly refracted waves, the following displacement potentials were adopted.

For the incidence of the P wave,

Displacement potential of the incident wave:

$$\phi_{10}=e^{ih_1(V_{p1}t+z\cos\alpha+x\sin\alpha)}\ ,$$

For the incidence of the SV wave,

Displacement potential of the incident wave:

$$\psi_{10}=e^{i\sqrt{\sigma_1}h_1(V_{s1}t+z\cos\beta+x\sin\beta)}\ ,$$

For both cases,

Displacement potential of regularly reflected wave:

$$\phi_1^{(0)}=A_0e^{ih_1(V_{p1}t-z\cos\alpha+x\sin\alpha)}\ ,$$

$$\psi_1^{(0)}=B_0e^{i\sqrt{\sigma_1}h_1(V_{s1}t-z\cos\beta+x\sin\beta)}\ ,$$

Displacement potential of a regularly refracted wave:

$$\phi_2^{(0)}=C_0e^{ih_2(V_{p2}t+z\cos\gamma+x\sin\gamma)}\ ,$$

$$\psi_2^{(0)}=D_0e^{i\sqrt{\sigma_2}h_2(V_{s2}t+z\cos\delta+x\sin\delta)}\ ,$$

where V_{p1}, V_{s1}, V_{p2} and V_{s2} are the velocities of P and S waves in the lower and the upper medium respectively. By Snell's law, the angle of reflection of the P wave, α, that of the S wave, β, the angle of refraction of the P wave, γ, and that of the S wave, δ, are connected as follows:

$$h_1 \sin\alpha = \sqrt{\sigma_1}\, h_1 \sin\beta = h_2 \sin\gamma = \sqrt{\sigma_2}\, h_2 \sin\delta \, .$$

Besides these regular waves, it is necessary to take into consideration the effect of the corrugation on reflections and refractions and to introduce the following displacement potentials of irregular waves:

(1) Displacement potentials of irregularly reflected waves with whose spectrum of the nth order being:

$$\phi_1^{(n)} = A_n e^{ih_1(V_{p1}t - z\cos\alpha_n + x\sin\alpha_n)} + A_n' e^{ih_1(V_{p1}t - z\cos\alpha_n' + x\sin\alpha_n')} \, ,$$

$$\psi_1^{(n)} = B_n e^{i\sqrt{\sigma_1}h_1(V_{s1}t - z\cos\beta_n + x\sin\beta_n)} + B_n' e^{i\sqrt{\sigma_1}h_1(V_{s1}t - z\cos\beta_n' + x\sin\beta_n')} \, .$$

(2) Displacement potentials of irregularly refracted waves whose spectrum of the nth order being:

$$\phi_2^{(n)} = C_n e^{ih_2(V_{p2}t + z\cos\gamma_n + x\sin\gamma_n)} + C_n' e^{ih_2(V_{p2}t + z\cos\gamma_n' + x\sin\gamma_n')} \, ,$$

$$\psi_2^{(n)} = D_n e^{i\sqrt{\sigma_2}h_2(V_{s2}t + z\cos\delta_n + x\sin\delta_n)} + D_n' e^{i\sqrt{\sigma_2}h_2(V_{s2}t + z\cos\delta_n' + x\sin\delta_n')} \, ,$$

where α_n, α_n', β_n, β_n', etc. are connected by the following relations:

$$\sin\alpha_n - \sin\alpha = np/h_1 \, , \qquad \sin\alpha_n' - \sin\alpha = -np/h_1 \, ,$$

$$\sin\beta_n - \sin\beta = np/(\sqrt{\sigma_1}\, h_1) \, , \qquad \sin\beta_n' - \sin\beta = -np/(\sqrt{\sigma_1}\, h_1) \, ,$$

$$\sin\gamma_n - \sin\gamma = np/h_2 \, , \qquad \sin\gamma_n' - \sin\gamma = -np/h_2 \, ,$$

$$\sin\delta_n - \sin\delta = np/(\sqrt{\sigma_2}\, h_2) \, , \qquad \sin\delta_n' - \sin\delta = -np/(\sqrt{\sigma_2}\, h_2) \, ,$$

$$h_1 \sin\alpha_n = \sqrt{\sigma_1}\, h_1 \sin\beta_n = h_2 \sin\gamma_n = \sqrt{\sigma_2}\, h_2 \sin\delta_n \, .$$

Superposing above displacement potentials, the resultant displacement potentials are obtained in the lower and in the upper medium respectively.

In the lower medium:

$$\phi_1 = e^{ih_1 x\sin\alpha}\Big[a e^{ih_1 z\cos\alpha} + A_0 e^{-ih_1 z\cos\alpha} + \sum_{n=1}^{\infty}\{A_n e^{i(npx - h_1 z\cos\alpha_n)} + A_n' e^{-i(npx + h_1 z\cos\alpha_n')}\}\Big] \, , \qquad (6)$$

$$\psi_1=e^{ih_1x\sin\alpha}\Big[be^{i\sqrt{\sigma_1}h_1z\cos\beta}+B_0e^{-i\sqrt{\sigma_1}h_1z\cos\beta}$$
$$+\sum_{n=1}^{\infty}\{B_ne^{i(npx-\sqrt{\sigma_1}h_1z\cos\beta_n)}+B'_ne^{-i(npx+\sqrt{\sigma_1}h_1z\cos\beta_n')}\}\Big], \qquad (7)$$

where $a=1$ and $b=0$ in the case of incidence of the P wave and $a=0$ and $b=1$ in the case of incidence of the SV wave.
In the upper medium:

$$\phi_2=e^{ih_1x\sin\alpha}\Big[C_0e^{ih_2z\cos\gamma}$$
$$+\sum_{n=1}^{\infty}\{C_ne^{i(h_2z\cos\gamma_n+npx)}+C'_ne^{i(h_2z\cos\gamma_n'-npx)}\}\Big], \qquad (8)$$

$$\psi_2=e^{ih_1x\sin\alpha}\Big[D_0e^{i\sqrt{\sigma_2}h_2z\cos\delta}$$
$$+\sum_{n=1}^{\infty}\{D_ne^{i(\sqrt{\sigma_2}h_2z\cos\delta_n+npx)}+D'_ne^{i(\sqrt{\sigma_2}h_2z\cos\delta_n'-npx)}\}\Big]. \qquad (9)$$

In the above formulae, the common time factor $e^{i\omega t}$ is overlooked.

§ 2. Boundary Conditions

The boundary conditions are given by the conditions of the continuity of stress and displacement on the boundary surface $z=\zeta$.

Substituting ϕ_i, ψ_i, and their derivatives into $(1+\zeta'^2)N_\nu^{(1)}=(1+\zeta'^2)N_\nu^{(2)}$ or $(1+\zeta'^2)T_\nu^{(1)}=(1+\zeta'^2)T_\nu^{(2)}$, we can get the following relations between A_0, B_0, C_0, D_0, A_n, B_n, etc.

$$-2\mu_1h_1^2\{(1-2\sin^2\alpha)\zeta'+(1-\zeta'^2)\sin\alpha\cos\alpha\}ae^{ih_1\zeta\cos\alpha}$$
$$-2\mu_1h_1^2\{(1-2\sin^2\alpha)\zeta'-(1-\zeta'^2)\sin\alpha\cos\alpha\}A_0e^{-ih_1\zeta\cos\alpha}$$
$$+2\mu_1\sum_{n=1}^{\infty}\{-h_1^2\zeta'+2\zeta'(h_1\sin\alpha+np)^2$$
$$+h_1(h_1\sin\alpha+np)(1-\zeta'^2)\cos\alpha_n\}A_ne^{i(npx-h_1\zeta\cos\alpha_n)}$$
$$+2\mu_1\sum_{n=1}^{\infty}\{-h_1^2\zeta'+2\zeta'(h_1\sin\alpha-np)^2$$
$$+h_1(h_1\sin\alpha-np)(1-\zeta'^2)\cos\alpha'_n\}A'_ne^{-i(npx+h_1\zeta\cos\alpha_n')}$$
$$+\mu_1h_1^2\{4\sqrt{\sigma_1}\zeta'\sin\alpha\cos\beta-(\sigma_1-2\sin^2\alpha)(1-\zeta'^2)\}be^{i\sqrt{\sigma_1}h_1\zeta\cos\beta}$$
$$-\mu_1h_1^2\{4\sqrt{\sigma_1}\zeta'\sin\alpha\cos\beta+(\sigma_1-2\sin^2\alpha)(1-\zeta'^2)\}B_0e^{-i\sqrt{\sigma_1}h_1\zeta\cos\beta}$$

$$
\begin{aligned}
&-\mu_1\sum_{n=1}^{\infty}[4\sqrt{\sigma_1}h_1\zeta'(h_1\sin\alpha+np)\cos\beta_n \\
&\qquad+\{\sigma_1h_1^2-2(h_1\sin\alpha+np)^2\}(1-\zeta'^2)]B_ne^{i(npx-\sqrt{\sigma_1}h_1\zeta\cos\beta_n)} \\
&-\mu_1\sum_{n=1}^{\infty}[4\sqrt{\sigma_1}h_1\zeta'(h_1\sin\alpha-np)\cos\beta_n' \\
&\qquad+\{\sigma_1h_1^2-2(h_1\sin\alpha-np)^2\}(1-\zeta'^2)]B_n'e^{-i(npx+\sqrt{\sigma_1}h_1\zeta\cos\beta_n')} \\
=&-2\mu_2\{(h_2^2-2h_1^2\sin^2\alpha)\zeta'+(1-\zeta'^2)h_1h_2\sin\alpha\cos\gamma\}C_0e^{ih_2\zeta\cos\gamma} \\
&+\mu_2\{4\sqrt{\sigma_2}h_1h_2\zeta'\sin\alpha\cos\delta-(\sigma_2h_2^2-2h_1^2\sin^2\alpha)(1-\zeta'^2)\}D_0e^{i\sqrt{\sigma_2}h_2\zeta\cos\delta} \\
&-2\mu_2\sum_{n=1}^{\infty}[\{h_2^2-2(h_1\sin\alpha+np)^2\}\zeta' \\
&\qquad+(1-\zeta'^2)(h_1\sin\alpha+np)h_2\cos\gamma_n]C_ne^{i(npx+h_2\zeta\cos\gamma_n)} \\
&-2\mu_2\sum_{n=1}^{\infty}[\{h_2^2-2(h_1\sin\alpha-np)^2\}\zeta' \\
&\qquad+(1-\zeta'^2)(h_1\sin\alpha-np)h_2\cos\gamma_n']C_n'e^{-i(npx-h_2\zeta\cos\gamma_n')} \\
&+\mu_2\sum_{n=1}^{\infty}[4\sqrt{\sigma_2}h_2\zeta'(h_1\sin\alpha+np)^2\cos\delta_n \\
&\qquad-(1-\zeta'^2)\{\sigma_2h_2^2-2(h_1\sin\alpha+np)^2\}]D_ne^{i(npx+\sqrt{\sigma_2}h_2\zeta\cos\delta_n)} \\
&+\mu_2\sum_{n=1}^{\infty}[4\sqrt{\sigma_2}h_2\zeta'(h_1\sin\alpha-np)^2\cos\delta_n' \\
&\qquad-(1-\zeta'^2)\{\sigma_2h_2^2-2(h_1\sin\alpha-np)^2\}]D_n'e^{-i(npx-\sqrt{\sigma_2}h_2\zeta\cos\delta_n')}\,, \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
&-h_1^2\{\lambda_1+2\mu_1\cos^2\alpha-4\mu_1\zeta'\sin\alpha\cos\alpha+\zeta'^2(\lambda_1+2\mu_1\sin^2\alpha)\}ae^{ih_1\zeta\cos\alpha} \\
&-h_1^2\{\lambda_1+2\mu_1\cos^2\alpha+4\mu_1\zeta'\sin\alpha\cos\alpha+\zeta'^2(\lambda_1+2\mu_1\sin^2\alpha)\}A_0e^{-ih_1\zeta\cos\alpha} \\
&-\sum_{n=1}^{\infty}[h_1^2(\lambda_1+2\mu_1\cos^2\alpha_n')+4\mu_1\zeta'h_1(h_1\sin\alpha-np)\cos\alpha_n' \\
&\qquad+\zeta'^2\{\lambda_1h_1^2+2\mu_1(h_1\sin\alpha+np)^2\}]A_ne^{i(npx-h_1\zeta\cos\alpha_n)} \\
&-\sum_{n=1}^{\infty}[h_1^2(\lambda_1+2\mu_1\cos^2\alpha_n')+4\mu_1\zeta'h_1(h_1\sin\alpha-np)\cos\alpha_n' \\
&\qquad+\zeta'^2\{\lambda_1h_1^2+2\mu_1(h_1\sin\alpha-np)^2\}]A_n'e^{-i(npx+h_1\zeta\cos\alpha_n')} \\
&+2\mu_1h_1^2\{\sqrt{\sigma_1}\sin\alpha\cos\beta+\zeta'(\sigma_1-2\sin^2\alpha) \\
&\qquad-\zeta'^2\sqrt{\sigma_1}\sin\alpha\cos\beta\}be^{i\sqrt{\sigma_1}h_1\zeta\cos\beta} \\
&+2\mu_1h_1^2\{-\sqrt{\sigma_1}\sin\alpha\cos\beta+\zeta'(\sigma_1-2\sin^2\alpha) \\
&\qquad+\zeta'^2\sqrt{\sigma_1}\sin\alpha\cos\beta\}B_0e^{-i\sqrt{\sigma_1}h_1\zeta\cos\beta}
\end{aligned}
$$

$$+2\mu_1\sum_{n=1}^{\infty}[-\sqrt{\sigma_1}h_1(h_1\sin\alpha+np)\cos\beta_n+\zeta'\{\sigma_1h_1^2-2(h_1\sin\alpha+np)^2\}$$

$$+\zeta'^2\sqrt{\sigma_1}h_1(h_1\sin\alpha+np)\cos\beta_n]B_ne^{i(npx-\sqrt{\sigma_1}h_1\zeta\cos\beta_n)}$$

$$+2\mu_1\sum_{n=1}^{\infty}[-\sqrt{\sigma_1}h_1(h_1\sin\alpha-np)\cos\beta_n'+\zeta'\{\sigma_1h_1^2-2(h_1\sin\alpha-np)^2\}$$

$$+\zeta'^2\sqrt{\sigma_1}h_1(h_1\sin\alpha-np)\cos\beta_n']B_n'e^{i(npx+\sqrt{\sigma_1}h_1\zeta\cos\beta_n')}$$

$$=\{-h_2^2(\lambda_2+2\mu_2\cos^2\gamma)+4\mu_2\zeta'h_1h_2\sin\alpha\cos\gamma$$

$$-\zeta'^2(\lambda_2h_2^2+2\mu_2h_1^2\sin^2\alpha)\}C_0e^{ih_2\zeta\cos\gamma}$$

$$-\sum_{n=1}^{\infty}[h_2^2(\lambda_2+2\mu_2\cos^2\gamma_n)-4\mu_2h_2\zeta'(h_1\sin\alpha+np)\cos\gamma_n$$

$$+\zeta'^2\{\lambda_2h_2^2+2\mu_2(h_1\sin\alpha+np)^2\}]C_ne^{i(npx+h_2\zeta\cos\gamma_n)}$$

$$-\sum_{n=1}^{\infty}[h_2^2(\lambda_2+2\mu_2\cos^2\gamma_n')-4\mu_2h_2\zeta'(h_1\sin\alpha-np)\cos\gamma_n'$$

$$+\zeta'^2\{\lambda_2h_2^2+2\mu_2(h_1\sin\alpha-np)^2\}]C_n'e^{-i(npx-h_2\zeta\cos\gamma_n')}$$

$$+2\mu_2\{\sqrt{\sigma_2}h_1h_2\sin\alpha\cos\delta+\zeta'(\sigma_2h_2^2-2h_1^2\sin^2\alpha)$$

$$-\zeta'^2\sqrt{\sigma_2}h_1h_2\sin\alpha\cos\delta\}D_0e^{i\sqrt{\sigma_2}h_2\zeta\cos\delta}$$

$$+2\mu_2\sum_{n=1}^{\infty}[\sqrt{\sigma_2}h_2(h_1\sin\alpha+np)\cos\delta_n+\zeta'\{\sigma_2h_2^2-2(h_1\sin\alpha+np)^2\}$$

$$-\zeta'^2\sqrt{\sigma_2}h_2(h_1\sin\alpha+np)\cos\delta_n]D_ne^{i(npx+\sqrt{\sigma_2}h_2\zeta\cos\delta_n)}$$

$$+2\mu_2\sum_{n=1}^{\infty}[\sqrt{\sigma_2}h_2(h_1\sin\alpha-np)\cos\delta_n'+\zeta'\{\sigma_2h_2^2-2(h_1\sin\alpha-np)^2\}$$

$$-\zeta'^2\sqrt{\sigma_2}h_2(h_1\sin\alpha-np)\cos\delta_n']D_n'e^{-i(npx-\sqrt{\sigma_2}h_2\zeta\cos\delta_n')} \tag{11}$$

From the condition of the continuity of displacement u and w, we have

$$h_1(ae^{ih_1\zeta\cos\alpha}+A_0e^{-ih_1\zeta\cos\alpha})\sin\alpha$$

$$+\sum_{n=1}^{\infty}\{(h_1\sin\alpha+np)A_ne^{i(npx-h_1\zeta\cos\alpha_n)}$$

$$+(h_1\sin\alpha-np)A_n'e^{-i(npx+h_1\zeta\cos\alpha_n')}\}$$

$$+\sqrt{\sigma_1}h_1(be^{i\sqrt{\sigma_1}h_1\zeta\cos\beta}-B_0e^{-i\sqrt{\sigma_1}h_1\zeta\cos\beta})\cos\beta$$

$$-\sqrt{\sigma_1}h_1\sum_{n=1}^{\infty}\{B_ne^{i(npx-\sqrt{\sigma_1}h_1\zeta\cos\beta_n)}\cos\beta_n+B_n'e^{-i(npx+\sqrt{\sigma_1}h_1\zeta\cos\beta_n')}\cos\beta_n'\}$$

$$=h_1C_0e^{ih_2\zeta\cos\gamma}\sin\alpha$$

$$+\sum_{n=1}^{\infty}\{(h_1\sin\alpha+np)C_ne^{i(npx+h_2\zeta\cos\gamma_n)}+(h_1\sin\alpha-np)C_n'e^{-i(npx-h_2\zeta\cos\gamma_n')}\}$$

$$+\sqrt{\sigma_2}h_2D_0e^{i\sqrt{\sigma_2}h_2\zeta\cos\delta}\cos\delta$$
$$+\sum_{n=1}^{\infty}\sqrt{\sigma_2}h_2\{D_ne^{i(npx+\sqrt{\sigma_2}h_2\zeta\cos\delta_n)}\cos\delta_n+D'_ne^{-i(npx-\sqrt{\sigma_2}h_2\zeta\cos\delta_n')}\cos\delta'_n\}\ , \tag{12}$$

$$h_1(ae^{ih_1\zeta\cos\alpha}-A_0e^{-ih_1\zeta\cos\alpha})\cos\alpha$$
$$-h_1\sum_{n=1}^{\infty}\{A_ne^{i(npx-h_1\zeta\cos\alpha_n)}\cos\alpha_n+A'_ne^{-i(npx+h_1\zeta\cos\alpha_n')}\cos\alpha'_n\}$$
$$-h_1(be^{i\sqrt{\sigma_1}h_1\zeta\cos\beta}+B_0e^{-i\sqrt{\sigma_1}h_1\zeta\cos\beta})\sin\alpha$$
$$-\sum_{n=1}^{\infty}\{(h_1\sin\alpha+np)B_ne^{i(npx-\sqrt{\sigma_1}h_1\zeta\cos\beta_n)}$$
$$+(h_1\sin\alpha-np)B'_ne^{-i(npx+\sqrt{\sigma_1}h_1\zeta\cos\beta_n')}\}$$
$$=h_2C_0e^{ih_2\zeta\cos\gamma}\cos\gamma+h_2\sum_{n=1}^{\infty}\{C_ne^{i(npx+h_2\zeta\cos\gamma_n)}\cos\gamma_n+C'_ne^{-i(npx-h_2\zeta\cos\gamma_n')}\cos\gamma'_n\}$$
$$-h_1D_0e^{i\sqrt{\sigma_2}h_2\zeta\cos\delta}\sin\alpha$$
$$-\sum_{n=1}^{\infty}\{(h_1\sin\alpha+np)D_ne^{i(npx+\sqrt{\sigma_2}h_2\zeta\cos\delta_n)}$$
$$+(h_1\sin\alpha-np)D'_ne^{-i(npx-\sqrt{\sigma_2}h_2\zeta\cos\delta_n')}\}\ . \tag{13}$$

§ 3. The Solutions of the First Approximation

As in the case of incidence of the SH wave, the amplitude of corrugation, ζ, is assumed to be very small so that the terms of an order higher than ζ are neglected.

First the solutions of the first approximation will be obtained by picking up the term independent of x and ζ in (10) through (13).

$$-2\mu_1h_1^2(a-A_0)\sin\alpha\cos\alpha+\mu_1h_1^2(2\sin^2\alpha-\sigma_1)(b+B_0)$$
$$+2\mu_2h_1h_2C_0\sin\alpha\cos\gamma-\mu_2D_0(2h_1^2\sin^2\alpha-\sigma_2h_2^2)=0\ ,$$
$$-h_1^2(\lambda_1+2\mu_1\cos^2\alpha)(a+A_0)+2\mu_1h_1^2\sqrt{\sigma_1}(b-B_0)\sin\alpha\cos\beta$$
$$+h_2^2(\lambda_2+2\mu_2\cos^2\gamma)C_0-2\mu_2\sqrt{\sigma_2}h_1h_2D_0\sin\alpha\cos\delta=0\ , \tag{14}$$
$$h_1(a+A_0)\sin\alpha+\sqrt{\sigma_1}h_1(b-B_0)\cos\beta-h_1C_0\sin\alpha-\sqrt{\sigma_2}h_2D_0\cos\delta=0\ ,$$
$$h_1(a-A_0)\cos\alpha-h_1(b+B_0)\sin\alpha-h_2C_0\cos\gamma+h_1D_0\sin\alpha=0$$

These equations are the same as those obtained in the case of the reflection and refraction of elastic waves at a plane boundary surface.

From the coefficients of e^{inpx} in (10) through (13), the following formulae determining the first approximation of A_n, B_n, C_n and D_n are obtained.

$$
\begin{aligned}
&2\mu_1 h_1 A_n(h_1 \sin\alpha + np)\cos\alpha_n - \mu_1 B_n\{\sigma_1 h_1^2 - 2(h_1\sin\alpha + np)^2\} \\
&\quad + 2\mu_2 h_2 C_n(h_1\sin\alpha + np)\cos\gamma_n - \mu_2 D_n\{-\sigma_2 h_2^2 + 2(h_1\sin\alpha + np)^2\} \\
&\quad = 2i\mu_1\zeta_n h_1^2(a + A_0)\{np(1 - 2\sin^2\alpha) + h_1\sin\alpha\cos^2\alpha\} \\
&\quad\quad - i\mu_1\sqrt{\sigma_1}\, h_1^2\zeta_n(b - B_0)\{4np\sin\alpha + h_1(2\sin^2\alpha - \sigma_1)\}\cos\beta \\
&\quad\quad - 2i\mu_2 C_0\zeta_n\{np(h_2^2 - 2h_1^2\sin^2\alpha) + h_1 h_2^2\sin\alpha\cos^2\gamma\} \\
&\quad\quad + i\mu_2 D_0\sqrt{\sigma_2}\, h_2\zeta_n\{4nph_1\sin\alpha - (\sigma_2 h_2^2 - 2h_1^2\sin^2\alpha)\}\cos\delta\ , \\
&-h_1^2 A_n(\lambda_1 + 2\mu_1\cos^2\alpha_n) - 2\mu_1\sqrt{\sigma_1}\, h_1 B_n(h_1\sin\alpha + np)\cos\beta_n \\
&\quad + h_2^2 C_n(\lambda_2 + 2\mu_2\cos^2\gamma_n) - 2\mu_2\sqrt{\sigma_2}\, h_2 D_n(h_1\sin\alpha + np)\cos\delta_n \\
&\quad = ih_1^2\zeta_n(a - A_0)[h_1(\lambda_1 + 2\mu_1\cos^2\alpha) - 4\mu_1 np\sin\alpha]\cos\alpha \\
&\quad\quad - 2i\mu_1 h_1^2\zeta_n(b + B_0)\{np(\sigma_1 - 2\sin^2\alpha) + \sigma_1 h_1\sin\alpha\cos^2\beta\} \\
&\quad\quad + iC_0 h_2\zeta_n\{-(\lambda_2 + 2\mu_2\cos^2\gamma)h_2^2 + 4np\mu_2 h_1\sin\alpha\}\cos\gamma \\
&\quad\quad + 2i\mu_2\zeta_n D_0\{\sigma_2 h_1 h_2^2\sin\alpha\cos^2\delta + np(\sigma_2 h_2^2 - 2h_1^2\sin^2\alpha)\}\ ,
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&(h_1\sin\alpha + np)A_n - \sqrt{\sigma_1}\, h_1 B_n\cos\beta_n - (h_1\sin\alpha + np)C_n - \sqrt{\sigma_2}\, h_2 D_n\cos\delta_n \\
&\quad = -ih_1^2\zeta_n(a - A_0)\sin\alpha\cos\alpha - i\sigma_1 h_1^2\zeta_n(b + B_0)\cos^2\beta \\
&\quad\quad + ih_1 h_2 C_0\zeta_n\sin\alpha\cos\gamma + i\sigma_2 h_2^2\zeta_n D_0\cos^2\delta\ , \\
&-h_1 A_n\cos\alpha_n - (h_1\sin\alpha + np)B_n - h_2 C_n\cos\gamma_n + (h_1\sin\alpha + np)D_n \\
&\quad = -ih_1^2\zeta_n(a + A_0)\cos^2\alpha + i\sqrt{\sigma_1}\, h_1^2\zeta_n(b - B_0)\sin\alpha\cos\beta \\
&\quad\quad + ih_2^2\zeta_n C_0\cos^2\gamma - i\sqrt{\sigma_2}\, h_1 h_2\zeta_n D_0\sin\alpha\cos\delta
\end{aligned}
\tag{16}
$$

From the coefficients of e^{-inpx} in (10) through (13), the following formulae concerning the first approximation of A'_n, B'_n, C'_n and D'_n are derived.

$$
\begin{aligned}
&2\mu_1 h_1 A'_n(h_1\sin\alpha - np)\cos\alpha'_n - \mu_1 B'_n\{\sigma_1 h_1^2 - 2(h_1\sin\alpha - np)^2\} \\
&\quad + 2\mu_2 h_2 C'_n(h_1\sin\alpha - np)\cos\gamma'_n + \mu_2 D'_n\{\sigma_2 h_2^2 - 2(h_1\sin\alpha - np)^2\} \\
&\quad = 2i\mu_1 h_1^2\zeta_{-n}(a + A_0)\{-np(1 - 2\sin^2\alpha) + h_1\sin\alpha\cos^2\alpha\} \\
&\quad\quad + i\mu_1 h_1^2\sqrt{\sigma_1}\,\zeta_{-n}(b - B_0)[4np\sin\alpha + (\sigma_1 - 2\sin^2\alpha)h_1]\cos\beta \\
&\quad\quad - 2i\mu_2 C_0\zeta_{-n}\{-np(h_2^2 - 2h_1^2\sin^2\alpha) + h_1 h_2^2\sin\alpha\cos^2\gamma\} \\
&\quad\quad - i\mu_2\sqrt{\sigma_2}\, h_2 D_0\zeta_{-n}\{4nph_1\sin\alpha + (\sigma_2 h_2^2 - 2h_1^2\sin^2\alpha)\}\cos\delta
\end{aligned}
$$

$$-h_1^2 A'_n(\lambda_1+2\mu_1\cos^2\alpha'_n)-2\mu_1\sqrt{\sigma_1}h_1B'_n(h_1\sin\alpha-np)\cos\beta'_n$$
$$+h_2^2C'_n(\lambda_2+2\mu_2\cos^2\gamma'_n)-2\mu_2\sqrt{\sigma_2}h_2D'_n(h_1\sin\alpha-np)\cos\delta'_n$$
$$=ih_1^2\zeta_{-n}(a-A_0)\{h_1(\lambda_1+2\mu_1\cos^2\alpha)+4\mu_1 np\sin\alpha\}\cos\alpha \tag{17}$$
$$-2i\mu_1h_1^2\zeta_{-n}(b+B_0)\{-np(\sigma_1-2\sin^2\alpha)+\sigma_1h_1\sin\alpha\cos^2\beta\}$$
$$-iC_0h_2\zeta_{-n}\{(\lambda_2+2\mu_2\cos^2\gamma)h_2^2+4np\mu_2h_1\sin\alpha\}\cos\gamma$$
$$+2i\mu_2\zeta_{-n}D_0\{\sigma_2h_1h_2^2\sin\alpha\cos^2\delta-(\sigma_2h_2^2-2h_1^2\sin^2\alpha)np\}\ ,$$

$$(h_1\sin\alpha-np)A'_n-\sqrt{\sigma_1}h_1B'_n\cos\beta'_n-(h_1\sin\alpha-np)C'_n-\sqrt{\sigma_2}h_2D'_n\cos\delta'_n$$
$$=-ih_1^2\zeta_{-n}(a-A_0)\sin\alpha\cos\alpha-i\sigma_1h_1^2\zeta_{-n}(b+B_0)\cos^2\beta$$
$$+ih_1h_2C_0\zeta_{-n}\sin\alpha\cos\gamma+i\sigma_2h_2^2\zeta_{-n}D_0\cos^2\delta\ ,$$

$$-h_1A'_n\cos\alpha'_n-(h_1\sin\alpha-np)B'_n-h_2C'_n\cos\gamma'_n+(h_1\sin\alpha-np)D'_n$$
$$=-ih_1^2\zeta_{-n}(a+A_0)\cos^2\alpha+i\sqrt{\sigma_1}h_1^2\zeta_{-n}(b-B_0)\sin\alpha\cos\beta$$
$$+ih_2^2C_0\zeta_{-n}\cos^2\gamma-i\sqrt{\sigma_2}h_1h_2\zeta_{-n}D_0\sin\alpha\cos\delta\ .$$

§ 4. The Solutions of the Second Approximations

If the terms, of an order higher than ζ^3, are omitted in the equations of boundary conditions (10), (11), (12) and (13),

$$-2\mu_1h_1^2(a-A_0)\sin\alpha\cos\alpha-2\mu_1h_1^2(a+A_0)\{(1-2\sin^2\alpha)\zeta'+ih_1\zeta\sin\alpha\cos^2\alpha\}$$
$$+2\mu_1\sum[-\zeta'\{h_1^2-2(h_1\sin\alpha+np)^2\}$$
$$+h_1(1-ih_1\zeta\cos\alpha_n)(h_1\sin\alpha+np)\cos\alpha_n]A_ne^{inpx}$$
$$+2\mu_1\sum[-\zeta'\{h_1^2-2(h_1\sin\alpha-np)^2\}$$
$$+h_1(1-ih_1\zeta\cos\alpha'_n)(h_1\sin\alpha-np)\cos\alpha'_n]A'_ne^{-inpx}$$
$$-\mu_1h_1^2(b+B_0)(\sigma_1-2\sin^2\alpha)$$
$$+\mu_1h_1^2(b-B_0)\{4\sqrt{\sigma_1}\zeta'\sin\alpha\cos\beta-i\sqrt{\sigma_1}h_1\zeta(\sigma_1-2\sin^2\alpha)\cos\beta\}$$
$$-\mu_1\sum[4\sqrt{\sigma_1}h_1\zeta'(h_1\sin\alpha+np)\cos\beta_n$$
$$+\{\sigma_1h_1^2-2(h_1\sin\alpha+np)^2\}(1-i\sqrt{\sigma_1}h_1\zeta\cos\beta_n)]B_ne^{inpx}$$
$$-\mu_1\sum[4\sqrt{\sigma_1}h_1\zeta'(h_1\sin\alpha-np)\cos\beta'_n$$
$$+\{\sigma_1h_1^2-2(h_1\sin\alpha-np)^2\}(1-i\sqrt{\sigma_1}h_1\zeta\cos\beta'_n)]B'_ne^{-inpx}$$
$$=-2\mu_2C_0\{(h_2^2-2h_1^2\sin^2\alpha)\zeta'+h_1h_2(1+ih_2\zeta\cos\gamma)\sin\alpha\cos\gamma\}$$
$$+\mu_2D_0\{4\sqrt{\sigma_2}h_1h_2\zeta'\sin\alpha\cos\delta-(\sigma_2h_2^2-2h_1^2\sin^2\alpha)(1+i\sqrt{\sigma_2}h_2\zeta\cos\delta)\}$$

$$
\begin{aligned}
&-2\mu_2 \sum[\{h_2^2-2(h_1 \sin\alpha+np)^2\}\zeta' \\
&\qquad +(h_1 \sin\alpha+np)(1+ih_2\zeta \cos\gamma_n)h_2 \cos\gamma_n]C_n e^{inpx} \\
&-2\mu_2 \sum[\{h_2^2-2(h_1 \sin\alpha-np)^2\}\zeta' \\
&\qquad +(h_1 \sin\alpha-np)(1+ih_2\zeta \cos\gamma_n')h_2 \cos\gamma_n']C_n' e^{-inpx} \\
&+\mu_2 \sum[4\sqrt{\sigma_2}\,h_2(h_1 \sin\alpha+np)\zeta' \cos\delta_n \\
&\qquad -\{\sigma_2 h_2^2-2(h_1 \sin\alpha+np)^2\}(1+i\sqrt{\sigma_2}\,h_2\zeta \cos\delta_n)]D_n e^{inpx} \\
&+\mu_2 \sum[4\sqrt{\sigma_2}\,h_2(h_1 \sin\alpha-np)\zeta' \cos\delta_n' \\
&\qquad -\{\sigma_2 h_2^2-2(h_1 \sin\alpha-np)^2\}(1+i\sqrt{\sigma_2}\,h_2\zeta \cos\delta_n')]D_n' e^{-inpx}
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
&-h_1^2(a+A_0)(\lambda_1+2\mu_1 \cos^2\alpha) \\
&\quad -h_1^2(a-A_0)\{ih_1(\lambda_1+2\mu_1^2 \cos^2\alpha)\zeta \cos\alpha-4\mu_1\zeta' \sin\alpha \cos\alpha\} \\
&\quad -h_1 \sum\{h_1(\lambda_1+2\mu_1 \cos^2\alpha_n)(1-ih_1\zeta \cos\alpha_n) \\
&\qquad +4\mu_1\zeta'(h_1 \sin\alpha+np)\cos\alpha_n\}A_n e^{inpx} \\
&\quad -h_1 \sum\{h_1(\lambda_1+2\mu_1 \cos^2\alpha_n')(1-ih_1\zeta \cos\alpha_n') \\
&\qquad +4\mu_1\zeta'(h_1 \sin\alpha-np)\cos\alpha_n'\}A_n' e^{-inpx} \\
&\quad +2\mu_1 h_1^2\sqrt{\sigma_1}(b-B_0) \sin\alpha \cos\beta \\
&\qquad +2\mu_1 h_1^2(b+B_0)\{(\sigma_1-2\sin^2\alpha)\zeta'+i\sigma_1 h_1\zeta \sin\alpha \cos^2\beta\} \\
&\quad +2\mu_1 \sum[-\sqrt{\sigma_1}\,h_1(h_1 \sin\alpha+np)(1-i\sqrt{\sigma_1}\,h_1\zeta \cos\beta_n)\cos\beta_n \\
&\qquad +\zeta'\{\sigma_1 h_1^2-2(h_1 \sin\alpha+np)^2\}]B_n e^{inpx} \\
&\quad +2\mu_1 \sum[-\sqrt{\sigma_1}\,h_1(h_1 \sin\alpha-np)(1-i\sqrt{\sigma_1}\,h_1\zeta \cos\beta_n')\cos\beta_n' \\
&\qquad +\zeta'\{\sigma_1 h_1^2-2(h_1 \sin\alpha-np)^2\}]B_n' e^{-inpx} \\
&=C_0\{-h_2^2(\lambda_2+2\mu_2 \cos^2\gamma)(1+ih_2\zeta \cos\gamma)+4\mu_2\zeta' h_1 h_2 \sin\alpha \cos\gamma\} \\
&\quad -\sum\{h_2^2(\lambda_2+2\mu_2 \cos^2\gamma_n)(1+ih_2\zeta \cos\gamma_n) \\
&\qquad -4\mu_2\zeta'(h_1 \sin\alpha+np)h_2 \cos\gamma_n\}C_n e^{inpx} \\
&\quad -\sum\{h_2^2(\lambda_2+2\mu_2 \cos^2\gamma_n')(1+ih_2\zeta \cos\gamma_n') \\
&\qquad -4\mu_2\zeta'(h_1 \sin\alpha-np)h_2 \cos\gamma_n'\}C_n' e^{-inpx} \\
&\quad +2\mu_2 D_0\{\sqrt{\sigma_2}\,h_1 h_2(1+i\sqrt{\sigma_2}\,h_2\zeta \cos\delta) \sin\alpha \cos\delta+\zeta'(\sigma_2 h_2^2-2h_1^2 \sin^2\alpha)\} \\
&\quad +2\mu_2 \sum[\sqrt{\sigma_2}\,h_2(h_1 \sin\alpha+np)(1+i\sqrt{\sigma_2}\,h_2\zeta \cos\delta_n)\cos\delta_n \\
&\qquad +\zeta'\{\sigma_2 h_2^2-2(h_1 \sin\alpha+np)^2\}]D_n e^{inpx}
\end{aligned}
$$

$$+2\mu_2\sum[\sqrt{\sigma_2}h_2(h_1\sin\alpha-np)(1+i\sqrt{\sigma_2}h_2\zeta\cos\delta_n')\cos\delta_n'$$
$$+\zeta'\{\sigma_2h_2^2-2(h_1\sin\alpha-np)^2\}]D_n'e^{-inpx} \tag{19}$$

$$h_1(a+A_0)\sin\alpha+ih_1\zeta(a-A_0)\sin\alpha\cos\alpha$$
$$+\sum\{(h_1\sin\alpha+np)(1-ih_1\zeta\cos\alpha_n)A_ne^{inpx}$$
$$+(h_1\sin\alpha-np)(1-ih_1\zeta\cos\alpha_n')A_n'e^{-inpx}\}$$
$$+\sqrt{\sigma_1}h_1(b-B_0)\cos\beta+i\sigma_1h_1^2\zeta(b+B_0)\cos^2\beta$$
$$-\sqrt{\sigma_1}h_1\sum\{(1-i\sqrt{\sigma_1}h_1\zeta\cos\beta_n)B_ne^{inpx}\cos\beta_n$$
$$+(1-i\sqrt{\sigma_1}h_1\zeta\cos\beta_n')B_n'e^{-inpx}\cos\beta_n'\}$$
$$=C_0h_1(1+ih_2\zeta\cos\gamma)\sin\alpha$$
$$+\sum\{(h_1\sin\alpha+np)(1+ih_2\zeta\cos\gamma_n)C_ne^{inpx}$$
$$+(h_1\sin\alpha-np)(1+ih_2\zeta\cos\gamma_n')C_n'e^{-inpx}\}$$
$$+\sqrt{\sigma_2}h_2D_0(1+i\sqrt{\sigma_2}h_2\zeta\cos\delta)\cos\delta$$
$$+\sqrt{\sigma_2}h_2\sum\{D_ne^{inpx}(1+i\sqrt{\sigma_2}h_2\zeta\cos\delta_n)\cos\delta_n$$
$$+D_n'e^{-inpx}(1+i\sqrt{\sigma_2}h_2\zeta\cos\delta_n')\cos\delta_n'\} \tag{20}$$

$$h_1(a-A_0)\cos\alpha+ih_1^2\zeta(a+A_0)\cos^2\alpha$$
$$-h_1\sum\{A_ne^{inpx}(1-ih_1\zeta\cos\alpha_n)\cos\alpha_n+A_n'e^{-inpx}(1-ih_1\zeta\cos\delta_n')\cos\delta_n'\}$$
$$-h_1(b+B_0)\sin\alpha-ih_1^2\sqrt{\sigma_1}\zeta(b-B_0)\sin\alpha\cos\beta$$
$$-\sum[(h_1\sin\alpha+np)(1-i\sqrt{\sigma_1}h_1\zeta\cos\beta_n)B_ne^{inpx}$$
$$+(h_1\sin\alpha-np)(1-i\sqrt{\sigma_1}h_1\zeta\cos\beta_n')B_n'e^{-inpx}]$$
$$=h_2C_0(1+ih_2\zeta\cos\gamma)\cos\gamma$$
$$+h_2\sum\{C_ne^{inpx}(1+ih_2\zeta\cos\gamma_n)\cos\gamma_n+C_n'e^{-inpx}(1+ih_2\zeta\cos\gamma_n')\cos\gamma_n'\}$$
$$-h_1D_0(1+i\sqrt{\sigma_2}h_2\zeta\cos\delta)\sin\alpha$$
$$-\sum\{(h_1\sin\alpha+np)(1+i\sqrt{\sigma_2}h_2\zeta\cos\delta_n)D_ne^{inpx}$$
$$+(h_1\sin\alpha-np)(1+i\sqrt{\sigma_2}h_2\zeta\cos\delta_n')D_n'e^{-inpx}\}\ . \tag{21}$$

