BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 39 (1961), pp. 85-95

4. *An Empirical Formula for the Spectrum of Strong Earthquake Motions.*

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(Read July 26, 1960.—Received Dec. 28, 1960.)

1. Introduction

The usual meaning of the term "earthquake motion" is the wave motion at ground surface transmitted from the earthquake origin through the earth's crust. If $O(T)$, $Q(T, q)$ and $G(T, q)$ represent, respectively, the vibrational characteristics of the earthquake origin, of the earth's crust, and of the ground at an observation station, the wave form of the earthquake motion at the station may be written as

$$u(T, q)=f\{O(T),\ Q(T, q),\ G(T, q)\} \tag{1}$$

in which T and q represent, respectively, wave period and wave attenuation. $O(T)$ depends on both the earthquake magnitude and the mechanism of seismic wave generation, while $Q(T, q)$ is determined by the reflection, refraction, scattering, and absorption of waves in the earth's crust.

In this paper, one of the empirical formulae for the spectrum of earthquake motions obtained in the previous paper[1] is somewhat improved, following which the formula is applied to one case in Japan and nineteen cases in the U.S.A. for which strong earthquake motion observations are available.

2. The spectrum of seismic waves at depth

The following empirical formula, which has previously been reported[2] describes the amplitude-period relation of seismic waves at depth of the epicentral distance Δ in km and a magnitude M:

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[1] K. KANAI, "A Study of Strong Earthquake Motions," *Bull. Earthq. Res. Inst.*, **36** (1958), 295-310.

[2] *loc. cit.*, 1), 297, equation (7).

$$\bar{u}=10^{0.61M-1.73\log\Delta-1.47}T \qquad (2)$$

$$[0.05,\ 0.1\,\mathrm{sec}<T<T_m]$$

in which u and T represent, respectively, the displacement amplitude in cm and the period in second of seismic waves at depth and 0.05, 0.1 sec and T_m[3] are represented schematically in Fig. 1.

It follows directly from (2) that

$$\bar{v}=10^{0.61M-1.73\log\Delta-0.67}, \qquad (3)$$

$$\bar{a}=\frac{10^{0.61M-1.73\log\Delta+0.13}}{T}, \qquad (4)$$

where $\bar{v}$ is the spectral velocity in cm/sec and $\bar{a}$ is the spectral acceleration in cm/sec^2 of seismic waves at depth.

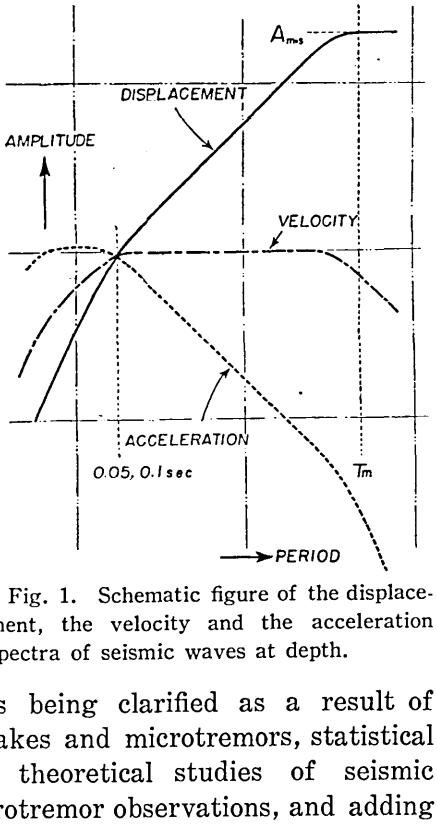

Fig. 1. Schematic figure of the displacement, the velocity and the acceleration spectra of seismic waves at depth.

3. The vibrational characteristics of the ground

Understanding of the seismic characteristics of the ground surface is being clarified as a result of observational investigations of earthquakes and microtremors, statistical analyses of earthquake damage and theoretical studies of seismic waves. Based on earthquake and microtremor observations, and adding theoretical considerations, it has been obtained a semi-empirical formula (5)[4], for the seismic characteristics of the ground consisting of a layer overlying a semi-infinite medium:

$$G(T,q)=\frac{1}{\sqrt{\left\{1-\left(\frac{T}{T_0}\right)^2\right\}^2+\left\{\frac{0.2}{\sqrt{T_0}}\frac{T}{T_0}\right\}^2}} \qquad (5)$$

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[3] K. KANAI and S. YOSHIZAWA, "The Amplitude and the Period of Earthquake Motions. II," *Bull. Earthq. Res. Inst.*, **36** (1958), 278, equation (5).

loc. cit., 1), 295, equation (1).

