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25. *Studies of the Thermal State of the Earth. The Fourth Paper: Terrestrial Heat Flows related to Possible Geophysical Events.*

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Summary

A subcrustal convection current having a linear dimension of a few thousand kilometers accounts for the high heat flows found on the East Pacific Rise provided a velocity of several *cm/year* and $1°C/km$ temperature gradient are assumed in the earth's mantle. A long-lived magma of high temperature and a phase transformation layer can give rise to only a fraction of the world average of heat flow for conceivable cases. In general, it is therefore difficult to presume what sort of geophysical events are prevailing in the earth only from the heat flow anomalies observed.

1. Introduction

Important bearing of terrestrial heat flow on geophysics has been recently emphasized. Heat flow values as measured at various localities all over the earth scatter in a wide range as are summarized in a brief form in Table 1.

One of the most surprising results is certainly the fact that a wide variety of heat flow values has been found on ocean floors. According to Herzen[20], the ratio of the maximum to the minimum exceeds 50. Systematic distributions of high and low values of heat flow in the south-eastern Pacific seem to be even more outstanding. A number of geophysicists are now inclined to suppose that such anomalies may be caused by some convection currents in the earth's mantle. The fact

Table 1. Heat flow values from various localities.

Locality	Heat flow (10^{-6} *cal/cm*^{2} *sec*)	Temp. gradient ($°C/100\ m$)	Authority
CANADA			
Ontario, Quebec	0.69–1.32	0.9–1.6	Misener et al.[1] (1951)
Resolute Bay	0.4–2.9	3.95	Misener[2] (1955)
U.S.A.			
Texas, New Mexico	1.1–2.0	0.77–3.4	Herrin & Clark[3] (1956) Birch & Clark[4] (1945) Richardson & Wells[5] (1931)
California	0.66–1.3	0.86	Benfield[6] (1947) Clark[7] (1957)
Michigan	0.9	1.8	Birch[8] (1954)
Colorado	1.6–1.9	2.0–2.4	〃 [9] (1950)
Georgia	1.4	1.7–2.2	〃 [10] (1947)
Kansas	1.4–1.7	2.8	〃 〃
GREAT BRITAIN	0.68–2.87	1.7–6.5	Benfield[11] (1939) Bullard & Niblett[12] (1951) Anderson[13] (1939) Chadwick[14] (1956)
HUNGARY	2.0–3.04	4.0–7.0	Boldizsar[15] (1956), [16] (1959)
IRAN	0.53–1.22	0.9–2.0	Coster[17] (1947)
SOUTH AFRICA	0.75–1.52	0.95–2.23	Bullard[18] (1939)
PACIFIC	0.14–8.09	4.0–7.0	Revelle & Maxwell[19] (1952) Herzen[20] (1959)

(to be continued)

1) A.D. MISENER, L.G.D. THOMPSON and R.J. UFFEN, *Trans. Amer. Geophys. Uni.*, **32** (1951), 729.
2) A. D. MISENER, *Trans. Amer. Geophys. Uni.*, **36** (1955), 1055.
3) E. HERRIN and S. P. CLARK JR., *Geophysics*, **21** (1956), 1087.
4) F. BIRCH and H. CLARK, *Amer. Jour. Sci.*, **243** -A (1945) 69.
5) L. T. RICHARDSON and R. C. WELLS, *Jour. Wash. Acad. Sci.*, **21** (1931), 243.
6) A. E. BENFIELD, *Amer. Jour. Sci.*, **245** (1947), 1.
7) S. P. CLARK JR., *Trans. Amer. Geophys. Uni.*, **38** (1957), 239.
8) F. BIRCH, *Amer. Jour. Sci.*, **252** (1954), 1.
9) F. BIRCH, *Bull. Geol. Soc. Amer.*, **61** (1950), 567.
10) F. BIRCH, *Amer. Jour. Sci.*,**245** (1947), 733,
11) A. E. BENFIELD, *Proc. Roy. Soc. A*, **173** (1939), 428.
12) E. C. BULLARD and E. R. NIBLETT, *Mon. Not. Roy. Astr. Soc., Geophys. Suppl.*, **6** (1951), 222.
13) E. M. ANDERSON, *Roy, Soc. Edinb. Pr.*, **60** (1939), 192.
14) P. CHADWICK, *Nature*, **178** (1956), 105.
15) T. BOLDIZSAR, *Geofis. Pura e Applic.*, **34** (1956), 66.
16) T. BOLDIZSAR, *Publ. Faculties Min. Geotechn., Technical Univ. Sopron*, (XX) (1959), 1.
17) H. P. COSTER, *Mon. Not. Roy. Astr. Soc. Geophys. Suppl.*, **5** (1947) 131.
18) E. C. BULLARD, *Proc. Roy. Soc. A*, **173** (1939), 473.
19) R. REVELLE and A. E. MAXWELL, *Nature*, **170** (1952), 199.
20) R. HERZEN, *Nature*, **183** (1959), 882.

