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15. *On the Period and the Damping of Vibration in Actual Buildings.*

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1. Introduction

The time is rapidly ripening for a reformation in earthquake resistant design techniques, that is, a reforming from static methods to dynamic methods, caused by the rapid development of not only numerical calculation techniques but also the observation system of strong earthquake motions.

At any rate, the principal factors of building in the dynamic method for earthquake resistant design are considered to be natural period, damping coefficient and ductility factor.

Therefore, so many results from experimental as well as theoretical studies concerning the natural period and damping coefficient of building have already been obtained, and there are various empirical formulae relating to the natural period of building.

Nevertheless it cannot be said that the features of building vibration, especially damping have already been clarified. On the other hand, it may be said that the study of the ductility problem in building is developing rapidly.

In the present investigation, a large number of measurement data of natural periods as well as damping coefficients of actual buildings more than several stories high of various types will be dealt with from some new view points.

2. The Values of the Fundamental Period of Actual Buildings

The relation among height (H), length parallel to vibration direction (D), effective stiffness (G), mass (m), natural period (T) of building and connection factor of building base to ground (ξ) can be written as follows:

$$T=f(H, D, G, m, \xi) . \qquad (1)$$

Now, for simplicity, it is assumed that the relationship of (1) may be rewritten as the following formula:

$$T=f_1(H)\cdot f_2(D)\,. \tag{2}$$

In that case the ratio of the period (T_1) for vibration parallel of the long side (D_1) to the one (T_2) of the short side (D_2) of a building

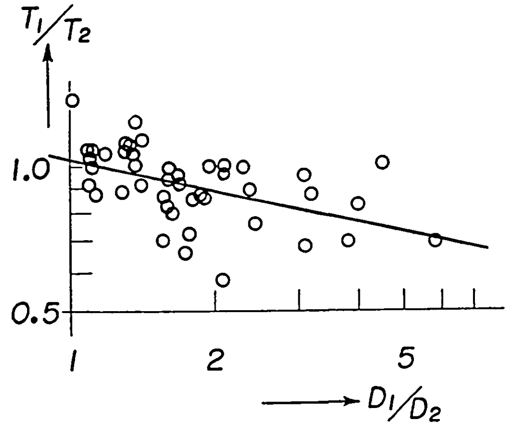

Fig. 1. Relation between the fundamental period and the depth parallel to the vibration direction of actual buildings in Japan. Suffix 1 and 2 represent the long and the short sides, respectively.

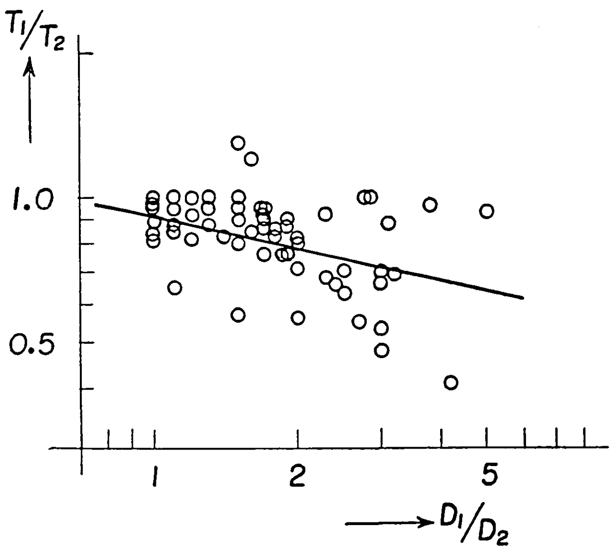

Fig. 2. Relation between the fundamental period and the depth parallel to the vibration direction of actual buildings in the U.S.A. Suffix 1 and 2 represent the long and the short sides, respectively.

becomes, as follows:

$$\frac{T_1}{T_2}=\frac{f_1(D_1)}{f_2(D_2)} \,. \qquad (3)$$

Under the consideration of (3), Figs. 1 and 2 were obtained from the results of vibration experiments of actual buildings including reinforced concrete, steel frame and reinforced concrete, steel-concrete composite constructions in Japan[1] and in the U.S.A.[2]

From Figs. 1 and 2, by means of the least square method, we get the empirical formulae as follows:

$$\text{Japan}\;; \quad \frac{T_1}{T_2}=1.03\left(\frac{D_1}{D_2}\right)^{-0.21}, \qquad (4)$$

$$\text{U.S.A.}\;; \quad \frac{T_1}{T_2}=0.92\left(\frac{D_1}{D_2}\right)^{-0.24}. \qquad (5)$$

It will be said from (4) and (5) that the empirical formulae of the period of building obtained from the data in Japan and in the U.S.A. coincide well within their errors.