[4] K. KANAI, "Semi-empirical Formula for the Seismic Characteristics of the Ground," *Bull. Earthq. Res. Inst.*, **35** (1957), 309-325.

K. KANAI, R. TAKAHASI and H. KAWASUMI, "Seismic Characteristics of Ground," *Proc. World Conf. Earthq. Engg., Berkeley* (1956), 31.

in which T and T_0 represent, respectively, in seconds, the period of seismic waves and the predominant period of the ground.

Strictly speaking, the stratum alone cannot be regarded as a conservative system, but the vibrations in that stratum should be considered as taking place in a dissipative system.

The expression of the following formula, (6), has been modified from (5) in order to make it more closely analogous to the expression for motion of a stratified visco-elastic body:

$$G(T,\ q)=1+\frac{1}{\sqrt{\left[\frac{1+\alpha}{1-\alpha}\left\{1-\left(\frac{T}{T_0}\right)^2\right\}\right]^2+\left\{\frac{0.3}{\sqrt{T_0}}\left(\frac{T}{T_0}\right)\right\}^2}} \tag{6}$$

in which

$$\alpha=\sqrt{\frac{\rho_1\mu_1}{\rho_2\mu_2}}\left(\equiv\frac{\rho_1 V_1}{\rho_2 V_2}\right) \tag{7}$$

and ρ_1, μ_1, V_1 and ρ_2, μ_2, V_2, are respectively the densities, elastic constants and velocities of the layer and the subjacent medium.

The following special cases are presented as illustrations of (6).

(i) Cases of no surface layer or extremely long wave periods:

$$T_0\rightarrow 0 \text{ or } T\rightarrow\infty;\ G(T,\ q)\rightarrow 1\ .$$

(ii) Cases of a thick surface layer or very short wave periods :

$$T_0\rightarrow\infty \text{ or } T\rightarrow 0;\ G(T,\ q)\rightarrow\frac{2}{1+\alpha}\ .$$

(iii) The case of resonance:

$$T=T_0;\ G(T,\ q)=1+\frac{\sqrt{T_0}}{0.3}\ .$$

In cases (i) and (ii) the ground magnifies the wave motion only very slightly. In case (iii) the values from (6) correspond very closely with those obtained from the previous formula (5).

4. Comparisons of computed and observed spectral values

Combining equations (2), (3) and (4) with (6) yields the following empirical formulae for ground surface earthquake motion spectra:

$$u=10^{0.61M-1.73\log \Delta-1.47}T\left[1+\frac{1}{\sqrt{\left[\frac{1+\alpha}{1-\alpha}\left\{1-\left(\frac{T}{T_0}\right)^2\right\}\right]^2+\left\{\frac{0.3}{\sqrt{T_0}}\left(\frac{T}{T_0}\right)\right\}^2}}\right], \tag{8}$$

$$v=10^{0.61M-1.73\log \Delta-0.67}\left[1+\frac{1}{\sqrt{\left[\frac{1+\alpha}{1-\alpha}\left\{1-\left(\frac{T}{T_0}\right)^2\right\}\right]^2+\left\{\frac{0.3}{\sqrt{T_0}}\left(\frac{T}{T_0}\right)\right\}^2}}\right], \tag{9}$$

$$a=\frac{10^{0.61M-1.73\log \Delta+0.13}}{T}\left[1+\frac{1}{\sqrt{\left[\frac{1+\alpha}{1-\alpha}\left\{1-\left(\frac{T}{T_0}\right)^2\right\}\right]^2+\left\{\frac{0.3}{\sqrt{T_0}}\left(\frac{T}{T_0}\right)\right\}^2}}\right], \tag{10}$$

$[0.05,\ 0.1\,\mathrm{sec}<T<T_m]$

where u, v and a are, respectively, the spectral displacement in cm, velocity in cm/sec and acceleration in cm/sec^2 of earthquake motions at the ground surface.

There are some records of destructive earthquakes in Japan including the 1923 Kwanto earthquake, but none of them was successful. They are either incomplete or unreliable.

A comparison of the values of acceleration calculated by (10) with the analytical results obtained from the incomplete seismogram of the 1923 Kwanto earthquake which was recorded at the University of Tokyo is shown in Table 1.