(continued)

Locality	Heat flow (10^{-6} *cal/cm*^{2} *sec*)	Temp. gradient (°*C*/100 *m*)	Authority
ATLANTIC			
N. Atlantic	0.58–1.42	0.95–2.23	Bullard[21] (1954)
Mid-Atlantic Ridge	6.0		Bullard[22] (1957)
JAPAN			
Sasago Tunnel	2.06	2.7	Tanakadate[23] (1903)
Tokyo Univ.	0.43	2.2	Uyeda, Yukutake & Tanaoka[24] (1958)
Mt. Sirane	10.4	24.2	
Hitachi (Copper mine)	0.63–0.67(N)	0.94	Hôrai[25] (1959)
	0.78–0.90(S)	1.21	〃
Oil-field			
Yabase	2.01	4.8	Teikoku-Sekiyu Co.[26] & Hôrai[25]
Innai		4.8	〃
Kotaki		7.3	〃
Mogami		7.1	〃
Nishiyama		4.8	〃
Matsunoyama		8.5	〃
Gas-field			
Utino		2.0	〃
Nagatoro		5.0	〃

that rises and deeps on the ocean bottom accompany respectively high and low heat flow values seems also favourable to the convection theory.

Turning to heat flows on land, however, the values observed cover a range from 0.4 to 3.0×10^{-6} *cal/cm*^{2} *sec*, no large differences such as found on ocean bottoms being reported except in volcanic or geothermal areas. The average value of heat flow on land has been believed to be 1.2×10^{-6} *cal/cm*^{2} *sec* though, according to recent accumulation of data, a slightly higher value, 1.4×10^{-6} *cal/cm*^{2} *sec* say, seems to be more likely.

Heat flow value observed at the earth's surface are affected by a number of conditions, physical and chemical, in the earth's crust. Influences of contents of radioactive matter in rocks, chemical action taking place in the crust, circulation of underground water, irregularity

21) E. C. BULLARD, *Proc. Roy. Soc.*, **222** (1954), 408.

22) E. C. BULLARD, *Geophysics*, **22** (1957), 432.

23) A. TANAKADATE, *Rep. Imp. Earthq. Invest. Committ.*, **45** (1903), 17.

24) S. UYEDA, T. YUKUTAKE and I. TANAOKA, *Bull. Earthq. Res. Inst.*, **36** (1958), 251.

25) K. HÔRAI, *Bull. Earthq. Res. Inst.*, **37** (1959), 571.

26) TEIKOKU-SEKIYU CO., *Unpublished data.*

of topography and such like on terrestrial heat flows have often been discussed though it seems difficult to estimate them in a general way. It is true that some of these conditions may account for high or low values of heat flow actually observed to some extent. However, it does not seem likely that all the anomalies found in heat flow values could be explained in this way.

In Japan, one of the countries where seismic and volcanic activities are most remarkable, it had been simply supposed that we should observe fairly large heat flows there prior to the actual observations. It turns out, however, that, as far as a few measurements that have been carried out so far are concerned, the heat flow values in Japan do not differ much from the world's average as can be seen in Table 1. Although the influences of the conditions discussed in the last paragraph on these Japanese values are not clear, some geophysicists are of the opinion that the terrestrial heat flows are unexpectedly small in Japan in spite of the geophysical and geological activities.

Under these circumstances, it would be of some use to estimate to what extent heat flows are affected by possible geophysical events such as thermal convection in the upper part of the mantle, existence of a magma reservoir of high temperature over a long period, occurrence of phase transformation in the earth's crust and so forth. It might be possible to explain some of the geothermal anomalies on the basis of these theoretical estimations.

The first study in this line has been already made by one of the writers[27] in relation to heat flow associated with magma intrusion though no account was taken of latent heat of solidification. It becomes clear that any intrusion of magma mass having a radius of a few kilometers does not produce a large anomaly of heat flow unless the intrusion occurs in a shallow depth, 1 *km* or so. It should be difficult to detect a high temperature mass of that size by measuring heat flow at the earth's surface provided the depth exceeds several kilometers. Since this sort of study seems to be useful for interpretations of heat flow anomalies as well as for presuming conditions of thermal processes taking place in the earth, it is intended in this paper to conduct a few studies of the sort which may be applied to possible geophysical events mentioned in the last paragraph.

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27) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **37** (1959), 233.

2. Heat flow accompanied by subcrustal convection currents

Subcrustal convection currents have long been supposed by a number of geologists[28)-32)] to be causes of orogenesis and associated phenomena. It is beyond the scope of this paper to deal with the details of the convection theory. We may refer to the modern reviews[33),34)] in order to see the outline of the theory.

