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10. *Reflection and Refraction of Elastic Waves at a Corrugated Boundary Surface. Part I. The Case of Incidence of SH Wave.*

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1. Introduction

To say nothing of the small scale geological structures near the earth's surface, we have good reason to believe that the crustal layers in the earth's interior are not restricted to plane surfaces. It seems to the writer that the following facts may be attributed at least in part to the undulations of the boundary surfaces of the earth's crustal layers.

1) Gravity anomalies which are well-explained by the Airy hypothesis of isostasy.

2) Irregular intensity distributions of earthquakes other than those attributable to the surface ground conditions.

3) The obliteration of reflected artificial earthquake waves from the Moho (Mohorovičić discontinuity) as observed by H. E. Tatel, L. H. Adams, and M. A. Tuve in western California[1] where the lower crustal boundary is considered to be roughly disturbed as is easily inferred from the isostatic view point by the topographical irregularity.

4) The larger attenuation of artificial seismic waves across the Andes as compared to the path parallel to the mountain range, as observed by H. E. Tatel, M. A. Tuve and their colleagues.[2]

In view of these facts, we are naturally led to the inference that the roughness or the irregular undulation of a boundary surface of the media, through which the seismic wave is propagated, plays a certain

1) H. E. TATEL, L. H. ADAMS, and M. A. TUVE, *Proc. Amer. Phil. Soc.*, **97** (1953), 658.

2) H. E. TATEL, M. A. TUVE, and COLLEAGUES, "Abstract of paper presented at 39th Annual Meeting of American Geophysical Union held in 1958," *Trans. Amer. Geophys. Un.*, **39** (1958), 533.

role in the energy distribution of reflected and refracted waves according to the relative situations to the boundary surface. It is therefore desirable to study the effect of the undulation of the boundary surface on the reflection and refraction of elastic waves theoretically. In this paper the writer intends to study this problem under a simplified assumption that the undulation of the boundary surface is only in one direction, and the incident wave is a plane wave whose plane of incidence is perpendicular to the boundary surface.

Although there may be various methods to deal with this sort of problem, in this paper, the writer adopted the method which was first used by Lord Rayleigh in his classical theory of gratings,[3] and recently applied by R. Sato in his theoretical study of reflection of elastic waves at an undùlatory free surface.[4]

In the present part, discussions are restricted to the case of incident SH waves, and the other cases of P and SV waves will be reported in the near future.

2. Equation of boundary surface

The coordinate axes were taken as shown in Fig. 1. That is, x- and y-axis are on the horizontal surface, and z-axis is taken vertically downwards. The equation of boundary surface is given by $z=\zeta$, where ζ is a periodic function of x and independent of y, the mean value of which being zero. Then ζ can be represented by Fourier's series as follows:

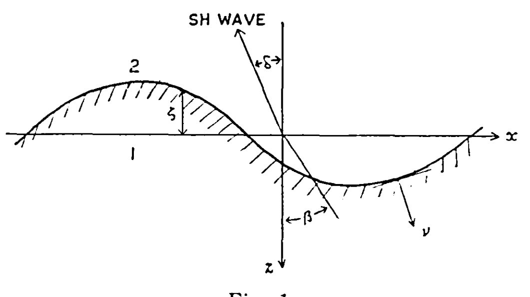

Fig. 1.

$$\zeta=\sum_{n=1}^{\infty}(\zeta_n e^{inpx}+\zeta_{-n}e^{-inpx})$$

$$=c_1\cos px+c_2\cos 2px+s_2\sin 2px+\cdots+c_n\cos npx+s_n\sin npx+\cdots,$$

where

$$\zeta_1=\zeta_{-1}=c_1/2\ ,$$

$$\zeta_{\pm n}=(c_n\mp is_n)/2\ .$$

3) LORD RAYLEIGH, *Proc. Roy. Soc. Lond.*, **A, 79** (1907), 399.

4) R. SATO, *Zisin*, [ii], **8** (1955), 8, (in Japanese).

When the boundary surface is expressible by one cosine term, i.e. $\zeta = c\cos px$, the wave length of the corrugation is $2\pi/p$.