From the above four equations we can obtain linear simultaneous equations determining the solutions of the second approximation for A_0, B_0, C_0 and D_0 by picking up the terms independent of x. They are as follows:

$$-2\mu_1 h_1^2(a-A_0)\sin\alpha\cos\alpha-\mu_1 h_1^2(\sigma_1-2\sin^2\alpha)(b+B_0)$$
$$+2\mu_2 C_0 h_1 h_2\sin\alpha\cos\gamma+\mu_2 D_0(\sigma_2 h_2^2-2h_1^2\sin^2\alpha)$$
$$=-2i\mu_1\zeta_{-n}A_n[np\{h_1^2-2(h_1\sin\alpha+np)^2\}-h_1^2(h_1\sin\alpha+np)\cos^2\alpha_n]$$
$$+2i\mu_1\zeta_n A'_n[np\{h_1^2-2(h_1\sin\alpha-np)^2\}+h_1^2(h_1\sin\alpha-np)\cos^2\alpha'_n]$$
$$-i\mu_1\sqrt{\sigma_1}\,h_1\zeta_{-n}B_n[4np(h_1\sin\alpha+np)+\{\sigma_1 h_1^2-2(h_1\sin\alpha+np)^2\}]\cos\beta_n$$
$$+i\mu_1\sqrt{\sigma_1}\,h_1\zeta_n B'_n[4np(h_1\sin\alpha-np)-\{\sigma_1 h_1^2-2(h_1\sin\alpha-np)^2\}]\cos\beta'_n$$
$$-2i\mu_2\zeta_{-n}C_n[-np\{h_2^2-2(h_1\sin\alpha+np)^2\}+h_2^2(h_1\sin\alpha+np)\cos^2\gamma_n]$$
$$-2i\mu_2\zeta_n C'_n[np\{h_2^2-2(h_1\sin\alpha-np)^2\}+h_2^2\cos^2\gamma'_n(h_1\sin\alpha-np)]$$
$$-i\mu_2\sqrt{\sigma_2}\,h_2\zeta_{-n}D_n[4np(h_1\sin\alpha+np)+\{\sigma_2 h_2^2-2(h_1\sin\alpha+np)^2\}]\cos\delta_n$$
$$+i\mu_2\sqrt{\sigma_2}\,h_2\zeta_n D'_n[4np(h_1\sin\alpha-np)-\{\sigma_2 h_2^2-2(h_1\sin\alpha-np)^2\}]\cos\delta'_n\,,$$

$$-h_1^2(\lambda_1+2\mu_1\cos^2\alpha)(a+A_0)+2\mu_1 h_1^2\sqrt{\sigma_1}(b-B_0)\sin\alpha\cos\beta$$
$$+C_0 h_2^2(\lambda_2+2\mu_2\cos^2\gamma)-2\mu_2\sqrt{\sigma_2}\,h_1 h_2 D_0\sin\alpha\cos\delta$$
$$=-ih_1\zeta_{-n}A_n\{h_1^2(\lambda_1+2\mu_1\cos^2\alpha_n)+4\mu_1 np(h_1\sin\alpha+np)\}\cos\alpha_n$$
$$+ih_1\zeta_n A'_n\{-h_1^2(\lambda_1+2\mu_1\cos^2\alpha'_n)+4\mu_1 np(h_1\sin\alpha-np)\}\cos\alpha'_n$$
$$-2i\mu_1\zeta_{-n}B_n[\sigma_1 h_1^2(h_1\sin\alpha+np)\cos^2\beta_n-np\{\sigma_1 h_1^2-2(h_1\sin\alpha+np)^2\}]$$
$$-2i\mu_1\zeta_n B'_n[\sigma_1 h_1^2(h_1\sin\alpha-np)\cos^2\beta'_n+np\{\sigma_1 h_1^2-2(h_1\sin\alpha-np)^2\}]$$
$$-ih_2\zeta_{-n}C_n[h_2^2(\lambda_2+2\mu_2\cos^2\gamma_n)+4np\mu_2(h_1\sin\alpha+np)]\cos\gamma_n$$
$$-ih_2\zeta_n C'_n[h_2^2(\lambda_2+2\mu_2\cos^2\gamma'_n)-4np\mu_2(h_1\sin\alpha-np)]\cos\gamma'_n$$
$$+2i\mu_2\zeta_{-n}D_n[\sigma_2 h_2^2(h_1\sin\alpha+np)\cos^2\delta_n-np\{\sigma_2 h_2^2-2(h_1\sin\alpha+np)^2\}]$$
$$+2i\mu_2\zeta_n D'_n[\sigma_2 h_2^2(h_1\sin\alpha-np)\cos^2\delta'_n+np\{\sigma_2 h_2^2-2(h_1\sin\alpha-np)^2\}]\,,$$

$$(a+A_0)h_1\sin\alpha+\sqrt{\sigma_1}\,h_1(b-B_0)\cos\beta-C_0 h_1\sin\alpha-\sqrt{\sigma_2}\,h_2 D_0\cos\delta$$
$$=ih_1\zeta_{-n}A_n(h_1\sin\alpha+np)\cos\alpha_n+ih_1\zeta_n A'_n(h_1\sin\alpha-np)\cos\alpha'_n$$
$$-i\sigma_1 h_1^2\zeta_{-n}B_n\cos^2\beta_n-i\sigma_1 h_1^2\zeta_n B'_n\cos^2\beta'_n+ih_2\zeta_{-n}C_n(h_1\sin\alpha+np)\cos\gamma_n$$
$$+ih_2\zeta_n C'_n(h_1\sin\alpha-np)\cos\gamma'_n+i\sigma_2 h_2^2\zeta_{-n}D_n\cos^2\delta_n+i\sigma_2 h_2^2\zeta_n D'_n\cos^2\delta'_n\,,$$

$$(a-A_0)h_1\cos\alpha-(b+B_0)h_1\sin\alpha-h_2 C_0\cos\gamma+h_1 D_0\sin\alpha$$
$$=-ih_1^2\zeta_{-n}A_n\cos^2\alpha_n-ih_1^2A'_n\zeta_n\cos^2\alpha'_n-i\sqrt{\sigma_1}\,h_1\zeta_{-n}B_n(h_1\sin\alpha+np)\cos\beta_n$$
$$-i\sqrt{\sigma_1}\,h_1\zeta_n B'_n(h_1\sin\alpha-np)\cos\beta'_n+ih_2^2\zeta_{-n}C_n\cos^2\gamma_n+ih_2^2\zeta_n C'_n\cos^2\gamma'_n$$
$$-i\sqrt{\sigma_2}\,h_2\zeta_{-n}D_n(h_1\sin\alpha+np)\cos\delta_n-i\sqrt{\sigma_2}\,h_2\zeta_n D'_n(h_1\sin\alpha-np)\cos\delta'_n$$

where for A_n, B_n, A'_n, B'_n, etc. those of the first approximation are used. Since A_n, A'_n, B_n, B'_n, etc. are of the order of ζ_n or ζ_{-n} as shown in the preceding section, A_0, B_0, C_0 and D_0 are of the order of ζ^2.

From the coefficients of e^{inpx} in (18) through (21) linear simultaneous equations determining A_n, B_n, C_n and D_n can be obtained as follows:

$$
\begin{aligned}
&-2i\mu_1 h_1^2\zeta_n(a+A_0)\{np(1-2\sin^2\alpha)+h_1\sin\alpha\cos^2\alpha\}\\
&\quad+2\mu_1\Big[-\sum_{j\neq n} iA_j(n-j)p\zeta_{n-j}\{h_1^2-2(h_1\sin\alpha+jp)^2\}\\
&\qquad+h_1A_n(h_1\sin\alpha+np)\cos\alpha_n-\sum_{j\neq n} ih_1^2A_j\zeta_{n-j}(h_1\sin\alpha+jp)\cos^2\alpha_j\Big]\\
&\quad-2i\mu_1\sum_{k=n+1}^{\infty}A'_{k-n}\zeta_k(kp[h_1^2-2\{h_1\sin\alpha-(k-n)p\}^2]\\
&\qquad+h_1^2\{h_1\sin\alpha-(k-n)p\}\cos^2\alpha'_{k-n})\\
&\quad+i\mu_1h_1^2\sqrt{\sigma_1}\zeta_n(b-B_0)\{4np\sin\alpha-h_1(\sigma_1-2\sin^2\alpha)\}\cos\beta\\
&\quad-\mu_1\Big[4\sqrt{\sigma_1}h_1\sum_{j\neq n} i(n-j)pB_j\zeta_{n-j}(h_1\sin\alpha+jp)\cos\beta_j\\
&\qquad+\{\sigma_1h_1^2-2(h_1\sin\alpha+np)^2\}B_n\\
&\qquad-i\sqrt{\sigma_1}h_1\sum_{j\neq n}\zeta_{n-j}B_j\{\sigma_1h_1^2-2(h_1\sin\alpha+jp)^2\}\cos\beta_j\Big]\\
&\quad-i\sqrt{\sigma_1}h_1\mu_1\sum_{k=n+1}^{\infty}(4kp\{h_1\sin\alpha-(k-n)p\}\\
&\qquad-[\sigma_1h_1^2-2\{h_1\sin\alpha-(k-n)p\}^2])\zeta_kB'_{k-n}\cos\beta'_{k-n}\\
&=-2i\zeta_n\mu_2C_0\{(h_2^2-2h_1^2\sin^2\alpha)np+h_1h_2^2\sin\alpha\cos^2\gamma\}\\
&\qquad+i\mu_2D_0\zeta_n\sqrt{\sigma_2}h_2\{4nph_1\sin\alpha-(\sigma_2h_2^2-2h_1^2\sin^2\alpha)\}\cos\delta\\
&\quad-2\mu_2\Big[\sum_{j\neq n} i(n-j)pC_j\zeta_{n-j}\{h_2^2-2(h_1\sin\alpha+jp)^2\}+C_n(h_1\sin\alpha+np)h_2\cos\gamma_n\\
&\qquad+\sum_{j\neq n} ih_2^2C_j\zeta_{n-j}(h_1\sin\alpha+jp)\cos^2\gamma_j\Big]\\
&\quad-2i\mu_2\sum_{k=n+1}^{\infty}\zeta_kC'_{k-n}[kp\{h_2^2-2(h_1\sin\alpha-(k-n)p)^2\}\\
&\qquad+h_2^2\{h_1\sin\alpha-(k-n)p\}\cos^2\gamma'_{k-n}]\\
&\quad+\mu_2\Big[4\sqrt{\sigma_2}h_2\sum_{j\neq n} ip\zeta_{n-j}D_j(h_1\sin\alpha+jp)(n-j)\cos\delta_j\\
&\qquad-\{\sigma_2h_2^2-2(h_1\sin\alpha+np)^2\}D_n\\
&\qquad-i\sqrt{\sigma_2}h_2\sum_{j\neq n}\zeta_{n-j}D_j\{\sigma_2h_2^2-2(h_1\sin\alpha+jp)^2\}\cos\delta_j\Big]\\
&\quad+i\mu_2\sqrt{\sigma_2}h_2\sum_{k=n+1}^{\infty}D'_{k-n}\zeta_k(4kp\{h_1\sin\alpha-(k-n)p\}\\
&\qquad-[\sigma_2h_2^2-2\{h_1\sin\alpha-(k-n)p\}^2])\cos\delta'_{k-n}\,,
\end{aligned}
$$

$$
\begin{aligned}
&-ih_1^2\zeta_n(a-A_0)\{h_1(\lambda_1+2\mu_1\cos^2\alpha)-4\mu_1 np\sin\alpha\}\cos\alpha\\
&\quad+2i\mu_1h_1^2\zeta_n\{np(\sigma_1-2\sin^2\alpha)+\sigma_1h_1\sin\alpha\cos^2\beta\}(b+B_0)\\
&\quad-h_1[h_1A_n(\lambda_1+2\mu_1\cos^2\alpha_n)-\sum_{j\neq n} ih_1^2\zeta_{n-j}A_j(\lambda_1+2\mu_1\cos^2\alpha_j)\cos\alpha_j\\
&\qquad+\sum_{j\neq n}4i\mu_1pA_j\zeta_{n-j}(n-j)(h_1\sin\alpha+jp)\cos\alpha_j]\\
&\quad+h_1\sum_{k=n+1}^{\infty} i\zeta_kA'_{k-n}[h_1^2(\lambda_1+2\mu_1\cos^2\alpha'_{k-n})\\
&\qquad-4kp\mu_1\{h_1\sin\alpha-(k-n)p\}]\cos\alpha'_{k-n}\\
&\quad+2\mu_1[-\sqrt{\sigma_1}h_1B_n(h_1\sin\alpha+np)\cos\beta_n\\
&\qquad+\sum_{j\neq n} i\sigma_1h_1^2\zeta_{n-j}B_j(h_1\sin\alpha+jp)\cos^2\beta_j\\
&\qquad+\sum_{j\neq n} ip\zeta_{n-j}B_j(n-j)\{\sigma_1h_1^2-2(h_1\sin\alpha+jp)^2\}]\\
&\quad+2i\mu_1\sum_{k=n+1}^{\infty}B'_{k-n}\zeta_k(\sigma_1h_1^2\{h_1\sin\alpha-(k-n)p\}\cos^2\beta'_{k-n}\\
&\qquad+kp[\sigma_1h_1^2-2\{h_1\sin\alpha-(k-n)p\}^2])\\
&=ih_2\zeta_nC_0\{-h_2^2(\lambda_2+2\mu_2\cos^2\gamma)+4np\mu_2h_1\sin\alpha\}\cos\gamma\\
&\qquad+2i\mu_2D_0\zeta_n\{\sigma_2h_1h_2^2\sin\alpha\cos^2\delta+np(\sigma_2h_2^2-2h_1^2\sin^2\alpha)\}\\
&\quad-[h_2^2C_n(\lambda_2+2\mu_2\cos^2\gamma_n)+\sum_{j\neq n} ih_2^3C_j\zeta_{n-j}(\lambda_2+2\mu_2\cos^2\gamma_j)\cos\gamma_j\\
&\qquad-\sum_{j\neq n}4i\mu_2h_2pC_j\zeta_{n-j}(n-j)(h_1\sin\alpha+jp)\cos\gamma_j]\\
&\quad-\sum_{k=n+1}^{\infty} ih_2\zeta_kC'_{k-n}[h_2^2(\lambda_2+2\mu_2\cos^2\gamma'_{k-n})-4\mu_2kp\{h_1\sin\alpha-(k-n)p\}]\cos\gamma'_{k-n}\\
&\quad+2\mu_2[\sqrt{\sigma_2}h_2D_n(h_1\sin\alpha+np)\cos\delta_n+\sum_{j\neq n} i\sigma_2h_2^2D_j\zeta_{n-j}(h_1\sin\alpha+jp)\cos^2\delta_j\\
&\qquad+\sum_{j\neq n} ipD_j\zeta_{n-j}(n-j)\{\sigma_2h_2^2-2(h_1\sin\alpha+jp)^2\}]\\
&\quad+2\mu_2\sum_{k=n+1}^{\infty} i\zeta_kD'_{k-n}(\sigma_2h_2^2\{h_1\sin\alpha-(k-n)p\}\cos^2\delta'_{k-n}\\
&\qquad+kp[\sigma_2h_2^2-2\{h_1\sin\alpha-(k-n)p\}^2])\,,
\end{aligned}
$$

$$
\begin{aligned}
&ih_1^2\zeta_n(a-A_0)\sin\alpha\cos\alpha+A_n(h_1\sin\alpha+np)-\sum_{j\neq n} ih_1A_j\zeta_{n-j}(h_1\sin\alpha+jp)\cos\alpha_j\\
&\quad-\sum_{k=n+1}^{\infty} ih_1\zeta_kA'_{k-n}\{h_1\sin\alpha-(k-n)p\}\cos\alpha'_{k-n}+i\sigma_1h_1^2\zeta_n(b+B_0)\cos^2\beta\\
&\quad-\sqrt{\sigma_1}h_1[B_n\cos\beta_n-i\sqrt{\sigma_1}h_1\sum_{j\neq n}\zeta_{n-j}B_j\cos^2\beta_j]+i\sigma_1h_1^2\sum_{k=n+1}^{\infty}\zeta_kB'_{k-n}\cos^2\beta'_{k-n}\\
&=iC_0h_1h_2\zeta_n\sin\alpha\cos\gamma+C_n(h_1\sin\alpha+np)+ih_2\sum_{j\neq n}\zeta_{n-j}C_j(h_1\sin\alpha+jp)\cos\gamma_j
\end{aligned}
$$

$$+\sum_{k=n+1}^{\infty} ih_2\zeta_k C'_{k-n}\{h_1 \sin\alpha-(k-n)p\}\cos\gamma'_{k-n}+i\sigma_2 h_2^2 D_0\zeta_n \cos^2\delta$$

$$+\sqrt{\sigma_2}h_2[D_n\cos\delta_n+\sum_{j\neq n} i\sqrt{\sigma_2}h_2 D_j\zeta_{n-j}\cos^2\delta_j]+\sum_{k=n+1}^{\infty} i\sigma_2 h_2^2 D'_{k-n}\zeta_k \cos^2\delta'_j\ ,$$

$$(a+A_0)ih_1^2\zeta_n\cos^2\alpha-h_1\{A_n\cos\alpha_n-ih_1\sum_{j\neq n}A_j\zeta_{n-j}\cos^2\alpha_j\}$$

$$+ih_1^2\sum_{k=n+1}^{\infty}A'_{k-n}\zeta_k\cos^2\alpha'_{k-n}-(b-B_0)i\sqrt{\sigma_1}h_1^2\zeta_n\sin\alpha\cos\beta$$

$$-(h_1\sin\alpha+np)B_n+i\sqrt{\sigma_1}h_1\sum_{j\neq n}\zeta_{n-j}B_j(h_1\sin\alpha+jp)\cos\beta_j$$

$$+i\sqrt{\sigma_1}h_1\sum_{k=n+1}^{\infty}B'_{k-n}\zeta_k\{h_1\sin\alpha-(k-n)p\}\cos\beta'_{k-n}$$

$$=ih_2^2C_0\zeta_n\cos^2\gamma+h_2\{C_n\cos\gamma_n+ih_2\sum_{j\neq n}C_j\zeta_{n-j}\cos^2\gamma_j\}$$

$$+ih_2^2\sum_{k=n+1}^{\infty}C'_{k-n}\zeta_k\cos^2\gamma'_{k-n}-i\sqrt{\sigma_2}h_1h_2D_0\zeta_n\sin\alpha\cos\delta$$

$$-(h_1\sin\alpha+np)D_n-i\sqrt{\sigma_2}h_2\sum_{j\neq n}D_j\zeta_{n-j}(h_1\sin\alpha+jp)\cos\delta_j$$

$$-i\sqrt{\sigma_2}h_2\sum_{k=n+1}^{\infty}D'_{k-n}\zeta_k\{h_1\sin\alpha-(k-n)p\}\cos\delta'_{k-n}$$

From the coefficients of e^{-inpx} in (18) through (21), the following equations to determine A'_n, B'_n, C'_n and D'_n of the second approximation are obtained.

$$-2i\mu_1 h_1^2\zeta_{-n}(a+A_0)\{-np(1-2\sin^2\alpha)+h_1\sin\alpha\cos^2\alpha\}$$

$$+2i\mu_1\sum_{k=n+1}^{\infty}\zeta_{-k}A_{k-n}(kp[h_1^2-2\{h_1\sin\alpha+(k-n)p\}^2]$$

$$-h_1^2\{h_1\sin\alpha+(k-n)p\}\cos^2\alpha_{k-n})$$

$$+2\mu_1[i\sum_{j\neq n}(n-j)p\zeta_{j-n}A'_j\{h_1^2-2(h_1\sin\alpha-jp)^2\}$$

$$+h_1A'_n(h_1\sin\alpha-np)\cos\alpha'_n-i\sum_{j\neq n}h_1^2\zeta_{j-n}A'_j(h_1\sin\alpha-jp)\cos^2\alpha'_j]$$

$$-i\mu_1\sqrt{\sigma_1}h_1^2\zeta_{-n}(b-B_0)\{4np\sin\alpha+h_1(\sigma_1-2\sin^2\alpha)\}\cos\beta$$

$$+i\mu_1\sqrt{\sigma_1}h_1\sum_{k=n+1}^{\infty}\zeta_{-k}B_{k-n}[4kp\{h_1\sin\alpha+(k-n)p\}$$

$$+(\sigma_1h_1^2-2\{h_1\sin\alpha+(k-n)p\}^2)]\cos\beta_{k-n}$$

$$-\mu_1[4\sqrt{\sigma_1}h_1\sum_{j\neq n}i(j-n)p\zeta_{j-n}B'_j(h_1\sin\alpha-jp)\cos\beta'_j$$

$$+\{\sigma_1h_1^2-2(h_1\sin\alpha-np)^2\}B'_n$$

$$-i\sqrt{\sigma_1}h_1\sum_{j\neq n}\{\sigma_1h_1^2-2(h_1\sin\alpha-jp)^2\}\zeta_{j-n}B'_j\cos\beta'_j]$$

$$
\begin{aligned}
=&-2i\mu_2\zeta_{-n}C_0\{-np(h_2^2-2h_1^2\sin^2\alpha)+h_1h_2^2\sin\alpha\cos^2\gamma\}\\
&-i\mu_2\sqrt{\sigma_2}\,h_2\zeta_{-n}D_0\{4nph_1\sin\alpha+\sigma_2h_2^2-2h_1^2\sin^2\alpha\}\cos\delta\\
&-2i\mu_2\sum_{k=n+1}^{\infty}\zeta_{-k}C_{k-n}(-[h_2^2-2\{h_1\sin\alpha+(k-n)p\}^2]kp\\
&\qquad+h_2^2\{h_1\sin\alpha+(k-n)p\}\cos^2\gamma_{k-n})\\
&-2\mu_2[\sum_{j\neq n}ip\zeta_{j-n}C'_j(j-n)\{h_2^2-2(h_1\sin\alpha-jp)^2\}\\
&\qquad+(h_1\sin\alpha-np)h_2C'_n\cos\gamma'_n+\sum_{j\neq n}ih_2^2C'_j\zeta_{j-n}(h_1\sin\alpha-jp)\cos^2\gamma'_j]\\
&-i\mu_2\sqrt{\sigma_2}\,h_2\sum_{k=n+1}^{\infty}D_{k-n}\zeta_{-k}(4kp\{h_1\sin\alpha+(k-n)p\}\\
&\qquad+[\sigma_2h_2^2-2\{h_1\sin\alpha+(k-n)p\}^2]\cos\delta_{k-n})\\
&+\mu_2[\sum_{j\neq n}4i\sqrt{\sigma_2}\,h_2p\zeta_{j-n}D'_j(j-n)(h_1\sin\alpha-jp)\cos\delta'_j\\
&\qquad-\{\sigma_2h_2^2-2(h_1\sin\alpha-np)^2\}D'_n\\
&\qquad-\sum_{j\neq n}i\sqrt{\sigma_2}\,h_2D'_j\zeta_{j-n}\{\sigma_2h_2^2-2(h_1\sin\alpha-jp)^2\}\cos\delta'_j]\ ,
\end{aligned}
$$

$$
\begin{aligned}
&-ih_1^2\zeta_{-n}(a-A_0)\{h_1(\lambda_1+2\mu_1\cos^2\alpha)+4\mu_1np\sin\alpha\}\cos\alpha\\
&\quad+2i\mu_1h_1^2\zeta_{-n}(b+B_0)\{-np(\sigma_1-2\sin^2\alpha)+\sigma_1h_1\sin\alpha\cos^2\beta\}\\
&\quad+ih_1\sum_{k=n+1}^{\infty}\zeta_{-k}A_{k-n}[(\lambda_1+2\mu_1\cos^2\alpha_{k-n})+4\mu_1kp\{h_1\sin\alpha+(k-n)p\}]\cos\alpha_{k-n}\\
&\quad-h_1[h_1A'_n(\lambda_1+2\mu_1\cos^2\alpha'_n)-\sum_{j\neq n}ih_1^2\zeta_{j-n}A'_j(\lambda_1+2\mu_1\cos^2\alpha'_j)\cos\alpha'_j\\
&\qquad-\sum_{j\neq n}4i\mu_1p\zeta_{j-n}A'_j(n-j)(h_1\sin\alpha-jp)\cos\alpha'_j]\\
&\quad+2\mu_1\sum_{k=n+1}^{\infty}i\zeta_{-k}B_{k-n}(\sigma_1h_1^2\{h_1\sin\alpha+(k-n)p\}\cos^2\beta_{k-n}\\
&\qquad-kp[\sigma_1h_1^2-2\{h_1\sin\alpha+(k-n)p\}^2])\\
&\quad+2\mu_1[-\sqrt{\sigma_1}\,h_1B'_n(h_1\sin\alpha-np)\cos\beta'_n\\
&\qquad+\sum_{j\neq n}i\sigma_1h_1^2\zeta_{j-n}B'_j(h_1\sin\alpha-jp)\cos^2\beta'_j\\
&\qquad-\sum_{j\neq n}ip\zeta_{j-n}B'_j(n-j)\{\sigma_1h_1^2-2(h_1\sin\alpha-jp)^2\}]\\
&=-ih_2C_0\zeta_{-n}\{h_2^2(\lambda_2+2\mu_2\cos^2\gamma)+4np\mu_2h_1\sin\alpha\}\cos\gamma\\
&\quad+2i\mu_2\zeta_{-n}D_0\{\sigma_2h_1h_2^2\sin\alpha\cos^2\delta-np(\sigma_2h_2^2-2h_1^2\sin^2\alpha)\}\\
&\quad-\sum_{k=n+1}^{\infty}ih_2\zeta_{-k}C_{k-n}[h_2^2(\lambda_2+2\mu_2\cos^2\gamma_{k-n})+4\mu_2kp\{h_1\sin\alpha+(k-n)p\}]\cos\gamma_{k-n}\\
&\quad-[h_2^2C'_n(\lambda_2+2\mu_2\cos^2\gamma'_n)+\sum_{j\neq n}ih_2^3\zeta_{j-n}C'_j(\lambda_2+2\mu_2\cos^2\gamma'_j)\cos\gamma'_j
\end{aligned}
$$

$$
\begin{aligned}
&+\sum_{j\neq n} 4ip\mu_2 h_2\zeta_{j-n}C'_j(n-j)(h_1\sin\alpha-jp)\cos\gamma'_j]\\
&+2i\mu_2\sum_{k=n+1}^{\infty}\zeta_{-k}D_{k-n}(\sigma_2 h_2^2\{h_1\sin\alpha+(k-n)p\}\cos^2\delta_{k-n}\\
&\quad -kp[\sigma_2 h_2^2-2\{h_1\sin\alpha+(k-n)p\}^2])\\
&+2\mu_2[\sqrt{\sigma_2}h_2 D'_n(h_1\sin\alpha-np)\cos\delta'_n+\sum_{j\neq n} i\sigma_2 h_2^2\zeta_{j-n}D'_j(h_1\sin\alpha-jp)\cos^2\delta'_j\\
&\quad -\sum_{j\neq n} ip\zeta_{j-n}D'_j(n-j)\{\sigma_2 h_2^2-2(h_1\sin\alpha-jp)^2\}]\ ,
\end{aligned}
$$

$$
\begin{aligned}
&ih_1^2\zeta_{-n}(a-A_0)\sin\alpha\cos\alpha-\sum_{k=n+1}^{\infty} ih_1\zeta_{-k}A_{k-n}\{h_1\sin\alpha+(k-n)p\}\cos\alpha_j\\
&\quad +A'_n(h_1\sin\alpha-np)-\sum_{j\neq n} ih_1(h_1\sin\alpha-jp)\zeta_{j-n}A'_j\cos\alpha'_j\\
&\quad +i\sigma_1 h_1\zeta_{-n}(b+B_0)\cos^2\beta+i\sigma_1 h_1^2\sum_{k=n+1}^{\infty}\zeta_{-k}B_{k-n}\cos^2\beta_{k-n}\\
&\quad -\sqrt{\sigma_1}h_1[B'_n\cos\beta'_n-i\sqrt{\sigma_1}h_1\sum_{j\neq n}\zeta_{j-n}B'_j\cos^2\beta'_j]\\
&=ih_1h_2C_0\zeta_{-n}\sin\alpha\cos\gamma+\sum_{k=n+1}^{\infty} ih_2C_{k-n}\zeta_{-k}\{h_1\sin\alpha+(k-n)p\}\cos\gamma_{k-n}\\
&\quad +(h_1\sin\alpha-np)C'_n+\sum_{j\neq n} ih_2\zeta_{j-n}C'_j(h_1\sin\alpha-jp)\cos\gamma'_j+i\sigma_2 h_2^2 D_0\zeta_{-n}\cos^2\delta\\
&\quad +i\sigma_2 h_2^2\sum_{k=n+1}^{\infty} D_{k-n}\zeta_{-k}\cos^2\delta_{k-n}\\
&\quad +\sqrt{\sigma_2}h_2[D'_n\cos\delta'_n+i\sqrt{\sigma_2}h_2\sum_{j\neq n} D'_j\zeta_{j-n}\cos^2\delta'_j]\ ,
\end{aligned}
$$

$$
\begin{aligned}
&(a+A_0)ih_1^2\zeta_{-n}\cos^2\alpha+ih_1^2\sum_{k=n+1}^{\infty} A_{k-n}\zeta_{-k}\cos^2\alpha_{k-n}\\
&\quad -h_1\{A'_n\cos\alpha'_n-ih_1\sum_{j\neq n} A'_j\zeta_{j-n}\cos^2\alpha'_j\}-i\sqrt{\sigma_1}h_1^2\zeta_{-n}(b-B_0)\sin\alpha\cos\beta\\
&\quad +i\sqrt{\sigma_1}h_1\sum_{k=n+1}^{\infty}\zeta_{-k}B_{k-n}\{h_1\sin\alpha+(k-n)p\}\cos\beta_{k-n}\\
&\quad -(h_1\sin\alpha-np)B'_n+i\sqrt{\sigma_1}h_1\sum_{j\neq n}\zeta_{j-n}B'_j(h_1\sin\alpha-jp)\cos\beta'_j\\
&=ih_2^2\zeta_{-n}C_0\cos^2\gamma+ih_2^2\sum_{k=n+1}^{\infty} C_{k-n}\zeta_{-k}\cos^2\gamma_{k-n}\\
&\quad +h_2\{C'_n\cos\gamma'_n+ih_2\sum_{j\neq n} C'_j\zeta_{j-n}\cos^2\gamma'_j\}-ih_1\sqrt{\sigma_2}h_2D_0\zeta_{-n}\sin\alpha\cos\delta\\
&\quad -i\sqrt{\sigma_2}h_2\sum_{k=n+1}^{\infty} D_{k-n}\zeta_{-k}\{h_1\sin\alpha+(k-n)p\}\cos\delta_{k-n}\\
&\quad -(h_1\sin\alpha-np)D'_n-i\sqrt{\sigma_2}h_2\sum_{j\neq n} D'_j\zeta_{j-n}(h_1\sin\alpha-jp)\cos\delta'_j\ .
\end{aligned}
$$

In these formulae for A_n, B_n, A'_n, B'_n, etc. the values of the first approximation are used for A_0, B_0, A_j, B_j, etc. As they are of the order of ζ, A_n, B_n, A'_n, B'_n, etc. are of the order of ζ^2.

§ 5. The Case of Normal Incidence on the Mean Surface of the Boundary $\zeta = c \cos px$

As in the incident SH wave, the case of normal incidence on the harmonic boundary surface given by the equation $\zeta = c \cos px$ was calculated. In this case, $\zeta_n = \zeta_{-n} = 0$ $(n \neq 1)$, $\zeta_1 = \zeta_{-1} = c/2$, $\alpha = \beta = \gamma = \delta = 0$, $\sin \alpha_n = np/h_1$, $\sin \alpha'_n = -np/h_1$, $\sin \beta_n = np/\sqrt{\sigma_1}h_1$, $\sin \beta'_n = -np/\sqrt{\sigma_1}h_1$, $\cos \alpha_n = \cos \alpha'_n$, $\cos \beta_n = \cos \beta'_n$, etc.

§ 5.1. The Case of Incidence of the P Wave

In this case, we get the following linear equations from which the solutions of the first approximation for A_0, B_0, C_0 and D_0 are obtained.

$$-\mu_1\sigma_1 h_1^2 B_0 + \mu_2\sigma_2 h_2^2 D_0 = 0 ,$$

$$-h_1^2 A_0(\lambda_1 + 2\mu_1) + h_2^2 C_0(\lambda_2 + 2\mu_2) = h_1^2(\lambda_1 + 2\mu_1) ,$$

$$\sqrt{\sigma_1}h_1 B_0 + \sqrt{\sigma_2}h_2 D_0 = 0 , \qquad h_1 A_0 + h_2 C_0 = h_1 .$$

Therefore, $B_0 = D_0 = 0$,

$$A_0 = \{h_2(\lambda_2 + 2\mu_2) - h_1(\lambda_1 + 2\mu_1)\} / \{h_2(\lambda_2 + 2\mu_2) + h_1(\lambda_1 + 2\mu_1)\} ,$$

$$C_0 = 2h_1^2(2\mu_1 + \lambda_1)/[h_2\{h_1(\lambda_1 + 2\mu_1) + h_2(\lambda_2 + 2\mu_2)\}] ,$$

as far as the solutions of the first approximation are concerned.

With regard to the solutions of the first approximation for A_1, B_1, C_1 and D_1, the following linear equations are obtained.

$$2\mu_1 h_1 p A_1 \cos \alpha_1 - \mu_1 B_1(\sigma_1 h_1^2 - 2p^2) + 2\mu_2 p h_2 C_1 \cos \gamma_1 + \mu_2 D_1(\sigma_2 h_2^2 - 2p^2)$$
$$= 2i\mu_1 h_1^2 p\zeta(1 + A_0) - 2i\mu_2 h_2^2 p\zeta_1 C_0 ,$$

$$-h_1^2 A_1(\lambda_1 + 2\mu_1 \cos^2 \alpha_1) - 2\mu_1\sqrt{\sigma_1}h_1 p B_1 \cos \beta_1$$
$$+ h_2^2 C_1(\lambda_2 + 2\mu_2 \cos^2 \gamma_1) - 2\mu_2\sqrt{\sigma_2}h_2 p D_1 \cos \delta_1$$
$$= i h_1^3 \zeta_1(\lambda_1 + 2\mu_1)(1 - A_0) - iC_0 h_2^3 \zeta_1(\lambda_2 + 2\mu_2) ,$$

$$pA_1 - \sqrt{\sigma_1}h_1 B_1 \cos \beta_1 - pC_1 - \sqrt{\sigma_2}h_2 D_1 \cos \delta_1 = 0 ,$$

$$-h_1 A_1 \cos \alpha_1 - pB_1 - h_2 C_1 \cos \gamma_1 + pD_1 = -i h_1^2 \zeta_1(1 + A_0) + i h_2^2 \zeta_1 C_0 .$$

We see that the right hand side of the first equation becomes zero by using A_0 and C_0 of the first approximation obtained above.

If this set of linear equations is written in the following form,

$$a_{11}A_1-a_{12}B_1+a_{13}C_1+a_{14}D_1=0\ ,$$

$$-a_{21}A_1-a_{22}B_1+a_{23}C_1-a_{24}D_1=l_2\ ,$$

$$a_{31}A_1-a_{32}B_1-a_{33}C_1-a_{34}D_1=0\ ,$$

$$-a_{41}A_1-a_{42}B_1-a_{43}C_1+a_{44}D_1=l_4\ ,$$

the simultaneous linear equations giving the solutions of the first approximation for A_1', B_1', etc. have the following forms:

$$-a_{11}A_1'-a_{12}B_1'-a_{13}C_1'+a_{14}D_1'=0\ ,$$

$$-a_{21}A_1'+a_{22}B_1'+a_{23}C_1'+a_{24}D_1'=l_2\ ,$$

$$-a_{31}A_1'-a_{32}B_1'+a_{33}C_1'-a_{34}D_1'=0\ ,$$

$$-a_{41}A_1'+a_{42}B_1'-a_{43}C_1'-a_{44}D_1'=l_4\ .$$

By a simple transformation of a set of determinants of the coefficients in the latter equations, the following relations between A_1, B_1, etc. and A_1', B_1', etc. can be obtained.

$$A_1=A_1'\ ,\qquad C_1=C_1'\ ,\qquad B_1=-B_1'\ ,\qquad D_1=-D_1'\ .$$

Since the case of normal incidence is under consideration, these relations are very reasonable and simplify the problem.

For A_0, B_0, etc. of the second approximation by using the relations above obtained, $A_1=A_1'$, $B_1=-B_1'$, etc., we have

$$\begin{aligned}
&-3\mu_1h_1^2A_0+3\mu_2h_2^2C_0\\
&\quad=3\mu_1h_1-2i\mu_1h_1\zeta_1A_1(3h_1^2+2p^2)\cos\alpha_1-4i\mu_1\zeta_1B_1p^3\\
&\qquad-2i\mu_2h_2\zeta_1C_1(3h_2^2+2p^2)\cos\gamma_1+4i\mu_2\zeta_1D_1p^3\ ,
\end{aligned}$$

$$\begin{aligned}
-h_1A_0-h_2C_0=&-h_1-2i\zeta_1A_1(h_1^2-p^2)-2i\sqrt{\sigma_1}\zeta_1B_1p\cos\beta_1\\
&+2i\zeta_1C_1(h_2^2-p^2)-2i\sqrt{\sigma_2}h_2\zeta_1D_1p\cos\delta_1\ ,
\end{aligned}$$

$$-\mu_1\sigma_1h_1^2B_0+\mu_2\sigma_2h_2^2D_0=0\ ,$$

$$-\sqrt{\sigma_1}h_1B_0-\sqrt{\sigma_2}h_2D_0=0\ .$$

From the last two equations, $B_0=D_0=0$.

With respect to A_1, B_1, etc. of the second approximation, the magnitude of the correction term to those of the first approximation is of the order of ζ^3. Thus as far as the present approximations are concerned, A_1, B_1, etc. of the second approximation are the same as those of the first approximation.