Next, for the present, let us assume that the fundamental periods of buildings (T) are proportional to their height (H). Under the above assumption, the relations between T/H and the length parallel to the vibration direction (D) of actual buildings in Japan and in the U.S.A. are shown in Figs. 3 and 4.

The empirical formula of the problem obtained from Fig. 3 is presented by (6) as well as by a straight line in Fig. 3.

$$\left.\begin{aligned} \frac{T}{H}&=0.034\,D^{-0.19}\,, \\ \text{or }\; T&=\frac{0.034H}{D^{0.19}}\,. \end{aligned}\right\} \qquad (6)$$

1) K. KANAI and S. YOSHIZAWA, "On the damping of vibration of actual buildings. I", *Bull. Earthq. Res. Inst.*, **30** (1952), 121-126.

T. NAITO and S. NASU, "Vibration tests of actual buildings", *Memoirs Faculty Sci. and Egg., Waseda Univ.*, **16** (1952), 54-61.

H. KAWASUMI and K. KANAI, "Vibrations of buildings in Japan", *Proc. World Conf. Earthq. Engg.*, (1956), Berkeley, 7_1-1-14.

K. NAKAGAWA, "Vibrational characteristics of reinforced concrete buildings existing in Japan", *Proc. II. World Conf. Earthq. Engg.*, (1960), Tokyo, 95-2-1-10.

2) *Earthq. Invest. in Calif., 1934-1935, U.S.C.G.S. Special Public.* **201**, p. 54, Table 3.

It can be said that the present assumption, in which the fundamental period of buildings is proportional to the height of the buildings embodies the conditions of actual buildings, because the degree of the power to D in (4) as well as (5) and (6) coincides approximately with one another.

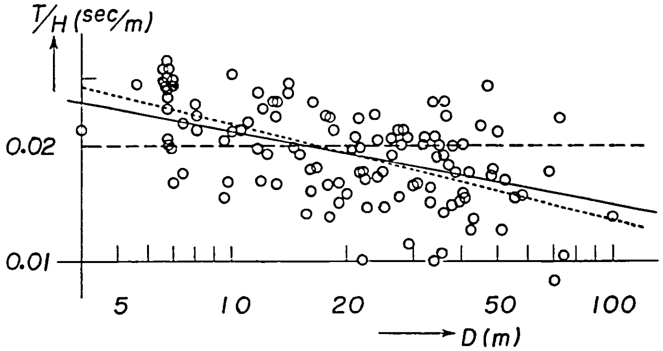

Fig. 3. Relation among the fundamental period, the height and the depth of actual buildings in Japan.

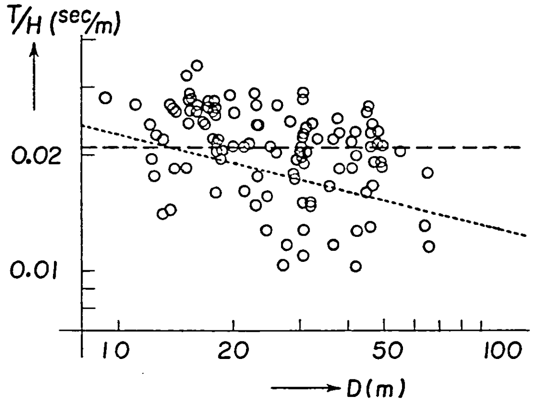

Fig. 4. Relation among the fundamental period, the height and the depth of actual buildings in the U.S.A.