3. Equations of motion, and incident, reflected and refracted wave

Let us consider the case when the plane SH wave with period of $2\pi/\omega$ is incident with an angle of β to z-axis from the lower to the upper medium. The quantities concerning the lower medium will be shown by suffix 1 and those of the upper medium, by suffix 2. If the displacement v of incident SH wave is parallel to the y-axis, the equations of motion are as follows:

$$(\nabla^2+\sigma_i h_i^2)v_i=0\ , \qquad (i=1,\ 2)$$

where
$$\nabla^2=\text{Laplacian},$$
$$\sigma_i=(\lambda_i+2\mu_i)/\mu_i\ ,$$
$$h_i^2=\rho_i\omega^2/(\lambda_i+2\mu_i)\ ,$$
$$\lambda_i,\ \mu_i=\text{Lamé's constants},$$
$$\rho_i=\text{density}.$$

It is sufficient to consider only the following solutions of above equations if the boundary surface is plane.

Incident wave: $v_1=e^{i\sqrt{\sigma_1}h_1(V_{s1}t+z\cos\beta+x\sin\beta)}$,
Regularly reflected wave: $v_1=B_0e^{i\sqrt{\sigma_1}h_1(V_{s1}t-z\cos\beta+x\sin\beta)}$,
Regularly refracted wave: $v_2=D_0e^{i\sqrt{\sigma_2}h_2(V_{s2}t+z\cos\delta+x\sin\delta)}$,

where V_{s1} and V_{s2} are the velocity of S waves in the lower and upper medium respectively and δ is the angle of refraction which is connected with incident angle β by Snell's law

$$\sqrt{\sigma_1}\,h_1\sin\beta=\sqrt{\sigma_2}\,h_2\sin\delta\ .$$

In the case of corrugated boundary surface it is necessary to take, in addition to these regular waves, the effect of corrugation on reflections and refractions of elastic waves into consideration and introduce the following irregular waves:

(1) Irregularly reflected waves whose spectrum of the nth order is:

$$v_1=B_n e^{i\sqrt{\sigma_1}h_1(V_{s1}t-z\cos\beta_n+x\sin\beta_n)}+B'_n e^{i\sqrt{\sigma_1}h_1(V_{s1}t-z\cos\beta'_n+x\sin\beta'_n)}$$

(2) Irregularly refracted waves whose spectrum of the nth order is:

$$v_2=D_n e^{i\sqrt{\sigma_2}h_2(V_{s2}t+z\cos\delta_n+x\sin\delta_n)}+D'_n e^{i\sqrt{\sigma_2}h_2(V_{s2}t+z\cos\delta'_n+x\sin\delta'_n)}$$

where β_n, β'_n, δ_n and δ'_n are given by the following spectrum theorem,

$$\sin\beta_n-\sin\beta=np/(\sqrt{\sigma_1}\,h_1)\,, \qquad \sin\beta'_n-\sin\beta=-np/(\sqrt{\sigma_1}\,h_1)\,,$$
$$\sin\delta_n-\sin\delta=np/(\sqrt{\sigma_2}\,h_2)\,, \qquad \sin\delta'_n-\sin\delta=-np/(\sqrt{\sigma_2}\,h_2)\,.$$

The above relations have physical meanings that only in the directions with difference of path of n wave lengths of SH wave the waves can propagate. The variation of v, for example for irregularly reflected waves, in the direction parallel to x-axis at a certain fixed level z in the case of normal incidence, i.e. $\beta=0$ is given by the sum of factors $e^{i\sqrt{\sigma_1}h_1x\sin\beta_n}=e^{inpx}$ and $e^{i\sqrt{\sigma_1}h_1x\sin\beta'_n}=e^{-inpx}$. That is to say, in the x-direction at a fixed level, the variation of v is a harmonic function of x. This agrees with that of diffraction of light.