Although the theory of convection currents in the earth's mantle has many advantages for the interpretation of geological and geophysical phenomena, no direct proofs of the existence of such currents have ever been put forward until recently. According to the measurements[18),19),20)] on ocean floors, however, high or low heat flow values which seem likely to be associated with some convection currents have been found as mentioned in the introduction. Although it is not quite certain whether or not these anomalies of heat flow are the direct evidence of subcrustal convection currents, it is surely worth-while to estimate theoretically what convection will accompany what heat flow at the earth's surface. Such a study may serve as some check of the convection theory together with the heat flow values observed. Let us suppose a steady convection current under the solid crust as schematically shown in Fig. 1 where everything is assumed to be independent of y.

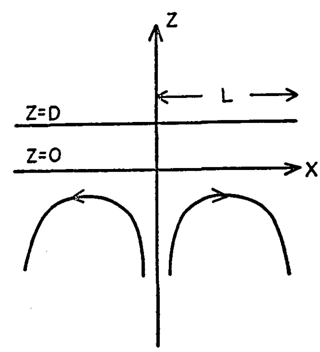

Fig. 1. Model of subcrustal convection currents.

By assuming that the fluid is incompressible, the x, y and z components of the velocity of such a convectional motion may be expressed as

$$\vec{v} = \begin{cases} \dfrac{AL}{l\pi}\left(1+\dfrac{z}{l}\right)\left(1+\dfrac{3z}{l}\right)\sin\dfrac{\pi x}{L} \\ 0 \\ -A\,\dfrac{z}{l}\left(1+\dfrac{z}{l}\right)^2\cos\dfrac{\pi x}{L}\,, \end{cases} \tag{1}$$

28) A. HOLMES, *Mining Mag.*, **40** (1929), 40.
29) D. GRIGGS, *Amer. Journ. Sci.*, **237** (1939), 611.
30) H. H. HESS, *Trans. Amer. Geophys. Un.*, **32** (1951), 528.
31) H. BROOKS, *Trans. Amer. Geophys. Un.*, **27** (1941), 548.
32) F. A. VENING MEINESZ, *Quart. Journ. Geol. Soc. Lond.*, **103** (1948), 191.
33) A. E. SCHEIDEGGER, *Principles of Geodynamics*, Springer (1958).
34) W. A. HEISKANEN & F. A. VENING MEINESZ, *The Earth and Its Gravity Field*, McGraw-Hill (1958).

which satisfies $\mathrm{div}\,\vec{v}=0$. It is seen from (1) that the velocity in the direction parallel to the z-axis vanishes at $z=0$ and $z=l$. Since this type of convection current is often found in experiments, it may be allowed to take (1) as a possible type of subcrustal convection currents if they existed.

The equation of heat transfer in fluid is

$$\frac{\partial U}{\partial t}+(\vec{v}\,\mathrm{grad})U=\kappa\nabla^2 U\,, \qquad (2)$$

where U and κ denote the temperature and thermal diffusivity respectively. U may be put

$$U=U_0+u\,, \qquad (3)$$

where U_0 does not depend on time. If we think of a portion of fluid being driven by the thermal convection, its temperature varies in a way nearly parallel to the temperature-depth curve for adiabatic gradient which is known to be smaller than the actual temperature gradient in the earth's mantle. A likely ratio[35] of the adiabatic gradient to the actual one has been suggested to be 1/8 or thereabout. In case L is so large that we can also ignore $\partial u/\partial x$, we may drop, for an approximate estimate, $(\vec{v}\,\mathrm{grad})\,u$ from equation (2) so long as $\vec{v}$ is not very large.

Denoting

$$\frac{\partial U_0}{\partial x}=0\,, \quad -\frac{\partial U_0}{\partial z}=\beta\,, \qquad (4)$$

(2) can be written as

$$\frac{\partial u}{\partial t}+A\beta\frac{z}{l}\left(1+\frac{z}{l}\right)^2\cos\frac{\pi x}{L}=\kappa\nabla^2 u\,. \qquad (5)$$

Solutions of (5) may be written as

$$u=u_2(z,\,t)\cos\frac{\pi x}{L}\,, \qquad (6)$$

where u_2 is a function of z and t only. Putting (6) into (5), we have

$$\frac{\partial u_2}{\partial t}+A\beta\frac{z}{l}\left(1+\frac{z}{l}\right)^2=\kappa\left\{\frac{\partial^2 u_2}{\partial z^2}-\left(\frac{\pi}{L}\right)^2 u_2\right\}\,. \qquad (7)$$

35) J. VERHOOGEN, *Physics & Chemistry of the Earth.* I, Pergamon (1956), 17.