Table 1. The values of acceleration of the 1923 Kwanto earthquake.

Earthquake		Station (Univ. of Tokyo)		Period (sec)	Acceleration (cm/sec^2)	
Date	M	Δ (km)	Orientation		Observed	Calculated
Sept. 1, '23	7.9	100	EW	1.35	100	110
				0.3	>250	290

Table 1 tells us that the agreement between calculated and observed values is rather good.

Table 2. Instrumental data of fourteen stations in U.S.A.

Station			Instrument		
No.	Location	No. of stories	Location	Orientation	Period (sec.)
I	Vernon, Calif., Centr. Manuf. Dist. Term.	6	basement	N08E	0.065
				N98E	0.065
II	Los Angeles, Calif., Subway Terminal Bldg.	13	subbasement	N39E	0.065
				N129E	0.065
III	El Centro, Calif., Irrig. Dist. Sub-station	1	ground level, slab	NS	0.065
				EW	0.065
IV	Helena, Montana, Carroll College	5	basement	NS	0.080
				EW	0.080
V	Ferndale, Calif., City Hall	2	slab	N45E	0.066
				N135E	0.065
VI	Santa Barbara, Calif., Court House	2～4	basement	N42E	0.064
				N132E	0.065
VII	Hollister, Calif., City Library	1	basement	N01E	0.066
				N91E	0.066
VIII	Olympia, Washington, Highway Test Lab.	1	slab	N80E	0.080
				N170E	0.081
IX	Seattle, Washington, Army Deport	1	shed	N02E	0.085
				N92E	0.085
X	San Francisco, Calif., Alexander Bldg.	16	basement	N81E	0.063
				N171E	0.065
XI	San Francisco, Calif., S. Pacific Bldg.	14	basement	N45E	0.067
				N135E	0.067
XII	San Francisco, Calif., State Bldg.	6	basement	N81E	0.065
				N171E	0.065
XIII	San Francisco, Calif., Golden Gate Park	1	slab	N10E	0.075
				N100E	0.077
XIV	Oakland, Calif., City Hall	16	basement	N26E	0.066
				N116E	0.068

Table 3. The values of periods of twelve destructive earthquakes in U.S.A.

Earthquake			Station		Period (sec.)	
No.	Date	M	No.	Δ(km)	Max. accel. recorded	Max. accel. of spectrum
1	Mar. 10, '33	6.3	I	45	0.3	0.3
			II	53	0.65	0.65, 0.3
2	Oct. 2, '33	5.3	I	27	0.35	0.3
			II	35	0.65	0.65, 0.3
3	Dec. 30, '34	6.5	III	56	0.5	0.5, 0.2
4	Oct. 31, '35	6.0	IV	24	0.4	0.4, 0.25
5	Sept. 11, '38	5.5	V	56	0.2	0.2, 0.4
6	May 18, '40	7.0	III	48	0.5	0.5, 0.2
7	Feb. 9, '41	6.6	V	121	0.3	0.3, 0.6
8	June 30, '41	5.9	VI	24	0.3	0.3, 0.6
9	Oct. 3, '41	6.4	V	80	0.3	0.3, 0.6
10	Mar. 9, '49	5.3	VII	16	0.35	0.35
11	Apr. 13, '49	7.1	VIII	72	0.35	——
			IX	88	0.9	0.9, 0.3
12	Mar. 22, '57	5.3	X	17	0.3	0.6
			XI	18	0.2	1.25
			XII	16	0.3	0.5
			XIII	13	0.2	0.5
			XIV	28	0.2	0.7

The instrumental records and spectra[5] of twelve United States earthquakes recorded by the United States Coast and Geodetic Survey in the period from 1933 to 1957 have been examined. The seismograph

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[5] J. L. ALFORD, G. W. HOUSNER and R. R. MARTEL, "Spectrum Analysis of Strong-motion Earthquakes," *1st Tech. Rep. Office Nav. Res., Contr. No. 6onr-244, Task Order 25, Project Design, NR-081-091*-(1951).

D. E. HUDSON and G. W. HOUSNER, "An Analysis of Strong-motion Accelerometer Data from the San Francisco Earthquake of March 22, 1957, "*Bull. Seism. Soc. Amer.*, **48** (1958), 253-268.

Table 4. The values of accelerations of twelve destructive earthquakes in U.S.A.