The total displacement v_1 in the lower medium is the sum of those of incident, regularly reflected and irregularly reflected waves. That is, by the spectrum theorem and neglecting common time factor $e^{i\omega t}$,

$$v_1=e^{i\sqrt{\sigma_1}h_1x\sin\beta}\Big[e^{i\sqrt{\sigma_1}h_1z\cos\beta}+B_0e^{-i\sqrt{\sigma_1}h_1z\cos\beta}+\sum_n B_ne^{inpx}e^{-i\sqrt{\sigma_1}h_1z\cos\beta_n}$$
$$+\sum_n B'_ne^{-inpx}e^{-i\sqrt{\sigma_1}h_1z\cos\beta'_n}\Big]\,.$$

Similarly,

$$v_2=e^{i\sqrt{\sigma_1}h_1x\sin\beta}\Big[D_0e^{i\sqrt{\sigma_2}h_2z\cos\delta}+\sum_n D_ne^{inpx}e^{i\sqrt{\sigma_2}h_2z\cos\delta_n}$$
$$+\sum_n D'_ne^{-inpx}e^{i\sqrt{\sigma_2}h_2z\cos\delta'_n}\Big]\,.$$

4. Boundary conditions

The boundary conditions are given by the condition that y-component of normal stress of boundary surface and the displacement are continuous at $z=\zeta$.

The y-component of normal stress of boundary surface Y_ν is given as

$$Y_\nu=[Y_z-Y_x\zeta']/\sqrt{1+\zeta'^2}\,.$$

In the case of SH wave, $Y_\nu=\mu(\partial v/\partial z-\partial v/\partial x\cdot\zeta')/\sqrt{1+\zeta'^2}$. Since ζ' is independent of medium, and depends only on the functional form of ζ, it is enough to ensure the continuity of $\sqrt{1+\zeta'^2}\cdot Y_\nu$ at $z=\zeta$ instead of the condition of continuity of Y_ν. From the condition of continuity of v at $z=\zeta$,

$$\begin{aligned}
&e^{i\sqrt{\sigma_1}h_1\zeta\cos\beta}+B_0e^{-i\sqrt{\sigma_1}h_1\zeta\cos\beta}+\sum_n B_ne^{inpx}e^{-i\sqrt{\sigma_1}h_1\zeta\cos\beta_n}\\
&\qquad+\sum_n B'_ne^{-inpx}e^{-i\sqrt{\sigma_1}h_1\zeta\cos\beta'_n}\\
&\qquad=D_0e^{i\sqrt{\sigma_2}h_2\zeta\cos\delta}+\sum_n D_ne^{inpx}e^{i\sqrt{\sigma_2}h_2\zeta\cos\delta_n}+\sum_n D'_ne^{-inpx}e^{i\sqrt{\sigma_2}h_2\zeta\cos\delta'_n}\,.
\end{aligned} \tag{1}$$

From the condition of continuity of $\sqrt{1+\zeta'^2}\cdot Y_\nu$ at $z=\zeta$,

$$\begin{aligned}
&\mu_1[\sqrt{\sigma_1}\,h_1(\cos\beta-\zeta'\sin\beta)e^{i\sqrt{\sigma_1}h_1\zeta\cos\beta}-\sqrt{\sigma_1}\,h_1(\cos\beta+\zeta'\sin\beta)B_0e^{-i\sqrt{\sigma_1}h_1\zeta\cos\beta}\\
&\quad-\sum_n\{\sqrt{\sigma_1}\,h_1\cos\beta_n+(\sqrt{\sigma_1}\,h_1\sin\beta+np)\zeta'\}B_ne^{inpx}e^{-i\sqrt{\sigma_1}h_1\zeta\cos\beta_n}\\
&\quad-\sum_n\{\sqrt{\sigma_1}\,h_1\cos\beta'_n+(\sqrt{\sigma_1}\,h_1\sin\beta-np)\zeta'\}B'_ne^{-inpx-i\sqrt{\sigma_1}h_1\zeta\cos\beta'_n}]\\
&=\mu_2[(\sqrt{\sigma_2}\,h_2\cos\delta-\sqrt{\sigma_1}\,h_1\zeta'\sin\beta)D_0e^{i\sqrt{\sigma_2}h_2\zeta\cos\delta}\\
&\quad+\sum_n\{\sqrt{\sigma_2}\,h_2\cos\delta_n-\zeta'(\sqrt{\sigma_1}\,h_1\sin\beta+np)\}D_ne^{inpx}e^{i\sqrt{\sigma_2}h_2\zeta\cos\delta_n}\\
&\quad+\sum_n\{\sqrt{\sigma_2}\,h_2\cos\delta'_n-\zeta'(\sqrt{\sigma_2}\,h_2\sin\beta-np)\}D'_ne^{-inpx}e^{i\sqrt{\sigma_2}h_2\zeta\cos\delta'_n}]\,.
\end{aligned} \tag{2}$$