Consequently, an empirical formula to obtain the fundamental period of building for practical use may be written as follows:

$$T=\frac{0.04\,H}{\sqrt[4]{D}}, \tag{7}$$

in which T is in sec and H and D are in m. The results of (7), as a reference, are shown by the dotted lines is Figs. 3 and 4.

Nevertheless, in general, the empirical formula to be taken regardless of D will be more practical because the degree of the power to D in the empirical formulae obtained above are fairly small. Then, when we adopt the mean values of ordinates in Figs. 3 and 4 (two values are about the same), the empirical formula shown by broken lines in the respective figures becomes as follows:

$$\left.\begin{aligned} T&=0.02\,H\,, \\ \text{or}\ \ T&=0.08\,N\,, \end{aligned}\right\} \tag{8}$$

in which N represents the number of stories.

3. Mathematical Interpretation

The results of experiments using a vibration generator attached to actual buildings showed us that, in general, there take place (a) a relative displacement at the base to the ground and (b) a rocking-like vibration at the base together with (c) elastic deformation of the upper structure. The ratios among (a), (b) and (c) depend mainly on the kinds of soil as well as the types of construction. For instance, a rigid structure standing on soft ground is has strong influence of (b), on the contrary, a flexible structure standing on hard ground acts mainly upon (c). Fig. 5 shows, as an example, the distribution of horizontal displacement in a reinforced concrete apartment-house with six metre concrete piles[3] which stands on crushed stone.

It seems from Fig. 5 that the boundary conditions at the joints of vertical and horizontal members of building may be adopted as follows. That is

$$x_n=l_n\,,\quad x_{n+1}=0\,;\qquad\qquad y_n=y_{n+1}, \tag{9}$$

$$\frac{\partial y_n}{\partial x_n}=0,\qquad \frac{\partial y_{n+1}}{\partial x_{n+1}}=0\,, \qquad (10),\ (11)$$

$$-E_nI_n\frac{\partial^3 y_n}{\partial x_n^3}+m_n\frac{\partial^2 y_n}{\partial t^2}=-E_{n+1}I_{n+1}\frac{\partial^3 y_{n+1}}{\partial x_{n+1}^3}\,, \tag{12}$$

in which x=the ordinate, y=the displacement, E=Young's modulus, I=the moment of inertia of the crosssection, m=the concentrated mass at the joint, l=floor height, n=the number of floor.

3) K. KANAI, T. TANAKA and T. SUZUKI, "Rocking and Elastic Vibrations of Actual Buildings. I", *Bull. Earthq. Res. Inst.*, **36** (1958), 193, Fig. 19.

Strictly speaking, the boundary conditions at the base of a building are not simple, although, it is considered that they are at least between the following alternative assumptions.

(i) $x_1=0$;

$$y_1=0, \quad \frac{\partial y_1}{\partial x_1}=0, \qquad (13),(14)$$

(ii) $x_1=0$;

$$y_1=0, \quad \frac{\partial^2 y_1}{\partial x_1^2}=0, \qquad (15),(16)$$

when $E_1I_1=E_2I_2=\ldots\ldots$, $m_1=m_2=\ldots\ldots$, $l_1=l_2=\ldots\ldots$, we have[4)]

(i) $$T\simeq 2\pi\sqrt{\frac{ml^3}{EI}}\times(0.18N+0.106), \qquad (17)$$

(ii) $$T\simeq 2\pi\sqrt{\frac{ml^3}{EI}}\times(0.295)(N+1), \qquad (18)$$

in which N represents the number of stories. It follows from these equations that the periods of free vibrations increase linearly as the number of stories increases. It follows, therefore, that the results of theoretical study are in good agreement with those obtained from experimental studies.