Earthquake		Station		Orientation	Acceleration (g)	
					Observed	Calculated
1	Mar. 10, '33	I	Vernon	N08E	0.10	0.12
				N98E	0.12	
		II	Los Angeles	N39E	0.04	0.06
				N129E	0.03	
2	Oct. 2, '33	I	Vernon	N08E	0.07	0.07
				N98E	0.08	
		II	Los Angeles	N39E	0.03	0.03
				N129E	0.02	
3	Dec. 30, '34	III	El Centro	NS	0.14	0.08
				EW	0.12	
4	Oct. 31, '35	IV	Helena	NS	0.11	0.19
				EW	0.12	
5	Sept. 11, '38	V	Ferndale	N45E	0.07	0.04
				N135E	0.06	
6	May 18, '40	III	El Centro	NS	0.28	0.21
				EW	0.17	
7	Feb. 9, '41	V	Ferndale	N45E	0.05	0.04
				N135E	0.03	
8	June 30, '41	VI	Santa Barbara	N42E	0.16	0.20
				N132E	0.12	
9	Oct. 3, '41	V	Ferndale	N45E	0.07	0.05
				N135E	0.07	
10	Mar. 9, '49	VII	Hollister	N01E	0.10	0.16
				N91E	0.14	

(To be continued)

(Table 4. Continued)

Earthquake		Station		Orientation	Acceleration (g)	
					Observed	Calculated
11	Apr. 13, '49	VIII	Olympia	N80E	0.18	0.15
				N170E	0.15	
		IX	Seattle	N02E	0.05	0.06
				N92E	0.05	
12	Mar. 22, '57	X	Alexander	N81E	0.04	0.08
				N171E	0.04	
		XI	S. Pacific	N45E	0.03	0.05
				N155E	0.03	
		XII	State Bldg.	N81E	0.05	0.10
				N171E	0.08	
		XIII	Golden Gate	N10E	0.07	0.14
				N100E	0.08	
		XIV	Oakland	N26E	0.03	0.04
				N116E	0.02	

constants and earthquake data appear in Tables 2 and 3. The resonance values of acceleration have been computed from (10), using as T_0 the periods of the maximum trace accelerations from Table 3. The calculated values are compared with values of the maximum recorded accelerations in Table 4 and Fig. 2. (The maximum recorded acceleration is the average of two waves in the vicinity of the maximum.) Again it will be seen in Table 4 as well as Fig. 2 that the comparison of the calculated and observed values is good.

In order to serve as a reference, the displacement, the velocity and the acceleration spectra in the cases where T_0 (predominant period of ground)$=0.2$, 0.4, 0.6, 0.8, 1.0 sec, Δ (epicentral distance)$=50$, 100, 150 km and M (magnitude)$=7$, 8, besides the condition where α (the impedance ratio of the surface layer to the subjacent medium)$=1/5$, are shown in Fig. 3. (The value of $\alpha=1/5$ is a plausible value in consideration of the average condition of the observation points in Japan and, in this case, the value of α has a slight influence upon the amplitude of the reso-