Subscript 2 is hereafter used for the quantities below the crust. If we write p in place of time-operator $\partial/\partial t$, (7) can be written as

$$\frac{d^2u_2}{dz^2}-\left[\left(\frac{\pi}{L}\right)^2+p\kappa_2^{-1}\right]u_2=A\beta\kappa_2^{-1}\frac{z}{l}\left(1+\frac{z}{l}\right)^2 . \tag{8}$$

The solution of (8), that does not become infinitely large at $z\to-\infty$, becomes

$$u_2=C_2e^{\sqrt{(\pi/L)^2+p\kappa_2^{-1}}\,z} -A\beta\kappa_2^{-1}\left[\left\{\left(\frac{z}{l}\right)^3+2\left(\frac{z}{l}\right)^2+\frac{z}{l}\right\}\Big/\left\{\left(\frac{\pi}{L}\right)^2+p\kappa_2^{-1}\right\} +\left\{6\left(\frac{z}{l}\right)+4\right\}\Big/\left\{\left\{\left(\frac{\pi}{L}\right)^2+p\kappa_2^{-1}\right\}^2l^2\right\}\right] . \tag{9}$$

In the solid crust, the conduction of heat is governed by a differential equation of the type

$$\frac{\partial u}{\partial t}=\kappa_1\nabla^2u , \tag{10}$$

the appropriate solution of which is given by

$$u=u_1(z)\cos\frac{\pi x}{L} , \tag{11}$$

where

$$u_1=C_1e^{\sqrt{(\pi/L)^2+p\kappa_1^{-1}}\,z}+\overline{C}_1e^{-\sqrt{(\pi/L)^2+p\kappa_1^{-1}}\,z} , \tag{12}$$

in which κ_1 is the thermal diffusivity and p stands for $\partial/\partial t$ as before.

The boundary conditions by which free constants C_1, $\overline{C_1}$ and C_2 are determined are given as

$$\left.\begin{array}{lll} u_1=u_2 , & K_1\dfrac{\partial u_1}{\partial z}=K_2\dfrac{\partial u_2}{\partial z} , & \text{at } z=0 \\ u_1=0 , & & \text{at } z=D \end{array}\right\} \tag{13}$$

where K_1 and K_2 are the thermal conductivities in the respective regions. The continuity of temperature at the mantle-crust boundary would not always hold for a sudden occurrence of convection current. In such a case, another boundary condition should be considered.

From (13), we have

$$\left.\begin{aligned}
&C_1 e^{\sqrt{(\pi/L)^2+p\kappa_1^{-1}}\,D}+\overline{C}_1 e^{-\sqrt{(\pi/L)^2+p\kappa_1^{-1}}\,D}=0\ ,\\
&C_1+\overline{C}_1=C_2-\frac{4A\beta\kappa_2^{-1}}{\{(\pi/L)^2+p\kappa_2^{-1}\}^2 l^2}\ ,\\
&K_1(C_1-\overline{C}_1)\sqrt{(\pi/L)^2+p\kappa_1^{-1}}\\
&\quad=K_2\Big[C_2\sqrt{(\pi/L)^2+p\kappa_2^{-1}}-\frac{A\beta\kappa_2^{-1}}{l\{(\pi/L)^2+p\kappa_2^{-1}\}}\\
&\qquad\qquad\times\Big(1+\frac{6}{\{(\pi/L)^2+p\kappa_2^{-1}\}l^2}\Big)\Big]\ .
\end{aligned}\right\}\tag{14}$$

Solving (14), C_1, $\overline{C}_1$ and C_2 are easily obtained. Hence we can obtain u_1 and u_2 from (12) and (9).

In order to get some idea concerning heat flow values associated with the convection currents, however, no general solutions would be needed at the moment. For an order-of-magnitude estimate, we are specially interested in the steady state which is given by putting $p\rightarrow 0$. For the sake of simplicity, we also assume

$$\kappa_1=\kappa_2=\kappa\ ,\qquad K_1=K_2=K\ .$$

In that case, (14) is simplified as

$$\left.\begin{aligned}
&C_1 e^{\pi D/L}+\overline{C}_1 e^{-\pi D/L}=0\ ,\\
&C_1+\overline{C}_1=C_2-\frac{4A\beta\kappa^{-1}}{(\pi/L)^4 l^2}\ ,\\
&C_1-\overline{C}_1=C_2-\frac{A\beta\kappa^{-1}}{(\pi/L)^3 l}\Big(1+\frac{6}{(\pi/L)^2 l^2}\Big)\ ,
\end{aligned}\right\}\tag{15}$$

from which we obtain

$$\left.\begin{aligned}
&C_1=\frac{A\beta\kappa^{-1}}{2(\pi/L)^4 l^2}e^{-2\pi D/L}\Big(4-\frac{\pi l}{L}-\frac{6L}{l\pi}\Big)\ ,\\
&\overline{C}_1=-\frac{A\beta\kappa^{-1}}{2(\pi/L)^4 l^2}\Big(4-\frac{\pi l}{L}-\frac{6L}{\pi l}\Big)\ .
\end{aligned}\right\}\tag{16}$$

With (16), the steady value of temperature gradient at the earth's surface is readily estimated. We have

$$\left(\frac{\partial u_1}{\partial z}\right)_{z=D}=\frac{\pi}{L}(C_1e^{(\pi/L)D}-\overline{C}_1e^{-(\pi/D)L}) , \tag{17}$$

which becomes

$$\left(\frac{\partial u_1}{\partial z}\right)_{z=D}=e^{-\pi D/L}\frac{A\beta\kappa^{-1}}{(\pi/L)^3l^2}\left(4-\frac{\pi l}{L}-\frac{6L}{\pi l}\right) . \tag{18}$$

If we simply assume $L=l$, (18) can be written as

$$\left(\frac{\partial u_1}{\partial z}\right)_{z=D}=-0.0339\,e^{-\pi D/L}A\beta\kappa^{-1}L , \tag{19}$$

so that the temperature gradient right above the ascending current can be estimated from (19).