5. The solutions of the first approximation

Now the extent of corrugation, ζ, is assumed as very small and by iteration the approximations are advanced. First of all, the solutions of the first approximation for B_0 and D_0 can be obtained by picking up the term independent of x and ζ in (1) and (2).

$$1+B_0=D_0$$

$$\mu_1\sqrt{\sigma_1}\,h_1(1-B_0)\cos\beta-\mu_2\sqrt{\sigma_2}\,h_2D_0\cos\delta=0\,.$$

These formulas give the amplitudes of reflected and refracted waves in the case of plane boundary surface. From these two formulas, B_0 and D_0 can be solved as follows:

$$B_0=(\mu_1\sqrt{\sigma_1}\,h_1\cos\beta-\mu_2\sqrt{\sigma_2}\,h_2\cos\delta)/(\mu_1\sqrt{\sigma_1}\,h_1\cos\beta+\mu_2\sqrt{\sigma_2}\,h_2\cos\delta)\ , \tag{3}$$

$$D_0=2\mu_1\sqrt{\sigma_1}\,h_1\cos\beta/(\mu_1\sqrt{\sigma_1}\,h_1\cos\beta+\mu_2\sqrt{\sigma_2}\,h_2\cos\delta)\ . \tag{4}$$

From the coefficients of e^{inpx} in (1) and (2), the solutions of the first approximation for B_n and D_n can be obtained as follows:

$$B_n-D_n=-i\sqrt{\sigma_1}\,h_1\zeta_n(1-B_0)\cos\beta+i\sqrt{\sigma_2}\,h_2\zeta_nD_0\cos\delta\ ,$$

$$\begin{aligned}&\mu_1B_n\sqrt{\sigma_1}\,h_1\cos\beta_n+\mu_2D_n\sqrt{\sigma_2}\,h_2\cos\delta_n\\&\quad=i\mu_1\sqrt{\sigma_1}\,h_1\zeta_n(1+B_0)(\sqrt{\sigma_1}\,h_1\cos^2\beta-np\sin\beta)\\&\qquad-i\mu_2\zeta_nD_0(\sigma_2h_2^2\cos^2\delta-\sqrt{\sigma_1}\,h_1np\sin\beta)\ .\end{aligned}$$

Similarly from the coefficients of e^{-inpx} in (1) and (2), the solutions of the first approximation for B'_n and D'_n can be obtained as follows:

$$B'_n-D'_n=-i(1-B_0)\sqrt{\sigma_1}\,h_1\zeta_{-n}\cos\beta+iD_0\sqrt{\sigma_2}\,h_2\zeta_{-n}\cos\delta\ ,$$

$$\begin{aligned}&\mu_1\sqrt{\sigma_1}\,h_1B'_n\cos\beta'_n+\mu_2\sqrt{\sigma_2}\,h_2D'_n\cos\delta'_n\\&\quad=i\mu_1\sqrt{\sigma_1}\,h_1\zeta_{-n}(1+B_0)(\sqrt{\sigma_1}\,h_1\cos^2\beta+np\sin\beta)\\&\qquad-i\mu_2\zeta_{-n}D_0(\sqrt{\sigma_1}\,h_1np\sin\beta+\sigma_2h_2^2\cos^2\delta)\ .\end{aligned}$$