In the second approximation, A_2, B_2, etc. must be taken into consideration and A_n, B_n, etc. $(n>2)$ are quantities of an order higher than ζ^3. Linear equations for A_2, B_2, etc. are as follows:

$$
\begin{aligned}
&4\mu_1 pA_2h_1\cos\alpha_2-\mu_1B_2(\sigma_1h_1^2-8p^2)+4\mu_2pC_2h_2\cos\gamma_2+\mu_2D_2(\sigma_2h_2^2-8p^2)\\
&\quad=2i\mu_1pA_1\zeta_1(2h_1^2-3p^2)+3i\mu_1\zeta_1B_1\sqrt{\sigma_1}\,h_1(2p^2-h_1^2)\cos\beta_1\\
&\qquad-2i\mu_2pC_1\zeta_1(2h_2^2-3p^2)+3i\mu_2\zeta_1D_1\sqrt{\sigma_2}\,h_2(2p^2-h_2^2)\cos\delta_1\ ,\\
&-\mu_1h_1^2A_1(1+2\cos^2\alpha_2)-4\mu_1\sqrt{\sigma_1}\,h_1B_2p\cos\beta_2\\
&\qquad+\mu_2h_2^2C_2(1+2\cos^2\gamma_2)-4\mu_2\sqrt{\sigma_2}\,h_2pD_2\cos\delta_2\\
&\quad=-3i\mu_1h_1\zeta_1A_1(h_1^2-2p^2)\cos\alpha_1-6i\mu_1p\zeta_1B_1(2h_1^2-p^2)\\
&\qquad-3i\mu_2h_2\zeta_1C_1(h_2^2-2p^2)\cos\gamma_1-6i\mu_2p\zeta_1D_1(2h_2^2-p^2)\ ,\\
&2pA_2-\sqrt{\sigma_1}\,h_1B_2\cos\beta_2-2pC_2-\sqrt{\sigma_2}\,h_2D_2\cos\delta_2\\
&\quad=ih_1\zeta_1A_1p\cos\alpha_1-i\sigma_1h_1^2\zeta_1B_1\cos^2\beta_1+ih_2\zeta_1C_1p\cos\gamma_1+i\sigma_2h_2^2\zeta_1D_1\cos^2\delta_1\ ,\\
&-h_1A_2\cos\alpha_2-2pB_2-h_2C_2\cos\gamma_2+2pD_2\\
&\quad=-ih_1^2A_1\zeta_1\cos^2\alpha_1-i\sqrt{\sigma_1}\,h_1\zeta_1B_1p\cos\beta_1\\
&\qquad+ih_2^2C_1\zeta_1\cos^2\gamma_1-i\sqrt{\sigma_2}\,h_2\zeta_1D_1p\cos\delta_1\ .
\end{aligned}
$$

Through the same procedure as already adopted, having obtained the relations $A_1=A_1'$, $B_1=-B_1'$, etc., it is proved readily that the relations $A_2=A_2'$, $B_2=-B_2'$, $C_2=C_2'$ and $D_2=-D_2'$ hold as in A_1, B_1, A_1', B_1', etc.

The numerical constants necessary for the calculations are assumed as follows:

$$V_{p1}/V_{p2}=V_{s1}/V_{s2}=6/8\ , \qquad \mu_2/\mu_1=2\ ,$$

on Poisson's hypothesis $\lambda_i=\mu_i$.

The formulae are expressed in terms of L/L_{p1}, that is, the ratio of the wave length of corrugation to that of the incident wave, and differ slightly from each other according to the intervals of L/L_{p1}. As an example these formulae for A_1, B_1, etc. are given in the following.

(i) $L/L_{p1} \geq 4/3$

In this range, A_1, B_1, etc. are purely imaginary numbers.

$$2A_1\sqrt{(L/L_{p1})^2-1}-B_1\{3(L/L_{p1})^2-2\}+2C_1(\mu_2/\mu_1)\sqrt{(h_2/h_1)^2(L/L_{p1})^2-1}$$
$$-D_1(\mu_2/\mu_1)\{2-3(h_2/h_1)^2(L/L_{p1})^2\}=0\ ,$$

$$-\{3(L/L_{p1})^2-2\}A_1-2B_1\sqrt{3(L/L_{p1})^2-1}+C_1(\mu_2/\mu_1)\{3(h_2/h_1)^2(L/L_{p1})^2-2\}$$
$$-2D_1(\mu_2/\mu_1)\sqrt{3(h_2/h_1)^2(L/L_{p1})^2-1}$$
$$=3\pi i(c/L_{p1})(L/L_{p1})^2\{1-A_0-C_0(\mu_2/\mu_1)(h_2/h_1)^3\}\ ,$$

$$A_1-B_1\sqrt{3(L/L_{p1})^2-1}-C_1-D_1\sqrt{3(h_2/h_1)^2(L/L_{p1})^2-1}=0\ ,$$
$$-A_1\sqrt{(L/L_{p1})^2-1}-B_1-C_1\sqrt{(h_2/h_1)^2(L/L_{p1})^2-1}+D_1$$
$$=-i\pi(c/L_{p1})(L/L_{p1})\{1+A_0-(h_2/h_1)^2C_0\}\ .$$

For L/L_{p1} smaller than 4/3, A_1, B_1, etc. are complex numbers and A_1 is written as $A_1=RA_1+iIA_1$ and so forth.

(ii) $4/3>L/L_{p1}\geq 1$

$$2RA_1\sqrt{(L/L_{p1})^2-1}-RB_1\{3(L/L_{p1})^2-2\}+2IC_1(\mu_2/\mu_1)\sqrt{1-(h_2/h_1)^2(L/L_{p1})^2}$$
$$-RD_1(\mu_2/\mu_1)\{2-3(h_2/h_1)^2(L/L_{p1})^2\}=0\ ,$$

$$2IA_1\sqrt{(L/L_{p1})^2-1}-IB_1\{3(L/L_{p1})^2-2\}-2RC_1(\mu_2/\mu_1)\sqrt{1-(h_2/h_1)^2(L/L_{p1})^2}$$
$$-ID_1(\mu_2/\mu_1)\{2-3(h_2/h_1)^2(L/L_{p1})^2\}=0\ ,$$

$$-\{3(L/L_{p1})^2-2\}RA_1-2RB_1\sqrt{3(L/L_{p1})^2-1}$$
$$+RC_1(\mu_2/\mu_1)\{3(h_2/h_1)^2(L/L_{p1})^2-2\}$$
$$-2RD_1(\mu_2/\mu_1)\sqrt{3(h_2/h_1)^2(L/L_{p1})^2-1}=0\ ,$$

$$-\{3(L/L_{p1})^2-2\}IA_1-2IB_1\sqrt{3(L/L_{p1})^2-1}$$
$$+IC_1(\mu_2/\mu_1)\{3(h_2/h_1)^2(L/L_{p1})^2-2\}$$
$$-2ID_1(\mu_2/\mu_1)\sqrt{3(h_2/h_1)^2(L/L_{p1})^2-1}$$
$$=3\pi(c/L_{p1})(L/L_{p1})^2\{1-A_0-C_0(\mu_2/\mu_1)(h_2/h_1)^3\}\ ,$$

$$RA_1-RB_1\sqrt{3(L/L_{p1})^2-1}-RC_1-RD_1\sqrt{3(h_2/h_1)^2(L/L_{p1})^2-1}=0\ ,$$

$$IA_1-IB_1\sqrt{3(L/L_{p1})^2-1}-IC_1-ID_1\sqrt{3(h_2/h_1)^2(L/L_{p1})^2-1}=0\ ,$$

$$-RA_1\sqrt{(L/L_{p1})^2-1}-RB_1-IC_1\sqrt{1-(h_2/h_1)^2(L/L_{p1})^2}+RD_1=0\ ,$$

$$-IA_1\sqrt{(L/L_{p1})^2-1}-IB_1+RC_1\sqrt{1-(h_2/h_1)^2(L/L_{p1})^2}+ID_1$$
$$=-\pi(c/L_{p1})(L/L_{p1})\{1+A_0-(h_2/h_1)^2C_0\}$$

(iii) $1>L/L_{p1}\geq 0.770$

In this interval, only the first, second, seventh and eighth equations are slightly different from those in the interval (ii). In the following, only dissimilar equations are given.

$$2IA_1\sqrt{1-(L/L_{p1})^2}-RB_1\{3(L/L_{p1})^2-2\}+2IC_1(\mu_2/\mu_1)\sqrt{1-(h_2/h_1)^2(L/L_{p1})^2}$$
$$-RD_1(\mu_2/\mu_1)\{2-3(h_2/h_1)^2(L/L_{p1})^2\}=0\ ,$$

$$-2RA_1\sqrt{1-(L/L_{p1})^2}-IB_1\{3(L/L_{p1})^2-2\}$$
$$-2RC_1(\mu_2/\mu_1)\sqrt{1-(h_2/h_1)^2(L/L_{p1})^2}$$
$$-ID_1(\mu_2/\mu_1)\{2-3(h_2/h_1)^2(L/L_{p1})^2\}=0\ ,$$

$$-IA_1\sqrt{1-(L/L_{p1})^2}-RB_1-IC_1\sqrt{1-(h_2/h_1)^2(L/L_{p1})^2}+RD_1=0\ ,$$

$$RA_1\sqrt{1-(L/L_{p1})^2}-IB_1+RC_1\sqrt{1-(h_2/h_1)^2(L/L_{p1})^2}+ID_1$$
$$=-\pi(c/L_{p1})(L/L_{p1})\{1+A_0-(h_2/h_1)^2C_0\}\ .$$

(iv) $0.770>L/L_{p1}\geq 0.578$

In this interval, only the third, fourth, fifth and sixth equations given below are slightly different from those in the interval (iii).

$$-\{3(L/L_{p1})^2-2\}RA_1-2RB_1\sqrt{3(L/L_{p1})^2-1}$$
$$+RC_1(\mu_2/\mu_1)\{3(h_2/h_1)^2(L/L_{p1})^2-2\}$$
$$-2ID_1(\mu_2/\mu_1)\sqrt{1-3(h_2/h_1)^2(L/L_{p1})^2}=0\ ,$$

$$-\{3(L/L_{p1})^2-2\}IA_1-2IB_1\sqrt{3(L/L_{p1})^2-1}$$
$$+IC_1(\mu_2/\mu_1)\{3(h_2/h_1)^2(L/L_{p1})^2-2\}+2RD_1(\mu_2/\mu_1)\sqrt{1-3(h_2/h_1)^2(L/L_{p1})^2}$$
$$=3\pi(c/L_{p1})(L/L_{p1})^2\{1-A_0-C_0(\mu_2/\mu_1)(h_2/h_1)^3\}\ ,$$

$$RA_1-RB_1\sqrt{3(L/L_{p1})^2-1}-RC_1-ID_1\sqrt{1-3(h_2/h_1)^2(L/L_{p1})^2}=0\ ,$$

$$IA_1-IB_1\sqrt{3(L/L_{p1})^2-1}-IC_1+RD_1\sqrt{1-3(h_2/h_1)^2(L/L_{p1})^2}=0\ .$$

(v) $0.578>L/L_{p1}$

In this interval, only the third, fourth, fifth and sixth equations given below are slightly different from those in the interval (iv).

$$-\{3(L/L_{p1})^2-2\}RA_1-2IB_1\sqrt{1-3(L/L_{p1})^2}$$
$$+RC_1(\mu_2/\mu_1)\{3(h_2/h_1)^2(L/L_{p1})^2-2\}$$
$$-2ID_1(\mu_2/\mu_1)\sqrt{1-3(h_2/h_1)^2(L/L_{p1})^2}=0\ ,$$
$$-\{3(L/L_{p1})^2-2\}IA_1+2RB_1\sqrt{1-3(L/L_{p1})^2}$$
$$+IC_1(\mu_2/\mu_1)\{3(h_2/h_1)^2(L/L_{p1})^2-2\}$$
$$+2RD_1(\mu_2/\mu_1)\sqrt{1-3(h_2/h_1)^2(L/L_{p1})^2}$$
$$=3\pi(c/L_{p1})(L/L_{p1})^2\{1-A_0-C_0(\mu_2/\mu_1)(h_2/h_1)^3\}\ ,$$
$$RA_1-IB_1\sqrt{1-3(L/L_{p1})^2}-RC_1-ID_1\sqrt{1-3(h_2/h_1)^2(L/L_{p1})^2}=0\ ,$$
$$IA_1+RB_1\sqrt{1-3(L/L_{p1})^2}-IC_1+RD_1\sqrt{1-3(h_2/h_1)^2(L/L_{p1})^2}=0\ .$$

In calculating these formulae, the parametron computor 'PC-1' of

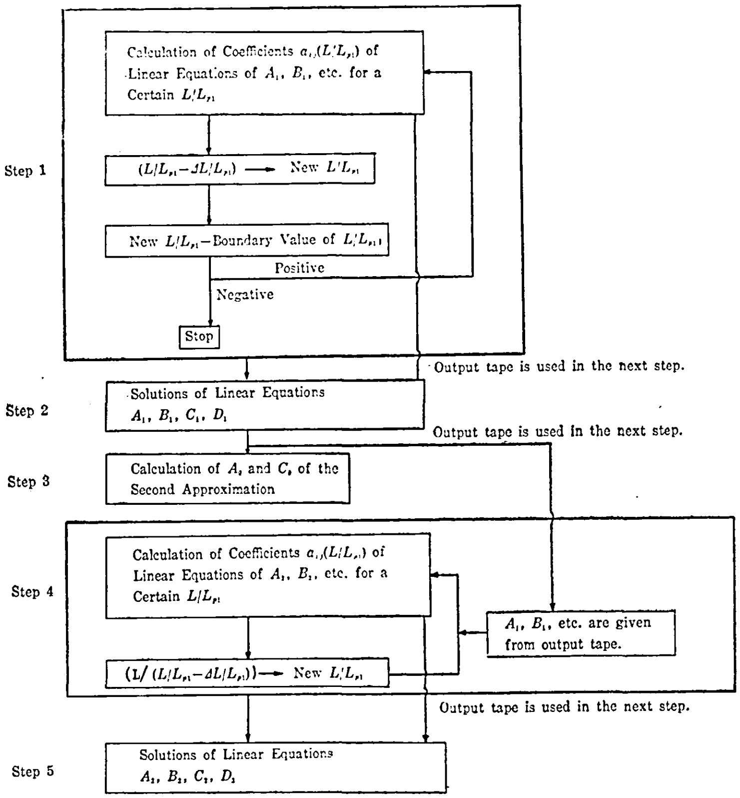

Fig. 2. Flow diagram.

Takahashi Laboratory, Institute of Physics, Faculty of Science, University of Tokyo was used. The memory capacity of this computor is only 256 in long word and is too small for these calculations. The reason why this computor was used depended mostly on the cost, although partly it is because the writer is familiar with the program of this computor. The flow diagram used in these calculations is shown in Fig. 2.

Step 1 is the step to calculate at appropriate intervals $\Delta(L/L_{p1})$ all coefficients of simultaneous linear equations from which A_1, B_1, etc. are determined. The tape on which the coefficients are punched in the output is used directly as an input tape in the next step, Step 2. In Step 2, a revised program of the complete program No. 3 in the PC–1 Library is used, which is a program for obtaining the solution of simultaneous linear equations by using the sweeping out method. The tape punched in the output of Step 2 is edited and used as input tape

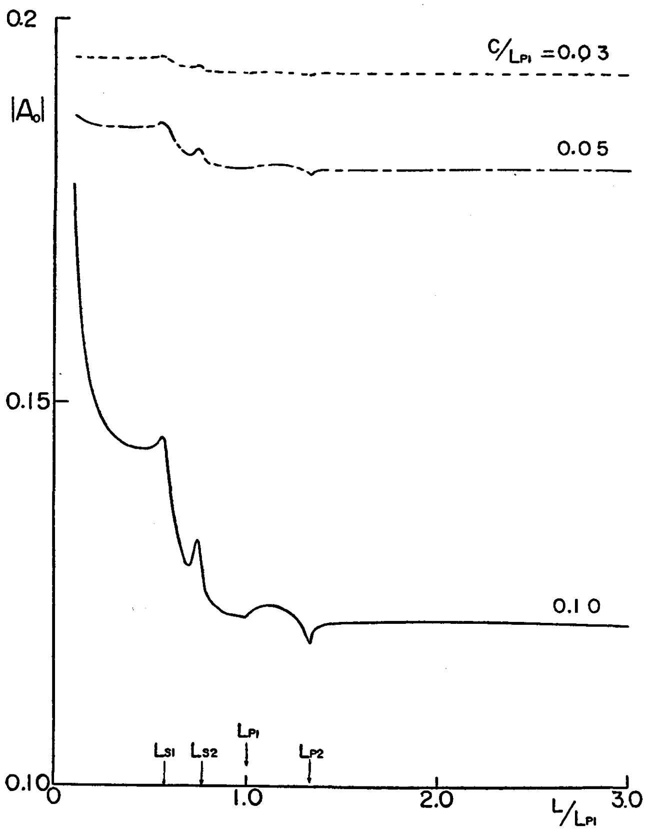

Fig. 3.

in Step 3 and Step 4. In Step 3, A_0 and C_0 of the second approximation are calculated. Step 4 is similar to Step 1, and is the step to obtain all the coefficients of simultaneous linear equations determining A_2, B_2, etc. In

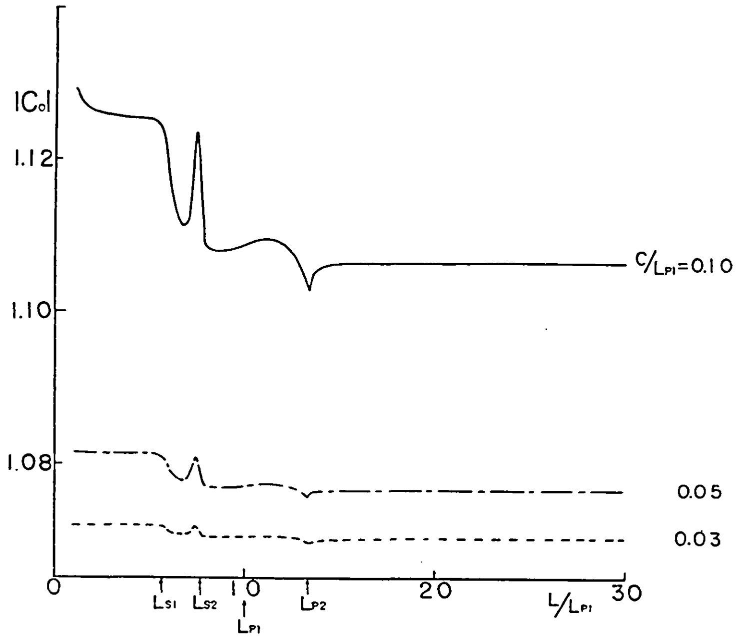

Fig. 4.

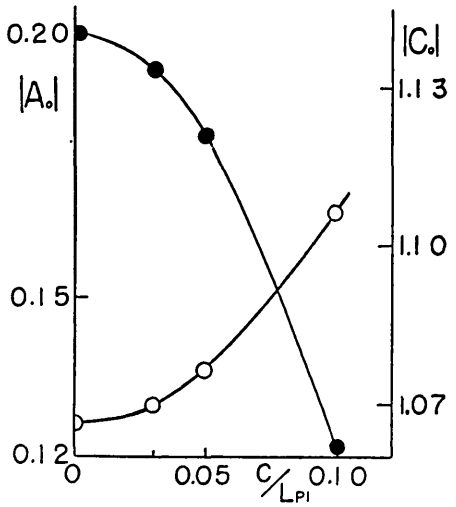

Fig. 5.

Step 5, A_2, B_2, etc. are obtained by means of the program used in Step 2.

The results obtained are given in Figs. 3–18 and Tables 1–5.

From Fig. 3 and Fig. 4, we can state the following points as to $|A_0|$ and $|C_0|$.

(1) The variations of $|A_0|$ and $|C_0|$ with L/L_{p1} are rather small, but in comparison with the case of incidence of the SH wave they are a little larger.

(2) The influence of corrugation is larger on reflection than on refraction. This is also shown

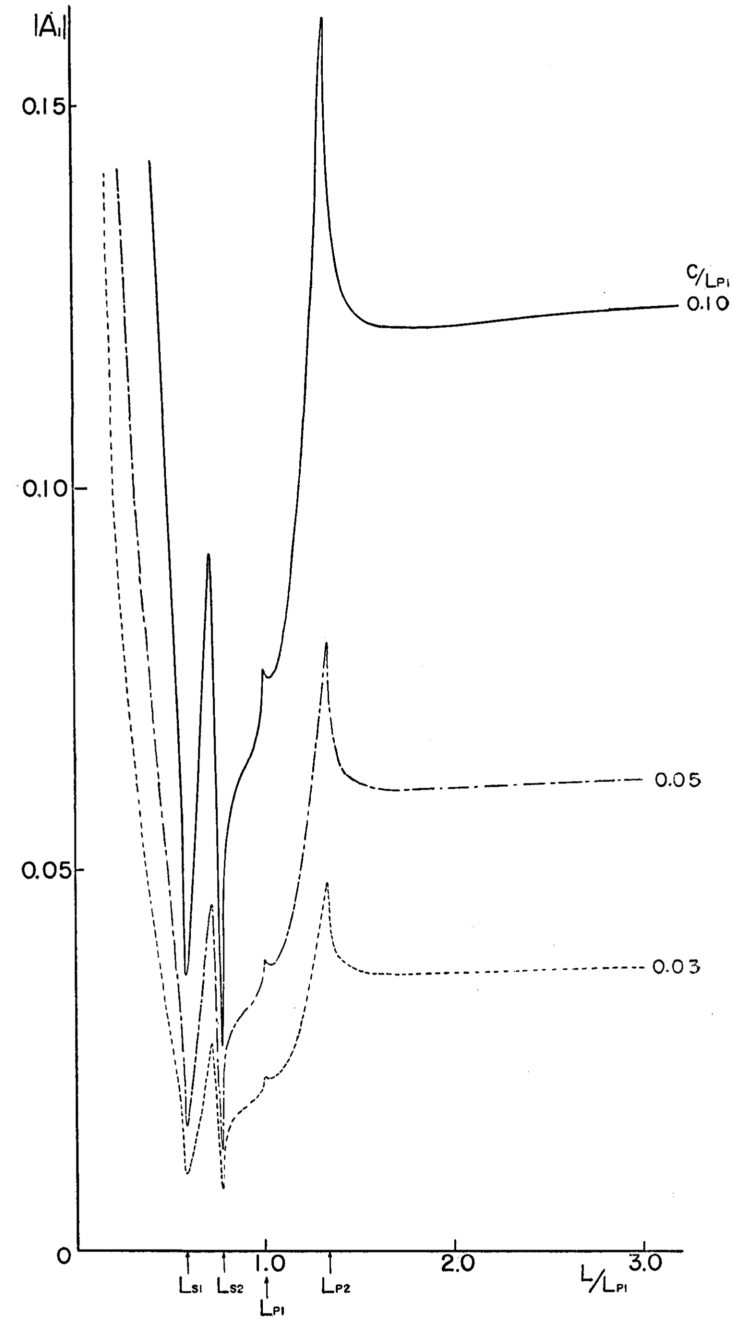

Fig. 6.

clearly in Fig. 5. We see that the variation of $|A_0|$ with the ratio of amplitude of corrugation to wave length of the incident wave, c/L_{p1}, is much larger than that of $|C_0|$. A similar phenomenon is also seen in Fig. 13.

(3) The effect of corrugation increases with increasing c/L_{p1} in such a manner that $|A_0|$ decreases and $|C_0|$ increases with increasing c/L_{p1}.

(4) For L/L_{p1} smaller than 4/3, an increasing tendency is generally observed, especially for $|A_0|$.

For L/L_{p1} at least smaller than 0.3 it seems that this method can

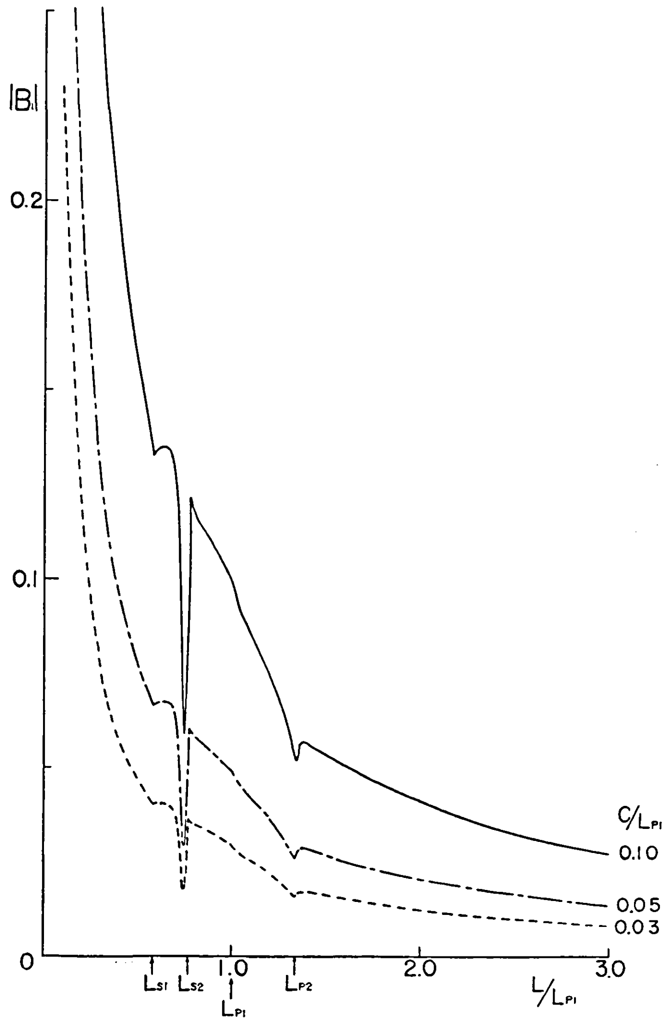

Fig. 7.

not be applied satisfactorily.

(5) At $L/L_{p1}=4/3$, that is, $L=L_{p2}$, minimum values of $|A_0|$ and $|C_0|$ are rather clearly perceived.

(6) In the neighbourhood of $L/L_{p1}=0.77$, i.e. $L=L_{s2}$, both $|A_0|$ and $|C_0|$ have a sharp maximum.

In Figs. 6 through 9, $|A_1|$, $|B_1|$, etc. are given.

(7) For L/L_{p1} larger than 4/3, $|A_1|$ and $|C_1|$ are almost constant.

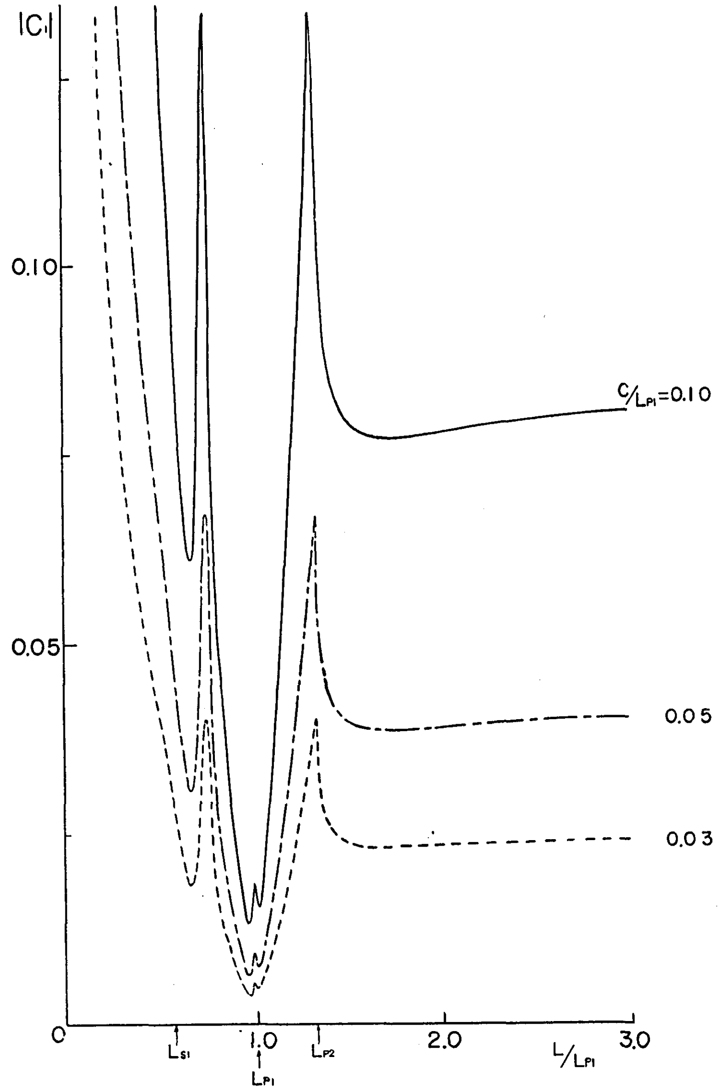

Fig. 8.

(8) At $L/L_{p1}=4/3$, i.e. $L=L_{p2}$, both $|A_1|$ and $|C_1|$ have a sharp maximum and for L/L_{p1} smaller than 4/3 they decrease very rapidly until $|A_1|$ has a sharp minimum at $L/L_{p1}=0.77$, i.e. $L=L_{s2}$ and $|C_1|$ has two minima near $L/L_{p1}=1$.

(9) $|B_1|$ and $|D_1|$, the S wave components of reflected and refracted waves, have an increasing tendency with decreasing L/L_{p1}. Therefore, the vertical component of displacement becomes very large. Since we are concerned here with the case of normal incidence, this result is very reasonable.

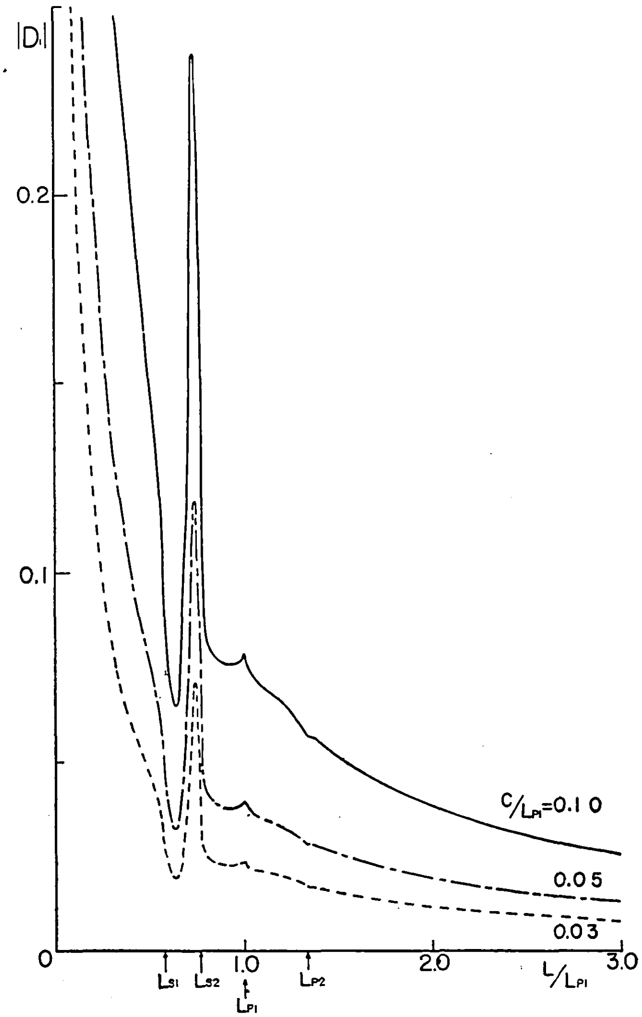

Fig. 9.

(10) It is remarkable that $|B_1|$ decreases very abruptly near $L/L_{p1}=0.77$, i.e. $L=L_{s2}$ and reaches a minimum at $L/L_{p1}=0.74$. Corresponding to this, $|A_1|$, $|C_1|$ and $|D_1|$, especially $|D_1|$ become very large and reach sharp peaks around $L/L_{p1}=0.74$.

(11) The abnormal feature at $L=L_{p1}$ and $L=L_{s1}$ is not as clear as those at $L=L_{p2}$ and $L=L_{s2}$.

(12) For L/L_{p1} smaller than about 0.3, all irregular waves become very large and this tendency impairs the accuracy of this method.

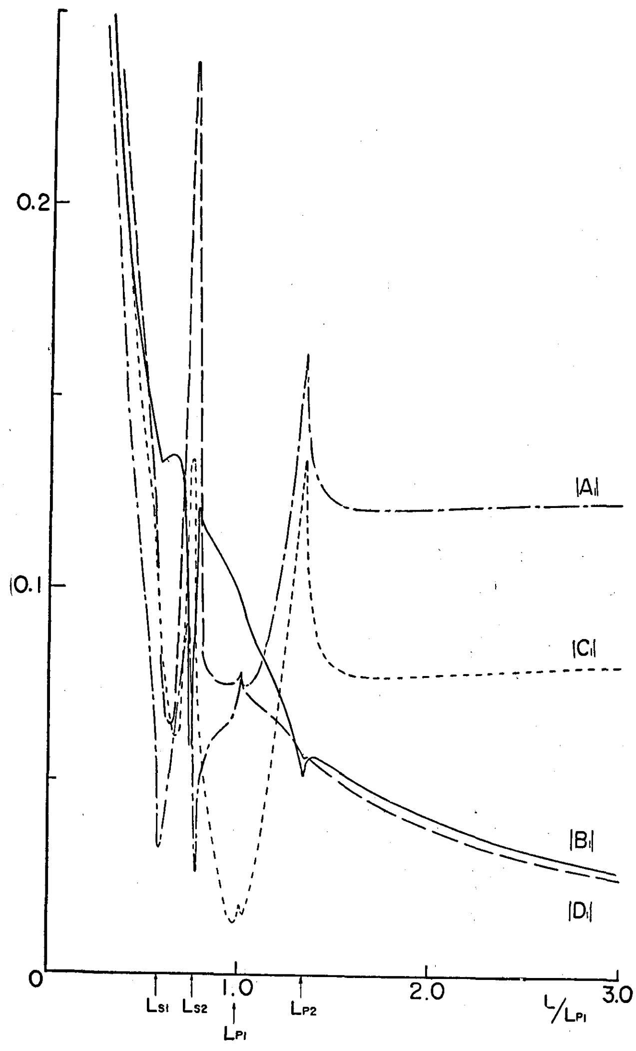

Fig. 10.

In Fig. 10, $|A_1|$, $|B_1|$, $|C_1|$ and $|D_1|$ of $c/L_{p1}=0.1$ are given together for comparison. From this figure, we can see clearly that the S wave component in both reflected and refracted waves increases with decreasing L/L_{p1} and near $L=L_{p1}$ it becomes comparable order to the P wave component. Furthermore, the abnormal character is very conspicuous in the neighbourhood of $L=L_{s2}$.

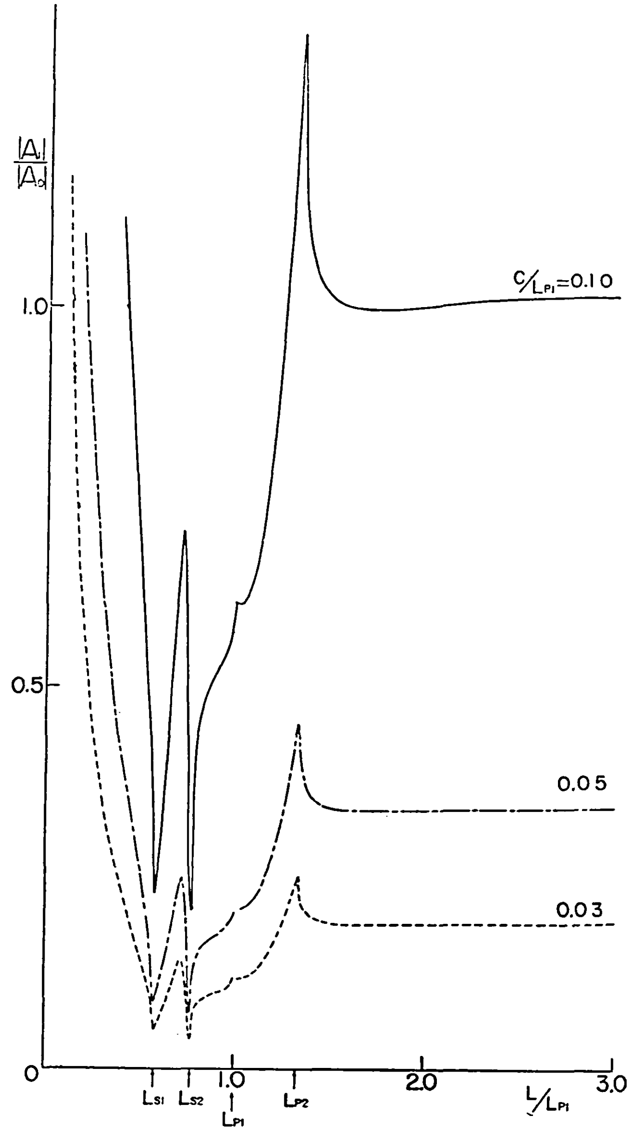

Fig. 11.

(13) According to the value of L/L_{p1}, some irregular waves become boundary waves. That is to say, their amplitude decreases exponentially with increasing $|z|$. The P wave component of an irregularly reflected wave which is represented by $|A_1|$ becomes like the boundary wave for L/L_{p1} smaller than 1, or for L smaller than L_{p1}.

Similar circumstances prevail for the following waves in the ranges of wave lengths specified below:

The S wave component of an irregularly reflected wave B_1, for L/L_{p1} smaller than 0.578 or for L smaller than L_{s1}, the P wave component

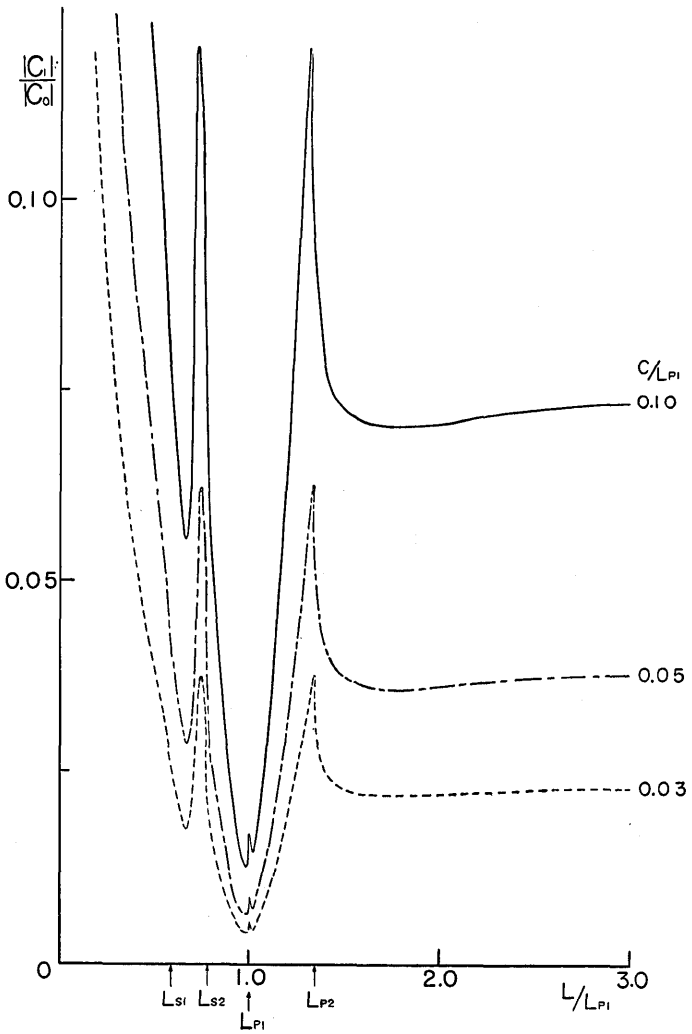

Fig. 12.

of an irregularly refracted wave C_1, for L/L_{p1} smaller than 4/3 or for L smaller than L_{p2}, and the S wave component of an irregularly refracted wave D_1, for L/L_{p1} smaller than 0.77 or for L smaller than L_{s2}.