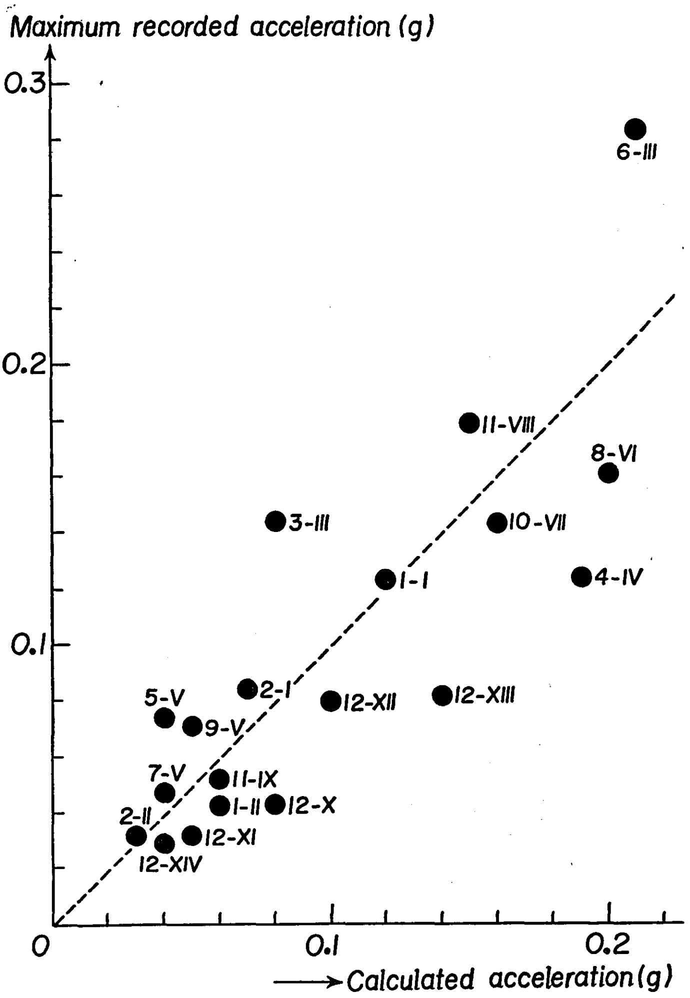

Fig. 2. Relation between the maximum recorded accelerations and the values of calculated accelerations obtained by using the empirical formula.

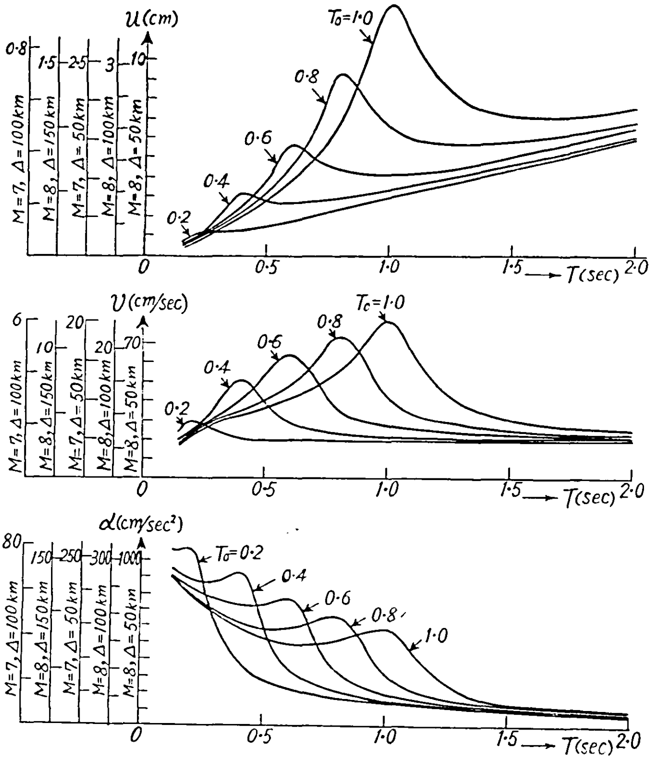

Fig. 3. The displacement, the velocity and the acceleration spectra obtained by using equations (8)~(10). T; period of waves, M; magnitude, Δ; epicentral distance, T_0; predominant period of ground.

nance condition.)

5. Conclusion

In determining earthquake ground motion characteristics for purposes of structural design, equations (8), (9) and (10) seem to be applicable. The magnitudes and epicentral distances of anticipated earthquakes may be estimated by the application of engineering judgment to statistical analyses of the seismicity, and there can be thus obtained an approximate spectrum of seismic waves in the bed rock. If this spectrum is combined with the vibration characteristics of the ground at the structure, the lateral force coefficient can be determined.

The author wishes to thank Professor C. M. Duke, University of California, Los Angeles, and Miss S. Yoshizawa who have assisted him in preparing this paper.

4. 強震動スペクトルに関する実験式

地震研究所　金　井　　清

前論文において，地震動スペクトルに関する実験式を，日本とアメリカ合衆国について，別々に求め，それらを，それぞれの国の代表的な破壊的地震にあてはめて最大加速度を計算し，それらの値が，実際に記録された値とよく合うことを示した．

今回は，前回，日本について求めた実験式に地盤の振動特性を加えたものを求め，関東地震および12ケのアメリカ合衆国の代表的な破壊的地震にあてはめて最大加速度を計算し，それらの値が，実際に記録された値と，前回以上によく合うことを示す．

今回の研究で，ある土地について，最悪の地震動と期待される，大地震のマグニチュードと震央距離を工学的判断によつて仮定し，更に地盤の卓越周期を加味して，その土地の耐震工学上の強震地動スペクトルを求める方法が，実用に供し得ることが一層はつきりしたことになる．