For B_0 and D_0 involved in the above formulas (3) and (4) are used. From the above formulas we can see that the solutions of the first approximation for B_n, D_n, B'_n, and D'_n are proportional to ζ_n or ζ_{-n} and of order of ζ_n or ζ_{-n} as in the case of reflection of elastic waves at a corrugated free surface.[5)]

6. The solutions of the second approximation

Next if the terms of higher order than ζ^2 in the equations (1) and (2) are neglected, the following two equations are obtained.

$$\begin{aligned}&1+i\sqrt{\sigma_1}\,h_1\zeta\cos\beta+B_0(1-i\sqrt{\sigma_1}\,h_1\zeta\cos\beta)\\&\quad+\sum_n B_ne^{inpx}(1-i\sqrt{\sigma_1}\,h_1\zeta\cos\beta_n)+\sum_n B'_ne^{-inpx}(1-i\sqrt{\sigma_1}\,h_1\zeta\cos\beta'_n)\\&=D_0(1+i\sqrt{\sigma_2}\,h_2\zeta\cos\delta)+\sum_n D_ne^{inpx}(1+i\sqrt{\sigma_2}\,h_2\zeta\cos\delta_n)\\&\quad+\sum_n D'_ne^{-inpx}(1+i\sqrt{\sigma_2}\,h_2\zeta\cos\delta'_n)\ ,\end{aligned}$$

$$\begin{aligned}&\mu_1[\sqrt{\sigma_1}\,h_1(1-B_0)\cos\beta+\sqrt{\sigma_1}\,h_1(1+B_0)(i\sqrt{\sigma_1}\,h_1\zeta\cos^2\beta-\zeta'\sin\beta)\\&\quad-\sum_n\{\sqrt{\sigma_1}\,h_1\cos\beta_n+(\sqrt{\sigma_1}\,h_1\sin\beta+np)\zeta'-i\sigma_1h_1^2\zeta\cos^2\beta_n\}B_ne^{inpx}\end{aligned}$$

5) R. SATO, *loc. cit.*, 4).

$$-\sum_n \{\sqrt{\sigma_1}\, h_1 \cos\beta'_n + (\sqrt{\sigma_1}\, h_1 \sin\beta - np)\zeta' - i\sigma_1 h_1^2 \zeta \cos^2\beta'_n\} B'_n e^{-inpx}]$$

$$=\mu_2[(\sqrt{\sigma_2}\, h_2 \cos\delta + i\sigma_2 h_2^2 \zeta \cos^2\delta - \sqrt{\sigma_1}\, h_1 \zeta' \sin\beta) D_0$$

$$+\sum_n \{\sqrt{\sigma_2}\, h_2 \cos\delta_n + i\sigma_2 h_2^2 \zeta \cos^2\delta_n - \zeta'(\sqrt{\sigma_1}\, h_1 \sin\beta + np)\} D_n e^{inpx}$$

$$+\sum_n \{\sqrt{\sigma_2}\, h_2 \cos\delta'_n + i\sigma_2 h_2^2 \zeta \cos^2\delta'_n - \zeta'(\sqrt{\sigma_1}\, h_1 \sin\beta - np)\} D'_n e^{-inpx}.$$

From above two equations we can obtain the solutions of the second approximation for B_0 and D_0 by picking up the terms independent of x. They are as follows:

$$(\mu_1\sqrt{\sigma_1}\, h_1 \cos\beta + \mu_2\sqrt{\sigma_2}\, h_2 \cos\delta) D_0$$

$$=2\mu_1\sqrt{\sigma_1}\, h_1 \cos\beta + i\mu_1 \sum_n B_n \zeta_{-n} \{np(\sqrt{\sigma_1}\, h_1 \sin\beta + np)$$

$$-\sigma_1 h_1^2 (\cos\beta - \cos\beta_n) \cos\beta_n\}$$

$$-i\mu_1 \sum_n B'_n \zeta_n \{np(\sqrt{\sigma_1}\, h_1 \sin\beta - np) + \sigma_1 h_1^2 (\cos\beta - \cos\beta'_n) \cos\beta'_n\}$$