The temperature gradient in the earth's mantle has been supposed by one of the writers[36] to be $1.4°C/km$. For an order-of-magnitude estimate, we may therefore assume $\beta=1°C/km$. $\kappa=0.01$ *c.g.s.* is also assumed. The temperature gradient at the earth's surface is then calculated with A and L as parameters. By multiplying by K, which is assumed to take a usual value for rocks, 5×10^{-3} *cal/cm sec °C* say, the heat flow values for various thickness of the crust are obtained as are shown in Fig. 2.

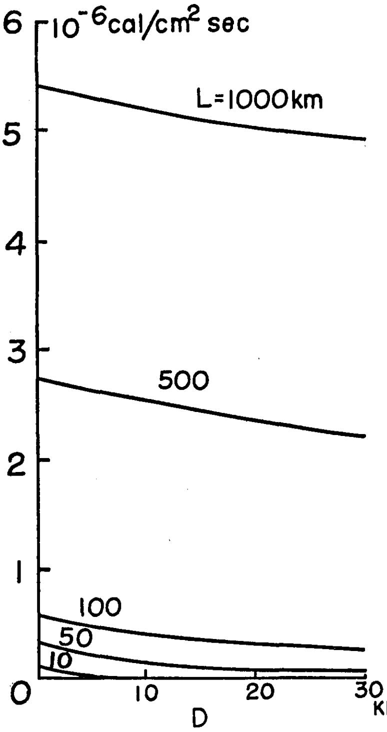

Fig. 2. Heat flows associated with the subcrustal convection currents for various thicknesses of the crust (D) and linear dimensions of the convection currents (L).

At a glance of Fig. 2, it is seen that the heat flow values are largely affected by the linear dimension of the convection current. If we suppose a convection in the earth's mantle, of which the linear scale exceeds 1000 *km* as has been supposed from the heat flow measurements in the south-eastern Pacific, we may well expect a heat flow amounting to 5×10^{-6} *cal/cm² sec* for $\beta=1°C/km$ and $A=10$ *cm/year*.

36) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **30** (1952), 13.

Since β may be able to take slightly larger values than those cited above, the heat flow anomalies found on the ocean floor are possibly interpreted as those caused by some subcrustal convection currents having a velocity of several *cm/year*.

Meanwhile, the thickness of the crust does not much affect the heat flow values except for convections of small scale. Hence an appreciable anomaly in heat flow must be observed even on the continent, where the crust is fairly thick, if there were subcrustal convection currents on an extensive scale.

The temperature-rise at the bottom of the crust is easily calculated for each case as given in Table 2. These values harmonize well with the heat flow values at the earth's surface. The reason why such high temperatures are associated with convections of large dimension is the fact that high temperature material at great depths comes up without losing much heat because we assume $l=L$.

Table 2. Temperature-rise at the bottom of the crust (°*C*).

L \ *D*	5 *km*	10 *km*	20 *km*	30 *km*
10 *km*	1.6	1.3	1.7	1.7
100 *km*	46	80	123	145
500 *km*	257	505	950	1460
1000 *km*	538	1030	2020	2940

3. Heat flow associated with long-lived magma of high temperature

In some volcanic areas, we have evidence to believe that the volcanic activity has been continually lasting in almost the same locality over a long period. In Izu District of Japan, where we find many tertiary and quarternary volcanoes, for instance, the volcanic activity seems to have been prevailing during the past few million years. According to H. Kuno[37], the fact may suggest that the source of the volcanic activity or a magma reservoir of high temperature has survived somehow in or under the earth's crust during the period. Such a long-lived magma of high temperature would possibly cause some anomaly in heat flow observed at the earth's surface. The purpose of this section is to estimate what anomaly will be found in heat flow when a

37) *Personal communication.*

certain area at some depth in the earth has been kept at a high temperature over a long period.