As in the case of the SH wave, the curves of $|A_1|/|A_0|$ and $|C_1|/|C_0|$ have almost the same tendencies as those of $|A_1|$ and $|C_1|$ themselves, because of small variations of $|A_0|$ and $|C_0|$ with L/L_{p1}. But since it is very interesting to know the values of these ratio, they are given in Figs. 11 through 12. From these figures, it is clear that the effect at $L=L_{p2}$ is most conspicuous in both $|A_1|/|A_0|$ and $|C_1|/|C_0|$. Furthermore, as seen from Fig. 13, $|A_1|/|A_0|$ is several times larger than $|C_1|/|C_0|$ and it may be inferred also from this figure that the effect of a corrugated boundary surface on reflection is larger than on refraction.

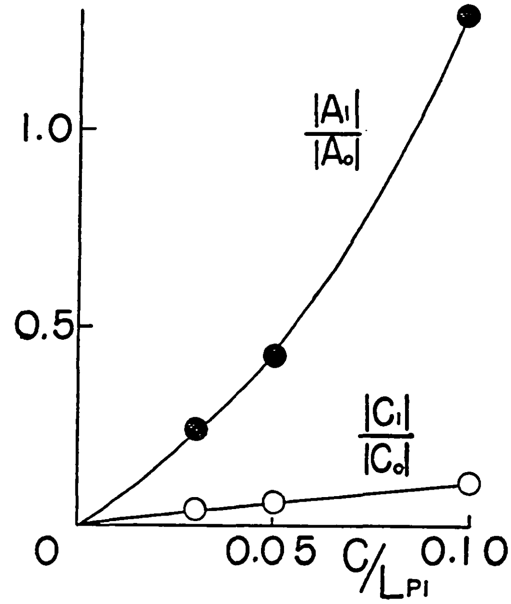

Fig. 13.

In order to see the degree of conversion of wave type, $|B_1|/|A_1|$ and $|D_1|/|C_1|$ are shown in Fig. 14. It is specially to be remarked that these values are independent of c/L_{p1} because both denominator and numerator are proportional to c/L_{p1}. It is clearly seen that the S wave component increases with decreasing L/L_{p1}. In this figure, however, it must be borne in mind that if $|A_1|$ or $|C_1|$, that is, the denominator becomes much smaller than the numerator, the ratio can be very large. In fact, two peaks near $L=L_{p1}$ in $|D_1|/|C_1|$ correspond to two minima of $|C_1|$.

Also two peaks of $|B_1|/|A_1|$ near $L=L_{s1}$ and $L=L_{s2}$ correspond to two minima of $|A_1|$. But it may be permissible to say that for L/L_{p1} below about 1, both $|D_1|/|C_1|$ and $|B_1|/|A_1|$ become comparable to or rather larger than 1.

As for A_2, B_2, etc. computations were carried out only for the case of $c/L_{p1}=0.1$. The results are shown in Figs. 15–18 together with those

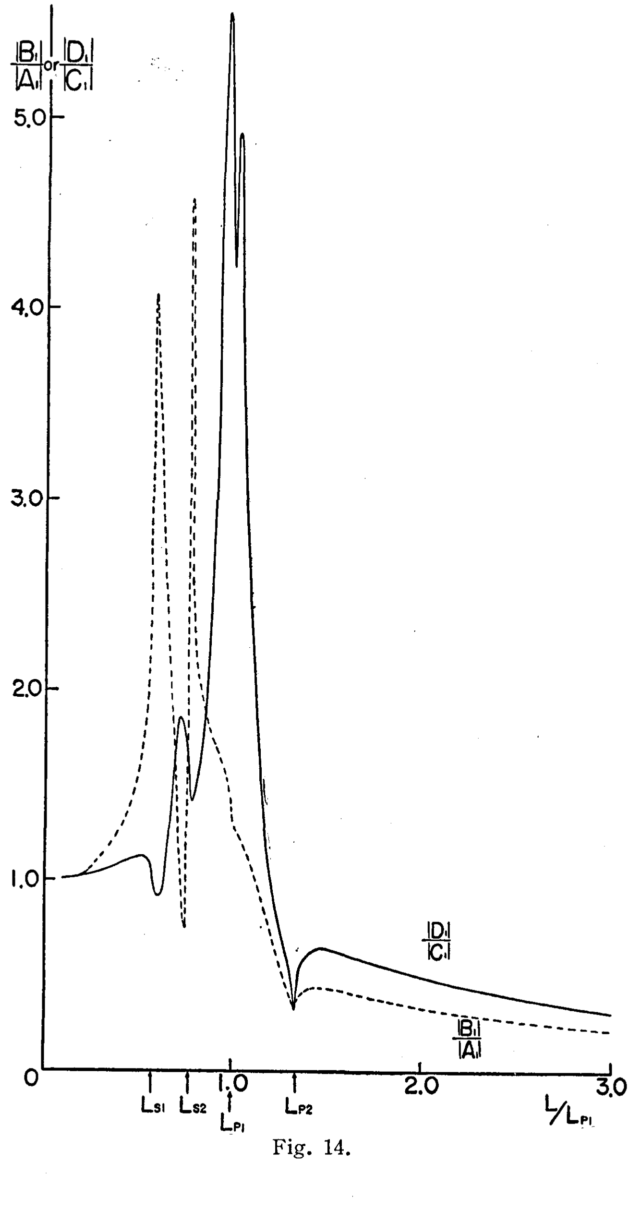

Fig. 14.

of the spectra of the first order, i.e. $|A_1|$, $|B_1|$, etc. for comparison.

(14) In general, for L/L_{p1} smaller than about 1, the values of $|A_2|$, $|B_2|$, etc. are comparable or larger than those of $|A_1|$, $|B_1|$, etc. and much smaller than those of $|A_0|$ and $|C_0|$ for almost all values of L/L_{p1}. Therefore, when we say something about $|A_0|$ or $|C_0|$, it is unnecessary to refer to $|A_2|$ or $|C_2|$. But when attention is paid to irregular waves for L/L_{p1} smaller than about 1, we must take the spectrum of the second order into account.

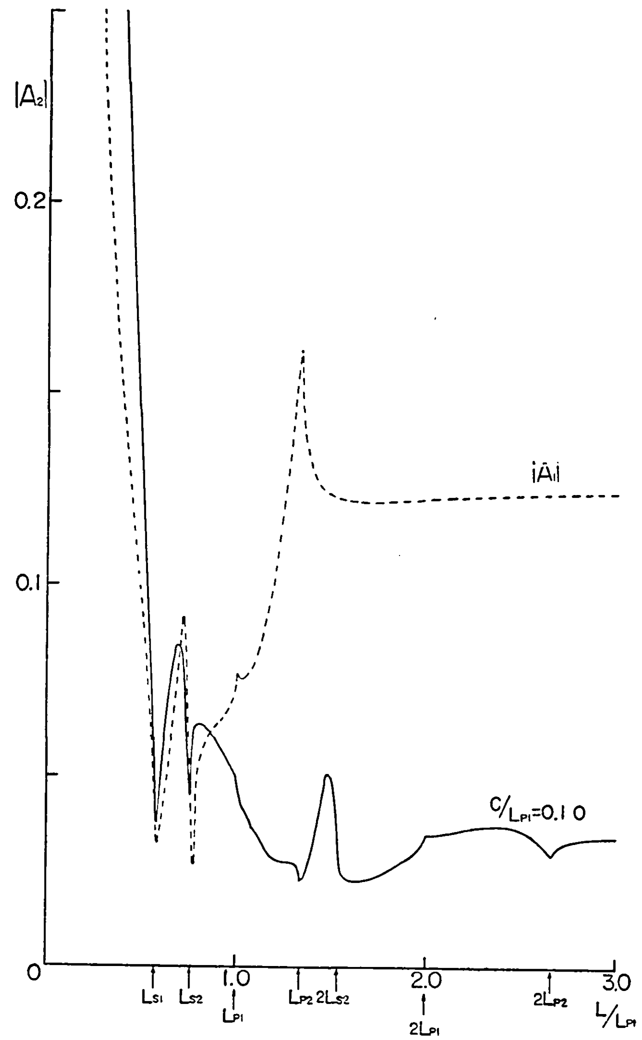

Fig. 15.

(15) As to the P wave component represented by A_n or C_n, $|A_1|$ is much larger than $|A_2|$ for L/L_{p1} larger than about 1; while $|C_1|$ is larger than $|C_2|$ for L/L_{p1} larger than about 1.2. However, the contrary is the case with respect to the S wave component, because both $|B_1|$ and $|D_1|$ are not so large in comparison with $|B_2|$ and $|D_2|$ as in the P wave component, but rather of the same order of magnitude with $|B_2|$ and $|D_2|$.

(16) A sharp peak exists in the neighbourhood of $L=2L_{s2}$ in each

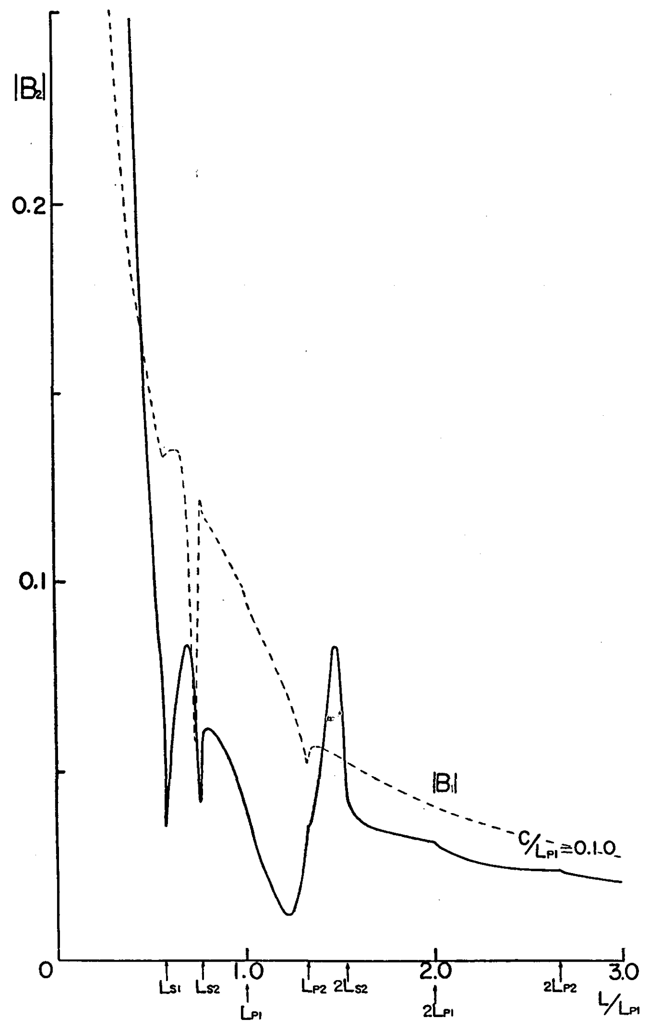

Fig. 16.

irregular wave of the second order. Especially in the S wave components the phenomena are notable.

(17) As a whole, the positions of the maximum or minimum of $|A_2|$, $|B_2|$, etc. especially for L/L_{p1} smaller than about 1 are almost the same as those of $|A_1|$, $|B_1|$, etc.

(18) For L smaller than $2L_{p2}$, some irregular waves become like boundary waves. Such circumstances occur in the ranges specified below: in waves represented by C_2 for L/L_{p1} smaller than 2.667 or for L smaller than $2L_{p2}$, in waves represented by A_2 for L/L_{p1} smaller than

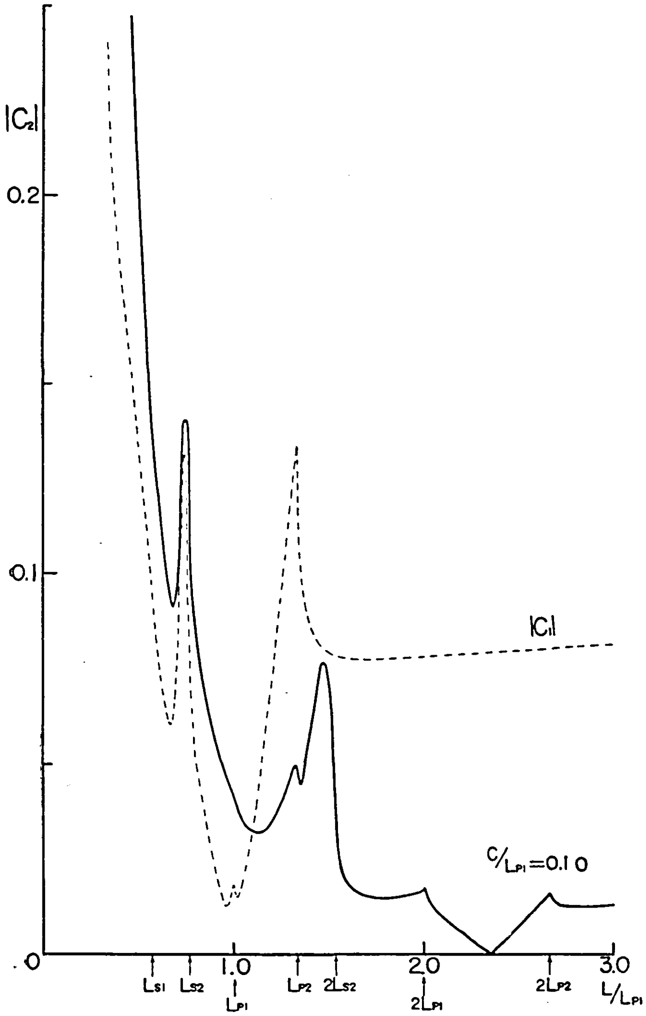

Fig. 17.

2 or for L smaller than $2L_{p1}$, in waves represented by D_2 for L/L_{p1} smaller than 1.539 or for L smaller than $2L_{s2}$, and in waves represented by B_2 for L/L_{p1} smaller than 1.156 or for L smaller than $2L_{s1}$. Therefore, from the practical point of view, it may be permitted to say that only the S wave component of irregularly reflected waves with spectra of the second order has significant contribution to displacement potentials in comparison with those of irregularly reflected and refracted waves with spectra of the first order.

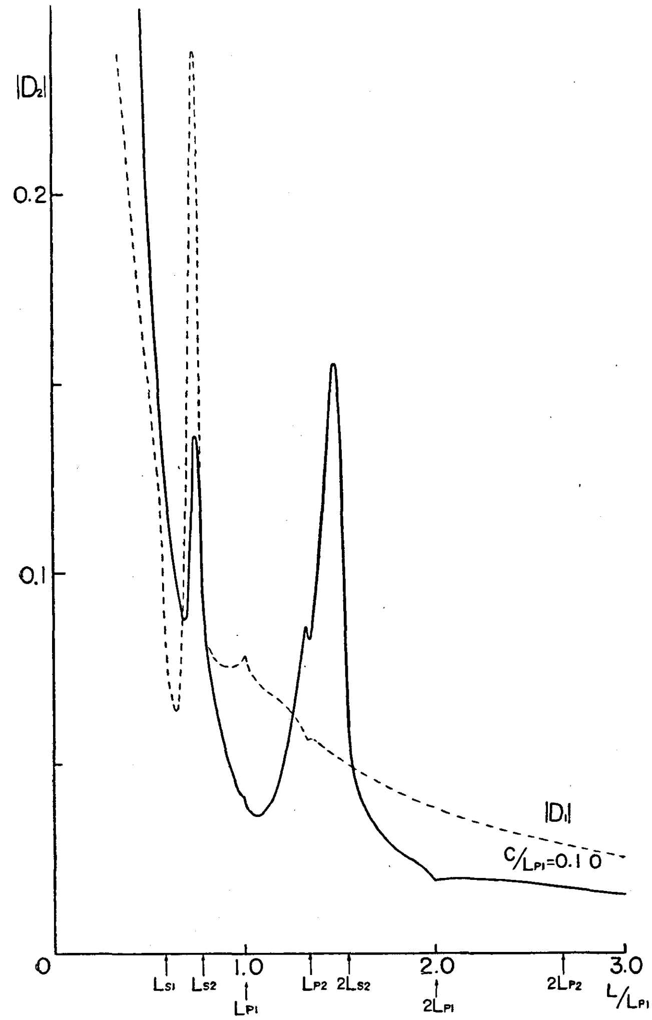

Fig. 18.

§ 5.2. The Case of Incidence of the SV Wave

Putting $a=0$ and $b=1$ in (14), the following simultaneous linear equations, determining solutions of the first approximation for A_0, B_0, C_0 and D_0 are obtained.

$$-B_0+\frac{\mu_2}{\mu_1}\cdot\frac{\sigma_2 h_2^2}{\sigma_1 h_1^2}D_0=1\ ,$$

$$B_0+\frac{\sqrt{\sigma_2}\,h_2}{\sqrt{\sigma_1}\,h_1}D_0=1\ ,$$

$$-h_1^2A_0(\lambda_1+2\mu_1)+h_2^2C_0(\lambda_2+2\mu_2)=0\ ,$$

$$-A_0h_1-C_0h_2=0$$

Then

$$A_0=C_0=0\ ,$$

$$B_0=1-2\Big/\left(1+\frac{\mu_2\sqrt{\sigma_2}\,h_2}{\mu_1\sqrt{\sigma_1}\,h_1}\right),$$

$$D_0=2\Big/\left\{\frac{\sqrt{\sigma_2}\,h_2}{\sqrt{\sigma_1}\,h_1}\left(1+\frac{\mu_2\sqrt{\sigma_2}\,h_2}{\mu_1\sqrt{\sigma_1}\,h_1}\right)\right\}.$$

From the following simultaneous linear equations we can obtain the solutions of the first approximation for A_1, B_1, C_1 and D_1,

$$\begin{aligned}
&2\mu_1h_1pA_1\cos\alpha_1-\mu_1B_1(\sigma_1h_1^2-2p^2)+2\mu_2h_2pC_1\cos\gamma_1+\mu_2D_1(\sigma_2h_2^2-2p^2)\\
&\quad=i\mu_1\sigma_1\sqrt{\sigma_1}\,h_1^3\zeta_1(1-B_0)-i\mu_2\sigma_2\sqrt{\sigma_2}\,h_2^3\zeta_1D_0\ ,\\
&-h_1^2A_1(\lambda_1+2\mu_1\cos^2\alpha_1)-2\mu_1\sqrt{\sigma_1}\,h_1pB_1\cos\beta_1\\
&\qquad+h_2^2C_1(\lambda_2+2\mu_2\cos^2\gamma_1)-2\mu_2\sqrt{\sigma_2}\,h_2pD_1\cos\delta_1\\
&\quad=-2i\mu_1h_1^2\zeta_1p\sigma_1(1+B_0)+2i\mu_2h_2^2\zeta_1p\sigma_2D_0\ ,\\
&pA_1-\sqrt{\sigma_1}\,h_1B_1\cos\beta_1-pC_1-\sqrt{\sigma_2}\,h_2D_1\cos\delta_1\\
&\quad=-i\sigma_1h_1^2\zeta_1(1+B_0)+i\sigma_2h_2^2\zeta_1D_0\ ,\\
&-h_1A_1\cos\alpha_1-pB_1-h_2C_1\cos\gamma_1+pD_1=0\ .
\end{aligned}$$

By using B_0 and D_0 of the first approximation obtained above, the right hand side of the second equation is proved to be zero.

As in the case of incidence of the P wave, it is shown readily that there exist the relations $A_1=-A_1'$, $B_1=B_1'$, $C_1=-C_1'$ and $D_1=D_1'$.

As to A_0, B_0, etc. of the second approximation, by using the relations obtained above $A_1=-A_1'$, $B_1=B_1'$, etc., we have

$$-\sigma_1\mu_1h_1^2B_0+\sigma_2\mu_2h_2^2D_0$$
$$=\sigma_1\mu_1h_1^2+4i\mu_1\zeta_{-1}A_1p^3-2i\mu_1\zeta_{-1}B_1\sqrt{\sigma_1}\,h_1(\sigma_1h_1^2+2p^2)\cos\beta_1$$
$$-4i\mu_2\zeta_{-1}C_1p^3-2i\mu_2\zeta_{-1}D_1\sqrt{\sigma_2}\,h_2(\sigma^2h_2^2+2p^2)\cos\delta_1\ ,$$
$$-\sqrt{\sigma_1}\,h_1B_0-\sqrt{\sigma_2}\,h_2D_0$$
$$=-\sqrt{\sigma_1}\,h_1+2ih_1\zeta_{-1}A_1p\cos\alpha_1-2i\zeta_{-1}B_1\sigma_1h_1^2\cos^2\beta_1$$
$$+2i\zeta_{-1}C_1ph_2\cos\gamma_1+2i\zeta_{-1}D_1\sigma_2h_2^2\cos^2\delta_1\ ,$$
$$-h_1^2A_0(\lambda_1+2\mu_1)+h_2^2C_0(\lambda_2+2\mu_2)=0\ ,$$
$$-h_1A_0-h_2C_0=0\ .$$

From the last two equations, $A_0=C_0=0_0$.

As in the case of incidence of the P wave, the magnitude of correction terms to A_1, B_1, etc. of the first approximation is of the order of ζ^3. Thus, as far as the present approximations are concerned, A_1, B_1, etc. of the second approximation are the same as those of the first approximation.

In the second approximation, A_2, B_2, etc. must be taken into consideration and A_n, B_n, etc. $(n>2)$ are quantities of an order higher than ζ^3. Simultaneous linear equations for A_2, B_2, etc. have the same expressions as corresponding equations in the case of incidence of the P wave.

In this case, the relations $A_2=-A_2'$, $B_2=B_2'$, $C_2=-C_2'$ and $D_2=D_2'$ are obtained by the same procedure as used in the case of incidence of the P wave.

The constants used are the same ones as in the case of incidence of the P wave.

In this case, the formulae are expressed in terms of L/L_{s1}, that is, the ratio of the wave length of corrugation to that of incident wave, and differ slightly one from another in respective intervals of L/L_{s1} concerned. In this case, as examples, these formulae for A_2, B_2, etc. are given in the following:

(i)-(1) $L/L_{s1}\geq 4.619$

In this interval, A_2, B_2, etc. are real numbers and as A_1, B_1, etc. are purely imaginary numbers, in the following A_1/i, B_1/i, etc. are written as A_1, B_1 and so on.

$$4\sqrt{3}\,A_2\sqrt{(L/L_{s1})^2-12}-3B_2\{(L/L_{s1})^2-8\}$$
$$+4\sqrt{3}\,C_2(\mu_2/\mu_1)\sqrt{(h_2/h_1)^2(L/L_{s1})^2-12}+3D_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$
$$=-\frac{\pi(c/L_{s1})}{(L/L_{s1})}[2A_1\{2(L/L_{s1})^2-9\}+3B_1\{6-(L/L_{s1})^2\}\sqrt{(L/L_{s1})^2-1}$$
$$-2C_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-9\}$$
$$+3D_1(\mu_2/\mu_1)\{6-(h_2/h_1)^2(L/L_{s1})^2\}\sqrt{(h_2/h_1)^2(L/L_{s1})^2-1}\,]\ ,$$

$$3A_2\{(L/L_{s1})^2-8\}+12B_2\sqrt{(L/L_{s1})^2-4}-3C_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$
$$+12(\mu_2/\mu_1)D_2\sqrt{(h_2/h_1)^2(L/L_{s1})^2-4}$$
$$=-\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[3A_1\{(L/L_{s1})^2-6\}\sqrt{(L/L_{s1})^2-3}+6\sqrt{3}\,B_1\{2(L/L_{s1})^2-3\}$$
$$+3C_1(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-6\}\sqrt{(h_2/h_1)^2(L/L_{s1})^2-3}$$
$$-6\sqrt{3}\,D_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-3\}\,]\ ,$$

$$2A_2-B_2\sqrt{(L/L_{s1})^2-4}-2C_2-D_2\sqrt{(h_2/h_1)^2(L/L_{s1})^2-4}$$
$$=-\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[A_1\sqrt{(L/L_{s1})^2-3}-\sqrt{3}\,B_1\{(L/L_{s1})^2-1\}$$
$$+C_1\sqrt{(h_2/h_1)^2(L/L_{s1})^2-3}+\sqrt{3}\,D_1\{(h_2/h_1)^2(L/L_{s1})^2-1\}\,]\ ,$$

$$A_2\sqrt{(L/L_{s1})^2-12}+2\sqrt{3}\,B_2+C_2\sqrt{(h_2/h_1)^2(L/L_{s1})^2-12}-2\sqrt{3}\,D_2$$
$$=-\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[A_1\{(L/L_{s1})^2-3\}+3B_1\sqrt{(L/L_{s1})^2-1}$$
$$-C_1\{(h_2/h_1)^2(L/L_{s1})^2-3\}+3D_1\sqrt{(h_2/h_1)^2(L/L_{s1})^2-1}\,]\ .$$

For the value of L/L_{s1} below 4.619, A_2, B_2, etc. become complex numbers and are written as $A_2=RA_2+iIA_2$, etc.

(i)-(2) $4.619>L/L_{s1}\geq 3.464$

$$4\sqrt{3}\,RA_2\sqrt{(L/L_{s1})^2-12}-3RB_2\{(L/L_{s1})^2-8\}$$
$$+4\sqrt{3}\,IC_2(\mu_2/\mu_1)\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}+3RD_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$
$$=-\frac{\pi(c/L_{s1})}{(L/L_{s1})}[2A_1\{2(L/L_{s1})^2-9\}+3B_1\{6-(L/L_{s1})^2\}\sqrt{(L/L_{s1})^2-1}$$
$$-2C_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-9\}$$
$$+3D_1(\mu_2/\mu_1)\{6-(h_2/h_1)^2(L/L_{s1})^2\}\sqrt{(h_2/h_1)^2(L/L_{s1})^2-1}\,]\ ,$$

$$4\sqrt{3}\,IA_2\sqrt{(L/L_{s1})^2-12}-3IB_2\{(L/L_{s1})^2-8\}$$
$$-4\sqrt{3}\,RC_2(\mu_2/\mu_1)\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}$$
$$+3ID_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}=0\ ,$$

$$3RA_2\{(L/L_{s1})^2-8\}+12RB_2\sqrt{(L/L_{s1})^2-4}-3RC_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$
$$+12RD_2(\mu_2/\mu_1)\sqrt{(h_2/h_1)^2(L/L_{s1})^2-4}$$
$$=-\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[3A_1\{(L/L_{s1})^2-6\}\sqrt{(L/L_{s1})^2-3}+6\sqrt{3}\,B_1\{2(L/L_{s1})^2-3\}$$
$$+3C_1(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-6\}\sqrt{(h_2/h_1)^2(L/L_{s1})^2-3}$$
$$-6\sqrt{3}\,D_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-3\}]\ ,$$

$$3IA_2\{(L/L_{s1})^2-8\}+12IB_2\sqrt{(L/L_{s1})^2-4}-3IC_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$
$$+12(\mu_2/\mu_1)ID_2\sqrt{(h_2/h_1)^2(L/L_{s1})^2-4}=0\ ,$$

$$2RA_2-RB_2\sqrt{(L/L_{s1})^2-4}-2RC_2-RD_2\sqrt{(h_2/h_1)^2(L/L_{s1})^2-4}$$
$$=-\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[A_1\sqrt{(L/L_{s1})^2-3}-\sqrt{3}\,B_1\{(L/L_{s1})^2-1\}$$
$$+C_1\sqrt{(h_2/h_1)^2(L/L_{s1})^2-3}+\sqrt{3}\,D_1\{(h_2/h_1)^2(L/L_{s1})^2-1\}]\ ,$$

$$2IA_2-IB_2\sqrt{(L/L_{s1})^2-4}-2IC_2-ID_2\sqrt{(h_2/h_1)^2(L/L_{s1})^2-4}=0\ ,$$

$$RA_2\sqrt{(L/L_{s1})^2-12}+2\sqrt{3}\,RB_2+IC_2\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}-2\sqrt{3}\,RD_2$$
$$=-\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[A_1\{(L/L_{s1})^2-3\}+3B_1\sqrt{(L/L_{s1})^2-1}$$
$$-C_1\{(h_2/h_1)^2(L/L_{s1})^2-3\}+3D_1\sqrt{(h_2/h_1)^2(L/L_{s1})^2-1}]\ ,$$

$$IA_2\sqrt{(L/L_{s1})^2-12}+2\sqrt{3}\,IB_2-RC_2\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}-2\sqrt{3}\,ID_2=0\ .$$

(i)-(3) $3.464>L/L_{s1}\geq 2.667$

In this interval, the first, second, seventh and eighth equations in (i)-(2) change slightly as follows:

$$4\sqrt{3}\,IA_2\sqrt{12-(L/L_{s1})^2}-3RB_2\{(L/L_{s1})^2-8\}$$
$$+4\sqrt{3}\,IC_2(\mu_2/\mu_1)\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}$$
$$+3RD_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$

$$= -\frac{\pi(c/L_{s1})}{(L/L_{s1})}[2A_1\{2(L/L_{s1})^2-9\} + 3B_1\{6-(L/L_{s1})^2\}\sqrt{(L/L_{s1})^2-1}$$

$$-2C_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-9\}$$

$$+3D_1(\mu_2/\mu_1)\{6-(h_2/h_1)^2(L/L_{s1})^2\}\sqrt{(h_2/h_1)^2(L/L_{s1})^2-1}] ,$$

$$-4\sqrt{3}RA_2\sqrt{12-(L/L_{s1})^2} - 3IB_2\{(L/L_{s1})^2-8\}$$

$$-4\sqrt{3}RC_2(\mu_2/\mu_1)\sqrt{12-(h_2/h_1)(L/L_{s1})^2}$$

$$+3ID_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\} = 0 ,$$

$$IA_2\sqrt{12-(L/L_{s1})^2} + 2\sqrt{3}RB_2 + IC_2\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2} - 2\sqrt{3}RD_2$$

$$= -\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[A_1\{(L/L_{s1})^2-3\} + 3B_1\sqrt{(L/L_{s1})^2-1}$$

$$-C_1\{(h_2/h_1)^2(L/L_{s1})^2-3\} + 3D_1\sqrt{(h_2/h_1)^2(L/L_{s1})^2-1}] ,$$

$$-RA_2\sqrt{12-(L/L_{s1})^2} + 2\sqrt{3}IB_2 - RC_2\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2} - 2\sqrt{3}ID_2 = 0.$$

(i)-(4) $2.667 > L/L_{s1} \geq 2.309$

In this interval, the third, fourth, fifth and sixth equations in (i)-(3) change slightly as follows:

$$3RA_2\{(L/L_{s1})^2-8\} + 12RB_2\sqrt{(L/L_{s1})^2-4} - 3RC_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$

$$+12ID_2(\mu_2/\mu_1)\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$

$$= -\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[3A_1\{(L/L_{s1})^2-6\}\sqrt{(L/L_{s1})^2-3} + 6\sqrt{3}B_1\{2(L/L_{s1})^2-3\}$$

$$+3C_1(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-6\}\sqrt{(h_2/h_1)^2(L/L_{s1})^2-3}$$

$$-6\sqrt{3}D_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-3\}] ,$$

$$3IA_2\{(L/L_{s1})^2-8\} + 12IB_2\sqrt{(L/L_{s1})^2-4} - 3IC_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$

$$-12RD_2(\mu_2/\mu_1)\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2} = 0 ,$$

$$2RA_2 - RB_2\sqrt{(L/L_{s1})^2-4} - 2RC_2 - ID_2\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$

$$= -\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[A_1\sqrt{(L/L_{s1})^2-3} - \sqrt{3}B_1\{(L/L_{s1})^2-1\}$$

$$+C_1\sqrt{(h_2/h_1)^2(L/L_{s1})^2-3} + \sqrt{3}D_1\{(h_2/h_1)^2(L/L_{s1})^2-1\}] ,$$

$$2IA_2 - IB_2\sqrt{(L/L_{s1})^2-4} - 2IC_2 + RD_2\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2} = 0 .$$

For L/L_{s1} smaller than 2.309 A_1, B_1, etc. become complex numbers and are written as $A_1 = RA_1 + iIA_1$, etc.

(ii)-(1) $2.309 > L/L_{s1} \geq 2$

$$
\begin{aligned}
&4\sqrt{3}\, IA_2 \sqrt{12-(L/L_{s1})^2} - 3RB_2\{(L/L_{s1})^2-8\} \\
&\quad + 4\sqrt{3}\, IC_2(\mu_2/\mu_1)\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2} \\
&\quad + 3RD_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\} \\
&= -\frac{\pi(c/L_{s1})}{(L/L_{s1})}[2IA_1\{2(L/L_{s1})^2-9\} + 3IB_1\{6-(L/L_{s1})^2\}\sqrt{(L/L_{s1})^2-1} \\
&\qquad - 2IC_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-9\} \\
&\qquad + 3ID_1(\mu_2/\mu_1)\{6-(h_2/h_1)^2(L/L_{s1})^2\}\sqrt{(h_2/h_1)^2(L/L_{s1})^2-1}\,],
\end{aligned}
$$

$$
\begin{aligned}
&-4\sqrt{3}\, RA_2 \sqrt{12-(L/L_{s1})^2} - 3IB_2\{(L/L_{s1})^2-8\} \\
&\quad - 4\sqrt{3}\, RC_2(\mu_2/\mu_1)\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2} \\
&\quad + 3ID_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\} \\
&= \frac{\pi(c/L_{s1})}{(L/L_{s1})}[2RA_1\{2(L/L_{s1})^2-9\} + 3RB_1\{6-(L/L_{s1})^2\}\sqrt{(L/L_{s1})^2-1} \\
&\qquad - 2RC_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-9\} \\
&\qquad + 3RD_1(\mu_2/\mu_1)\{6-(h_2/h_1)^2(L/L_{s1})^2\}\sqrt{(h_2/h_1)^2(L/L_{s1})^2-1}\,],
\end{aligned}
$$

$$
\begin{aligned}
&-3RA_2\{(L/L_{s1})^2-8\} - 12RB_2\sqrt{(L/L_{s1})^2-4} + 3RC_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\} \\
&\quad - 12(\mu_2/\mu_1)ID_2\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2} \\
&= \frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[3IA_1\{(L/L_{s1})^2-6\}\sqrt{(L/L_{s1})^2-3} + 6\sqrt{3}\, IB_1\{2(L/L_{s1})^2-3\} \\
&\qquad - 3RC_1(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-6\}\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2} \\
&\qquad - 6\sqrt{3}\, ID_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-3\}\,],
\end{aligned}
$$

$$
\begin{aligned}
&-3IA_2\{(L/L_{s1})^2-8\} - 12IB_2\sqrt{(L/L_{s1})^2-4} + 3IC_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\} \\
&\quad + 12RD_2(\mu_2/\mu_1)\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2} \\
&= \frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[-3RA_1\{(L/L_{s1})^2-6\}\sqrt{(L/L_{s1})^2-3} \\
&\qquad - 6\sqrt{3}\, RB_1\{2(L/L_{s1})^2-3\}
\end{aligned}
$$

$$-3IC_1(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-6\}\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2}$$
$$+6\sqrt{3}RD_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-3\}],$$

$$2RA_2-RB_2\sqrt{(L/L_{s1})^2-4}-2RC_2-ID_2\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$
$$=\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[-IA_1\sqrt{(L/L_{s1})^2-3}+\sqrt{3}IB_1\{(L/L_{s1})^2-1\}$$
$$+RC_1\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2}-\sqrt{3}ID_1\{(h_2/h_1)^2(L/L_{s1})^2-1\}],$$

$$2IA_2-IB_2\sqrt{(L/L_{s1})^2-4}-2IC_2+RD_2\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$
$$=\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[RA_1\sqrt{(L/L_{s1})^2-3}-\sqrt{3}RB_1\{(L/L_{s1})^2-1\}$$
$$+IC_1\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2}+\sqrt{3}RD_1\{(h_2/h_1)^2(L/L_{s1})^2-1\}],$$

$$IA_2\sqrt{12-(L/L_{s1})^2}+2\sqrt{3}RB_2+IC_2\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}-2\sqrt{3}RD_2$$
$$=-\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[IA_1\{(L/L_{s1})^2-3\}+3IB_1\sqrt{(L/L_{s1})^2-1}$$
$$-IC_1\{(h_2/h_1)^2(L/L_{s1})^2-3\}+3ID_1\sqrt{(h_2/h_1)^2(L/L_{s1})^2-1}],$$

$$-RA_2\sqrt{12-(L/L_{s1})^2}+2\sqrt{3}IB_2-RC_2\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}-2\sqrt{3}ID_2$$
$$=\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[RA_1\{(L/L_{s1})^2-3\}+3RB_1\sqrt{(L/L_{s1})^2-1}$$
$$-RC_1\{(h_2/h_1)^2(L/L_{s1})^2-3\}+3RD_1\sqrt{(h_2/h_1)^2(L/L_{s1})^2-1}].$$

(ii)-(2) $2>L/L_{s1}\geq 1.732$

In this interval, the third, fourth, fifth and sixth equations in (ii)-(1) change slightly as follows:

$$-3RA_2\{(L/L_{s1})^2-8\}-12IB_2\sqrt{4-(L/L_{s1})^2}+3RC_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$
$$-12ID_2(\mu_2/\mu_1)\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$
$$=\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[3IA_1\{(L/L_{s1})^2-6\}\sqrt{(L/L_{s1})^2-3}+6\sqrt{3}IB_1\{2(L/L_{s1})^2-3\}$$
$$-3RC_1(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-6\}\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2}$$
$$-6\sqrt{3}ID_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-3\}],$$

$$-3IA_2\{(L/L_{s1})^2-8\}+12RB_2\sqrt{4-(L/L_{s1})^2}+3IC_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$
$$+12RD_2(\mu_2/\mu_1)\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$

$$=\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[-3RA_1\{(L/L_{s1})^2-6\}\sqrt{(L/L_{s1})^2-3}$$
$$-6\sqrt{3}RB_1\{2(L/L_{s1})^2-3\}$$
$$-3IC_1(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-6\}\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2}$$
$$+6\sqrt{3}RD_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-3\}]\ ,$$

$$2RA_2-IB_2\sqrt{4-(L/L_{s1})^2}-2RC_2-ID_2\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$
$$=\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[-IA_1\sqrt{(L/L_{s1})^2-3}+\sqrt{3}IB_1\{(L/L_{s1})^2-1\}$$
$$+RC_1\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2}-\sqrt{3}ID_1\{(h_2/h_1)^2(L/L_{s1})^2-1\}]\ ,$$

$$2IA_2+RB_2\sqrt{4-(L/L_{s1})^2}-2IC_2+RD_2\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$
$$=\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[RA_1\sqrt{(L/L_{s1})^2-3}-\sqrt{3}RB_1\{(L/L_{s1})^2-1\}$$
$$+IC_1\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2}+\sqrt{3}RD_1\{(h_2/h_1)^2(L/L_{s1})^2-1\}]\ .$$

(iii) $1.732>L/L_{s1}\geq 4/3$

The third, fourth, fifth and sixth equations in (ii)-(2) change slightly as in the preceding interval.