$$-i \sum_n D_n \zeta_{-n} \{\mu_2 np(\sqrt{\sigma_1}\, h_1 \sin\beta + np)$$

$$+\sqrt{\sigma_2}\, h_2 (\mu_1\sqrt{\sigma_1}\, h_1 \cos\beta + \mu_2\sqrt{\sigma_2}\, h_2 \cos\delta_n) \cos\delta_n\}$$

$$+i \sum_n D'_n \zeta_n \{\mu_2 np(\sqrt{\sigma_1}\, h_1 \sin\beta - np)$$

$$-\sqrt{\sigma_2}\, h_2 (\mu_1\sqrt{\sigma_1}\, h_1 \cos\beta + \mu_2\sqrt{\sigma_2}\, h_2 \cos\delta'_n) \cos\delta'_n\}\ ,$$

$$B_0 - D_0 = -1 + \sum_n iB_n \zeta_{-n} \sqrt{\sigma_1}\, h_1 \cos\beta_n + \sum_n iB'_n \zeta_n \sqrt{\sigma_1}\, h_1 \cos\beta'_n$$

$$+\sum_n iD_n \zeta_{-n} \sqrt{\sigma_2}\, h_2 \cos\delta_n + \sum_n iD'_n \zeta_n \sqrt{\sigma_2}\, h_2 \cos\delta'_n\ ,$$

where for B_n, D_n, etc. those of the first approximation are used.

From the coefficients of e^{inpx} the solutions of the second approximation for B_n and D_n can be obtained as follows:

$$B_n - D_n = -i\sqrt{\sigma_1}\, h_1 \zeta_n (1 - B_0) \cos\beta + i\sqrt{\sigma_2}\, h_2 D_0 \zeta_n \cos\delta$$

$$+ i\left(\sum_{j=1}^{n-1} + \sum_{j=n+1}^{\infty}\right) B_j \zeta_{n-j} \sqrt{\sigma_1}\, h_1 \cos\beta_j + i \sum_{j=n+1}^{\infty} B'_{j-n} \zeta_j \sqrt{\sigma_1}\, h_1 \cos\beta'_{j-n}$$

$$+ i\left(\sum_{j=1}^{n-1} + \sum_{j=n+1}^{\infty}\right) D_j \zeta_{n-j} \sqrt{\sigma_2}\, h_2 \cos\delta_j + i \sum_{j=n+1}^{\infty} D'_{j-n} \zeta_j \sqrt{\sigma_2}\, h_2 \cos\delta'_{j-n}\ ,$$

$$\mu_1 B_n \sqrt{\sigma_1}\, h_1 \cos\beta_n + \mu_2 D_n \sqrt{\sigma_2}\, h_2 \cos\delta_n$$

$$= i\mu_1\sqrt{\sigma_1}\, h_1 \zeta_n (1 + B_0)(\sqrt{\sigma_1}\, h_1 \cos^2\beta - np \sin\beta)$$

$$- i\mu_2 \zeta_n D_0 (\sigma_2 h_2^2 \cos^2\delta - \sqrt{\sigma_1}\, h_1\, np \sin\beta)$$

$$-i\mu_1\left(\sum_{j=1}^{n-1}+\sum_{j=n+1}^{\infty}\right)\{p(n-j)(\sqrt{\sigma_1}\,h_1\sin\beta+jp)-\sigma_1h_1^2\cos^2\beta_j\}\zeta_{n-j}B_j$$

$$-i\mu_1\sum_{j=n+1}^{\infty}[jp\{\sqrt{\sigma_1}\,h_1\sin\beta+(n-j)p\}-\sigma_1h_1^2\cos^2\beta'_{j-n}]\zeta_jB'_{j-n}$$

$$-i\mu_2\left(\sum_{j=1}^{n-1}+\sum_{j=n+1}^{\infty}\right)\{\sigma_2h_2^2\cos^2\delta_j-p(n-j)(\sqrt{\sigma_1}\,h_1\sin\beta+jp)\}D_j\zeta_{n-j}$$

$$-i\mu_2\sum_{j=n+1}^{\infty}[\sigma_2h_2^2\cos^2\delta'_{j-n}-jp\{\sqrt{\sigma_1}\,h_1\sin\beta-(j-n)p\}]\zeta_jD'_{j-n}\,.$$