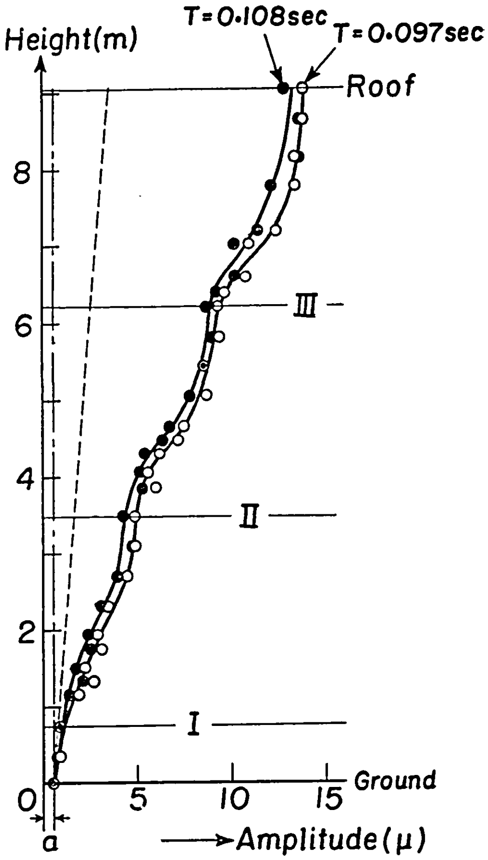

Fig. 5. Amplitude distuributions of the horizontal component of an apartment-house in the case of vibration experiments.

According to the theory concerning shearing vibrations of a building, the period of such vibrations is denoted by[5)]

$$T=4H\sqrt{\frac{\rho}{\mu}}, \qquad (19)$$

4) K. SEZAWA and K. KANAI, "Some New Problems of Free Vibrations of a Structure", *Bull. Earthq. Res. Inst.*, **12** (1934), 816, equation (48).

5) K. SUYEHIRO, "The Theory of the Vibration of Structures and a Method of Measuring It", *Journ. Inst. Japanese Archit.*, **40** (1926), 531-559, (in Japanese).

where H, μ and ρ represent the height, the shearing modulus and the density of the building, respectively. Comparing (19) with (17) or (18) we find that μ and ρ are respectively equivalent to

$$\text{(i)} \quad \rho=\frac{m}{al}, \qquad \mu \simeq 12.4\frac{EI}{al^2}, \tag{20}$$

$$\text{(ii)} \quad \rho=\frac{m}{al}, \qquad \mu \simeq 4.7\frac{EI}{al^2}. \tag{21}$$

4. Damping Coefficient of Actual Buildings

Roughly speaking, the damping factor of building, represented by the fraction of critical damping, h, can be written as follows:

$$h=\frac{T}{2\pi}\xi \cdot f(m, H), \tag{22}$$

in which T and ξ represent the period and damping coefficient of building and $f(m, H)$ means the function of mass and height of building. In that case, the ratio of the damping factor for vibration parallel to the long side of a building (suffix 1) to that of the short side of it (suffix 2) become as follows:

$$\frac{h_1}{h_2}=\frac{T_1\xi_1}{T_2\xi_2}, \tag{23}$$

or

$$\frac{h_1T_2}{h_2T_1}=\frac{\xi_1}{\xi_2}. \tag{23'}$$

On the other hand, the relation between h_1T_2/h_2T_1 and D_1/D_2 of actual buildings was obtained from Table 1 and is shown in Fig. 6[6]. An empirical formula obtained from Fig. 6 becomes as follows:

$$\frac{h_1T_2}{h_2T_1}=1.1\frac{D_1}{D_2}. \tag{24}$$

From (23′) and (24), we get

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6) Original data are presented in the following:

K. KANAI and S. YOSHIZAWA, "On the Damping of Vibration of Actual Buildings. I", *Bull. Earthq. Res. Inst.*, **30** (1959), 125, Table 2.

K. NAKAGAWA, "Vibrational Characteristics of Buildings", *Proc. II Word Conf. Earthq. Engg.*, (1960), Tokyo, 95-2-6, Table 1.

$$\frac{\xi_1}{\xi_2}=1.1\frac{D_1}{D_2}\,. \tag{25}$$

Table 1.