Let us consider an infinitely wide plate that is regarded as a model of the crust. The temperature at the upper surface is always kept at zero, while the part of the bottom surface defined by $-L/2<x<L/2$ is always kept at a constant temperature Θ. For the sake of mathematical simplicity, these conditions are assumed to repeat periodically with an interval $2l$ in the x direction as can be seen in Fig. 3. If things do not depend on y, the temperature in the plate satisfies

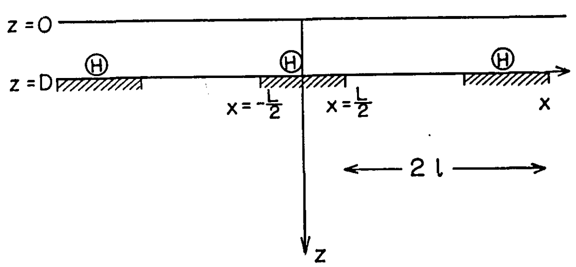

Fig. 3. The model of the crust, the hatched parts being kept at a constant temperature Θ.

$$\frac{\partial u}{\partial t}=\kappa\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right), \tag{20}$$

which can be written as

$$pu=\kappa\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}\right), \tag{21}$$

by replacing $\partial/\partial t$ by time-operator p, while the initial temperature is assumed to be zero throughout the plate. κ denotes the thermal diffusivity as usual.

The typical solution of (21), that satisfies $u=0$ at $z=0$, is given by

$$u=C_m \cos\frac{m\pi x}{l}\sinh\sqrt{(m\pi/l)^2+\kappa^{-1}p}\, z\,. \tag{22}$$

Since the temperature at $z=D$ should be

$$u_{z=D}=\Theta\left[\frac{L}{2l}+\frac{2}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}\sin\frac{m\pi L}{2l}\cos\frac{m\pi x}{l}\right], \tag{23}$$

C_m is chosen so as to satisfy (23). In that case, the solution, that satisfies all the boundary conditions becomes

$$u=\Theta\left[\frac{L}{2l}\frac{\sinh \kappa^{-1/2}p^{1/2}z}{\sinh \kappa^{-1/2}p^{1/2}D}\right.$$
$$\left.+\frac{2}{\pi}\sum_{m=1}^{\infty}\frac{\sin(m\pi L/2l)\sinh\{(m\pi/l)^2+\kappa^{-1}p\}^{1/2}z}{m\sinh\{(m\pi/l)^2+\kappa^{-1}p\}^{1/2}D}\cos\frac{m\pi x}{l}\right]. \quad (24)$$

Putting $p\rightarrow 0$, let us hereafter consider the steady state which is given as

$$u=\Theta\left[\frac{Lz}{2lD}+\frac{2}{\pi}\sum_{m=1}^{\infty}\frac{\sin(m\pi L/2l)\sinh(m\pi z/l)}{m\sinh\frac{(m\pi D)}{l}}\cos\frac{m\pi x}{l}\right]. \quad (25)$$

From (25), the steady temperature gradient at $z=0$ and $x=0$ becomes

$$-\left(\frac{\partial u}{\partial z}\right)_{x=0,\,z=0}=-\Theta\left[\frac{L}{2lD}+\frac{2}{l}\sum_{m=1}^{\infty}\frac{\sin(m\pi L/2l)}{\sinh(m\pi D/l)}\right].$$

Assuming $\Theta=1000°C$ and $l=100\,km$, the temperature gradient is calculated for various combinations of L and D as given in Table 3.

Table 3. Temperature gradient at $z=0$ and $x=0$ in units of $°C/km$.

L \ D	5 km	10 km	15 km	20 km
5 km	131.2	37.4	17.1	9.7
10 km	181.3	65.1	31.9	18.7
20 km	194.2	89.6	51.0	32.3
30 km	196.1	96.7	60.4	41.0

If the thermal conductivity is assumed to take a usual value for rocks, $5\times10^{-3}\,cal/cm\,sec\,°C$ say, the gradient values in Table 3 give heat flows as shown in Fig. 4. It is seen in the figure that appreciable heat flows can be only found in the cases of shallow magma. If we take a set of likely values, $L=10\,km$ and $D=20\,km$ say, the heat flow amounts to only $0.5\times10^{-6}\,cal/sec\,cm^2$ which is about half as large as the mean value of general heat flow. It is of difficulty, therefore, to detect a magma reservoir of $1000°C$ in temperature by observing heat flow anomaly unless the magma is situated at an extremely shallow depth, 10 km or less. It should be also noted that the heat flow

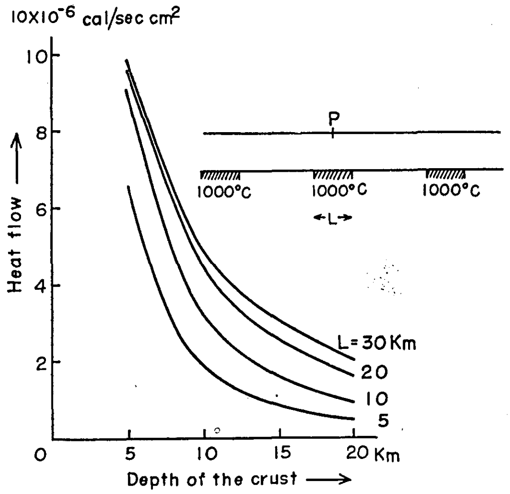

Fig. 4. Steady heat flows at the earth's surface right above the hot magma.

values depend on the width of the hot region rather largely.