From the coefficients of e^{-inpx}, the following equations which determine B'_n and D'_n of the second approximation are obtained.

$$B'_n-D'_n=-i\sqrt{\sigma_1}\,h_1\zeta_{-n}(1-B_0)\cos\beta+i\sqrt{\sigma_2}\,h_2\zeta_{-n}D_0\cos\delta$$

$$+i\sqrt{\sigma_1}\,h_1\sum_{j=n+1}^{\infty}\zeta_{-j}B_{j-n}\cos\beta_{j-n}+i\sqrt{\sigma_1}\,h_1\left(\sum_{j=1}^{n-1}+\sum_{j=n+1}^{\infty}\right)B'_j\zeta_{j-n}\cos\beta'_j$$

$$+i\sqrt{\sigma_2}\,h_2\sum_{j=n+1}^{\infty}D_{j-n}\zeta_{-j}\cos\delta_{j-n}+i\sqrt{\sigma_2}\,h_2\left(\sum_{j=1}^{n-1}+\sum_{j=n+1}^{\infty}\right)D'_j\zeta_{j-n}\cos\delta'_j\,,$$

$$\mu_1B'_n\sqrt{\sigma_1}\,h_1\cos\beta'_n+\mu_2D'_n\sqrt{\sigma_2}\,h_2\cos\delta'_n$$

$$=i\mu_1\sqrt{\sigma_1}\,h_1\zeta_{-n}(1+B_0)(\sqrt{\sigma_1}\,h_1\cos^2\beta+np\sin\beta)$$

$$-i\mu_2\zeta_{-n}D_0(\sigma_2h_2^2\cos^2\delta+\sqrt{\sigma_1}\,h_1\,np\sin\beta)$$

$$+i\mu_1\sum_{j=n+1}^{\infty}[jp\{\sqrt{\sigma_1}\,h_1\sin\beta+(j-n)p\}+\sigma_1h_1^2\cos^2\beta_{j-n}]\zeta_{-j}B_{j-n}$$

$$+i\mu_1\left(\sum_{j=1}^{n-1}+\sum_{j=n+1}^{\infty}\right)\{p(n-j)(\sqrt{\sigma_1}\,h_1\sin\beta-jp)+\sigma_1h_1^2\cos^2\beta'_j\}\zeta_{j-n}B'_j$$

$$-i\mu_2\sum_{j=n+1}^{\infty}[\sigma_2h_2^2\cos^2\delta_{j-n}+jp\{\sqrt{\sigma_1}\,h_1\sin\beta+(j-n)p\}]\zeta_{-j}D_{j-n}$$

$$-i\mu_2\left(\sum_{j=1}^{n-1}+\sum_{j=n+1}^{\infty}\right)\{\sigma_2h_2^2\cos^2\delta'_j+p(n-j)(\sqrt{\sigma_1}\,h_1\sin\beta-jp)\}\zeta_{-n}D'_j\,.$$

By iterating this procedure, we can get solutions of sufficient approximation.

7. The case of normal incidence on the boundary surface $\zeta=c\cos px$

For the simplicity's sake, the case of normal incidence on the boundary surface given by the equation $\zeta=c\cos px$ was calculated. In this case, $\zeta_n=\zeta_{-n}=0\ (n\neq1)$, $\zeta_1=\zeta_{-1}=c/2$, $\beta=\delta=0$, $\cos\beta_1=\cos\beta'_1$, and $\cos\delta_1=\cos\delta'_1$. Then the solutions of the first approximation for B_0, D_0, B_1, and D_1 are given as follows:

$$B_0=(\mu_1\sqrt{\sigma_1}\,h_1-\mu_2\sqrt{\sigma_2}\,h_2)/(\mu_1\sqrt{\sigma_1}\,h_1+\mu_2\sqrt{\sigma_2}\,h_2)\ ,$$