$$-3RA_2\{(L/L_{s1})^2-8\}-12IB_2\sqrt{4-(L/L_{s1})^2}+3RC_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$
$$-12ID_2(\mu_2/\mu_1)\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$
$$=-\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[3RA_1\{(L/L_{s1})^2-6\}\sqrt{3-(L/L_{s1})^2}$$
$$-6\sqrt{3}IB_1\{2(L/L_{s1})^2-3\}$$
$$+3RC_1(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-6\}\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2}$$
$$+6\sqrt{3}ID_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-3\}]\ ,$$

$$-3IA_2\{(L/L_{s1})^2-8\}+12RB_2\sqrt{4-(L/L_{s1})^2}+3IC_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$
$$+12RD_2(\mu_2/\mu_1)\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$
$$=-\frac{\pi(c/L_{s1})}{\sqrt{3}(L/L_{s1})}[3IA_1\{(L/L_{s1})^2-6\}\sqrt{3-(L/L_{s1})^2}$$
$$+6\sqrt{3}RB_1\{2(L/L_{s1})^2-3\}$$
$$+3IC_1(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-6\}\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2}$$

$$-6\sqrt{3}\,RD_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-3\}\,]\;,$$

$$2RA_2-IB_2\sqrt{4-(L/L_{s1})^2}-2RC_2-ID_2\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$

$$=\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[RA_1\sqrt{3-(L/L_{s1})^2}+\sqrt{3}\,IB_1\{(L/L_{s1})^2-1\}$$

$$+RC_1\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2}-\sqrt{3}\,ID_1\{(h_2/h_1)^2(L/L_{s1})^2-1\}\,]\;,$$

$$2IA_2+RB_2\sqrt{4-(L/L_{s1})^2}-2IC_2+RD_2\sqrt{4-(h_2/h_1)^2(L/L_{s1})^2}$$

$$=\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[IA_1\sqrt{3-(L/L_{s1})^2}-\sqrt{3}\,RB_1\{(L/L_{s1})^2-1\}$$

$$+IC_1\sqrt{3-(h_2/h_1)^2(L/L_{s1})^2}+\sqrt{3}\,RD_1\{(h_2/h_1)^2(L/L_{s1})^2-1\}\,]\;,$$

(iv) $4/3>L/L_{s1}\geq 1$

In this interval, the first, second, seventh and eighth equations in (iii) change slightly as follows:

$$4\sqrt{3}\,IA_2\sqrt{12-(L/L_{s1})^2}-3RB_2\{(L/L_{s1})^2-8\}$$

$$+4\sqrt{3}\,IC_2(\mu_2/\mu_1)\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}$$

$$+3RD_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$

$$=-\frac{\pi(c/L_{s1})}{(L/L_{s1})}[2IA_1\{2(L/L_{s1})^2-9\}+3IB_1\{6-(L/L_{s1})^2\}\sqrt{(L/L_{s1})^2-1}$$

$$-2IC_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-9\}$$

$$-3RD_1(\mu_2/\mu_1)\{6-(h_2/h_1)^2(L/L_{s1})^2\}\sqrt{1-(h_2/h_1)^2(L/L_{s1})^2}\,]\;,$$

$$-4\sqrt{3}\,RA_2\sqrt{12-(L/L_{s1})^2}-3IB_2\{(L/L_{s1})^2-8\}$$

$$-4\sqrt{3}\,RC_2(\mu_2/\mu_1)\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}$$

$$+3ID_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$

$$=\frac{\pi(c/L_{s1})}{(L/L_{s1})}[2RA_1\{2(L/L_{s1})^2-9\}+3RB_1\{6-(L/L_{s1})^2\}\sqrt{(L/L_{s1})^2-1}$$

$$-2RC_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-9\}$$

$$+3ID_1(\mu_2/\mu_1)\{6-(h_2/h_1)^2(L/L_{s1})^2\}\sqrt{1-(h_2/h_1)^2(L/L_{s1})^2}\,]\;,$$

$$IA_2\sqrt{12-(L/L_{s1})^2}+2\sqrt{3}\,RB_2+IC_2\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}-2\sqrt{3}\,RD_2$$

$$=-\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[IA_1\{(L/L_{s1})^2-3\}+3IB_1\sqrt{(L/L_{s1})^2-1}$$

$$-IC_1\{(h_2/h_1)^2(L/L_{s1})^2-3\}-3RD_1\sqrt{1-(h_2/h_1)^2(L/L_{s1})^2}\,]\,,$$

$$-RA_2\sqrt{12-(L/L_{s1})^2}+2\sqrt{3}\,IB_2-RC_2\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}-2\sqrt{3}\,ID_2$$

$$=\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[RA_1\{(L/L_{s1})^2-3\}+3RB_1\sqrt{(L/L_{s1})^2-1}$$

$$-RC_1\{(h_2/h_1)^2(L/L_{s1})^2-3\}+3ID_1\sqrt{1-(h_2/h_1)^2(L/L_{s1})^2}\,]\,.$$

(v) $1>L/L_{s1}$

In this interval, the first, second, seventh and eighth equations in (iv) change slightly as follows:

$$4\sqrt{3}\,IA_2\sqrt{12-(L/L_{s1})^2}-3RB_2\{(L/L_{s1})^2-8\}$$

$$+4\sqrt{3}\,IC_2(\mu_2/\mu_1)\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}$$

$$+3RD_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$

$$=-\frac{\pi(c/L_{s1})}{(L/L_{s1})}[2IA_1\{2(L/L_{s1})^2-9\}-3RB_1\{6-(L/L_{s1})^2\}\sqrt{1-(L/L_{s1})^2}$$

$$-2IC_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-9\}$$

$$-3RD_1(\mu_2/\mu_1)\{6-(h_2/h_1)^2(L/L_{s1})^2\}\sqrt{1-(h_2/h_1)^2(L/L_{s1})^2}\,]\,,$$

$$-4\sqrt{3}\,RA_2\sqrt{12-(L/L_{s1})^2}-3IB_2\{(L/L_{s1})^2-8\}$$

$$-4\sqrt{3}\,RC_2(\mu_2/\mu_1)\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}$$

$$+3ID_2(\mu_2/\mu_1)\{(h_2/h_1)^2(L/L_{s1})^2-8\}$$

$$=\frac{\pi(c/L_{s1})}{(L/L_{s1})}[2RA_1\{2(L/L_{s1})^2-9\}+3IB_1\{6-(L/L_{s1})^2\}\sqrt{1-(L/L_{s1})^2}$$

$$-2RC_1(\mu_2/\mu_1)\{2(h_2/h_1)^2(L/L_{s1})^2-9\}$$

$$+3ID_1(\mu_2/\mu_1)\{6-(h_2/h_1)^2(L/L_{s1})^2\}\sqrt{1-(h_2/h_1)^2(L/L_{s1})^2}\,]\,,$$

$$IA_2\sqrt{12-(L/L_{s1})^2}+2\sqrt{3}\,RB_2+IC_2\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}-2\sqrt{3}\,RD_2$$

$$=-\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[IA_1\{(L/L_{s1})^2-3\}-3RB_1\sqrt{1-(L/L_{s1})^2}$$

$$-IC_1\{(h_2/h_1)^2(L/L_{s1})^2-3\}-3RD_1\sqrt{1-(h_2/h_1)^2(L/L_{s1})^2}\,]\,,$$

$$-RA_2\sqrt{12-(L/L_{s1})^2}+2\sqrt{3}\,IB_2-RC_2\sqrt{12-(h_2/h_1)^2(L/L_{s1})^2}-2\sqrt{3}\,ID_2$$

$$=\frac{\pi(c/L_{s1})}{\sqrt{3}\,(L/L_{s1})}[RA_1\{(L/L_{s1})^2-3\}+3IB_1\sqrt{1-(L/L_{s1})^2}$$

$$-RC_1\{(h_2/h_1)^2(L/L_{s1})^2-3\}+3ID_1\sqrt{1-(h_2/h_1)^2(L/L_{s1})^2}\,]\,.$$

Results obtained are given in Figs. 19 through 34 and Tables 6 through 10.

In Fig. 19 and Fig. 20, the following tendencies concerning $|B_0|$ and $|D_0|$ are clearly seen.

(1) The variations of $|B_0|$ and $|D_0|$ with L/L_{s1} (that is, the ratio of the wave length of a corrugated boundary surface to that of an incident wave) are small but larger than in the case of incidence of the SH or P wave.

(2) It is also ascertained in this case, as in the cases of the incidence of SH and P waves, that the influence of corrugation is larger on reflected waves than on refracted waves. This is also shown clearly in Fig. 21. From Fig. 21, we can see that the variation of $|B_0|$ with c/L_{s1}, the ratio of amplitude of corrugation to the wave length of the incident wave, is much larger than that of $|D_0|$. This result is also confirmed by Fig. 29.

(3) The effect of corrugation increases with increasing c/L_{s1} in such a manner that $|B_0|$ decreases and $|D_0|$ increases with increasing c/L_{s1}.

(4) Both $|B_0|$ and $|D_0|$ have a general tendency to increase with decreasing L/L_{s1}. Moreover, the larger the amplitude of the corrugated boundary surface the more remarkable is this increasing tendency. This feature is also observed in the case of incidence of the SH wave.

(5) At $L/L_{s1}=2.309$ or at $L=L_{p2}$, both $|B_0|$ and $|D_0|$ have a steep peak.

(6) In the neighbourhood of $L/L_{s1}=2$, both $|B_0|$ and $|D_0|$ reach minimum.

(7) In the interval between $L/L_{s1}=2$, i.e. $L=2L_{s1}$ and $L/L_{s1}=1.732$, i.e. $L=L_{p1}$, both $|B_0|$ and $|D_0|$ increase rapidly especially for $c/L_{s1}=0.10$.

(8) At $L/L_{s1}=4/3$, i.e. $L=L_{s2}$ both $|B_0|$ and $|D_0|$, especially $|B_0|$ have a steep peak.

(9) It is conspicuous that a very sharp minimum appears in both $|B_0|$ and $|D_0|$ at the value of L just below L_{s2}. This is also common to the cases of the incidence of SH and SV waves and is more remarkable in this case than in the case of incidence of the SH wave. Corresponding to this minimum, all irregular waves have a sharp maximum.

(10) For L/L_{s1} below about 1, the increasing tendency appears in both $|B_0|$ and $|D_0|$.

From Figs. 22 through 25, we can see the following tendencies in $|A_1|$, $|B_1|$, $|C_1|$ and $|D_1|$.

(11) The sharp maxima common to all irregular waves with a

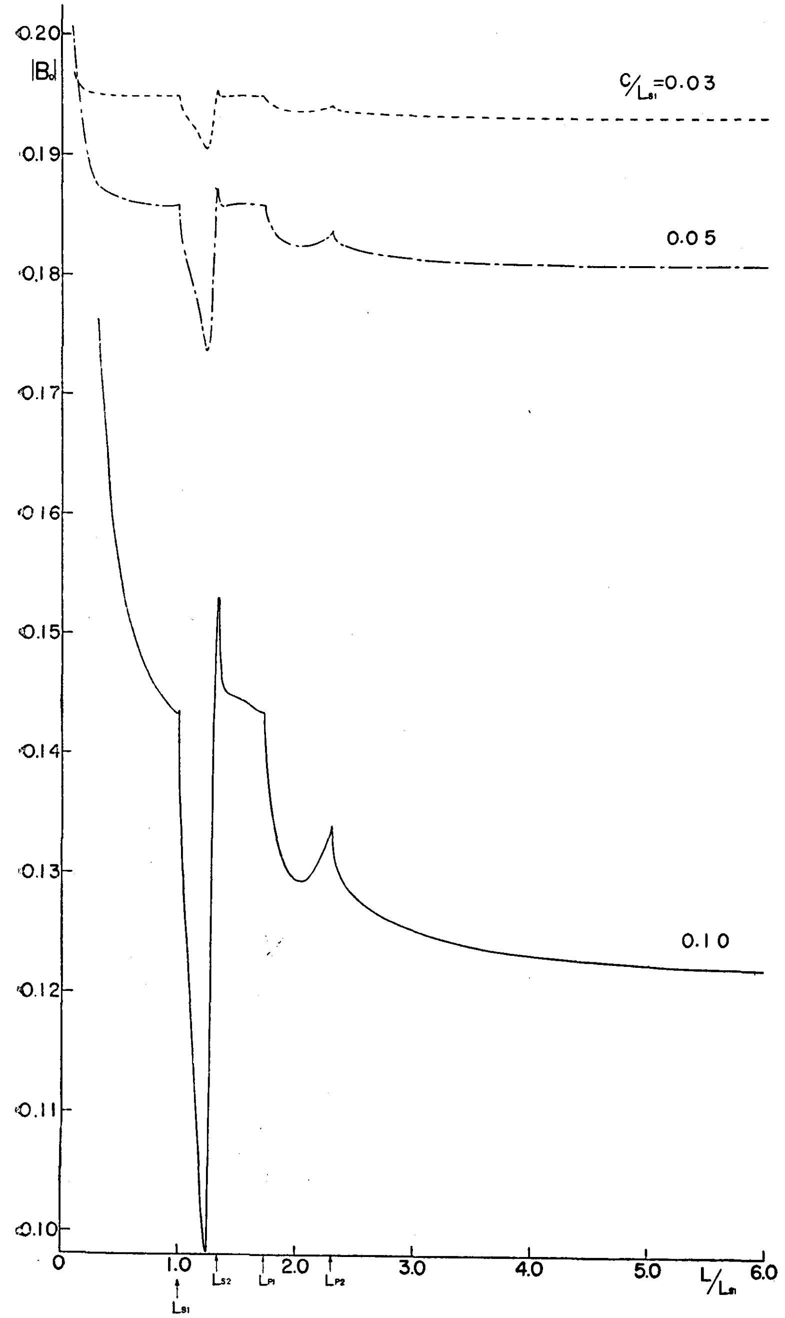
0.20
$|B_0|$
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.10
0
1.0
2.0
3.0
4.0
5.0
6.0
L/L_{s1}
L_{s1}
L_{s2}
L_{p1}
L_{p2}
c/L_{s1}=0.03
0.05
0.10

Fig. 19.

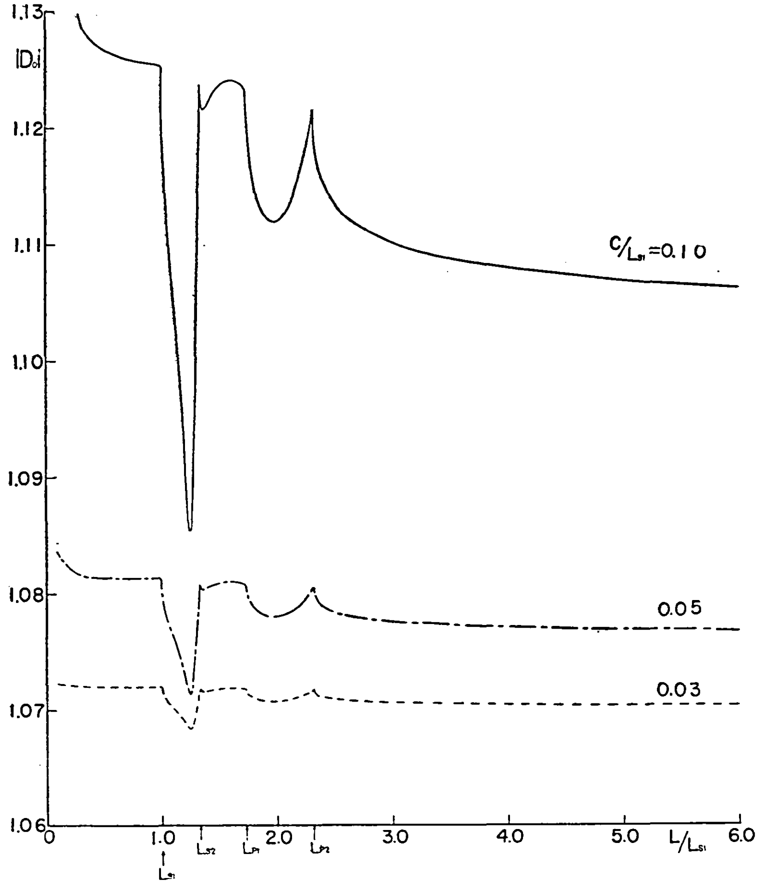
1.13
|D₀|
1.12
1.11
1.10
1.09
1.08
1.07
1.06
C/Ls1=0.10
0.05
0.03
0
1.0
2.0
3.0
4.0
5.0
6.0
L/Ls1
Ls1
Ls2
Lp1
Lp2

Fig. 20.

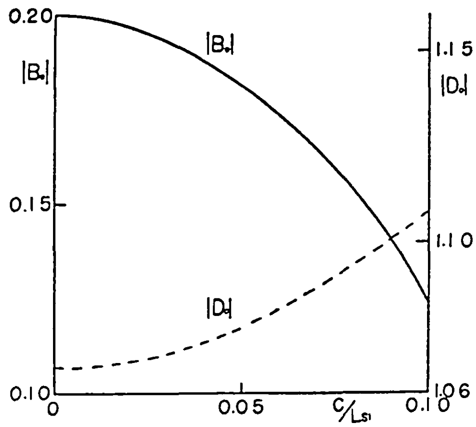
0.20
|B₀|
0.15
0.10
|B₀|
|D₀|
1.15
|D₀|
1.10
1.06
0
0.05
0.10
C/Ls1

Fig. 21.

spectrum of the first order in the neighbourhood of $L/L_{s1}=4/3$, i.e. $L=L_{s2}$ are quite remarkable and they correspond to the sharp minima of $|B_0|$ and $|D_0|$.

(12) $|A_1|$ and $|C_1|$ show similar tendencies to increase with decreasing L/L_{s1} except for certain ranges of L/L_{s1}. It is very interesting to compare this tendency of $|A_1|$ and $|C_1|$ with that of $|B_1|$ and $|D_1|$ in the case of incidence of the P wave. That is to say, irregular waves different in type to the incident wave have an increasing tendency in

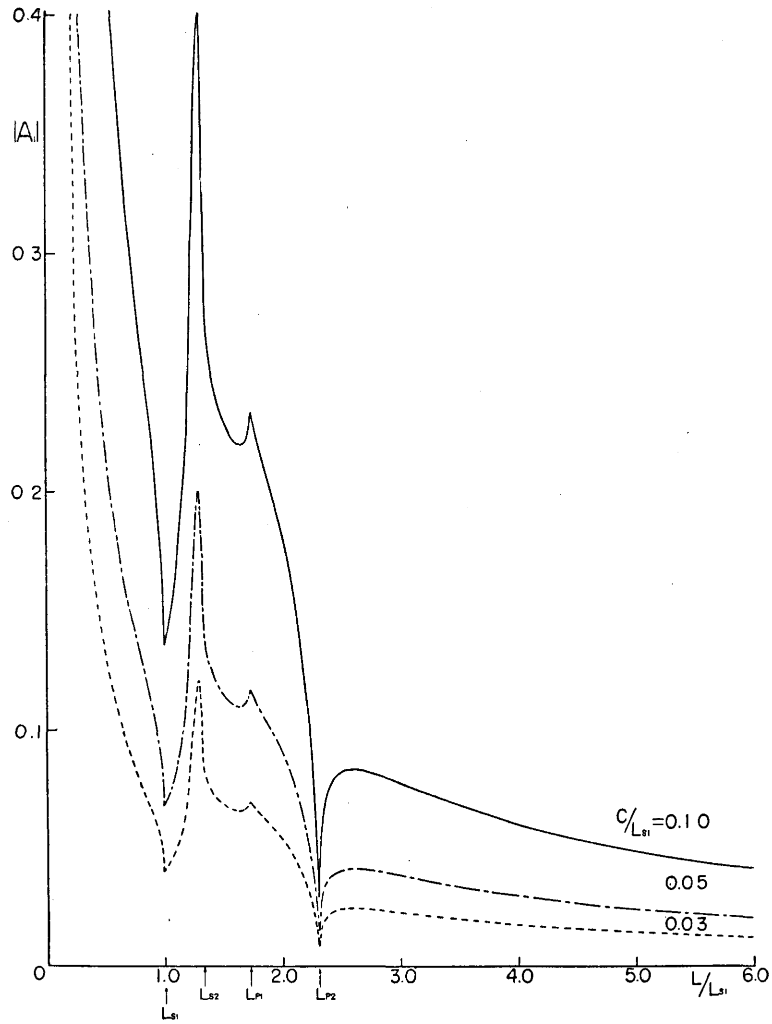

Fig. 22.

general with decreasing L/L_{s1}.

(13) On the contrary, $|B_1|$ and $|D_1|$ decrease as a whole with decreasing L/L_{s1} down to $L/L_{s1}=4/3$ or to $L=L_{s2}$ and to about $L/L_{s1}=1.732$ or to about $L=L_{p1}$ respectively.

(14) For L/L_{s1} smaller than about 1, all irregular waves increase monotonously. This tendency will eventually impair the accuracy of this calculation.

(15) $|A_1|$ has two sharp minima at $L=L_{p2}$ and $L=L_{s1}$ and another small peak at $L=L_{p1}$.

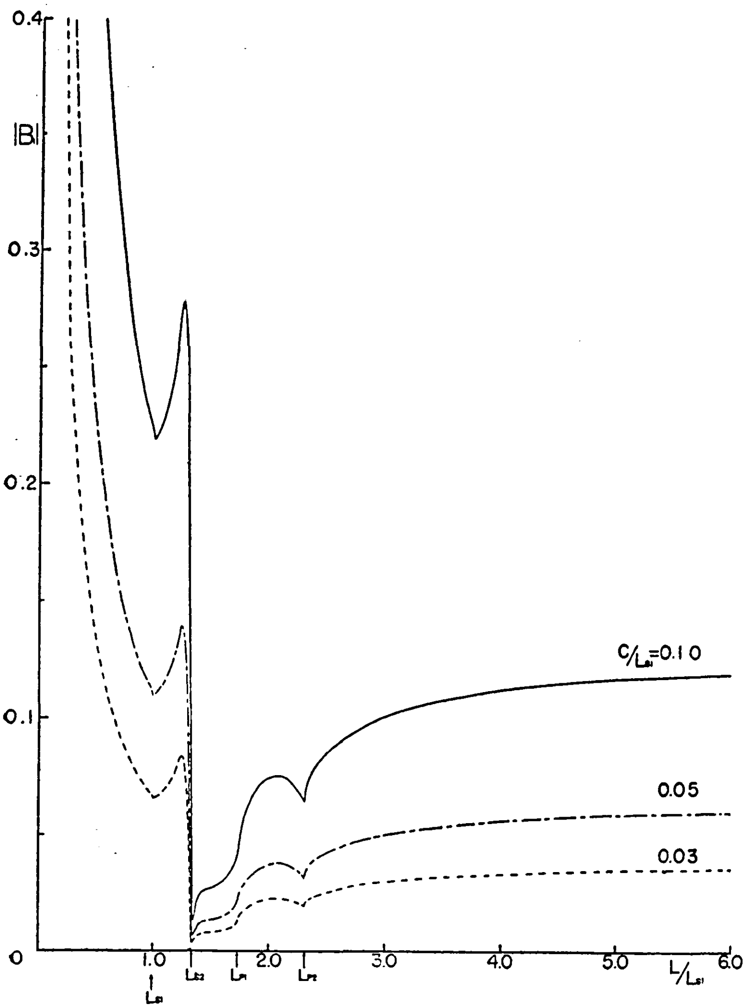

Fig. 23.

(16) $|B_1|$ has a sharp minima at $L=L_{p2}$, $L=L_{s2}$ and $L=L_{s1}$. The minimum at $L=L_{s2}$ is most remarkable and its value is very small.

(17) $|C_1|$ has a sharp maximum at $L=L_{p2}$ and becomes zero at $L=L_{p1}$. In addition to these two abnormal points $|C_1|$ has a sharp minima at $L=L_{s2}$ and near $L=L_{s1}$.

(18) $|D_1|$ is rather simple in comparison with $|A_1|$, $|B_1|$ and $|C_1|$, and has minima in the neighbourhood of $L=L_{p1}$ and $L=L_{s1}$.

(19) For L below L_{p2} some irregular waves become boundary

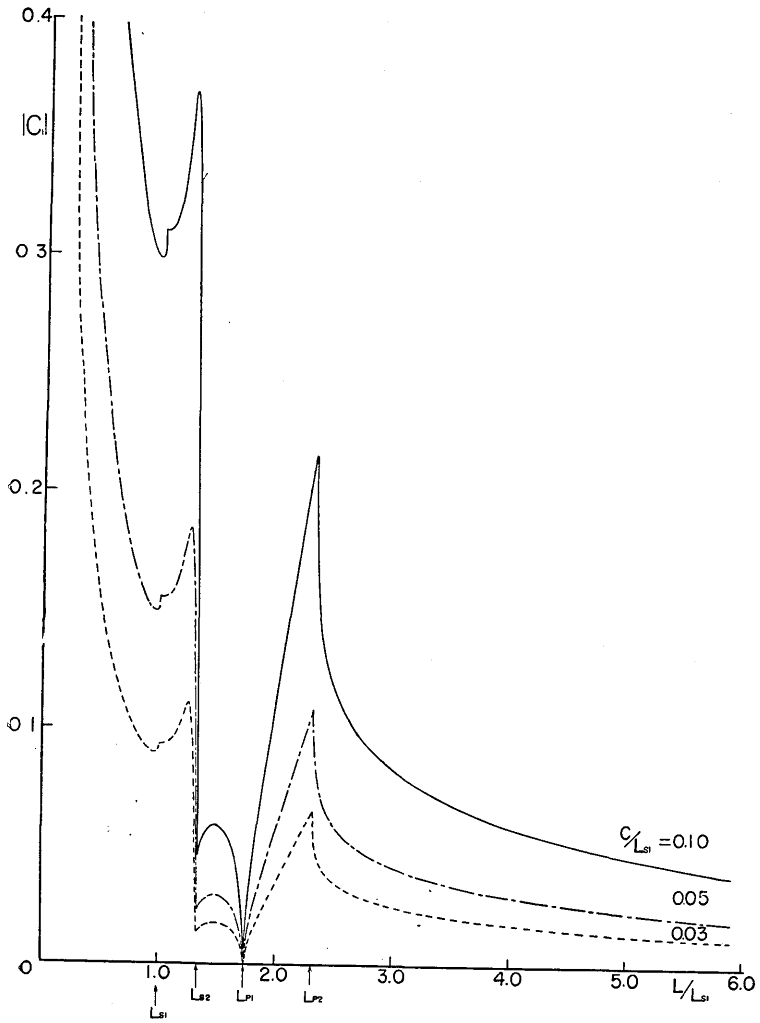

Fig. 24.

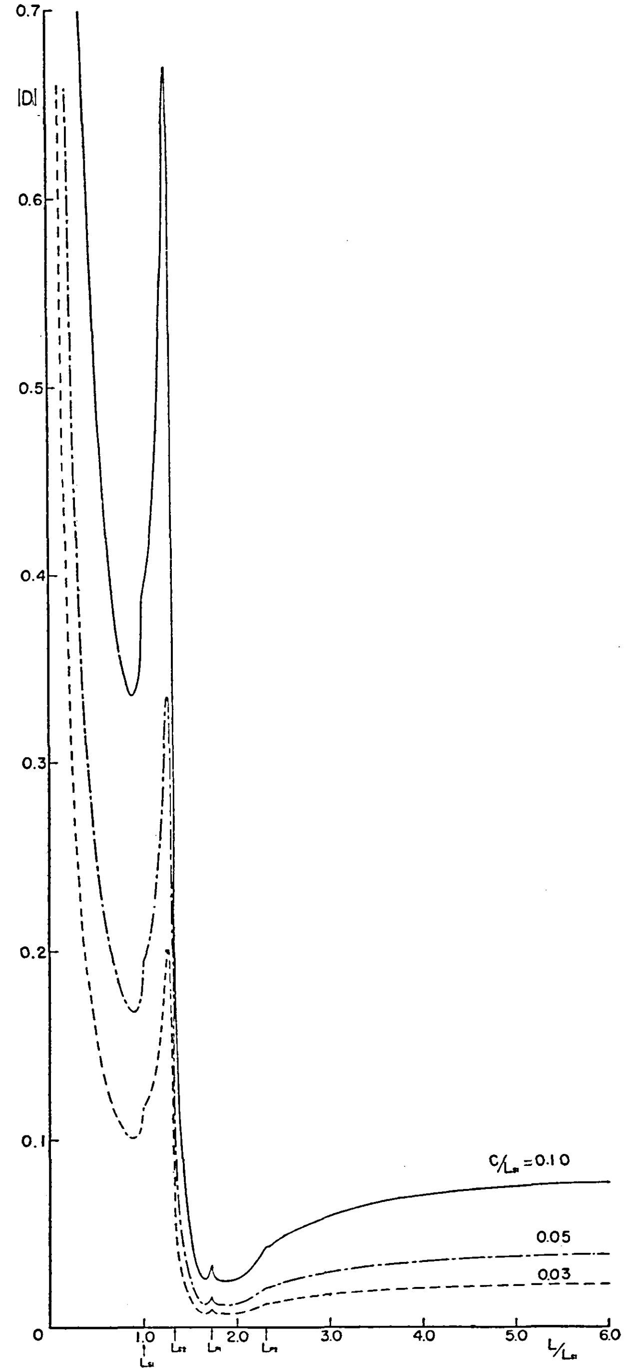

Fig. 25

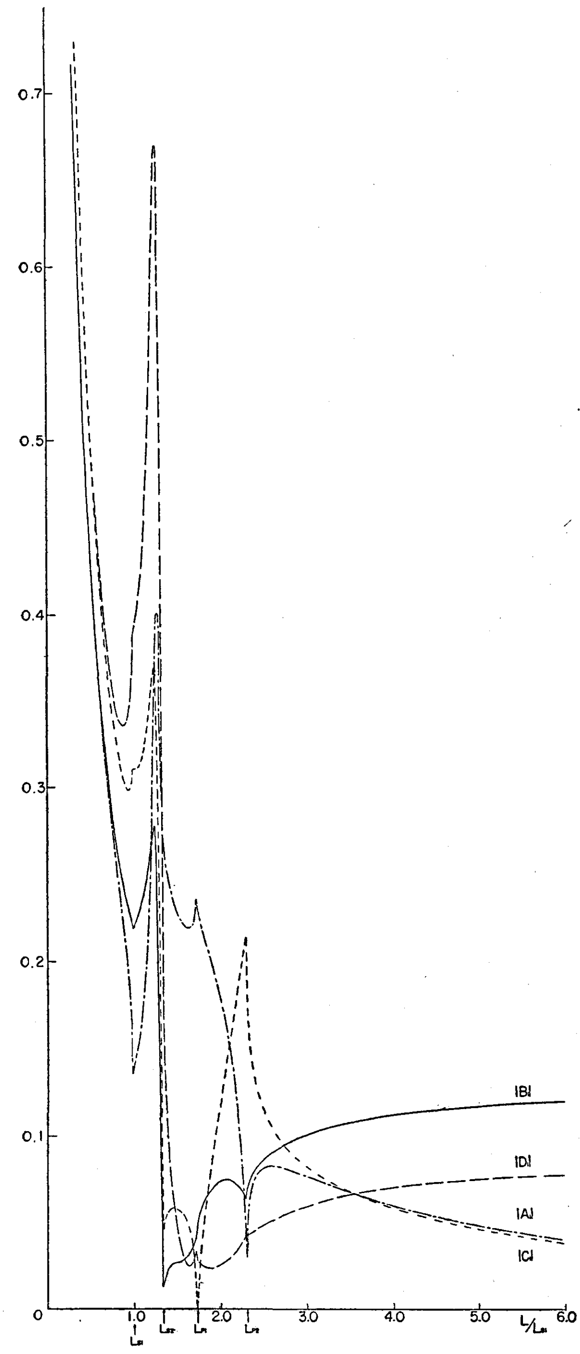

Fig. 26.

waves. If we dispense with the value of L/L_{s1} in expressing the range in which relevant waves become boundary waves but by the wave length of the P or S wave itself, the range in the present case is the same as that in the case of incidence of the P wave. For example, the wave represented by C_1 becomes a boundary wave for L smaller than L_{p2} in both cases.

In order to gain some perspective of the variations of $|A_1|$, $|B_1|$,

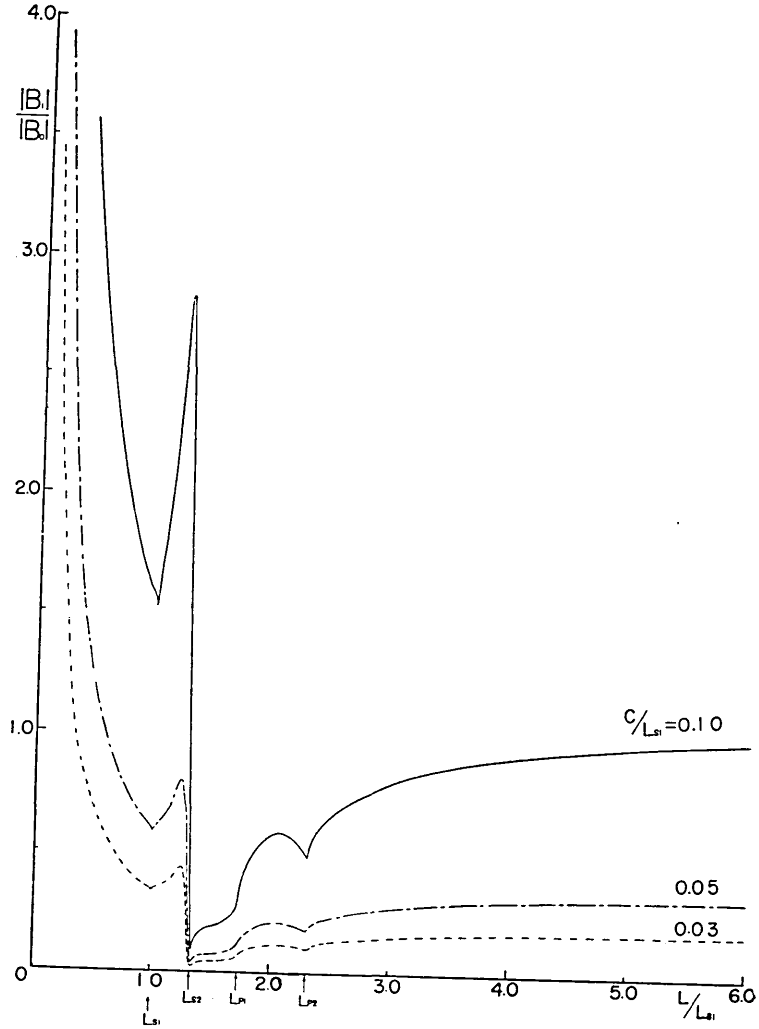

Fig. 27.

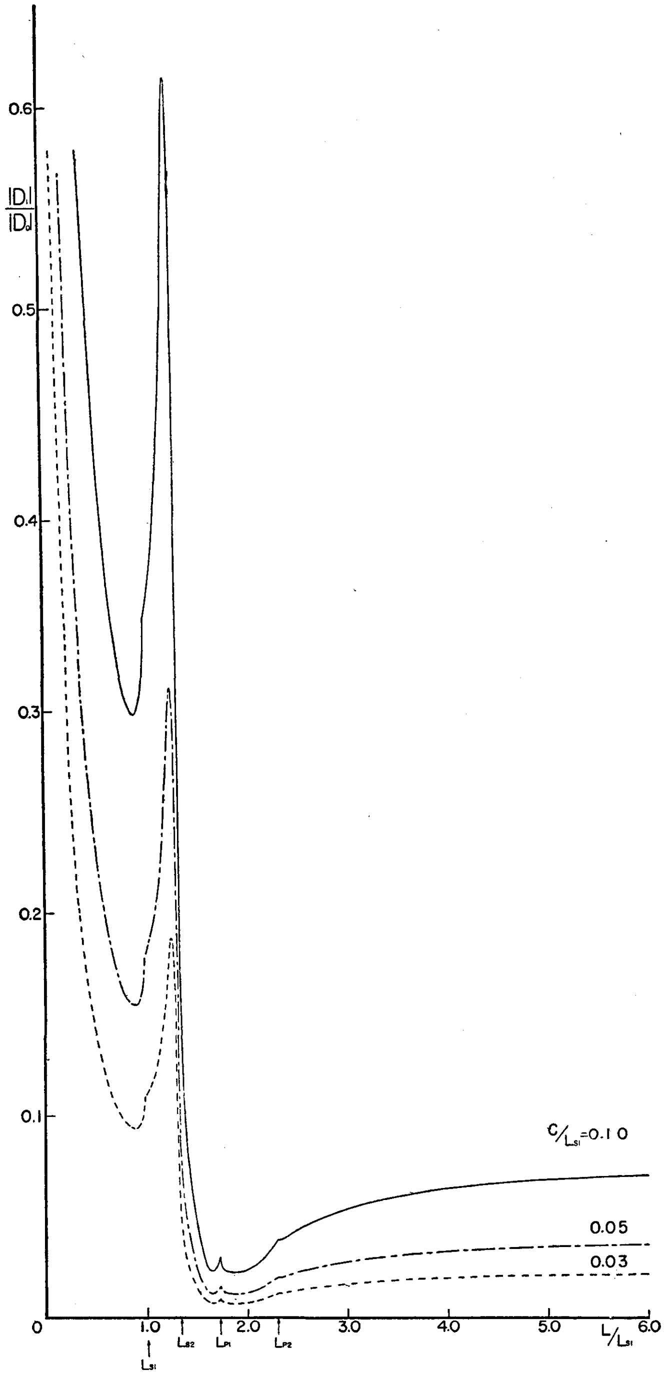
0.6
|D₁|/|D₀|
0.5
0.4
0.3
0.2
0.1
C/L_S1=0.10
0.05
0.03
0
1.0
2.0
3.0
4.0
5.0
6.0
L/L_S1
L_S1
L_S2
L_P1
L_P2

Fig. 28.

etc., they are shown together in Fig. 26 for $c/L_{s1}=0.1$. The notable points are as follows:

(a) a peak of $|C_1|$ at $L=L_{p2}$ and zero of $|C_1|$ at $L=L_{p1}$ corresponding to the peak of $|A_1|$,

(b) peaks of all irregular waves in the neighbourhood of $L=L_{s2}$,

(c) minima of all irregular waves at or near $L=L_{s1}$,

(d) monotonous increase of all irregular waves with L decreasing below L_{s1}.

(e) the tendency of P and S components to become comparable in order of magnitude to each other around $L/L_{s1}=3$.

The last point is also seen from Fig. 30.

As to the ratio of irregular waves to regular waves, the readers are referred to Figs. 27 through 29. It is to be noted as in the case of the incidence of P or SH waves, that the features of the curves are similar to those of irregular waves themselves because the variations of regular ones are small. Furthermore, no new features different from those of $|B_1|$ and $|D_1|$ appear.

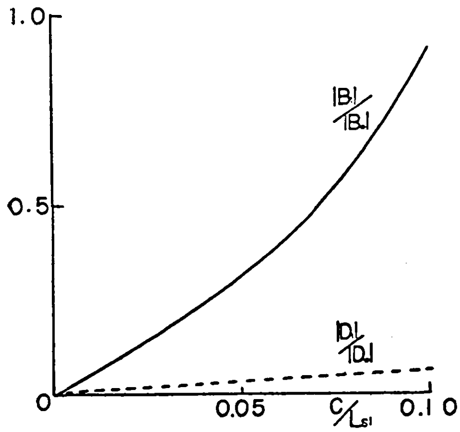

Fig. 29.

(20) The remarkable feature is that in spite of small values of $|B_1|/|B_0|$ for L a little larger than L_{s2} and of $|D_1|/|D_0|$ near $L=L_{p1}$, both these ratios become very large at L a little smaller than L_{s2}. The values of $|B_1|/|B_0|$ become large for L below L_{s2}. As seen in Fig. 29, the variations of $|B_1|/|B_0|$ with c/L_{s1} are much larger than those of $|D_1|/|D_0|$ and this shows the extent of the effect of corrugation on the reflection and refraction of waves. That is, Fig. 29 confirms together with Fig. 21 that the effect of corrugation is larger on reflection than on refraction.