4. Phase transformation and heat flow

Liberation or absorption of latent heat takes place in the earth at times of crystallization or fusion. Strictly speaking, therefore, problems related to cooling of intrusive magma must be studied with proper considerations of latent heat. It is extremely difficult, however, to take into account the movement of the solid-liquid boundary in solving the differential equation of heat conduction. Only a few studies in this line have been made by Jaeger[38] by introducing the temperature of "contact". According to Jaeger, the initial contact temperature immediately after the intrusion of magma, of which the latent heat is assumed as 100 *cal/gm* would be larger than that for a hypothetical liquid of zero latent heat by a factor of 1.2 or so. For an order-of-magnitude estimate of heat flow associated with magma intrusion, therefore, the effect of phase change does not seem to play a very important role.

In this section, let us consider a subterranean sheet in which some phase transformation is occurring. Ignoring the displacement of the liquid-solid boundary, the rate of heat generation is assumed in the

38) J. C. JAEGER, *Amer. Journ. Sci.*, **255** (1957), 306, **257** (1959), 44.

sheet. Although such a treatment should be regarded as a crude approximation, the heat flow mainly due to phase transformation could be investigated to some extent. Since the coupling between the displacement of boundary and heat conduction is ignored in this case, the total heat flow is given by the sum of that due to the conduction and that due to the phase transformation.

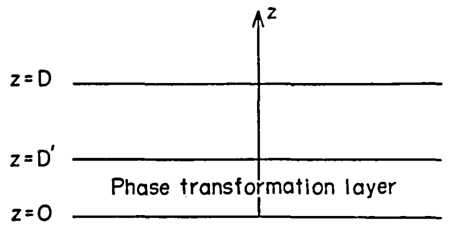

Fig. 5. The model of the crust underlain by a layer in which some phase transformation is taking place.

Suppose that an infinitely extending layer of phase transformation as can be seen in Fig. 5. At the lower boundary of the layer, we simply assume that the temperature is kept at a constant denoted by Θ. Meanwhile at the earth's surface, the temperature is always zero.

In the region between $z=D$ and $z=D'$, where we have no phase change, the temperature is governed by the usual differential equation of heat conduction and given as

$$u_1=C_1 e^{\kappa_1^{-1/2} p^{1/2} z}+\overline{C}_1 e^{-\kappa_1^{-1/2} p^{1/2} z}, \tag{27}$$

where the notations have the usual meanings. The equation in the phase transformation layer becomes

$$\frac{1}{\kappa_2}\frac{\partial u_2}{\partial t}=\frac{\partial^2 u_2}{\partial z^2}+\frac{A_0}{K_2}, \tag{28}$$

where A_0 is the heat produced in a unit volume per unit time. The solution of (28) is given as

$$u_2=C_2 e^{\kappa_2^{-1/2} p^{1/2} z}+\overline{C}_2 e^{-\kappa_2^{-1/2} p^{1/2} z}-\frac{A_0}{2K_2}z^2 . \tag{29}$$

The boundary conditions which are to be satisfied by u_1 and u_2 are

$$\left.\begin{aligned} &z=0: u_2=\Theta , \\ &z=D': u_1=u_2 , \quad K_1\frac{\partial u_1}{\partial z}=K_2\frac{\partial u_2}{\partial z} , \\ &z=D: u_1=0 . \end{aligned}\right\} \tag{30}$$

From these four conditions, the constants in (27) and (29) are determined getting, for instance,

$$\left.\begin{matrix} C_1 \\ \overline{C}_1 \end{matrix}\right\} = \mp e^{-\kappa^{-1/2}p^{1/2}D}$$

$$\times \frac{\Theta - \dfrac{A_0 D'}{2K}\left[\left(\dfrac{D'}{2} - \kappa^{1/2}p^{-1/2}\right)e^{\kappa^{-1/2}p^{1/2}D'} + \left(\dfrac{D'}{2} + \kappa^{1/2}p^{-1/2}\right)e^{-\kappa^{-1/2}p^{1/2}D'}\right]}{e^{\kappa^{1/2}p^{1/2}D} - e^{-\kappa^{-1/2}p^{1/2}D}}, \quad (31)$$

while $\kappa_1 = \kappa_2 = \kappa$ and $K_1 = K_2 = K$ are assumed again.