S	Name	No.	H		D	T	h	h_1T_2/h_2T_1
a	Yotsuya	2	11.0	D_1	56.0	0.36	—	4.5
				D_2	10.0	0.37	—	
b	Honmura-cho	4	11.1	D_1	32.3	0.175	—	3.7
				D_2	6.8	0.225	—	
c	Ōkura	5(1)	21.8	D_1	33.7	0.22	0.0609	—
				D_2	16.4	0.38	0.0376	
d	Mantetsu	6(1)	25.4	D_1	70.0	0.50	—	2.3
				D_2	28.0	0.44	—	
e	Tachikawa	4(1)	13.2	D_1	11.6	0.20	—	2.8
				D_2	4.4	0.31	—	
f	Kanzai	3(1)	14.7	D_1	28.8	0.31	0.0666	—
				D_2	18.4	0.32	0.0275	
g	Manpei	4(2)	17.8	D_1	44	0.40	—	1.6
				D_2	21	0.41	—	
h	Sannō-B	4(1)	12.7	D_1	21	0.21	—	1.7
				D_2	13	0.19	—	
i	Nonomiya	7(1)	24.3	D_1	16.5	0.39	—	1.5
				D_2	10.5	0.48	—	
j	Fujiginkō	6(1)	27.7	D_1	105	0.56	—	1.4
				D_2	75	0.54	—	
l	Sannō-A	4(1)	12.9	D_1	12	0.24	—	1.3
				D_2	10	0.275	—	
m	Itō-chū	8(2)	29.9	D_1	31.9	0.63	0.217	—
				D_2	27.7	0.60	0.159	

S=symbol of building, No.=no. of stories above (under) G. L., H=height in m, D=depth in m, suffix 1 and 2 represent the long and the short sides, respectively, T=period in sec, h=fraction of critical damping.

Now, if the vibration damping of a building is caused by resistance proportional to the velocity of a rocking-like movement at the base of the building, the damping coefficient of the building (ξ') can be obtained as follows:

$$\xi' = 2\int_0^{D/2} \xi_0 Lxdx = \frac{1}{4}\xi_0 LD^2 , \tag{26}$$

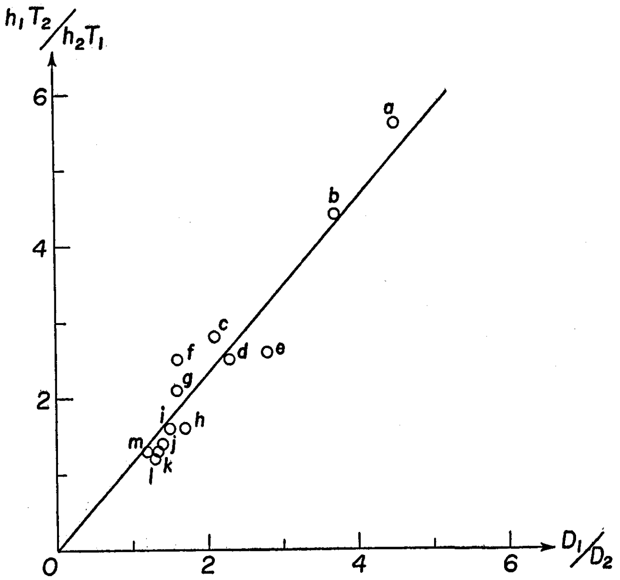

Fig. 6. Relation between the damping factor and the depth parallel to vibration direction of actual buildings. Suffix 1 and 2 repesent the long and the short sides, respectively.

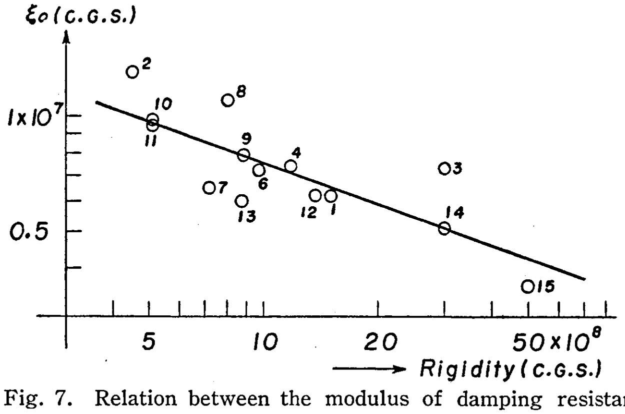

Fig. 7. Relation between the modulus of damping resistance and the rigidity modulus of various kinds of subsoil.

in which ξ_0, x, D and L represent the resistance constant of unit area, distance from the central axis of the rocking-like movement of the base, length parallel to vibration direction and length normal to the above. From (26) we can find easily the ratio of ξ' for the vibration parallel to the long side (suffix 1) to the one of the short side (suffix 2) of a building as follows

$$\frac{\xi_1'}{\xi_2'}=\frac{D_1}{D_2}. \tag{27}$$

The relation between (25) and (27) tells us that the assumption on which (27) was derived satisfies fairly the damping condition of vibration in actual buildings. The number 1.1 instead of 1.0 in (25) is considered as the resistance effect caused by the relative movement of the base to the ground at both sides of the vibration direction.