In Fig. 30, the ratios of P wave components to S wave components of irregular waves are given.

(21) As to $|A_1|/|B_1|$, it increases monotonously with decreasing L/L_{s1} down to L_{s2} as a whole. The minimum at $L=L_{p2}$ corresponds to that of $|A_1|$. The sharp peak at $L=L_{s2}$ is remarkable, but it must be borne in mind that this corresponds to the values of L/L_{s1} where the maximum

of $|A_1|$ and the minimum of $|B_1|$ appear. For L below L_{s1}, $|A_1|/|B_1|$ tends to 1.

(22) $|C_1|/|D_1|$ increases monotonously with decreasing L/L_{s1} in the interval of L larger than L_{p2} and decreases as a whole with decreasing L/L_{s1} in the interval of L between L_{p2} and L_{s2}. At $L=L_{p1}$, $|C_1|/|D_1|$ becomes 0 corresponding to the zero of $|C_1|$. For L smaller than L_{s2}, $|C_1|/|D_1|$ increases with decreasing L/L_{s1} and tends to 1.

(23) In comparison with the case of incidence of the P wave, the P wave component becomes comparable in order of magnitude to the S

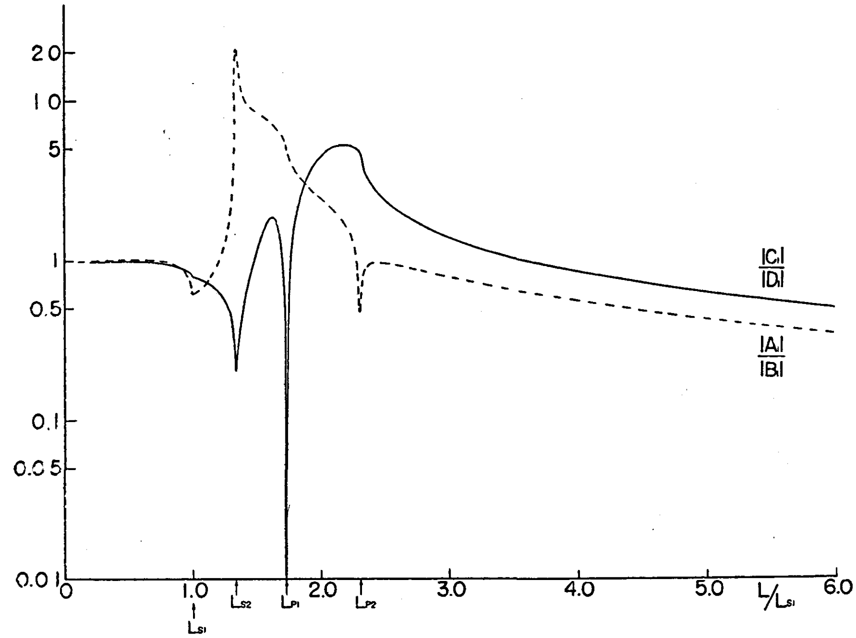

Fig. 30.

wave component from larger values of L, especially in irregularly refracted waves.

From Figs. 31–34, the following features can be seen concerning $|A_2|$, $|B_2|$, etc.

(24) In general, $|A_2|$, $|B_2|$, etc. are much smaller than $|A_1|$, $|B_1|$, etc. respectively. This result is similar to the case of incidence of the SH wave and differs from the case of incidence of the P wave. However, attention must be paid to the ranges of L/L_{s1} where any of $|A_1|$, $|B_1|$, etc. become small.

(25) It is common to all irregular waves that at L, just below L_{s2} there exists a sharp peak, while at $L \approx L_{s1}$ a sharp minimum appears. Furthermore, these features are the same as those with spectra of the first order.

(26) The variations of $|A_2|$, $|B_2|$, etc. with L/L_{s1} are much more complex than those with spectra of the first order.

(27) In this case also, as in the cases of incidence of P and SH waves, some irregular waves become boundary waves. Concerning $|A_2|$,

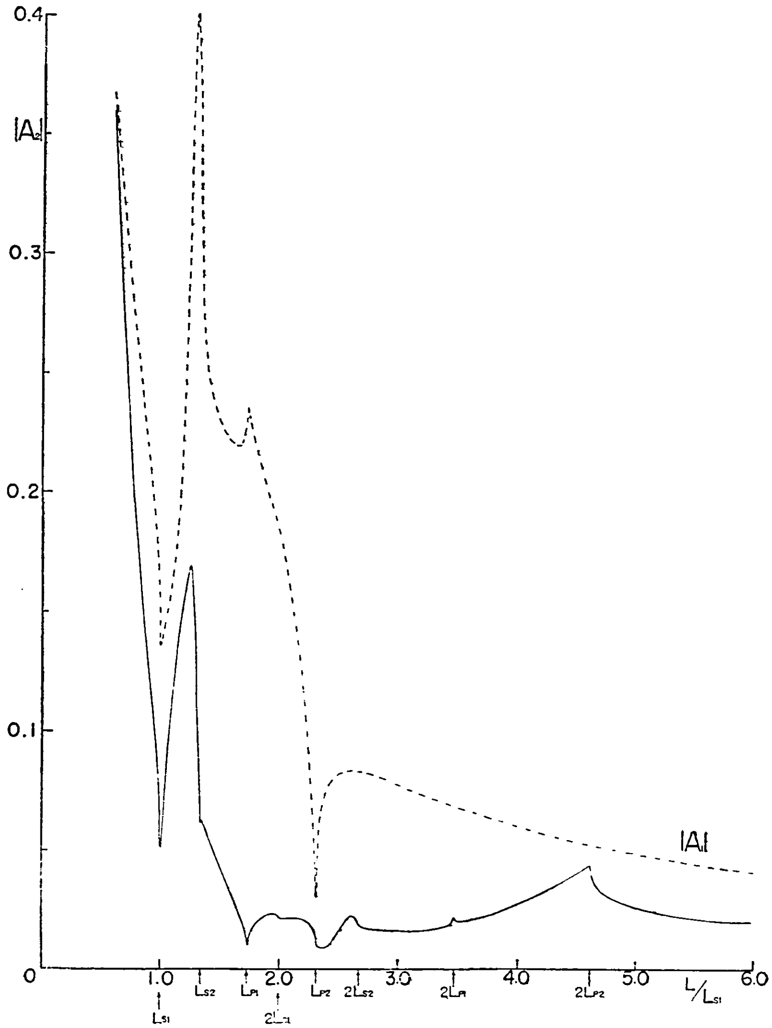

Fig. 31.

$|B_2|$, etc., the range where a certain kind of wave becomes a boundary wave is the same as in the case of incidence of the P wave, if it is expressed in terms of the wave length of P or S waves.

Chapter 4. Discussions and Conclusions

§ 1. Discussions

Now let us first consider the displacement itself. This is very im-

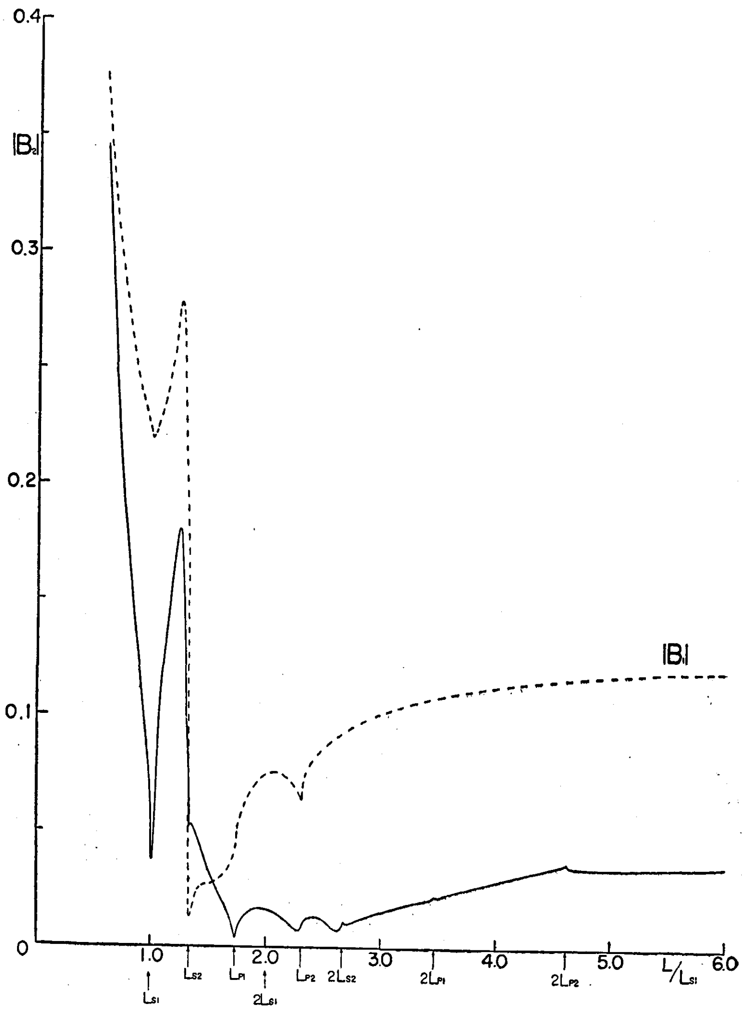

Fig. 32.

portant from the practical point of view since displacement potentials ϕ and ψ can not be observed as they are. But in our above mentioned calculations in cases of the incidence of P and SV waves, only the values of the displacement potentials are given. Therefore some remarks on the relations between displacement and potentials, as given hitherto in the figures, will be set down in the following.

As is well known, the displacement is defined by the relations $u=\partial\phi/\partial x+\partial\psi/\partial z$ and $w=\partial\phi/\partial z-\partial\psi/\partial x$. Because of the difference in the

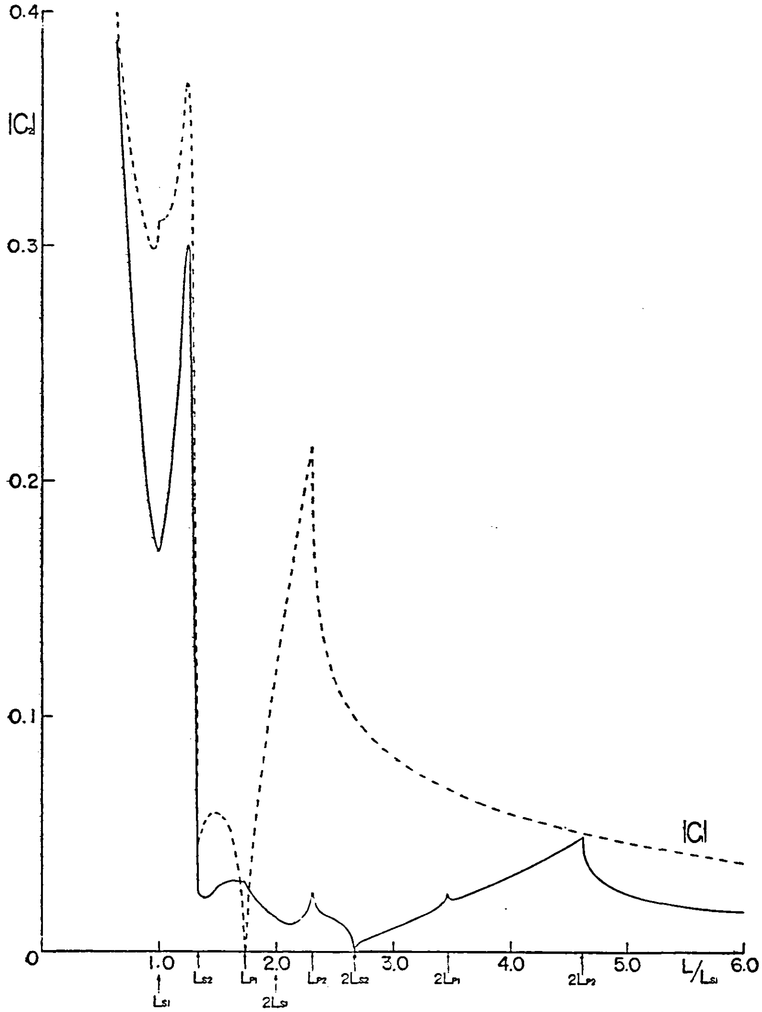

Fig. 33.

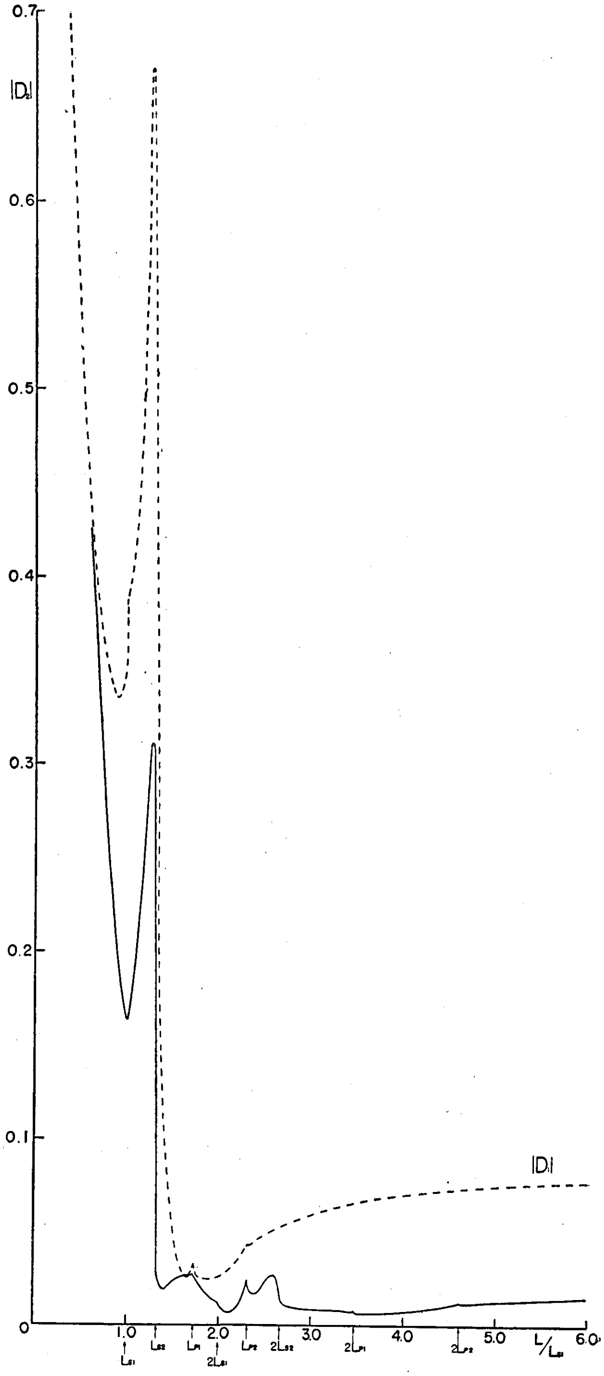
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
$|D_2|$
$|D_1|$
1.0
2.0
3.0
4.0
5.0
6.0
L/L_{S1}
L_{S1}
L_{S2}
L_{P1}
$2L_{S1}$
L_{P2}
$2L_{S2}$
$2L_{P1}$
$2L_{P2}$

Fig. 34.

velocity of propagation between P and S waves we shall consider separately the contributions of ϕ and ψ towards displacement. Let us take the case of normal incidence of the P wave as an example. In this case, final displacement potentials are written explicitly as follows, neglecting the time factor $e^{i\omega t}$.

$$\phi_1 = e^{ih_1z} + A_0 e^{-ih_1z} + \sum A_n \{e^{i(npx - h_1z\cos\alpha_n)} + e^{-i(npx + h_1z\cos\alpha_n)}\} ,$$

$$\psi_1 = \sum B_n \{e^{i(npx - \sqrt{\sigma_1}h_1z\cos\beta_n)} - e^{-i(npx + \sqrt{\sigma_1}h_1z\cos\beta_n)}\} ,$$

$$\phi_2 = C_0 e^{ih_2z} + \sum C_n \{e^{i(npx + h_2z\cos\gamma_n)} + e^{-i(npx - h_2z\cos\gamma_n)}\} ,$$

$$\psi_2 = \sum D_n \{e^{i(npx + \sqrt{\sigma_2}h_2z\cos\delta_n)} - e^{-i(npx - \sqrt{\sigma_2}h_2z\cos\delta_n)}\} .$$

Then the absolute amplitudes of displacement U are readily obtained as follows:

(1) Incident wave: $|U| = 2\pi/L_{p1}$

(2) Regular waves:

Reflected wave: $|U| = (2\pi/L_{p1}) \cdot |A_0|$

Refracted wave: $|U| = (2\pi/L_{p2}) \cdot |C_0|$

(3) Irregular waves:

P wave component:

Reflected wave: $|U_+| = (2\pi/L_{p1}) \cdot |A_n|$

Refracted wave: $|U_+| = (2\pi/L_{p2}) \cdot |C_n|$

S wave component:

Reflected wave: $|U_+| = (2\pi/L_{s1}) \cdot |B_n|$

Refracted wave: $|U_+| = (2\pi/L_{s2}) \cdot |D_n|$

where U_+ corresponds to the term with the factor e^{-inpx}. It is thus to be observed that the absolute amplitudes of the P or SV waves are $2\pi/L'$ times the amplitudes of the displacement potentials, where L' is the wave length of the waves concerned in their respective media, so that the relative amplitudes of these waves can be known only by dividing the amplitudes of the displacement potentials by the respective velocities of the waves concerned. The components of these amplitudes along the coordinate axes are readily ascertained by the nature of the wave and the angles of incidence, reflection and refraction of these waves. We can obtain the following displacement ratio of irregular waves to regular ones from the above formulae.

Reflected wave: displacement ratio $=|A_n|/|A_0|$

Refracted wave: displacement ratio $=|C_n|/|C_0|$

Thus from Figs. 3, 4, 11 and 12 it may be said that for L larger than L_{p1} and for large value of c/L_{p1} there may exist a case where the increase in irregularly reflected P waves surpasses the decrease in regularly reflected P waves. Since for L below L_{p1}, an irregularly reflected P wave with a spectrum of the first order becomes a boundary wave, it can be said from the practical point of view that the reflected P wave decreases with increasing c/L_{p1}. For refracted waves the effect of corrugated boundary surface is rather small.

Now in the whole discussion given above, the phase change which may be induced in reflection and refraction is not referred to. But in order to obtain a more complete and exact knowledge of the phenomena, it must be taken into consideration in discussing the resultant motion caused by all the waves concerned and some of those interpretations may have to be changed. Such a point is left for future study.

In all cases calculated, the existence of boundary waves was indicated the amplitudes of which decrease exponentially with increasing $|z|$. Each of the irregular waves is composed of two waves propagated in plus as well as minus directions of the x-axis which constitute, as a result, a standing wave in a stationary state. This is a common particular feature in the normal incidence of all waves.

Another particular feature, common to all irregular waves is the following:

The factor depending on z, for example in the P wave component of an irregularly reflected wave with a spectrum of n-th order in the case of incidence of the P wave is

$$\phi_n \propto 2A_n e^{-ik_1 z \cos \alpha_n} .$$

Therefore

$$\phi_n \propto 2A_n e^{-i(2\pi/L)z\sqrt{(L/L_{p1})^2-n^2}} \qquad \text{when } L/L_{p1} \text{ is larger than } n ,$$

or

$$\phi_n \propto 2A_n e^{-(2\pi/L)z\sqrt{n^2-(L/L_{p1})^2}} \qquad \text{when } L/L_{p1} \text{ is smaller than } n .$$

From the above formulae, we can perceive that the exponential factor of ϕ_n depends on wave lengths of corrugation and incident wave and the order n of spectrum, in such a way that

(a) the smaller the wave length L of a corrugated boundary surface,
(b) the higher the order n of spectrum, and
(c) the larger the wave length of incident wave L_{p1},
the larger is the rate of decrease in ϕ_n with increase in the distance z from the mean boundary.

As an example, the rate of decrease of the exponential factor of ϕ_1 is estimated, with the result that at a fixed level $z=L/2$, (that is, the level at half a wave length of corrugated boundary surface below the mean boundary surface,) for a wave with $L/L_{p1}=0.7$, the exponential factor becomes only 0.1 approximately, independent of the roughness c/L_{p1}. Therefore, from the practical point of view, it is unnecessary to take these boundary waves into consideration.

It is seen clearly from almost all figures given above that the increasing tendency of amplitudes for the smaller L/L_{p1} or L/L_{s1} is the more prominent. As mentioned previously, this tendency may, in extremity, impair the accuracy of this method of calculation, deteriorating the convergence of series. The amplitude of irregular waves with spectra of the second order gives the measure of the convergence. In cases of the incidence of both SH and SV waves, not only $|B_0|$ and $|D_0|$, but also $|A_1|$ etc. are in general much larger than $|A_2|$, $|B_2|$, etc. Even in these cases where the accuracy is better than in the case of incidence of the P wave $|A_2|$, $|B_2|$, etc. become comparable order to or larger than $|A_1|$, $|B_1|$, etc. for L/L_{s1} below about 0.5. In the case of incidence of the P wave, we must take into consideration the contributions of irregular waves with spectra of the second order from larger values of L/L_{p1} than in the two cases of S waves. However, since fortunately, characteristic features of waves with spectra of the first and second order, that is, the positions of peaks and minima are almost the same, with each other, it is unnecessary to change the results obtained about $|A_1|$, $|B_1|$, etc. by taking $|A_2|$, $|B_2|$, etc. into consideration. Needlessto say, the higher the approximation is advanced, the more information concerning the corrugation of boundary surface can be obtained.

We have had no time to study examples other than the case in which the velocity ratio $V_{p1}/V_{p2}=6/8$ and rigidity ratio $\mu_2/\mu_1=2$. It would therefore be better to abstain from drawing too wide a conclusion, but we cannot refrain from giving a few notices. From figures given above, we have seen that almost all the abnormal points are related to the wavelength of P and S waves. Therefore, since the wave length is closely connected with velocity itself, many theoretically interesting changes of features will be expected to accompany variations in velocity ratio. Considerations on various cases of velocity contrast will be carried out in the near future. Moreover, the results may depend more or less upon the adopted value of the rigidity ratio and the determining which of the two numerical constants ρ_1/ρ_2 and μ_2/μ_1 has the more effect on these phenomena is a very interesting investigation.

The computations were carried out only for cases of normal incidence on the mean surface of a harmonic boundary $\zeta=c\cos px$. One of the important problems to be considered is the case of the oblique incidence of waves and another is the problem concerning reflection and refraction of waves at an arbitrary form of boundary surface, especially at an isolated trough of large amplitude. If the results obtained above for various L are superposed, we can obtain in principle the solutions to an arbitrary form of boundary surface although the assumptions of small amplitude and periodicity concerning the corrugated boundary surface cannot be dispensed with. To do this will require a very tedious procedure and other appropriate methods may be applied much more easily. It is very interesting to solve the same problems by using other methods and to compare the results with one another and it is hoped that some tests will be carried out on these problems.

§ 2. Conclusions

In the present paper the effect of corrugation of boundary surface on reflection and refraction of elastic waves was considered by the method introduced by Lord Rayleigh in his problem of sound and light. The solutions were obtained for the cases of incidence of P and SV waves with an arbitrary angle of incidence on a corrugated periodic boundary surface which can be expanded in Fourier's series.

In Chapter 2 of the present paper, the fundamental equations such as equations of boundary surface, equations of motion and formulae of boundary conditions were presented. In Chapter 3, the solutions of the case of incidence of P and SV waves were obtained to the order of magnitude of ζ^2. That is to say, the solutions of the first and second approximation were obtained. Numerical calculations were carried out for the case of normal incidence of all waves on the mean surface of a harmonic boundary $\zeta=c\cos px$ by using the electronic computor PC–1. The results thus obtained were given in Figs. 3–18 and Tables 1–5 for

Table 1. $|A_0|$, $|C_0|$, etc. for $c/L_{p1}=0.03$ in the case of incidence of P wave.

L/L_{p1}	$\|A_0\|$	$\|C_0\|$	$\|A_1\|$	$\|B_1\|$	$\|C_1\|$	$\|D_1\|$	$\|A_1\|/\|A_0\|$	$\|C_1\|/\|C_0\|$
3.00	0.19291	1.07025	0.037083	0.008157	0.024288	0.007646	0.19223	0.022694
2.95	0.192910	1.070234	0.037064	0.008296	0.024261	0.007776		
2.90	0.192911	1.070235	0.037043	0.008439	0.024233	0.007910		
2.85	0.192911	1.070235	0.037022	0.008587	0.024204	0.008048		
2.80	0.192912	1.070236	0.037000	0.008740	0.024173	0.008192		
2.75	0.192912	1.070236	0.036976	0.008899	0.024140	0.008341		
2.70	0.192913	1.070237	0.036952	0.009064	0.024106	0.008496		
2.65	0.192914	1.070238	0.036926	0.009235	0.024071	0.008656		
2.60			0.036899	0.009412	0.024034	0.008823		
2.55	0.192915	1.070239	0.036871	0.009597	0.023995	0.008996		
2.50	0.19293	1.07025	0.036842	0.009788	0.023954	0.009176		
2.45	0.192916	1.070240	0.036811	0.009988	0.023911	0.009363		
2.40	0.192916	1.070240	0.036779	0.010196	0.023867	0.009558		
2.35	0.192918	1.070242	0.036746	0.010412	0.023820	0.009761		
2.30	0.192919	1.070242	0.036711	0.010638	0.023772	0.009974		
2.25	0.192920	1.070243	0.036675	0.010873	0.023721	0.010196		
2.20	0.19292	1.07024	0.036638	0.011120	0.023669	0.010428	0.18991	0.022116
2.15	0.192922	1.070245	0.036599	0.011377	0.023615	0.010671		
2.10	0.192927	1.070254	0.036560	0.011646	0.023560	0.010925		
2.05	0.192929	1.070255	0.036520	0.011929	0.023504	0.011192		
2.00	0.192931	1.070255	0.036480	0.012224	0.023447	0.011473		
1.95	0.192933	1.070255	0.036441	0.012534	0.023392	0.011767		
1.90	0.192936	1.070255	0.036404	0.012860	0.023338	0.012078		
1.85	0.192939	1.070254	0.036370	0.013202	0.023289	0.012405		
1.80	0.192943	1.070252	0.036341	0.013561	0.023249	0.012751		
1.75			0.036323	0.013938	0.023221	C.013117		
1.70	0.192953	1.070245	0.036320	0.014334	0.023215	0.013505		
1.65			0.036342	0.014748	0.023243	0.013917		
1.60	0.192969	1.070227	0.036404	0.015180	0.023327	0.014355		
1.55	0.192981	1.070209	0.036535	0.015626	0.023506	0.014823		
1.50	0.192999	1.070181	0.036786	0.016078	0.023855	0.015322	0.19060	0.022290
1.45	0.193028	1.070132	0.037275	0.016515	0.024538	0.015856		
1.40	0.193082	1.070035	0.038323	0.016869	0.026007	0.016424	0.19848	0.024305
1.35	0.193246	1.069762	0.041575	0.016797	0.030552	0.016993	0.21514	0.028560
1.333334	0.192680	1.069902	0.048559	0.015559	0.040177	0.016974	0.25202	0.037552
1.32	0.192748	1.069996	0.046455	0.016379	0.038159	0.017366	0.24101	0.035663
1.30	0.192838	1.070122	0.043330	0.017529	0.035057	0.017896	0.22470	0.032760
1.28	0.192913	1.070226	0.040379	0.018575	0.032037	0.018358		
1.26	0.192974	1.070311	0.037626	0.019532	0.029115	0.018762		
1.24	0.193022	1.070379	0.035089	0.020414	0.026304	0.019117	0.18179	0.024574
1.20	0.193085	1.070469	0.030712	0.021995	0.021046	0.019713	0.15906	0.019661

1.18	0.193103	1.070496	0.028887	0.022714	0.018610	0.019969		
1.16	0.193114	1.070512	0.027308	0.023397	0.016307	0.020208		
1.14	0.193118	1.070521	0.025971	0.024052	0.014139	0.020438	0.13448	0.013208
1.12	0.193117	1.070523	0.024869	0.024689	0.012108	0.020668		
1.10	0.193112	1.070519	0.023989	0.025315	0.010220	0.020907	0.12422	0.009547
1.08	0.193102	1.070512	0.023317	0.025941	0.008482	0.021168	0.12075	0.007923
1.06	0.193088	1.070501	0.022839	0.026580	0.006916	0.021468	0.11828	0.006461
1.04	0.193068	1.070488	0.022545	0.027254	0.005565	0.021833	0.11677	0.005199
1.02	0.193040	1.070472	0.022446	0.028014	0.004567	0.022329	0.11628	0.004266
1.00			0.022935	0.029393	0.005591	0.023530		
0.98	0.192985	1.070436	0.020834	0.030124	0.004091	0.022817	0.107957	0.003822
0.96	0.192995	1.070422	0.020107	0.030692	0.003952	0.022664	0.104184	
0.94	0.193002	1.070407	0.019640	0.031253	0.004747	0.022609	0.101761	0.004435
0.92	0.193010	1.070394	0.019294	0.031810	0.006012	0.022610	0.099964	0.005617
0.90	0.193021	1.070384	0.018994	0.032364	0.007477	0.022661	0.098404	0.006985
0.88			0.018689	0.032909	0.009043	0.022773		0.008436
0.86	0.193058	1.070377	0.018329	0.033440	0.010690	0.022974	0.094940	0.009987
0.84	0.193087	1.070383	0.017855	0.033947	0.012444	0.023329	0.092471	0.011626
0.82	0.193125	1.070398	0.017174	0.034425	0.014388	0.023980	0.088927	0.013442
0.80	0.193175	1.070425	0.016088	0.034882	0.016738	0.025286	0.083282	0.015637
0.78	0.193240	1.070465	0.013871	0.035542	0.020338	0.028668	0.071781	0.018999
0.769			0.008002	0.036430	0.028618	0.042816		
0.76	0.193706	1.071294	0.009992	0.027078	0.036107	0.059843	0.051583	0.033704
0.749			0.018038	0.018063	0.040085	0.070224		
0.74	0.193908	1.071796	0.023585	0.017514	0.039469	0.071235	0.121630	0.036825
0.729			0.026985	0.024551	0.034912	0.064440		
0.72	0.193664	1.071176	0.027418	0.030131	0.030181	0.055961	0.141575	0.028176
0.709			0.026412	0.034686	0.025113	0.045773		
0.70	0.193577	1.070752	0.025103	0.036963	0.022083	0.038697	0.129680	0.020624
0.689			0.023329	0.038632	0.019697	0.031806		
0.66	0.193771	1.070684	0.018966	0.040277	0.018425	0.021439	0.097878	0.017209
0.64			0.016335	0.040467	0.019760	0.019308		
0.62	0.194147	1.070969	0.013936	0.040381	0.021813	0.019924	0.071781	0.020368
0.60	0.194419	1.071207	0.011704	0.040172	0.024239	0.022266	0.060200	0.022628
0.58	0.194910	1.071651	0.009800	0.039909	0.027158	0.026473	0.050280	0.025342
0.55	0.195059	1.071873	0.020228	0.042017	0.033703	0.037693	0.103703	0.031443
0.50	0.194919	1.071942	0.028835	0.045806	0.040460	0.045437	0.147934	0.037745
0.45			0.036655	0.050687	0.047624	0.052661		
0.40	0.194873	1.071954	0.045072	0.056831	0.055651	0.060343	0.231290	0.051915
0.35	0.194866	1.071957	0.054971	0.064838	0.065675	0.069911	0.282096	0.061266
0.30	0.194864	1.071960	0.067362	0.075559	0.078612	0.082314	0.345688	0.073335
0.25	0.194870	1.071962	0.083930	0.090600	0.096286	0.099408	0.430700	0.089822
0.20	0.194888	1.071966	0.107948	0.113188	0.122302	0.124820	0.553899	0.114091
0.15	0.194929	1.071974	0.146983	0.150861	0.165049	0.166947	0.754030	0.153967
0.10	0.195057	1.071995	0.223674	0.226237	0.249668	0.250937	1.146710	0.232900

Table 2. $|A_0|$, $|C_0|$, etc. for $c/L_{p1}=0.05$ in the case of incidence of P wave.

L/L_{p1}	$\lvert A_0\rvert$	$\lvert C_0\rvert$	$\lvert A_1\rvert$	$\lvert B_1\rvert$	$\lvert C_1\rvert$	$\lvert D_1\rvert$	$\lvert A_1\rvert/\lvert A_0\rvert$	$\lvert C_1\rvert/\lvert C_0\rvert$
3.00	0.18030	1.07658	0.061806	0.013596	0.040481	0.012743	0.34281	0.037601
2.95	0.180306	1.076577	0.061773	0.013826	0.040436	0.012959		
2.90	0.180307	1.076579	0.061739	0.014064	0.040389	0.013183		
2.85	0.180309	1.076580	0.061703	0.014311	0.040339	0.013414		
2.80	0.180310	1.076581	0.061666	0.014567	0.040288	0.013654		
2.75	0.180312	1.076583	0.061627	0.014832	0.040234	0.013902		
2.70	0.180314	1.076584	0.061586	0.015106	0.040177	0.014159		
2.65	0.180316	1.076586	0.061543	0.015391	0.040118	0.014427		
2.60	0.180317	1.076588	0.061499	0.015687	0.040056	0.014704		
2.55	0.180319	1.076589	0.061452	0.015994	0.039991	0.014993		
2.50	0.18033	1.07659	0.061403	0.016314	0.039923	0.015293		
2.45	0.180323	1.076593	0.061352	0.016646	0.039852	0.015605		
2.40	0.180324	1.076593	0.061299	0.016993	0.039778	0.015930		
2.35	0.180328	1.076597	0.061243	0.017353	0.039700	0.016270		
2.30	0.180330	1.076599	0.061185	0.017730	0.039619	0.016624		
2.25	0.180333	1.076601	0.061125	0.018122	0.039536	0.016993		
2.20	0.18038	1.07663	0.061063	0.018533	0.039449	0.017380	0.33862	0.036643
2.15	0.180338	1.076605	0.060999	0.018962	0.039359	0.017785		
2.10	0.180341	1.076607	0.060933	0.019411	0.039267	0.018209		
2.05	0.180343	1.076610	0.060867	0.019881	0.039173	0.018654		
2.00			0.060800	0.020374	0.039079	0.019121		
1.95	0.180349	1.076614	0.060735	0.020891	0.038986	0.019612		
1.90			0.060673	0.021433	0.038897	0.020130		
1.85	0.180354	1.076617	0.060616	0.022003	0.038816	0.020676		
1.80	0.180356	1.076618	0.060569	0.022601	0.038748	0.021252		
1.75	0.180357	1.076618	0.060538	0.023230	0.038702	0.021862		
1.70			0.060534	0.023890	0.038691	0.022509		
1.65	0.180357	1.076613	0.060570	0.024580	0.038738	0.023195		
1.60	0.180354	1.076607	0.060674	0.025300	0.038878	0.023925		
1.55	0.180347	1.076596	0.060891	0.026044	0.039177	0.024704		
1.50	0.180333	1.076575	0.061310	0.026797	0.039759	0.025536	0.33998	0.036931
1.45	0.180307	1.076538	0.062124	0.027524	0.040896	0.026426		
1.40	0.180250	1.076459	0.063871	0.028115	0.043345	0.027374	0.35435	0.040266
1.35	0.180071	1.076212	0.069292	0.027995	0.050921	0.028321	0.38436	0.047135
1.333334	0.179667	1.075654	0.080932	0.025931	0.066961	0.028290	0.450456	0.062251
1.32	0.179856	1.075915	0.077425	0.027299	0.063598	0.028944	0.43048	0.059110
1.30	0.180106	1.076264	0.072216	0.029216	0.058428	0.029826	0.40096	0.054288
1.28	0.180315	1.076554	0.067299	0.030959	0.053395	0.030579		
1.26	0.180483	1.076791	0.062710	0.032554	0.048526	0.031271		
1.24	0.180617	1.076978	0.058481	0.034023	0.043840	0.031862	0.32378	0.040706
1.20	0.180792	1.077229	0.051186	0.036658	0.035078	0.032854	0.28312	0.032563

1.18	0.180843	1.077303	0.048145	0.037856	0.031017	0.033281		
1.16	0.180872	1.077349	0.045514	0.038994	0.027179	0.033680		
1.14	0.180884	1.077373	0.043285	0.040087	0.023565	0.034064	0.23930	0.021873
1.12	0.180882	1.077378	0.041448	0.041148	0.020181	0.034447		
1.10	0.180866	1.077369	0.039982	0.042192	0.017033	0.034846	0.22106	0.015810
1.08	0.180839	1.077347	0.038862	0.043235	0.014137	0.035280	0.21490	0.013122
1.06	0.180800	1.077317	0.038066	0.044300	0.011526	0.035779	0.21054	0.010699
1.04	0.180746	1.077280	0.037574	0.045423	0.009275	0.036389	0.20788	0.008610
1.02	0.180666	1.077237	0.037410	0.046689	0.007612	0.037214	0.20707	0.007066
1.00			0.038224	0.048988	0.009318	0.039216		
0.98	0.180513	1.077138	0.034724	0.050207	0.006819	0.038028	0.19236	0.006331
0.96	0.180541	1.077098	0.033511	0.051154	0.006586	0.037773		
0.94	0.180561	1.077057	0.032734	0.052088	0.007911	0.037681	0.18129	0.007345
0.92	0.180584	1.077021	0.032156	0.053017	0.010021	0.037683		
0.90	0.180615	1.076994	0.031657	0.053940	0.012462	0.037769	0.17527	0.011571
0.88	0.180658	1.076977	0.031149	0.054849	0.015072	0.037954	0.17242	0.013995
0.86	0.180718	1.076975	0.030548	0.055733	0.017816	0.038290		
0.84	0.180799	1.076992	0.029758	0.056579	0.020740	0.038882	0.16459	0.019257
0.82	0.180906	1.077033	0.028623	0.057375	0.023979	0.039966		
0.80	0.181046	1.077108	0.026814	0.058137	0.027897	0.042142	0.14811	0.025900
0.78	0.181229	1.077218	0.023118	0.059236	0.033896	0.047780	0.12756	0.031466
0.769			0.013335	0.060722	0.047697	0.071366		
0.76	0.182527	1.079522	0.016654	0.045130	0.060178	0.099738	0.091241	0.055745
0.749			0.030076	0.030110	0.066811	0.117040		
0.74	0.183079	1.080916	0.039308	0.029191	0.065780	0.118722	0.214705	0.060856
0.729			0.044984	0.040922	0.058189	0.107401		
0.72	0.182400	1.079193	0.045697	0.050219	0.050301	0.093267	0.250532	0.046610
0.709			0.044033	0.057812	0.041868	0.076290		
0.70	0.182161	1.078016	0.041838	0.061606	0.036805	0.064495	0.229676	0.034141
0.689			0.038895	0.064388	0.032837	0.053009		
0.66	0.182700	1.077825	0.031611	0.067129	0.030708	0.035730	0.173021	0.028491
0.64	0.183170	1.078145	0.027225	0.067445	0.032934	0.032180	0.148632	0.030547
0.62	0.183747	1.078618	0.023227	0.067301	0.036356	0.033206	0.126408	0.033707
0.60	0.184503	1.079279	0.019506	0.066953	0.040399	0.037110	0.105722	0.037431
0.58	0.185865	1.080512	0.016333	0.066515	0.045263	0.044122	0.087876	0.041890
0.55	0.186325	1.081128	0.033713	0.070029	0.056172	0.062821	0.180939	0.051956
0.50	0.185895	1.081320	0.048059	0.076343	0.067434	0.075729	0.258525	0.062363
0.45			0.061091	0.084479	0.079373	0.087769		
0.40	0.185794	1.081357	0.075120	0.094718	0.092751	0.100572	0.404321	0.085773
0.35	0.185797	1.081369	0.091618	0.108064	0.109458	0.116519	0.493110	0.101222
0.30	0.185830	1.081381	0.112271	0.125931	0.131020	0.137189	0.604157	0.121160
0.25	0.185907	1.081397	0.146750	0.159106	0.193091	0.165680	0.789377	0.178557
0.20	0.186067	1.081423	0.179913	0.188647	0.203837	0.208033	0.966928	0.188489
0.15	0.186428	1.081477	0.244971	0.251436	0.275082	0.278246	1.314026	0.254358
0.10	0.187480	1.081622	0.372790	0.377061	0.416113	0.418229	1.988424	0.384712

Table 3. $|A_0|$, $|C_0|$, etc. for $c/L_{p1}=0.10$ in the case of incidence of P wave.