With (31) the temperature gradient at the earth's surface is calculated as

$$\left(\frac{\partial u_1}{\partial z}\right)_{z=D} = -\frac{\kappa^{-1/2}p^{1/2}}{\sinh(\kappa^{-1/2}p^{1/2}D)}\left[\Theta - \frac{A_0 D'}{2K}\left\{D' \cosh(\kappa^{-1/2}p^{1/2}D') - \frac{2\sinh(\kappa^{-1/2}p^{1/2}D'}{\kappa^{-1/2}p^{1/2}}\right\}\right]. \quad (32)$$

Putting $p \to 0$, the steady gradient is given as

$$\left(\frac{\partial u_1}{\partial z}\right)_{z=D} = -\frac{\Theta}{D} - \frac{A_0 D'^2}{2KD}. \quad (33)$$

Since the first term of (33) is led from the condition that the temperature is kept at Θ at the bottom of the layer, only the second term is important in order to see the influence of the phase transformation.

Suppose that the latent heat amounts to 100 *cal/gm*, a likely value for magma, and that f *years* are required to solidify unit volume of magma, A_0 amounts to $f^{-1} \times 0.86 \times 10^{-6}$ *cal/cm³ sec* provided the density is taken at 2.7 *gm/cm³*. The surface heat flow is then given by $0.43 \times 10^{-6} \times (D'/D) \times f^{-1}D'$ *cal/cm² sec* from which we may estimate heat flow values for some combinations of D, D' and f as are given in Table 4.

Table 4. Heat flow values associated with solidification of magma in units of *cal/cm² sec*.

D'/D \ $f^{-1}D'$ (*km/year*)	10^{-5}	10^{-4}	10^{-3}	10^{-2}
1	4.3×10^{-7}	4.3×10^{-6}	4.3×10^{-5}	4.3×10^{-4}
0.1	4.3×10^{-8}	4.3×10^{-7}	4.3×10^{-6}	4.3×10^{-5}
0.01	4.3×10^{-9}	4.3×10^{-8}	4.3×10^{-7}	4.3×10^{-6}

We can see in the table that an appreciable heat flow, larger than $10^{-6}\,cal/cm^2\,sec$ say, takes place for $f^{-1}D'>10^{-4}\,km/year$ even in the case of $D'/D=1$. For smaller values of D'/D, much larger $f^{-1}D'$ is required. It has been known, however, that the isothermals are displaced with a speed of the order of $1\,cm/year$ or $10^{-5}\,km/year$ in the cases of cooling of magma mass having a linear scale of a few kilometers. The figure would become smaller if a bigger mass of magma were supposed. We therefore see that the heat flow considered here hardly reaches $10^{-6}\,cal/cm^2\,sec$. Even in a favourable case, heat flows of this origin would only be a fraction of the general one.

5. Concluding remarks

Terrestrial heat flows related to three possible events, namely subcrustal convection, long-living magma reservoir and phase transformation layer are estimated.

It turns out that a steady subcrustal convection current having a linear dimension of a few thousand kilometers can give rise to a heat flow several times as large as the world's average provided the velocity of several *cm/year* and temperature gradient of some $1°C/km$ are assumed in the earth's mantle. It is then conceivable that the high heat flow values observed on the East Pacific Rise may be caused by some convection currents in the earth's mantle. The high temperature suggested in this study at the bottom of the crust seems to harmonize well with the low velocity of seismic waves[39].

The possible heat flows associated with a high-temperature magma reservoir supposed in the earth's crust are not very large. It might not be possible to detect the magma by an observation of anomaly in terrestrial heat flow unless its depth does not exceed $10\,km$ or so.

It seems also likely that no conceivable phase transformation layers give a considerable heat flow anomaly. But some fractions of heat flow values may be caused by subcrustal convection of a small scale, underground magma mass being kept at a high temperature over a long period or presence of a layer in which some crystallization is taking place. It is not practicable to correlate these geophysical events with the small anomalies observed in heat flow. The only thing that seems quite certain is the fact that the high heat flows such as found on the East Pacific Rise cannot be explained unless we suppose

39) G. C. SHOR, *Information Bulletin, Pacific Sci. Ass.*, **11** No. 6 (1959), 12.

a large-scale convection current under the earth's crust.

There are many other geophysical events which may be associated with anomalies in heat flow. One of the most important problems is the possible coupling between mechanical strain and heat flow. Although it is no easy matter to tackle such an irresversible thermodynamical process, extensive investigations in this line are strongly recommened to perform on the basis of well-observed data of heat flow, crustal deformation and seismic activity.

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25. 地球熱学 (第4報) マントル内熱対流その他に伴なう熱流量

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地球マントル内の定常熱対流に伴なう地表面熱流量を計算し，通常想像されているように温度勾配を 1°*C*/*km*, 流速を数 *cm*/*year* 程度と仮定すれば，波長 1000 *km* 程度以上の対流においては，East Pacific Rise 海底で観測されるような大きな熱流量の値が期待されることを示した.

つぎに地殻内に高温マグマや相転移層が長期間存在する場合の定常熱伝導を調べたが，熱源がきわめて地表に近い場合を除けば，10^{-6} *cal*/*cm*^{2} *sec* 以上の熱流量を期待することはほとんど不可能であり，このような場合には熱流量の異常を調べて，地下の状態を推定することは困難である.