The unit resistance constant (ξ_0) in question, calculated from the results[7] of the observation of actual earthquakes in three and four storied reinforced-concrete apartment-houses which stand on various kinds of ground is shown in Fig. 7.

The values of the modulus of rigidity of ground at 4, 6 and 14 in Fig. 7 are the results of the experiments and those at other points are estimated by the predominant periods of the ground.

The constants of buildings treated here, together with the values of the unit resistance constants obtained by calculation are shown in Table 2.

From Fig. 7 the empirical formula can be written as follows:

$$\xi_0=1.1\times10^{10}\times\mu^{-0.35} \quad C.G.S., \tag{28}$$

in which μ is the modulus of rigidity of the ground.

It will be seen in (28) that the unit resistance constant (ξ_0) in question is about inversely proportionate to the velocity of the S-waves of the ground surrounding the base of a building. In other words, (28) tells us that the softer the ground, the larger the vibration damping of the building.

The result obtained here is in good agreement with the dissipation

7) K. KANAI, T. SUZUKI and S. YOSHIZAWA, "Relation between the Property of Building Vibration and the Nature of the Ground. III", *Bull. Earthq. Res. Inst.*, **34** (1956), 61-86.

K. KANAI, T. TANAKA and T. SUZUKI, "Rocking and Elastic Vibrations of Actual Buildings. II", *Bull. Earthq. Res. Inst.*, **36** (1958), 212-226.

theory which is based upon the idea that at the time of an earthquake, the vibration energy of a building dissipates to the ground again as the elastic waves that start from the base of it[8].

Table 2.

No.	Location	No.	Dimension (m)			M (ton)	T (sec)	Roof/Base	$\xi_0 \times 10^{-6}$ (C.G.S.)
			D	L	H				
1	Komagome	4	6.76	32.3	11.1	1400	0.32	3.7	6.2
2	Senju-hashido-chō	〃	〃	〃	〃	〃	0.28	2.0	13.1
3	Ishikawa-chō	〃	〃	〃	〃	〃	0.22	4.0	8.3
4	Totsuka	〃	〃	〃	〃	〃	0.33	3.0	7.4
6	Gōtokuji	〃	〃	〃	〃	〃	0.32	3.2	7.2
7	Taishidō	〃	〃	〃	〃	〃	0.32	3.5	6.5
8	Kyōdō	〃	〃	〃	〃	〃	0.32	2.1	10.9
9	Dairokuten-machi	〃	〃	〃	〃	〃	0.31	3.0	7.9
10	Nakarokugō	〃	〃	〃	〃	〃	0.30	2.5	9.8
11	Kugahara	〃	〃	〃	〃	〃	0.35	2.2	9.5
12	Sakae-chō	〃	〃	〃	〃	〃	0.35	3.4	6.2
13	Maeno-chō	〃	〃	〃	〃	〃	0.37	3.3	6.0
14	Hitachi	3	6.8	30.0	9.04	1000	0.18	4.0	5.1
15	Hitachi	〃	〃	〃	〃	〃	0.097 0.108	9.6	4.0 3.6

No.=number of stories, D=length parallel to vibration direction, L=length normal to vibration direction, H=height, M=total mass, T=fundamental period, Roof/Base=amplitude ratio of roof to base.

At any rate, the results mentioned above tell us that the upper parts of a building of any type of construction, seem to have slight influence upon the vibration damping.

Next, the relations between the damping factor of the buildings represented by a fraction of critical damping which were obtained by artificial vibration experiments, h[9], and the ones which were calculated from the results of observation of actual earthquakes, h'[10], are shown in Fig. 8.