L/L_{p1}	$\|A_0\|$	$\|C_0\|$	$\|A_1\|$	$\|B_1\|$	$\|C_1\|$	$\|D_1\|$	$\|A_1\|/\|A_0\|$	$\|C_1\|/\|C_0\|$
3.00	0.12128	1.10633	0.123611	0.027191	0.080962	0.025487	0.34281	0.037601
2.95	0.121223	1.106310	0.123546	0.027652	0.080871	0.025919		
2.90	0.121229	1.106315	0.123478	0.028129	0.080777	0.026365		
2.85	0.121235	1.106320	0.123407	0.028622	0.080678	0.026828		
2.80	0.121242	1.106326	0.123332	0.029134	0.080575	0.027307		
2.75	0.121248	1.106331	0.123254	0.029663	0.080467	0.027804		
2.70			0.123172	0.030212	0.080354	0.028319		
2.65	0.121262	1.106344	0.123087	0.030782	0.080236	0.028853		
2.60	0.121270	1.106350	0.122997	0.031374	0.080112	0.029409		
2.55	0.121277	1.106357	0.122904	0.031989	0.079982	0.029985		
2.50			0.122806	0.032628	0.079846	0.030585		
2.45	0.121294	1.106372	0.122704	0.033293	0.079704	0.031210		
2.40	0.121294	1.106372	0.122597	0.033985	0.079555	0.031861		
2.35	0.121312	1.106387	0.122486	0.034707	0.079401	0.032539		
2.30	0.121321	1.106395	0.122370	0.035459	0.079239	0.033247		
2.25	0.121331	1.106404	0.122250	0.036245	0.079071	0.033987		
2.20			0.122126	0.037066	0.078897	0.034760	0.33862	0.036643
2.15	0.121352	1.106421	0.121998	0.037924	0.078718	0.035569		
2.10	0.121362	1.106430	0.121867	0.038821	0.078533	0.036417		
2.05	0.121373	1.106439	0.121734	0.039762	0.078346	0.037307		
2.00	0.121384	1.106447	0.121601	0.040747	0.078158	0.038242		
1.95	0.121395	1.106455	0.121470	0.041781	0.077972	0.039225		
1.90	0.121405	1.106462	0.121345	0.042866	0.077794	0.040260		
1.85	0.121415	1.106467	0.121232	0.044006	0.077632	0.041351		
1.80	0.121423	1.106471	0.121138	0.045203	0.077495	0.042504	0.99765	0.070038
1.75	0.121429	1.106471	0.121077	0.046460	0.077403	0.043724		
1.70			0.121067	0.047779	0.077382	0.045017		
1.65	0.121446	1.106448	0.121140	0.049160	0.077475	0.046390		
1.60	0.121415	1.106428	0.121348	0.050600	0.077756	0.047851		
1.55			0.121783	0.052088	0.078354	0.049409		
1.50	0.121334	1.106301	0.122620	0.053595	0.079518	0.051073	0.33998	0.036931
1.45	0.121228	1.106150	0.124248	0.055048	0.081793	0.052852		
1.40	0.121000	1.105834	0.127742	0.056231	0.086689	0.054748	1.05572	0.078392
1.35	0.120283	1.104848	0.138584	0.055990	0.101841	0.056642	1.152150	0.092176
1.333334	0.118667	1.102617	0.161864	0.051862	0.133922	0.056581	1.364014	0.121458
1.32	0.119435	1.103664	0.154850	0.054598	0.127196	0.057887	1.29652	0.11525
1.30	0.120450	1.105061	0.144433	0.058431	0.116856	0.059652	1.19911	0.10575
1.28	0.121286	1.106223	0.134598	0.061918	0.106790	0.061194		
1.26	0.121960	1.107168	0.125421	0.065108	0.097052	0.062542		
1.24	0.122490	1.107917	0.116962	0.068047	0.087681	0.063725	0.95487	0.079140
1.20	0.123184	1.108918	0.102372	0.073316	0.070155	0.065709	0.83105	0.063264

1.18	0.123381	1.109213	0.096291	0.075712	0.062035	0.066563		
1.16	0.123496	1.109399	0.091027	0.077989	0.054357	0.067360		
1.14	0.123543	1.109494	0.086571	0.080174	0.047130	0.068127	0.70074	0.042479
1.12	0.123530	1.109514	0.082896	0.082296	0.040361	0.068894		
1.10	0.123467	1.109475	0.079963	0.084383	0.034066	0.069691	0.64765	0.030705
1.08	0.123357	1.109390	0.077725	0.086470	0.028275	0.070561	0.63008	0.025487
1.06	0.123200	1.109270	0.076131	0.088600	0.023052	0.071559	0.61795	0.020781
1.04	0.122984	1.109122	0.075149	0.090846	0.018549	0.072777	0.61105	0.016724
1.02	0.122666	1.108949	0.074821	0.093378	0.015223	0.074429	0.60996	0.013727
1.00			0.076448	0.097977	0.018635	0.078433		
0.98	0.122058	1.108554	0.069448	0.100415	0.013637	0.076056	0.56898	0.012302
0.96	0.122172	1.108391	0.067023	0.102308	0.013173	0.075547		
0.94	0.122257	1.108229	0.065468	0.104175	0.015823	0.075362	0.53549	0.014278
0.92	0.122353	1.108086	0.064312	0.106035	0.020042	0.075367		
0.90	0.122482	1.107975	0.063314	0.107880	0.024925	0.075538	0.51692	0.022496
0.88	0.122663	1.107908	0.062297	0.109698	0.030143	0.075909	0.50787	0.027207
0.86	0.122914	1.107900	0.061097	0.111466	0.035632	0.076580		
0.84	0.123252	1.107969	0.059515	0.113158	0.041479	0.077765	0.48287	0.037437
0.82	0.123700	1.108137	0.057245	0.114750	0.047958	0.079932		
0.80	0.124290	1.108438	0.053627	0.116274	0.055794	0.084285	0.43147	0.050336
0.78			0.046236	0.118472	0.067793	0.095560	0.26338	0.058757
0.769			0.026697	0.121448	0.095403	0.142735		
0.76	0.130375	1.118140	0.033308	0.090260	0.120356	0.199477	0.255478	0.107639
0.749			0.060161	0.060234	0.133627	0.234083		
0.74	0.132365	1.123665	0.078616	0.058382	0.131561	0.237447		
0.729			0.089973	0.081852	0.116385	0.214806		
0.72	0.129614	1.116783	0.091394	0.100437	0.100603	0.186537	0.705127	0.090083
0.709			0.088069	0.115627	0.083741	0.152584		
0.70	0.128681	1.112069	0.083676	0.123212	0.073610	0.128990	0.650261	0.066192
0.689			0.077796	0.128781	0.065679	0.106022		
0.66	0.130900	1.111300	0.063221	0.134258	0.061417	0.071462	0.482971	0.055266
0.64	0.132797	1.112586	0.054450	0.134889	0.065868	0.064361	0.410024	0.059203
0.62	0.135117	1.114478	0.046454	0.134602	0.072712	0.066413	0.343806	0.065243
0.60	0.138137	1.117128	0.039012	0.133906	0.080797	0.074219	0.282416	0.072326
0.58	0.143557	1.122062	0.032666	0.133030	0.090526	0.088244	0.227548	0.080678
0.55	0.145135	1.124515	0.067427	0.140057	0.112343	0.125642	0.464582	0.099904
0.50	0.143790	1.125292	0.096117	0.152687	0.134868	0.151457	0.668454	0.119852
0.45			0.122182	0.168958	0.158745	0.175538		
0.40	0.144074	1.125502	0.150241	0.189435	0.185502	0.201145	1.042799	0.164817
0.35	0.144685	1.125606	0.183236	0.216127	0.218916	0.233037	1.266452	0.194487
0.30	0.145741	1.125742	0.224541	0.251862	0.262041	0.274379	1.540680	0.232771
0.25	0.147567	1.125951	0.279768	0.301999	0.320953	0.331361	1.895875	0.285050
0.20	0.150944	1.126318	0.359827	0.377295	0.407673	0.416066	2.383836	0.361952
0.15	0.158060	1.127102	0.489942	0.502871	0.550164	0.556491	3.099721	0.488123
0.10	0.176877	1.129317	0.745580	0.754122	0.832226	0.836457	4.215247	0.736928

Table 4. $|B_1|/|A_1|$ and $|D_1|/|C_1|$ in the case of incidence of P wave.

L/L_{p1}	$\|B_1\|/\|A_1\|$	$\|D_1\|/\|C_1\|$	L/L_{p1}	$\|B_1\|/\|A_1\|$	$\|D_1\|/\|C_1\|$
3.00	0.21997	0.31479	0.96	1.59129	4.76311
2.50	0.26568	0.38306	0.94	1.52643	
2.20	0.30351	0.44057	0.92	1.70391	3.76040
2.00	0.33509	0.48929	0.90	1.76088	2.51818
1.80	0.37316	0.54847	0.86	1.90126	1.87473
1.75	0.38372		0.82	2.16820	1.51063
1.65	0.40581		0.80	2.56232	1.40961
1.60	0.41699		0.76	2.7099	1.65738
1.55	0.42770		0.74	0.74262	1.80483
1.50	0.43707	0.64227	0.72	1.09896	1.85418
1.45	0.44306	0.64618	0.70	1.47249	1.75234
1.40	0.44017	0.63154	0.66	2.12360	1.16354
1.35	0.40402	0.55618	0.64	2.47732	0.97711
1.32	0.35258	0.37649	0.62	2.89753	0.91336
1.30	0.40455	0.51047	0.60	3.43243	0.91859
1.24	0.58178	0.72678	0.58	4.07243	0.97479
1.20	0.71617	0.93660	0.55	2.07717	1.11838
1.14	0.92611	1.44553	0.50	1.58855	1.12300
1.10	1.05528	2.04579	0.45	1.38284	1.10578
1.08	1.11254	2.49558	0.40	1.26088	1.08433
1.06	1.16380	3.10420	0.35	1.17950	1.06451
1.04	1.20887	3.92334	0.30	1.12168	1.04708
1.02	1.24806	4.88886	0.20	1.04855	1.02059
1.00	1.28158	4.20863	0.15	1.02639	1.01150
0.98	1.44591	5.57677	0.10	1.01146	1.00508

Table 5. $|A_2|$, $|B_2|$, etc. for $c/L_{p1}=0.10$ in the case of incidence of P wave.

L/L_{p1}	$\|A_2\|$	$\|B_2\|$	$\|C_2\|$	$\|D_2\|$
3.00	0.034329	0.020961	0.013187	0.015756
2.95	0.034128	0.021302	0.012989	0.015991
2.90	0.033901	0.021657	0.012802	0.016231
2.85	0.033634	0.022029	0.012643	0.016476
2.80	0.033299	0.022426	0.012548	0.016725
2.75	0.032827	0.022861	0.012610	0.016975
2.70	0.031987	0.023099	0.013225	0.016977
2.65	0.030214	0.024152	0.015567	0.017440
2.60	0.032687	0.024061	0.012709	0.017829
2.55	0·034455	0.024061	0.009962	0.018152
2.50	0.035671	0.024154	0.007340	0.018419
2.45	0.036445	0.024343	0.004848	0.018641
2.40	0.036860	0.024624	0.002485	0.018827
2.35	0.036983	0.024997	0.000244	0.018988
2.30	0.036871	0.025459	0.001886	0.019129
2.25	0.036568	0.026011	0.003926	0.019258
2.20	0.036118	0.026655	0.005902	0.019378
2.15	0.035559	0.027404	0.007861	0.019489
2.10	0.034938	0.028284	0.009891	0.019582
2.05	0.034337	0.029378	0.012218	0.019615
2.00	0.034462	0.031623	0.017514	0.019060
1.99	0.032700	0.031794	0.016733	0.020179
1.97	0.031246	0.032014	0.016200	0.021181
1.95	0.030198	0.032214	0.015854	0.021980
1.90	0.028208	0.032699	0.015272	0.023759
1.85	0.026633	0.033191	0.014902	0.025521
1.80	0.025281	0.033718	0.014711	0.027431

(to be continued)

Table 5. (continued)

L/L_{p1}	$\|A_2\|$	$\|B_2\|$	$\|C_2\|$	$\|D_2\|$
1.75	0.024096	0.034313	0.014750	0.029642
1.65	0.022401	0.036036	0.016126	0.036113
1.60	0.022415	0.037653	0.018344	0.042052
1.58	0.022879	0.038686	0.019972	0.045852
1.56	0.024021	0.040241	0.022625	0.051723
1.54	0.028442	0.043798	0.030074	0.067743
1.53	0.038501	0.059316	0.046294	0.100602
1.51	0.047055	0.074757	0.063112	0.133084
1.50	0.049597	0.079779	0.069462	0.144790
1.49	0.050802	0.082603	0.073986	0.152627
1.47	0.049189	0.081325	0.076727	0.155397
1.45	0.044051	0.073552	0.072803	0.145077
1.40	0.030657	0.050705	0.055976	0.107705
1.35	0.023199	0.036563	0.044614	0.082836
1.333334	0.022607	0.035882	0.046651	0.083223
1.33	0.025194	0.034077	0.049271	0.085968
1.32	0.026654	0.030193	0.049459	0.084414
1.30	0.027307	0.023678	0.047166	0.078058
1.28	0.027437	0.018415	0.044319	0.071289
1.26	0.027531	0.014499	0.041540	0.064891
1.24	0.027697	0.012203	0.039000	0.059038
1.20	0.028368	0.012658	0.034862	0.049067
1.18	0.028948	0.014448	0.033299	0.044921
1.16	0.029985	0.016573	0.032061	0.041344
1.14	0.032648	0.019216	0.031990	0.040219
1.12	0.034154	0.021839	0.032001	0.038579
1.10	0.035695	0.024455	0.032369	0.037358
1.08	0.036505	0.026436	0.032606	0.036207
1.06	0.039137	0.029725	0.034373	0.036595
1.04	0.041169	0.032490	0.036071	0.037165
1.02	0.043672	0.035579	0.038334	0.038486
1.00	0.049192	0.041257	0.042047	0.041438
0.98	0.051371	0.044990	0.044010	0.042227
0.96	0.053131	0.047636	0.046706	0.044316
0.94	0.054894	0.050122	0.049825	0.047012
0.92	0.056646	0.052494	0.053339	0.050233
0.90	0.058354	0.054744	0.057253	0.053953
0.88	0.059967	0.056837	0.061594	0.058176
0.86	0.061420	0.058714	0.066426	0.062951
0.84	0.062609	0.060281	0.071874	0.068390
0.82	0.063364	0.061371	0.078208	0.074753
0.80	0.063331	0.061636	0.086105	0.082712
0.78	0.061367	0.059961	0.098148	0.094864
0.77	0.056954	0.055912	0.113927	0.110809
0.76	0.044418	0.042371	0.138446	0.134647
0.74	0.064207	0.062096	0.140502	0.136401
0.72	0.082516	0.081298	0.112367	0.108664
0.70	0.084281	0.083557	0.094923	0.091578
0.66	0.075114	0.074675	0.095668	0.092781
0.64	0.068544	0.068071	0.102692	0.099974
0.62	0.060521	0.059901	0.111396	0.108826
0.60	0.050240	0.049272	0.121322	0.118886
0.58	0.037502	0.035477	0.132566	0.130257
0.575	0.051392	0.049196	0.138078	0.135816
0.55	0.085516	0.083900	0.158268	0.156155
0.50	0.130900	0.129934	0.199855	0.197988
0.45	0.182550	0.182042	0.257916	0.256257
0.40	0.249145	0.249011	0.332279	0.330768
0.35	0.345645	0.345768	0.448616	0.447209
0.30	0.494434	0.494702	0.635275	0.633909
0.25	0.744708	0.744971	0.963989	0.962578

Table 6. $|B_0|$, $|D_0|$, etc. for $c/L_{s1}=0.03$ in the case of incidence of SV wave.

L/L_{s1}	$\lvert B_0\rvert$	$\lvert D_0\rvert$	$\lvert A_1\rvert$	$\lvert B_1\rvert$	$\lvert C_1\rvert$	$\lvert D_1\rvert$	$\lvert B_1\rvert/\lvert B_0\rvert$	$\lvert D_1\rvert/\lvert D_0\rvert$
5.900	0.192978	1.070298	0.012372	0.035926	0.011780	0.023334	0.186169	0.021801
5.800	0.192981	1.070301	0.012582	0.035863	0.011988	0.023271		
5.700	0.192984	1.070304	0.012799	0.035796	0.012204	0.023204		
5.600	0.192987	1.070307	0.013025	0.035726	0.012428	0.023133		
5.500	0.192991	1.070311	0.013258	0.035651	0.012660	0.023059	0.184731	0.021544
5.400	0.192995	1.070314	0.013499	0.035572	0.012902	0.022980		
5.300	0.192999	1.070318	0.013749	0.035488	0.013153	0.022897		
5.200	0.193003	1.070322	0.014008	0.035400	0.013415	0.022809		
5.100	0.193008	1.070327	0.014277	0.035305	0.013688	0.022715		
5.000	0.193013	1.070331	0.014557	0.035205	0.013972	0.022616	0.182400	0.021130
4.900	0.193018	1.070336	0.014847	0.035098	0.014269	0.022510		
4.800	0.193023	1.070342	0.015148	0.034984	0.014580	0.022398		
4.700	0.193029	1.070347	0.015462	0.034863	0.014905	0.022278		
4.600	0.193036	1.070353	0.015788	0.034732	0.015245	0.022150		
4.500	0.193043	1.070360	0.016128	0.034593	0.015603	0.022013	0.179199	0.020566
4.400	0.193050	1.070367	0.016482	0.034443	0.015979	0.021867		
4.300	0.193058	1.070375	0.016851	0.034281	0.016374	0.021709		
4.200	0.193067	1.070383	0.017235	0.034107	0.016792	0.021540		
4.100	0.193076	1.070392	0.017637	0.033919	0.017233	0.021358		
4.000	0.193086	1.070402	0.018055	0.033716	0.017700	0.021162	0.174616	0.019770
3.900	0.193097	1.070413	0.018492	0.033495	0.018196	0.020950		
3.800	0.193109	1.070425	0.018948	0.033254	0.018725	0.020720		
3.700	0.193123	1.070438	0.019424	0.032991	0.019290	0.020470		
3.600	0.193138	1.070452	0.019920	0.032703	0.019896	0.020199		
3.500	0.193154	1.070468	0.020436	0.032386	0.020549	0.019902	0.167673	0.018592
3.400	0.193172	1.070486	0.020973	0.032037	0.021257	0.019578		
3.300	0.193192	1.070506	0.021527	0.031649	0.022028	0.019222		
3.200	0.193215	1.070529	0.022098	0.031217	0.022877	0.018831		
3.100	0.193241	1.070555	0.022679	0.030733	0.023820	0.018399		
3.000	0.193270	1.070585	0.023262	0.030187	0.024882	0.017920	0.156191	0.016738
2.900	0.193304	1.070621	0.023830	0.029564	0.026099	0.017387		
2.800	0.193345	1.070663	0.024351	0.028846	0.027532	0.016791		
2.700	0.193393	1.070716	0.024768	0.028006	0.029281	0.016123		
2.600	0.193453	1.070783	0.024957	0.026996	0.031544	0.015371		
2.500	0.193532	1.070876	0.024617	0.025727	0.034772	0.014521	0.132937	0.013560
2.400	0.193652	1.071029	0.022739	0.023948	0.040418	0.013569	0.123667	0.012669
2.390	0.193669	1.071051	0.022346	0.023715	0.041286	0.013470		
2.380	0.193687	1.071076	0.021881	0.023463	0.042257	0.013371		

2.370	0.193707	1.071104	0.021324	0.023190	0.043360	0.013273		
2.360	0.193730	1.071135	0.020646	0.022889	0.044634	0.013176		
2.350	0.193756	1.071172	0.019803	0.022551	0.046141	0.013081		0.012212
2.340	0.193788	1.071217	0.018717	0.022158	0.047988	0.012993		
2.330	0.193827	1.071273	0.017237	0.021681	0.050384	0.012915		
2.320	0.193882	1.071355	0.014969	0.021036	0.053868	0.012863		
2.310	0.194007	1.071544	0.009218	0.019670	0.062127	0.012949	0.101388	0.012085
2.300			0.012677	0.019484	0.064287	0.012786		
2.280	0.193965	1.071473	0.019859	0.020039	0.062340	0.012269	0.103314	0.011451
2.260	0.193911	1.071383	0.024936	0.020535	0.060372	0.011776	0.105900	0.010991
2.240	0.193862	1.071299	0.029028	0.020974	0.058387	0.011306	0.108193	0.010554
2.220	0.193817	1.071221	0.032505	0.021358	0.056390	0.010863	0.110199	0.010140
2.200	0.193776	1.071148	0.035546	0.021689	0.054384	0.010446	0.111929	
2.160	0.193710	1.071020	0.040695	0.022197	0.050360	0.009696	0.114589	0.009053
2.140	0.193684	1.070965	0.042918	0.022377	0.048347	0.009363		
2.120	0.193663	1.070916	0.044956	0.022509	0.046336	0.009059	0.116230	0.008459
2.100			0.046254		0.044559	0.008832		
2.080	0.193636	1.070836	0.048580	0.022635	0.042322	0.008534	0.116897	0.007970
2.060	0.193630	1.070805	0.050204	0.022630	0.040321	0.008313		
2.040	0.193629	1.070780	0.051726	0.022580	0.038322	0.008117	0.116619	
2.020	0.193633	1.070761	0.053156	0.022486	0.036325	0.007945		0.007421
2.000	0.193643	1.070748	0.054508	0.022348	0.034327	0.007797	0.115409	0.007283
1.980	0.193658	1.070742	0.055791	0.022164	0.032327	0.007671		
1.960	0.193678	1.070741	0.057014	0.021934	0.030319	0.007565	0.113252	0.007066
1.940	0.193705	1.070747	0.058187	0.021657	0.028299	0.007479		
1.920	0.193738	1.070760	0.059319	0.021331	0.021331	0.007411		
1.900	0.193778	1.070781	0.060417	0.020952	0.024195	0.007362	0.108126	0.006875
1.880	0.193825	1.070809	0.061491	0.020517	0.022090	0.007331		0.006847
1.860	0.193881	1.070846	0.062551	0.020021	0.019931	0.007323	0.103265	0.006839
1.840	0.193947	1.070893	0.063608	0.019454	0.017694	0.007342		0.006856
1.820	0.194025	1.070952	0.064675	0.018806	0.015345	0.007399		0.006909
1.800	0.194119	1.071026	0.065773	0.018057	0.012827	0.007512	0.093021	0.007015
1.780	0.194236	1.071122	0.066934	0.017172	0.010035	0.007723	0.088408	0.007211
1.760	0.194389	1.071252	0.068224	0.016074	0.006728	0.008125	0.082692	0.007585
1.740	0.194632	1.071465	0.069884	0.014483	0.009069	0.009069	0.074414	0.008465
1.738	0.194670	1.071499	0.070111	0.014249	0.001255	0.009253		
1.737	0.194692	1.071519	0.070235	0.014118	0.000851	0.009361		
1.736	0.194715	1.071540	0.070370	0.013976	0.000404	0.009485		
1.735	0.194743	1.071565	0.070520	0.013815	0.000103	0.009630		

(to be continued)

Table 6. (continued)

L/L_{r1}	$\|B_0\|$	$\|D_0\|$	$\|A_1\|$	$\|B_1\|$	$\|C_1\|$	$\|D_1\|$	$\|B_1\|/\|B_0\|$	$\|D_1\|/\|D_0\|$
1.734	0.194775	1.071594	0.070692	0.013627	0.000708	0.009809		
1.733	0.194817	1.071633	0.070911	0.013384	0.001502	0.010055	0.068703	0.009383
1.730	0.195022	1.071621	0.069878	0.012406	0.004193	0.010078	0.063616	0.009405
1.728	0.195081	1.071651	0.069325	0.012223	0.004879	0.009839	0.062660	0.009182
1.726	0.195190	1.071712	0.068922	0.012080	0.005465	0.009655	0.061891	0.009009
1.720	0.194904	1.071769	0.068090	0.011749	0.006869	0.009239	0.060281	0.008621
1.700	0.194905	1.071794	0.066704	0.010991	0.009890	0.008386	0.056392	0.007825
1.680	0.194909	1.071810	0.066096	0.010424	0.011847	0.007908	0.053484	0.007378
1.660	0.194913	1.071821	0.065848	0.009952	0.013291	0.007704	0.051060	0.007188
1.640	0.194917	1.071828	0.065827	0.009548	0.014413	0.007771	0.048987	0.007251
1.620	0.194920	1.071832	0.065969	0.009203	0.015306	0.008115		0.007572
1.600	0.194922	1.071833	0.066240	0.008912	0.016023	0.008737		
1.580	0.194922	1.071830	0.066620	0.008675	0.016595	0.009628	0.044505	0.008984
1.560	0.194921	1.071824	0.067096	0.008487	0.017044	0.010778		
1.540	0.194917	1.071815	0.067661	0.008347	0.017381	0.012180	0.042824	0.011364
1.520	0.194912	1.071802	0.068315	0.008246	0.017616	0.013837		
1.500	0.194903	1.071785	0.069059	0.008174	0.017752	0.015766	0.041942	0.014710
1.480	0.194893	1.071764	0.069902	0.008115	0.017791	0.017997		
1.460	0.194880	1.071739	0.070856	0.008045	0.017727	0.020583	0.041286	0.019205
1.440	0.194864	1.071710	0.071944	0.007929	0.017555	0.023607		
1.420	0.194848	1.071676	0.073201	0.007713	0.017259	0.027205		
1.400	0.194833	1.071638	0.074690	0.007308	0.016816	0.031601	0.037511	0.029489
1.380	0.194825	1.071600	0.076521	0.006560	0.016184	0.037223		
1.360	0.194843	1.071570	0.078923	0.005183	0.015281	0.045042	0.026604	0.042034
1.340	0.194981	1.071603	0.082581	0.003993	0.013968	0.058752	0.020482	0.054826
1.338	0.195022	1.071621	0.083105	0.004460	0.013818	0.061115	0.022872	
1.336	0.195081	1.071651	0.083696	0.005383	0.013683	0.064078	0.027597	
1.334	0.195190	1.071712	0.084422	0.007401	0.013618	0.068612	0.037919	0.064021
1.333	0.195303	1.071767	0.086788	0.011689	0.016672	0.078076	0.059854	0.072848
1.328	0.195150	1.071617	0.094590	0.019818	0.028435	0.098393	0.101554	0.091818
1.323	0.194958	1.071452	0.099984	0.026154	0.037056	0.112738	0.134152	0.105220
1.320	0.194817	1.071336	0.102907	0.029835	0.041987	0.120740	0.153143	
1.300	0.193388	1.070252	0.117567	0.053786	0.073461	0.167806	0.278129	0.156791
1.290			0.120528	0.064354	0.087112	0.185164		
1.280	0.191625	1.069023	0.119865	0.072754	0.097830	0.196230	0.379671	0.183560
1.270			0.116019	0.078521	0.105061	0.200797		
1.260	0.190605	1.068386	0.110058	0.081820	0.109074	0.200057	0.429266	0.187252
1.255	0.190506	1.068338	0.106654	0.082713	0.110105	0.198255		

1.250	0.190457	1.068324	0.103134	0.083210	0.110633	0.195797	0.436899	0.183275
1.245	0.190450	1.068338	0.099595	0.083390	0.110760	0.192875		
1.240	0.190474	1.068373	0.096109	0.083320	0.110578	0.189651	0.437436	0.177514
1.220	0.190762	1.068629	0.083492	0.081598	0.108120	0.175873	0.427752	
1.200	0.191156	1.068940	0.073535	0.079048	0.104798	0.163262	0.413529	0.152733
1.180	0.191541	1.069235	0.065951	0.076555	0.101722	0.152844		
1.160	0.191891	1.069499	0.060168	0.074368	0.099182	0.144500	0.387553	0.135110
1.140	0.192207	1.069738	0.055699	0.072519	0.097192	0.137864		
1.120	0.192498	1.069958	0.052184	0.070973	0.095692	0.132582		
1.100	0.192774	1.070170	0.049367	0.069682	0.094608	0.128366	0.361474	0.119950
1.080	0.193045	1.070382	0.047068	0.068601	0.093873	0.124994		
1.060	0.193327	1.070606	0.045158	0.067690	0.093429	0.122293		
1.040	0.193637	1.070859	0.043539	0.066920	0.093228	0.120123		
1.020	0.194021	1.071179	0.042128	0.066265	0.093218	0.118352		
1.000	0.194902	1.071944	0.040683	0.065704	0.093151	0.116445	0.337118	0.108630
0.990	0.194873	1.071951	0.048148	0.066788	0.090619	0.109152	0.342726	0.101826
0.980			0.051303	0.067334	0.089958	0.106491		
0.970			0.053751	0.067861	0.089662	0.104704		
0.960			0.055839	0.068406	0.089579	0.103407		
0.950			0.057703	0.068978	0.089645	0.102443		
0.940			0.059414	0.069580	0.089826	0.101730		
0.930			0.061016	0.070212	0.090100	0.101215		
0.920			0.062536	0.070876	0.090455	0.100865		
0.910			0.063993	0.071570	0.090881	0.100655		
0.900	0.194854	1.071958	0.065403	0.072295	0.091369	0.100565	0.371021	0.093815
0.850	0.194854	1.071959	0.072061	0.076365	0.094604	0.101546		0.094730
0.800	0.194857	1.071960	0.078565	0.081183	0.098965	0.104343	0.416630	0.097339
0.750	0.194860	1.071962	0.085327	0.086797	0.104375	0.108606		
0.700	0.194865	1.071963	0.092638	0.093310	0.110887	0.114248	0.478847	0.106578
0.650	0.194871	1.071964	0.100767	0.100883	0.118644	0.121331		
0.600	0.194879	1.071966	0.110014	0.109748	0.127884	0.130041	0.563160	0.121311
0.550	0.194890	1.071967	0.120750	0.120230	0.138958	0.140691		
0.500	0.194904	1.071970	0.133473	0.132797	0.152374	0.153767	0.681345	0.143444
0.450	0.194923	1.071973	0.148886	0.148126	0.168881	0.169998		
0.400	0.194951	1.071977	0.168026	0.167241	0.189611	0.190503	0.857863	0.177712
0.350	0.194991	1.071983	0.192521	0.191755	0.216352	0.217059		
0.300	0.195054	1.071992	0.225068	0.224357	0.252090	0.252645		
0.250	0.195157	1.072007	0.270521	0.269891	0.302206	0.302634	1.382941	0.282306
0.150	0.195761	1.072091	0.451863	0.451455	0.503011	0.503239		
0.100	0.196934	1.072261	0.678233	0.677955	0.754226	0.754372	3.442552	0.703534

Table 7. $|B_0|$, $|D_0|$, etc. for $c/L_{s1}=0.05$ in the case of incidence of SV wave.

L/L_{s1}	$\lvert B_0\rvert$	$\lvert D_0\rvert$	$\lvert A_1\rvert$	$\lvert B_1\rvert$	$\lvert C_1\rvert$	$\lvert D_1\rvert$	$\lvert B_1\rvert/\lvert B_0\rvert$	$\lvert D_1\rvert/\lvert D_0\rvert$
5.900	0.180494	1.076755	0.020620	0.059877	0.019633	0.038890	0.331742	0.036118
5.800	0.180503	1.076763	0.020970	0.059772	0.019980	0.038785		
5.700	0.180512	1.076772	0.021333	0.059661	0.020340	0.038674		
5.600	0.180521	1.076781	0.021708	0.059543	0.020713	0.038556		
5.500			0.022096	0.059419	0.021101	0.038432		
5.400	0.180542	1.076800	0.022499	0.059287	0.021504	0.038301		0.035569
5.300	0.180553	1.076811	0.022915	0.059148	0.021923	0.038162		
5.200	0.180565	1.076822	0.023348	0.059000	0.022359	0.038015		
5.100	0.180578	1.076834	0.023796	0.058843	0.022813	0.037859		
5.000	0.180591	1.076847	0.024261	0.058676	0.023288	0.037694	0.324910	0.035004
4.900	0.180606	1.076861	0.024745	0.058498	0.023783	0.037518		
4.800	0.180621	1.076876	0.025247	0.058308	0.024300	0.037330		
4.700	0.180638	1.076891	0.025770	0.058105	0.024842	0.037131		
4.600	0.180656	1.076908	0.026314	0.057888	0.025409	0.036917		
4.500	0.180675	1.076927	0.026880	0.057655	0.026005	0.036689	0.319110	0.034068
4.400			0.027470	0.057405	0.026631	0.036445		
4.300	0.180718	1.076968	0.028085	0.057136	0.027291	0.036182		
4.200	0.180742	1.076991	0.028726	0.056846	0.027986	0.035901		
4.100	0.180768	1.077016	0.029395	0.056533	0.028722	0.035597		
4.000	0.180796	1.077043	0.030093	0.056193	0.029500	0.035270	0.310811	0.032747
3.900	0.180827	1.077073	0.030821	0.055825	0.030328	0.034916		
3.800	0.180860	1.077106	0.031581	0.055423	0.031209	0.034533		
3.700	0.180898	1.077142	0.032374	0.054985	0.032150	0.034117		
3.600	0.180893	1.077182	0.033200	0.054505	0.033160	0.033665		
3.500	0.180984	1.077227	0.034061	0.053977	0.034249	0.033171	0.298246	0.030793
3.400	0.181034	1.077276	0.034955	0.053395	0.035428	0.032631		
3.300	0.181090	1.077332	0.035879	0.052749	0.036714	0.032038		
3.200	0.181153	1.077396	0.036830	0.052029	0.038129	0.031385		
3.100	0.181225	1.077468	0.037799	0.051223	0.039700	0.030665		
3.000	0.181307	1.077552	0.038771	0.050312	0.041470	0.029866	0.277496	0.027717
2.900	0.181402	1.077651	0.039716	0.049274	0.043499	0.028978		
2.800	0.181513	1.077769	0.040585	0.048078	0.045887	0.027986		
2.700	0.181648	1.077915	0.041280	0.046677	0.048803	0.026872		
2.600	0.181814	1.078102	0.041595	0.044994	0.052574	0.025618		
2.500	0.182034	1.078361	0.041028	0.042879	0.057954	0.024202	0.235556	0.022443
2.400	0.182368	1.078785	0.037898	0.039914	0.067363	0.022616	0.218866	0.020964
2.390	0.182414	1.078847	0.037243	0.039525	0.068810	0.022451	0.216677	0.020810
2.380	0.182465	1.078915	0.034684	0.039106	0.070429	0.022286		

2.370	0.182521	1.078992	0.035540	0.038651	0.072267	0.022121	0.211764	0.020502
2.360	0.182585	1.079080	0.034411	0.038149	0.074390	0.021960		
2.350	0.182658	1.079182	0.033006	0.037585	0.076902	0.021803	0.205770	0.020203
2.340	0.182744	1.079306	0.031196	0.036931	0.079980	0.021655		
2.330	0.182853	1.079464	0.028729	0.036135	0.083973	0.021525	0.197619	0.019940
2.320	0.183006	1.079690	0.024948	0.035060	0.089781	0.021439		
2.310	0.183353	1.080214	0.015364	0.032783	0.103546	0.021583	0.178799	0.019980
2.300	0.183400	1.080279	0.021129	0.032473	0.107145	0.021310	0.177064	0.019726
2.280	0.183239	1.080018	0.033099	0.033398	0.103901	0.020449	0.182268	
2.260	0.183090	1.079770	0.041561	0.034225	0.100612	0.019626	0.186933	
2.240	0.182953	1.079537	0.048381	0.034957	0.097312	0.018844	0.191073	
2.220	0.182828	1.079319	0.054176	0.035597	0.093983	0.018105	0.194704	
2.200	0.182716	1.079117	0.059244	0.036148	0.090641	0.017410	0.197840	0.016134
2.160	0.182532	1.078762	0.067826	0.036995	0.083934	0.016160	0.202677	
2.140	0.182460	1.078609	0.071530	0.037295	0.080579	0.015605	0.204401	0.014468
2.120	0.182401	1.078473	0.074927	0.037515	0.077226	0.015099		
2.080	0.182326	1.078250	0.080966	0.037725	0.070537	0.014224	0.206913	0.013192
2.060	0.182309	1.078164	0.083674	0.037717	0.067201	0.013854	0.206882	
2.040	0.182306	1.078094	0.086210	0.037634	0.063870	0.013528		0.012548
2.020	0.182317	1.078042	0.088594	0.037478	0.060542	0.013243		
2.000	0.182343	1.078006	0.090847	0.037246	0.057213	0.012996	0.204266	0.012056
1.980	0.182385	1.077987	0.092985	0.036940	0.053878	0.012786		
1.960	0.182442	1.077986	0.095024	0.036557	0.050532	0.012609	0.200379	0.011697
1.940	0.182515	1.078003	0.096979	0.036095	0.047165	0.012465		
1.920	0.182606	1.078039	0.098865	0.035551	0.043768	0.012352		
1.900	0.182717	1.078096	0.100695	0.034920	0.040325	0.012270	0.191118	0.011381
1.880	0.182848	1.078174	0.102486	0.034196	0.036817	0.012219		0.011333
1.860	0.183003	1.078276	0.104252	0.033368	0.033218	0.012205		0.011319
1.840	0.183187	1.078407	0.106013	0.032424	0.029490	0.012237	0.177002	0.011347
1.820	0.183404	1.078571	0.107792	0.031344	0.025575	0.012331	0.170901	0.011433
1.800	0.183666	1.078777	0.109622	0.030095	0.021378	0.012521	0.163858	0.011606
1.780	0.183990	1.079042	0.111556	0.028620	0.016725	0.012872	0.155551	
1.760	0.184415	1.079403	0.113708	0.026790	0.011214	0.013541	0.145273	
1.740	0.185089	1.079996	0.116474	0.024139	0.003290	0.015116	0.130417	0.013996
1.738	0.185195	1.080091	0.116852	0.023748	0.002092	0.015422		
1.737	0.185255	1.080145	0.117059	0.023531	0.001418	0.015603		
1.736	0.185321	1.080205	0.117284	0.023293	0.000673	0.015808		
1.735	0.185397	1.080273	0.117533	0.023025	0.000173	0.016050		
1.734	0.185487	1.080355	0.117821	0.022711	0.001180	0.016349		

(to be continued)

Table 7. (continued)