Fig. 8 shows us that the values of h' are about five times larger than those of h. The facts mentioned above can be interpreted in various ways. However, the principal reason for it is considered as

8) K. SEZAWA and K. KANAI, "Decay in the Seismic Vibrations of a Structure by Dissipation of their Energy into the Ground", *Proc. Impr. Acad., Japan*, **11** (1935) 174-176.
9) K. NAKAGAWA, *loc. cit.*, 6), 95-2-9, Table 3.
K. KANAI and others, *loc. cit.*, 3).
10) K. KANAI, and others, *loc. cit.*, 7).

that the number of successive actual seismic waves, in which the periods closely approximate the natural period of building, was unsatisfactory.

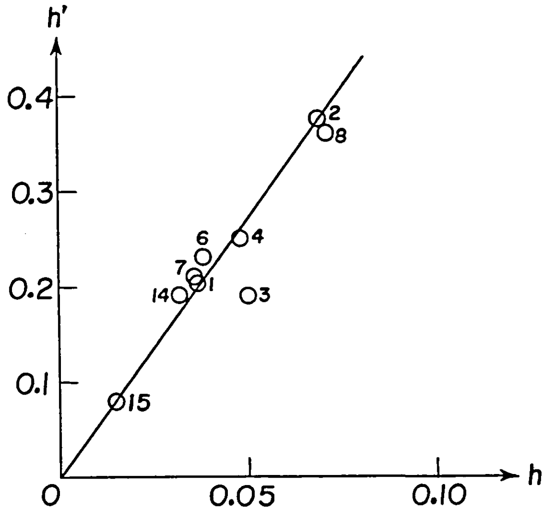

Fig. 8. Relation between the fraction of critical damping of buildings obtained by actual earthquake observations(h') and that obtained by artificial vibration experiments(h).

In other words, it seems that the synchronization of seismic waves to actual buildings was not sufficient.

5. Conclusion

From a statistical investigation of the fundamental periods of actual buildings more than several stories high, including various types which were obtained in Japan as well as in the U.S.A., the empirical formula obtained is as follows:

$$T=\frac{0.04\,H}{\sqrt[4]{D}}. \qquad (7)$$

And, in practical use, the empirical formula in question may be taken as follows:

$$\left.\begin{array}{r} T=0.02\,H, \\ \text{or } T=0.08\,N. \end{array}\right\} \qquad (8)$$

The present investigation also showed us that the vibration damping of actual buildings depends mostly upon the resistance between their base and the ground on which they stand. And the empirical formula concerning the unit resistance constant at the base of a building was obtained as follows:

$$\xi_0 = 1.1 \times 10^{10} \times \mu^{-0.35} \quad C.G.S. \tag{28}$$

At any rate, from the present investigation, it may be said that the period of a building more than several stories high, depends mostly on the height and is slightly subject to the effect of the foundation, on the contrary, the conditions concerning the damping are just the reverse of those mentioned above.

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15. 実在建物の周期と減衰

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日本およびアメリカ合衆国の実在建物の振動実験結果を統計的にしらべて，次のような実験式を得た．即ち，固有周期の式としては，

$$T = \frac{0.04H}{\sqrt[4]{D}}, \tag{7}$$

但し，T は秒，H と D は高さと振動方向の巾で，単位は m である．(7) から次の実用式も得られた．

$$\left.\begin{array}{ll} & T=0.02\,H, \\ \text{又は} & T=0.08\,N, \end{array}\right\} \tag{8}$$

ここに，N は層数である．

又，減衰の大部分は基礎部分の傾斜運動による，基礎と土との間の抵抗に原因することがわかり，その抵抗常数として次の値を得た．

$$\xi_0 = 1.1 \times 10^{10} \times \mu^{-0.35} \quad C.G.S., \tag{28}$$

但し，μ は土の剛性率である．即ち，この結果は，地震時に建物の振動勢力が地中に逸散するという考から出た理論結果と定性的には，よく合う．

とにかく，この研究で，数階建以上の建物の周期は上部構造できまり，基礎の影響は少く，反対に，減衰は基礎と土との間で主におこり，上部構造では，あまり生じないことがわかつた．