L/L_{s1}	$\|B_0\|$	$\|D_0\|$	$\|A_1\|$	$\|B_1\|$	$\|C_1\|$	$\|D_1\|$	$\|B_1\|/\|B_0\|$	$\|D_1\|/\|D_0\|$
1.733	0.185605	1.080463	0.118185	0.022307	0.002503	0.016759		0.015510
1.730	0.185859	1.080767	0.116464	0.020677	0.006989	0.016797	0.111253	0.015542
1.728	0.185853	1.080788	0.115541	0.020372	0.008132	0.016399	0.109617	0.015173
1.726	0.185850	1.080804	0.114870	0.020134	0.009108	0.016091	0.108335	
1.720	0.185847	1.080840	0.113483	0.019581	0.011448	0.015399	0.105364	0.014247
1.700	0.185852	1.080912	0.111173	0.018318	0.016484	0.013977	0.098564	0.012931
1.680	0.185865	1.080957	0.110160	0.017374	0.019745	0.013180	0.093477	0.012192
1.660	0.185880	1.080987	0.109748	0.016587	0.022152	0.012840	0.089237	0.011878
1.640	0.185894	1.081007	0.109712	0.015914	0.024022	0.012952	0.085608	0.011981
1.620	0.185906	1.081018	0.109949	0.015338	0.025511	0.013526	0.082508	0.012512
1.600	0.185914	1.081020	0.110401	0.014854	0.026706	0.014563	0.079901	0.013471
1.580	0.185919	1.081013	0.111034	0.014458	0.027659	0.016048	0.077768	
1.560	0.185919	1.080997	0.111826	0.014146	0.028406	0.017964	0.076089	0.016618
1.540	0.185914	1.080971	0.112769	0.013912	0.028969	0.020300		
1.520	0.185903	1.080936	0.113859	0.013743	0.029361	0.023062	0.073930	
1.500	0.185886	1.080890	0.115099	0.013624	0.029588	0.026276	0.073293	0.024310
1.480	0.185863	1.080833	0.116504	0.013526	0.029651	0.029995		
1.460	0.185834	1.080764	0.118093	0.013409	0.029546	0.034305	0.072159	
1.440	0.185800	1.080684	0.119906	0.013216	0.029258	0.039346		
1.420	0.185765	1.080591	0.122002	0.012855	0.028765	0.045341	0.069201	
1.400	0.185735	1.080480	0.124484	0.012180	0.028027	0.052669		0.048745
1.380	0.185729	1.080385	0.127535	0.010933	0.026973	0.062039	0.058867	
1.360	0.185799	1.080304	0.131540	0.008639	0.025469	0.075071		
1.340	0.186213	1.080397	0.137635	0.006656	0.023280	0.097920	0.035745	0.090633
1.338	0.186330	1.080449	0.138508	0.007343	0.023031	0.101858	0.039898	
1.336	0.186499	1.080533	0.139494	0.008972	0.022805	0.106797	0.048111	
1.334	0.186810	1.080702	0.140703	0.012335	0.022697	0.114354	0.066033	0.105814
1.333	0.187138	1.080857	0.144646	0.019483	0.027787	0.130126	0.104110	
1.328	0.186763	1.080445	0.157650	0.033030	0.047393	0.163988	0.176857	0.151779
1.323	0.186269	1.079989	0.166640	0.043590	0.061760	0.187897	0.234015	
1.320	0.185903	1.079671	0.171512	0.049725	0.069979	0.201235	0.267479	0.186385
1.300	0.182054	1.076667	0.195946	0.089644	0.122436	0.279677	0.492406	0.259762
1.290			0.200881	0.107257	0.145187	0.308607		
1.280		1.073246	0.199775	0.121257	0.163051	0.327050		0.304730
1.270			0.193365	0.130868	0.175103	0.334663		
1.260	0.174120	1.071461	0.183431	0.136367	0.181790	0.333428	0.783179	0.311190
1.255	0.173809	1.071325	0.177757	0.137856	0.183509	0.330425		
1.250	0.173642	1.071284	0.171890	0.138684	0.184389	0.326329	0.798682	0.304615

1.245	0.173593	1.071320	0.165992	0.138983	0.184600	0.321459		
1.240	0.173639	1.071415	0.160183	0.138867	0.184297	0.316085	0.799750	0.295017
1.220	0.174383	1.072122	0.139154	0.135998	0.180200	0.293122	0.779882	0.273404
1.200	0.175452	1.072984	0.122559	0.131747	0.174664	0.272104	0.750900	
1.180	0.176514	1.073802	0.109918	0.127591	0.169537	0.254740		
1.160	0.177482	1.074536	0.100281	0.123947	0.165303	0.240833	0.698362	0.224128
1.140	0.178358	1.075198	0.092832	0.120865	0.161987	0.229774		
1.120	0.179163	1.075810	0.086973	0.118289	0.159487	0.220970	0.660231	
1.100	0.179928	1.076398	0.082279	0.116137	0.157681	0.213944		0.198760
1.080	0.180683	1.076987	0.078447	0.114335	0.156455	0.208324	0.632795	
1.060	0.181464	1.077609	0.075264	0.112818	0.155715	0.203822		
1.040	0.182327	1.078312	0.072565	0.111534	0.155380	0.200206	0.611725	0.185666
1.020	0.183392	1.079201	0.070213	0.110442	0.155363	0.197253		
1.000	0.185840	1.081327	0.067805	0.109508	0.155252	0.194076	0.589260	0.179480
0.990	0.185765	1.081347	0.080246	0.111313	0.151033	0.181920	0.599216	0.168235
0.980			0.085506	0.112223	0.149930	0.177486		
0.970			0.089586	0.113103	0.149438	0.174508		
0.960			0.093065	0.114011	0.149299	0.172345		
0.950			0.096172	0.114964	0.149408	0.170739		
0.940			0.099024	0.115966	0.149710	0.169550		
0.930			0.101694	0.117021	0.150168	0.168693		
0.920			0.104226	0.118127	0.150759	0.168109		
0.910			0.106655	0.119284	0.151468	0.167758		
0.900	0.185748	1.081372	0.109005	0.120491	0.152282	0.167609	0.648683	0.154997
0.850			0.120102	0.127275	0.157673	0.169244		
0.800			0.130942	0.135305	0.164943	0.173905		
0.750			0.142212	0.144662	0.173959	0.181010		
0.700			0.154397	0.155517	0.184811	0.190413		
0.650			0.167945	0.168139	0.197740	0.202219		
0.600	0.186007	1.081418	0.183356	0.182913	0.213141	0.216735	0.983369	0.200417
0.550			0.201250	0.200384	0.231597	0.234486		
0.500			0.222456	0.221328	0.253957	0.256278		
0.450			0.248143	0.246877	0.281469	0.283330		
0.400			0.280044	0.278735	0.316019	0.317505		
0.350			0.320868	0.319592	0.360587	0.361765		
0.300	0.187431	1.081615	0.375114	0.373928	0.420151	0.420175	1.995015	0.389302
0.250			0.450868	0.449819	0.503677	0.504390		
0.150			0.753105	0.752425	0.838352	0.838731		
0.100	0.201984	1.083737	1.130388	1.129924	1.257043	1.257286		

Table 8. $|B_0|$, $|D_0|$, etc. for $c/L_{s1}=0.10$ in the case of incidence of SV wave.

L/L_{s1}	$\lvert B_0\rvert$	$\lvert D_0\rvert$	$\lvert A_1\rvert$	$\lvert B_1\rvert$	$\lvert C_1\rvert$	$\lvert D_1\rvert$	$\lvert B_1\rvert/\lvert B_0\rvert$	$\lvert D_1\rvert/\lvert D_0\rvert$
5.900	0.121979	1.107022	0.041240	0.119755	0.039267	0.077781	0.981772	0.070262
5.800	0.122013	1.107054	0.041941	0.119544	0.039961	0.077570		
5.700	0.122049	1.107088	0.042666	0.119322	0.040681	0.077348		
5.600	0.122086	1.107124	0.043416	0.119087	0.041427	0.077113		
5.500	0.122127	1.107162	0.044193	0.118838	0.042203	0.076865	0.973073	0.069425
5.400	0.122169	1.107202	0.044998	0.118575	0.043008	0.076603		
5.300	0.122214	1.107245	0.045831	0.118296	0.043846	0.076325		
5.200	0.122262	1.107290	0.046696	0.118000	0.044719	0.076031		
5.100	0.122313	1.107339	0.047592	0.117686	0.045627	0.075719		
5.000	0.122367	1.107390	0.048523	0.117352	0.046576	0.075388	0.959020	0.068077
4.900	0.122425	1.107445	0.049490	0.116996	0.047566	0.075036		
4.800	0.122487	1.107504	0.050495	0.116616	0.048601	0.074661		
4.700	0.122553	1.107568	0.051541	0.116210	0.049684	0.074262		
4.600	0.122624	1.107636	0.052628	0.115776	0.050819	0.073835		
4.500	0.122701	1.107709	0.053761	0.115310	0.052011	0.073378	0.939770	0.066243
4.400	0.122783	1.107787	0.054940	0.114810	0.053263	0.072890		
4.300	0.122872	1.107873	0.056170	0.114272	0.054582	0.072365		
4.200	0.122968	1.107965	0.057452	0.113692	0.055973	0.071802		
4.100	0.123072	1.108065	0.058790	0.113066	0.057444	0.071195		
4.000	0.123185	1.108175	0.060186	0.112387	0.059001	0.070541	0.912344	0.063655
3.900	0.123309	1.108294	0.061642	0.111650	0.060656	0.069833		
3.800	0.123444	1.108426	0.063162	0.110847	0.062418	0.069067		
3.700	0.123592	1.108570	0.064748	0.109971	0.064301	0.068235		
3.600	0.123755	1.108730	0.066401	0.109011	0.066321	0.067330		
3.500	0.123936	1.108908	0.068122	0.107955	0.068498	0.066342	0.871062	0.059827
3.400	0.124137	1.109107	0.069910	0.106790	0.070857	0.065262		
3.300	0.124361	1.109331	0.071759	0.105499	0.073429	0.064076		
3.200	0.124614	1.109584	0.073661	0.104059	0.076253	0.062771		
3.100	0.124901	1.109875	0.075599	0.102446	0.079400	0.061330		
3.000	0.125229	1.110211	0.077542	0.100624	0.082940	0.059733	0.803521	0.053803
2.900	0.125609	1.110606	0.079433	0.098542	0.086999	0.057856		
2.800	0.126056	1.111080	0.081171	0.096156	0.091775	0.055972		
2.700	0.126592	1.111662	0.082561	0.093354	0.097606	0.053745		
2.600	0.127259	1.112411	0.083191	0.089989	0.105148	0.051237		
2.500	0.128139	1.113447	0.082056	0.085759	0.115909	0.048404	0.669265	0.043473
2.400	0.129473	1.115141	0.075797	0.079828	0.134727	0.045232	0.616565	0.040562
2.390	0.129659	1.115389	0.074487	0.079050	0.137620	0.044902	0.609679	0.040257
2.380	0.129863	1.115663	0.072936	0.078213	0.140859	0.044572		

2.370	0.130088	1.115971	0.071080	0.077303	0.144535	0.044243	0.594237	0.039645
2.360	0.130341	1.116322	0.068822	0.076299	0.148781	0.043920		
2.350	0.130632	1.116731	0.066012	0.075170	0.153805	0.043606	0.575440	0.039048
2.340	0.130978	1.117226	0.062393	0.073863	0.159960	0.043310		
2.330	0.131414	1.117857	0.057458	0.072270	0.167946	0.043050	0.549945	0.038512
2.320	0.132025	1.118761	0.049897	0.070121	0.179563	0.042878		
2.310	0.133414	1.120860	0.030729	0.065567	0.207092	0.043166	0.491456	0.038511
2.300	0.133614	1.121122	0.042259	0.064947	0.214291	0.042621	0.486081	0.038016
2.280	0.132994	1.120081	0.066198	0.066797	0.207803	0.040899	0.502261	
2.260	0.132415	1.119094	0.083122	0.068451	0.201242	0.039253	0.516945	
2.240	0.131879	1.118165	0.096763	0.069915	0.194625	0.037689	0.530142	
2.220	0.131390	1.117296	0.108352	0.071195	0.187967	0.036210	0.541859	
2.200	0.130948	1.116491	0.118488	0.072297	0.181281	0.034821	0.552105	0.031188
2.160	0.130214	1.115071	0.135653	0.073990	0.167868	0.032320	0.568219	
2.140	0.129924	1.114460	0.143061	0.074590	0.161158	0.031211	0.574106	0.028005
2.100	0.130266	1.114243	0.154182		0.148531	0.029441		
2.080	0.129373	1.113023	0.161933	0.075451	0.141075	0.028448	0.583205	0.025559
2.060	0.129299	1.112677	0.167349	0.075435	0.134403	0.027709	0.583412	0.024903
2.040	0.129281	1.112397	0.172420	0.075269	0.127741	0.027057		
2.020	0.129320	1.112184	0.177189	0.074955	0.121084	0.026486		
2.000	0.129418	1.112040	0.181694	0.074493	0.114426	0.025992	0.575602	0.023374
1.980	0.129577	1.111963	0.185969	0.073881	0.107757	0.025572		
1.960	0.129798	1.111958	0.190048	0.073115	0.101064	0.025219	0.563296	0.022680
1.940	0.130087	1.112024	0.193958	0.072191	0.094331	0.024931		
1.920	0.130446	1.112167	0.197730	0.071103	0.087536	0.024705		
1.900	0.130883	1.112391	0.201391	0.069841	0.080650	0.024540	0.533616	0.022060
1.880	0.131405	1.112702	0.204972	0.068392	0.073634	0.024439		0.021964
1.860	0.132023	1.113111	0.208505	0.066736	0.066436	0.024411		0.021931
1.840	0.132754	1.113631	0.212027	0.064849	0.058980	0.024474	0.488490	0.021977
1.820	0.133622	1.114286	0.215585	0.062688	0.051150	0.024663	0.469145	0.022133
1.800	0.134667	1.115111	0.219244	0.060190	0.042757	0.025042	0.446957	0.022457
1.780	0.135961	1.116171	0.223113	0.057240	0.033451	0.025745	0.420811	0.023065
1.760	0.137663	1.117614	0.227415	0.053581	0.022429	0.027083	0.389222	0.024233
1.740	0.140358	1.119984	0.232948	0.048278	0.006581	0.030232	0.343961	0.026993
1.738	0.140782	1.120365	0.233704	0.047497	0.004185	0.030845		
1.737	0.141022	1.120581	0.234118	0.047063	0.002837	0.031206		
1.736	0.141287	1.120821	0.234568	0.046586	0.001347	0.031617	0.329731	0.028209
1.735	0.141589	1.121095	0.235066	0.046051	0.000346	0.032101		
1.734	0.141948	1.121422	0.235642	0.045423	0.002360	0.032699		

(to be continued)

Table 8. (continued)

L/L_{s1}	$\|B_0\|$	$\|D_0\|$	$\|A_1\|$	$\|B_1\|$	$\|C_1\|$	$\|D_1\|$	$\|B_1\|/\|B_0\|$	$\|D_1\|/\|D_0\|$
1.733	0.142420	1.121855	0.236370	0.044615	0.005007	0.033518	0.313263	0.029877
1.730	0.143441	1.123071	0.232928	0.041355	0.013978	0.033595	0.288306	0.029914
1.728	0.143428	1.123155	0.231083	0.040745	0.016264	0.032798	0.284083	0.029201
1.726	0.143423	1.123220	0.229740	0.040268	0.018217	0.032183	0.280768	0.028652
1.720	0.143427	1.123363	0.226967	0.039163	0.022897	0.030798	0.273053	0.027416
1.700	0.143512	1.123657	0.222346	0.036637	0.032968	0.027955	0.255288	0.024879
1.680	0.143630	1.123844	0.220321	0.034748	0.039491	0.026359	0.241930	0.023455
1.660	0.143759	1.123974	0.219496	0.033174	0.044304	0.025681	0.230765	0.022848
1.640	0.143889	1.124061	0.219425	0.031828	0.048045	0.025904	0.221199	0.023045
1.620	0.144018	1.124111	0.219899	0.030677	0.051022	0.027052	0.213012	0.024065
1.600	0.144141	1.124127	0.220803	0.029709	0.053412	0.029126	0.206115	0.025909
1.580	0.144257	1.124108	0.222068	0.028917	0.055319	0.032096	0.200457	0.028552
1.560	0.144364	1.124054	0.223653	0.028293	0.056813	0.035928	0.195984	0.031963
1.540	0.144619	1.123963	0.225539	0.027824	0.057939	0.040601		
1.520	0.144549	1.123833	0.227718	0.027487	0.058722	0.046125	0.190161	0.041043
1.500	0.144629	1.123663	0.230199	0.027248	0.059176	0.052553	0.188403	0.046769
1.480	0.144702	1.123449	0.233008	0.027052	0.059303	0.059990		
1.460	0.144774	1.123191	0.236187	0.026819	0.059092	0.068610	0.185250	0.061085
1.440	0.144856	1.122886	0.239813	0.026432	0.058517	0.078692		
1.420	0.144969	1.122538	0.244005	0.025710	0.057531	0.090683	0.177351	0.080784
1.400	0.145153	1.122155	0.248968	0.024361	0.056055	0.105338		
1.380	0.145504	1.121768	0.255070	0.021867	0.053946	0.124078	0.150283	0.110609
1.360	0.146281	1.121486	0.263079	0.017278	0.050939	0.150143		
1.340	0.148676	1.121922	0.275271	0.013312	0.046561	0.195840	0.089539	0.174558
1.338	0.149244	1.122141	0.277017	0.014868	0.046063	0.203717	0.099625	0.181543
1.336	0.150028	1.122484	0.278989	0.017945	0.045610	0.213594	0.119612	0.190287
1.334	0.151394	1.123176	0.281407	0.024671	0.045395	0.228708	0.162962	0.203626
1.333	0.153010	1.123822	0.289293	0.038965	0.055574	0.260253	0.254661	0.231578
1.328	0.152762	1.122256	0.315299	0.066061	0.094786	0.327977	0.432443	0.292248
1.323	0.151840	1.120496	0.333280	0.087180	0.123521	0.375794	0.574157	0.335382
1.320	0.151009	1.119257	0.343024	0.099450	0.139958	0.402470	0.658570	0.359586
1.300	0.139606	1.107400	0.391893	0.179289	0.244873	0.559355	1.284253	0.505106
1.290			0.401762	0.214515	0.290374	0.617215		
1.280	0.120304	1.093593	0.399550	0.242515	0.326102	0.654100	2.015843	0.598120
1.270			0.386730	0.261737	0.350206	0.669326		
1.260	0.104186	1.086184	0.366862	0.272735	0.363581	0.666857	2.617759	0.613944
1.255	0.101758	1.085582	0.355514	0.275712	0.367019	0.660851		
1.250	0.199998	1.085369	0.343781	0.277369	0.368778	0.652658	2.773752	0.601323

1.245	0.098840	1.085470	0.331985	0.277966	0.369201	0.642919		
1.240	0.098200	1.085815	0.320366	0.277735	0.368595	0.632170	2.828261	0.582208
1.220	0.099094	1.088559	0.278308	0.271996	0.360401	0.586245	2.744808	0.538551
1.200	0.102500	1.091974	0.245118	0.263495	0.349329	0.544209	2.570688	0.498371
1.180	0.106378	1.095229	0.219837	0.255183	0.339075	0.509481		
1.160	0.110090	1.098158	0.200562	0.247894	0.330606	0.481667	2.251730	0.438613
1.140	0.113515	1.100800	0.185665	0.241731	0.323975	0.459548		
1.120	0.116700	1.103247	0.173947	0.236578	0.318975	0.441941	2.027238	0.400582
1.100	0.119738	1.105596	0.164558	0.232275	0.315362	0.427889		0.387021
1.080	0.128442	1.113036	0.153666	0.228603				
1.060		1.110440	0.150528	0.225636	0.311431	0.407644		0.367101
1.040	0.129311	1.113248	0.145130	0.223068	0.310761	0.400412	1.725049	0.359679
1.020	0.133570	1.116804	0.140427	0.220884	0.310727	0.394507		
1.000	0.143361	1.125311	0.135610	0.219016	0.310505	0.388153	1.527718	0.344929
0.990	0.143187	1.125402	0.160493	0.222627	0.302066	0.363841	1.554801	0.323299
0.980			0.171013	0.224447	0.299860	0.354972		
0.970			0.179171	0.226206	0.298876	0.349016		
0.960			0.186130	0.228022	0.298598	0.344691		
0.950	0.143513	1.125510	0.192344	0.229928	0.298817	0.341479	1.602133	0.303399
0.940			0.198049	0.231933	0.299420	0.339101		
0.930			0.203388	0.234042	0.300336	0.337386		
0.920			0.208453	0.236254	0.301519	0.336218		
0.910			0.213311	0.238568	0.302936	0.335516		
0.900	0.144080	1.125594	0.218011	0.240983	0.304564	0.335219	1.672564	0.297815
0.850			0.240205	0.254551	0.315347	0.338489		
0.800			0.261884	0.270611	0.329886	0.347810		
0.750			0.284425	0.289324	0.347919	0.362020		
0.700			0.308795	0.311035	0.369623	0.380826		
0.650			0.335891	0.336279	0.395481	0.404438		
0.600	0.149823	1.126229	0.366713	0.365827	0.426283	0.433470	2.441732	0.384886
0.550			0.402501	0.400769	0.463195	0.468972		
0.500			0.444912	0.442656	0.507915	0.512557		
0.450			0.496286	0.493754	0.562938	0.566660		
0.400	0.161316	1.127494	0.560089	0.557471	0.632038	0.635010	3.455760	0.563204
0.350			0.641737	0.639184	0.721174	0.723531		
0.300	0.176071	1.129238	0.750228	0.747856	0.840303	0.842151	4.247456	0.745768

Table 9. $|A_1|/|B_1|$ and $|C_1|/|D_1|$ in the case of incidence of SV wave.

L/L_{s1}	$\lvert A_1\rvert/\lvert B_1\rvert$	$\lvert C_1\rvert/\lvert D_1\rvert$	L/L_{s1}	$\lvert A_1\rvert/\lvert B_1\rvert$	$\lvert C_1\rvert/\lvert D_1\rvert$
5.800	0.350843	0.515165	1.720	5.79540	0.743466
5.500	0.371879	0.549052	1.700	6.06891	1.179315
5.000	0.413488	0.617812	1.660		1.725168
4.500	0.466229	0.708805	1.640	6.89401	1.854708
4.000	0.535524	0.836417	1.620		1.886049
3.500	0.631025	1.032497	1.600	7.43202	1.833826
3.000	0.770611	1.388504	1.500	8.44813	1.126023
2.700	0.884386	1.816076	1.460		0.861276
2.600	0.924458	2.052164	1.420	9.49052	0.634423
2.500	0.956831		1.400	10.21977	
2.400	0.949505	2.978540	1.380	11.66464	0.43478
2.390	0.942277		1.360	15.22550	
2.360	0.902000	3.387550	1.340	20.67780	0.237752
2.340	0.844715		1.338	18.63113	
2.330	0.795049	3.901118	1.336	15.54668	
2.320	0.711589		1.334	11.40614	0.198485
2.310	0.468679	4.797511	1.333	7.42428	0.213541
2.300	0.650675		1.328	4.77286	0.289002
2.280	0.991028		1.323	3.82289	0.328694
2.260	1.214331	5.126709	1.320	3.44921	
2.220		5.190949	1.300	2.18581	
2.200	1.638900	5.206064	1.280	1.64753	0.498551
2.160		5.193938	1.260	1.34512	
2.140	1.917959	5.163477	1.200	0.930258	0.641902
2.060	2.218462	4.850389	1.100	0.708461	0.737017
2.000	2.439059	4.402231	1.060	0.667126	
1.980	2.517155	4.213867	1.020	0.635750	
1.940	2.686715		1.000	0.619181	0.789956
1.900	2.88356	3.286443	0.990	0.720907	0.830210
1.860	3.12429	2.721508	0.950	0.836541	0.875068
1.820	3.43900		0.900	0.904672	0.908552
1.800	3.64251	1.707372	0.800	0.967754	0.948466
1.760	4.24430	0.828148	0.700	0.992799	0.970582
1.740	4.82515	0.217690	0.600	1.002421	0.983418
1.738		0.135698	0.550	1.004321	
1.737		0.090940	0.500	1.005096	
1.736		0.042609	0.450	1.005129	
1.735	5.10443	0.010788	0.400	1.004695	
1.734		0.072195	0.300	1.003171	0.997806
1.733	5.29799	0.149409	0.200	1.000904	
1.730	5.63241	0.416077	0.100	1.000410	0.999807

Table 10. $|A_2|$, $|B_2|$, etc. for $c/L_{s1}=0.10$ in the case of incidence of SV wave.

L/L_{s1}	$\|A_2\|$	$\|B_2\|$	$\|C_2\|$	$\|D_2\|$
5.800	0.019907	0.035127	0.017583	0.014945
5.700	0.020410	0.035000	0.018045	0.014765
5.600	0.020955	0.034871	0.018644	0.014573
5.500	0.021552	0.034740	0.019313	0.014370
5.400	0.022213	0.034611	0.020068	0.014152
5.300	0.022955	0.034485	0.020933	0.013921
5.200	0.023801	0.034368	0.021943	0.013673
5.100	0.024792	0.034266	0.023154	0.013408
5.000	0.025988	0.034192	0.024653	0.013124
4.900	0.027508	0.034170	0.026607	0.012820
4.800	0.029605	0.034255	0.029375	0.012497
4.700	0.033067	0.034611	0.034053	0.012170
4.639001	0.037661	0.035342	0.040347	0.012028
4.619001	0.043073	0.036420	0.047783	0.012154
4.618	0.043813	0.036531	0.048786	0.012157
4.600	0.043301	0.036294	0.048302	0.011984
4.500	0.040447	0.034978	0.045570	0.011067
4.400	0.037628	0.033675	0.042801	0.010240
4.300	0.034896	0.032395	0.040047	0.009511
4.000	0.027632	0.028700	0.032216	0.007899
3.900	0.025619	0.027495	0.029833	0.007529
3.800	0.023854	0.026278	0.027599	0.007225
3.700	0.022363	0.025024	0.025546	0.006980
3.600	0.021187	0.023682	0.023752	0.006797
3.500	0.020443	0.022090	0.022541	0.006759
3.84001	0.020405	0.021756	0.022526	0.006802
3.460	0.021676	0.022398	0.024588	0.008216
3.440	0.019303	0.020900	0.021405	0.007637
3.420	0.018790	0.020788	0.020516	0.007803
3.400	0.018439	0.020448	0.019808	0.007850
3.300	0.017203	0.019368	0.016776	0.008117
3.200	0.016589	0.018150	0.014288	0.008278
3.100	0.016349	0.016807	0.012051	0.008438
3.000	0.016378	0.015339	0.009949	0.008646
2.900	0.016610	0.013745	0.007917	0.008962
2.800	0.017020	0.012018	0.005883	0.009549
2.700	0.017737	0.010265	0.003531	0.011275
2.687001	0.017897	0.010123	0.003105	0.011857
2.667001	0.018341	0.010907	0.002259	0.014341
2.660	0.019681	0.009660	0.004131	0.018056
2.640	0.021559	0.007044	0.006749	0.022444
2.600	0.022950	0.000701	0.010379	0.027163
2.500	0.015882	0.011731	0.013901	0.023143
2.400	0.009791	0.013598	0.015780	0.016878
2.390	0.009562	0.013497	0.016170	0.016774
2.370	0.009285	0.013187	0.017123	0.016914
2.350	0.009244	0.012717	0.018390	0.017609
2.330	0.009485	0.012019	0.020236	0.019162
2.310	0.010740	0.010554	0.024907	0.024393
2.300	0.014719	0.008003	0.021760	0.020298
2.260	0.018749	0.007492	0.016770	0.013987
2.220	0.020480	0.009324	0.014227	0.010672
2.140	0.021713	0.013023	0.011851	0.006876
2.100	0.021487	0.014125	0.011932	0.006413
2.060	0.021670	0.015325	0.012326	0.007576
2.020	0.021547	0.015919	0.013592	0.009854
2.000001	0.021899	0.016078	0.014836	0.012297

(to be continued)

Table 10. (continued)

L/L_{s1}	$\|A_2\|$	$\|B_2\|$	$\|C_2\|$	$\|D_2\|$
1.980	0.023187	0.016475	0.015437	0.012825
1.940	0.023503	0.016702	0.016817	0.014219
1.900	0.023134	0.016496	0.018485	0.015878
1.860	0.022198	0.015780	0.020445	0.017810
1.820	0.020593	0.014375	0.022720	0.020048
1.780	0.018007	0.011854	0.025397	0.022699
1.740	0.013228	0.006310	0.028992	0.026343
1.737	0.012597	0.005420	0.029401	0.026771
1.735	0.012094	0.004652	0.029727	0.027115
1.733	0.011429	0.003520	0.030164	0.027581
1.730	0.012894	0.003493	0.030317	0.027634
1.728	0.013779	0.004263	0.030228	0.027483
1.726	0.014464	0.004885	0.030177	0.027383
1.720	0.016032	0.006350	0.030112	0.027204
1.680	0.022311	0.012408	0.030276	0.026908
1.640	0.027038	0.017040	0.030398	0.026694
1.620	0.029270	0.019239	0.030348	0.026488
1.580	0.033698	0.023625	0.029941	0.025776
1.540	0.038219	0.028131	0.029042	0.024581
1.520	0.040550	0.030464	0.028384	0.023786
1.480	0.045396	0.035334	0.026663	0.021869
1.440	0.050536	0.040518	0.024605	0.019942
1.400	0.055970	0.045988	0.023039	0.019458
1.360	0.061338	0.051293	0.023767	0.022958
1.340	0.062819	0.052672	0.024849	0.026317
1.338	0.062689	0.052553	0.024904	0.026736
1.336	0.062356	0.052261	0.024986	0.027288
1.334	0.061444	0.051503	0.025384	0.028453
1.333	0.061824	0.052335	0.030022	0.033465
1.328	0.070449	0.061230	0.047711	0.049893
1.323	0.077473	0.068240	0.064136	0.065982
1.320	0.081650	0.072391	0.074788	0.076659
1.300	0.111538	0.104066	0.163634	0.168718
1.290	0.127783	0.123534	0.212580	0.220355
1.280	0.143247	0.143252	0.254408	0.264671
1.270	0.155821	0.159934	0.283180	0.295175
1.260	0.164173	0.171440	0.297909	0.310706
1.255	0.166727	0.175137	0.300818	0.313707
1.250	0.168316	0.177599	0.301425	0.314240
1.245	0.169075	0.178991	0.300238	0.312849
1.240	0.169142	0.179488	0.297713	0.310022
1.200	0.157744	0.167722	0.262483	0.271205
1.160	0.142597	0.150384	0.234160	0.239706
1.120	0.128482	0.133977	0.216533	0.219564
1.080	0.109112	0.111727	0.193341	0.193540
1.000	0.050506	0.037413	0.169576	0.163377
0.990	0.071027	0.058021	0.170064	0.163890
0.970	0.086704	0.073772	0.174423	0.168265
0.950	0.098486	0.085596	0.179983	0.173834
0.900	0.124187	0.111351	0.197493	0.191358
0.850	0.149952	0.137138	0.219829	0.213709
0.800	0.178445	0.165641	0.247537	0.241439
0.750	0.211458	0.198662	0.281770	0.275703
0.700	0.250873	0.238088	0.324304	0.318283
0.650	0.299078	0.286311	0.377737	0.371780
0.600	0.359373	0.346638	0.445904	0.440035
0.550	0.436568	0.423882	0.534599	0.528850
0.500	0.537989	0.525378	0.652861	0.647276
0.450	0.675311	0.662814	0.815434	0.810076

the case of incidence of the P wave, and in Figs. 19–34 and Tables 6–10 for the case of incidence of the SV wave. The last section of Chapter 3 was devoted to a study of the detailed features of the figures concerned.

Since these detailed features can be referred to last sections of Part 1 and Chapter 3, the writer will only mention the characteristics common to the three cases in the remaining part of this section.

(1) The larger the amplitude of a corrugated boundary surface, the larger is its effect on reflection and refraction. The effect is such that with respect to regular waves the reflected wave decreases and the refracted one increases with the increase in the ratio of amplitude of corrugation to the wave length of the incident wave.

(2) $|A_1|$, $|B_1|$, etc. which represent irregular waves with a spectrum of the first order are proportional to the amplitude of the corrugated boundary surface in the present approximation and they vary with the wave length of the corrugated boundary surface.

(3) In general, the effect of corrugation on reflection is larger than on refraction.

(4) The S component of both irregularly reflected and refracted waves increases in general with decreasing L/L_{p1} in the case of incidence of the P wave. The P component of both irregularly reflected and refracted waves also increases as a whole with decreasing L/L_{s1} in the case of incidence of the SV wave.

(5) When the wave length of corrugation is equal to or near one of the wave length as of P or S waves in any medium, abnormal features appear in both regular and irregular waves. Furthermore it is very interesting to note that in general the abnormal features are more sensitive to the wave length of P or S waves in the refracted side than to that of P or S waves in the incident side. Whether this feature depends on adopted velocity ratio or not, will be of much interest and will be studied in detail in the near future.

(6) In a certain range of L there exist boundary waves of which the amplitude decreases exponentially with the distance from the boundary surface. Its rate of decrease depends on the wave length of the corrugated boundary surface and incident wave and the order of spectrum. Although they become standing waves in a stationary state as far as cases of normal incidences are considered, it will be inferred that there is a possibility of the existence of pure boundary waves caused by corrugation in the case of oblique incidence.

(7) When the wave length of the corrugated boundary surface is smaller than that of the incident wave, almost all irregular waves become boundary waves. Therefore, from the practical point of view it is enough to take only regular waves into practical consideration. In the present cases, the reflected wave becomes the smaller and the refracted wave the larger as the corrugation becomes larger.

The possibility of obliteration of the reflected wave from the Mohorovičić discontinuity in mountainous regions as suggested by H. E. Tatel and others[11] may be inferred from the results just obtained. That is, if the roughness of the boundary surface is interpreted as a corrugation of the boundary surface of a shorter wave length than that of the incident wave, it is possible for reflected waves to be rendered too small to be observed for the instruments of given sensitivity. In Japan, the Research Group for Explosion Seismology has several times carried out observations of reflected waves from the Mohorovičić discontinuity. The success in catching them may depend on the technical circumstances and the ground conditions near the observation stations. But there are a few examples under almost the same observational circumstances concerning observers and instruments. The first example is given by the fifth experiment carried out in 1953 by the Group in north-eastern Japan[12] and the experiment carried out by the Group in 1954 near Kamaisi Mine in north-eastern Japan[13]. The chief observer and instruments used were the same in both cases although the observation stations were in different places. Another difference was the amount of explosives used. Explosives of 42 tons were used in the former case and in the latter experiment explosives of only 1 or 0.1 ton were used. In the former experiment it succeeded in catching reflected waves, while in the latter it failed, although the experiment was repeated four times under slightly different conditions of shooting. The failure may be partly attributed to the decrease of reflected wave caused by a corrugated boundary surface in addition to the smaller quantity of explosives used in the latter experiment. The next example seems to be better than the first. It is supplied from the observation at Sano in the Kanto District on the occasion of the second Hokoda

11) H. E. TATEL, L. H. ADAMS and M. A. TUVE, *loc. cit.*, 3).

12) THE RESEARCH GROUP FOR EXPLOSION SEISMOLOGY, *Bull. Earthq. Res. Inst.*, **33** (1955), 699.

T. MATUZAWA, *Bull. Earthq. Res. Inst.*, **37** (1959), 123.

13) THE RESEARCH GROUP FOR EXPLOSION SEISMOLOGY, *Bull. Earthq. Res. Inst.*, **37** (1959), 89.

explosion in 1957[14] and the observation at Kamioka in central Japan on the occasion of the third Miboro explosion in 1958[15]. In this example also the chief observer and instruments used were the same at both sites and the same as in the first example. The different conditions were the positions of the observation stations and shot point, quantities of explosives, method of detonation, etc. In the former explosion, explosives of one ton were detonated simultaneously and in the latter, explosives of 100 tons were not detonated simultaneously, but almost at the one time by means of delayed shot. In the former observation it was successful in catching manifested reflected wave from the Mohorovičić discontinuity, whereas in the latter the reflected wave observed was not so clear although the amount of explosives used was much larger. The former station was in the Kanto Plain and the latter station was set up in the mountainous region of Japan. Therefore, this may be interpreted by the results obtained. That is to say, one may interpret the difference in two cases as being partly due to the decrease of reflected wave caused by the roughness of the Mohorovičić discontinuity. From these speculations it may be said that the experiment would be better if carried out on plain, and if not, excessive amounts of explosives must be used in the observation of reflected wave from deep seated discontinuities which are expected to be rough. Concerning these respects there are not yet sufficient relevant data.

We can now say with confidence that if elaborate experiments are carried out and knowledge of the variation in amplitude of reflected or refracted waves is further increased, it is possible to gain knowledge about the character of the boundary surface concerned.

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14) THE RESEARCH GROUP FOR EXPLOSION SEISMOLOGY, *Bull. Earthq. Res. Inst.*, **37** (1959), 495.
T. MATUZAWA, T. MATUMOTO and S. ASANO, *Bull. Earthq. Res. Inst.*, **37** (1959), 509.
15) THE RESEARCH GROUP FOR EXPLOSION SEISMOLOGY, *Bull. Earthq. Res. Inst.*, **39** (1961), 285.
T. MIKUMO *et al.*, *Bull. Earthq. Res. Inst.*, **39** (1961), 321.

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13. 波型の境界面における弾性波の反射，屈折　第2報

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本報では，第1報で用いた Rayleigh の方法により，Fourier 級数に展開可能な境界面に，任意の入射角で P 及び SV 波が入射する場合の解を主に述べた．境界面の振巾と入射波の波長の比が小さいとして，この2乗の程度まで近似をたかめた．計算に用いた構造は SH 波の場合と同じく，$\lambda_l=\mu_l$，$V_{p1}/V_{p2}=3/4$，$\mu_2/\mu_1=2$ なる値を用い，低速側から高速側へ入射する場合に対して計算した．簡単のために，第1報と同様に，境界面は $\zeta=c\cos px$ なる形にとり，平面 P 波或いは SV 波が垂直に入射することを仮定した．

結果は，P 波入射の場合には Figs. 3-18, Tables 1-5, SV 波入射の場合には Figs. 19-34, Tables 6-10 に与えられている．

細かい点を除き，P, SH, SV 波に共通している結果は大体，次の通りである．

（1） 凹凸振巾が大きくなると，効果は大となる．regular wave について反射波は減少し，屈折波は増加する．

（2） 凹凸の効果は屈折よりも反射に大きい．

（3） irregular wave について凹凸と同じ波長で変化するもの A_1, B_1, … はこの程度の近似では凹凸振巾に比例する．

（4） 反射波，屈折波とも P 波入射の時は irregular wave の S 波の成分が凹凸の波長が小さくなるにつれて，増加する一般的傾向を有し，SV 波入射の時は irregular wave の P 波の成分が増加する一般的傾向を有する．

（5） 凹凸の波長が入射，或いは屈折側の P 波或いは S 波の波長に等しくなるか，或いは近くなると regular wave, irregular wave とも異常を示す．興味深い点はこのモデルに関する限り入射側より屈折側の量に，より敏感である点である．

（6） 凹凸波長の比によるが，境界面から離れるにつれ，指数関数的に急に減少する境界波が存在する．

（7） 凹凸の波長が入射波の波長に比べて小さい場合には，irregular wave は殆んどすべて指数関数的に減衰する境界波となるので，実際的には regular wave のみ考えれば十分である．すなわち，凹凸が増すと反射波は減少し屈折波は増加する．

若し，H. E. Tatel や M. A. Tuve がいう粗な境界面とは，入射波の波長に比べて凹凸の波長が小さい場合と解釈するならば (7) より反射波は減少するから Moho 面よりの反射波が観測されないことはあり得ると推察される．また，これらの結果は地震探鉱法にも参考になるものと思われる．

以上，述べた結果は1つのモデルに対する簡単な場合であるが，弾性波動の立場からはこのような結果が弾性定数の相異や，斜め入射の場合，更には境界面の形が異なった場合等により，どの程度，修正が必要かを調べることは，極めて興味深いので近い将来，報告する予定である．