$$D_0=2\mu_1\sqrt{\sigma_1}\,h_1/(\mu_1\sqrt{\sigma_1}\,h_1+\mu_2\sqrt{\sigma_2}\,h_2)\ ,$$

$$B_1-D_1=-i\sqrt{\sigma_1}\,h_1\zeta_1(1-B_0)+i\sqrt{\sigma_2}\,h_2\zeta_1D_0\ ,$$

$$\mu_1B_1\sqrt{\sigma_1}\,h_1\cos\beta_1+\mu_2D_1\sqrt{\sigma_2}\,h_2\cos\delta_1=i\mu_1\zeta_1(1+B_0)\sigma_1h_1^2-i\mu_2\zeta_1D_0\sigma_2h_2^2\ ,$$

$$B_1'=B_1\ ,\qquad D_1'=D_1\ .$$

The solutions of the second approximation for B_0, D_0, B_1, D_1, B_2 and D_2 are given as follows:

$$(\mu_1\sqrt{\sigma_1}\,h_1+\mu_2\sqrt{\sigma_2}\,h_2)D_0=2\mu_1\sqrt{\sigma_1}\,h_1+2i\mu_1B_1\zeta_{-1}\{p^2-\sigma_1h_1^2(1-\cos\beta_1)\cos\beta_1\}$$
$$-2iD_1\zeta_{-1}\{\mu_2p^2+\sqrt{\sigma_2}\,h_2(\mu_1\sqrt{\sigma_1}\,h_1+\mu_2\sqrt{\sigma_2}\,h_2\cos\delta_1)\cos\delta_1\}\ ,$$

$$B_0-D_0=-1+2iB_1\zeta_{-1}\sqrt{\sigma_1}\,h_1\cos\beta_1+2iD_1\zeta_{-1}\sqrt{\sigma_2}\,h_2\cos\delta_1\ ,$$

$$B_1-D_1=-i\sqrt{\sigma_1}\,h_1\zeta_1(1-B_0)+i\sqrt{\sigma_2}\,h_2D_0\zeta_1\ ,$$

$$\mu_1B_1\sqrt{\sigma_1}\,h_1\cos\beta_1+\mu_2D_1\sqrt{\sigma_2}\,h_2\cos\delta_1=i\mu_1\sigma_1h_1^2\zeta_1(1+B_0)-i\mu_2\sigma_2h_2^2\zeta_1D_0\ ,$$

$$B_2-D_2=iB_1\zeta_1\sqrt{\sigma_1}\,h_1\cos\beta_1+iD_1\zeta_1\sqrt{\sigma_2}\,h_2\cos\delta_1\ ,$$

$$\mu_1B_2\sqrt{\sigma_1}\,h_1\cos\beta_2+\mu_2D_2\sqrt{\sigma_2}\,h_2\cos\delta_2$$
$$=-i\mu_1\zeta_1B_1(p^2-\sigma_1h_1^2\cos^2\beta_1)+i\mu_2\zeta_1D_1(p^2-\sigma_2h_2^2\cos^2\delta_1)\ ,$$

$$B_1'=B_1\ ,\qquad D_1'=D_1\ ,\qquad B_2'=B_2\ ,\qquad D_2'=D_2\ .$$

In the above formulas, B_1 and D_1 are the same to those of the first approximation because terms of higher order than ζ are of order of ζ^3.

The constants necessary for the calculations are taken as follows:

$$V_{s1}/V_{s2}=3.5/4.6=V_{p1}/V_{p2}=6.0/8.0\ ,\qquad \mu_2/\mu_1=2.1\ ,$$

where V_{p1} and V_{p2} are the velocity of P waves in the lower and upper medium respectively, and Poisson's relation $\lambda_i=\mu_i$ is assumed. The results are given in Tables 1, 2, and 3, and Figs. 2–9.

From Figs. 2 and 3, we can see following trends as to B_0 and D_0. (1) The variations of B_0 and D_0 with the wave length L of corrugation are very small. (2) The influence of corrugation is larger on reflection than on refraction. As seen from Fig. 2(a) and Fig. 3(a), the difference of $|B_0|$ is larger than that of $|D_0|$ for different c/L_{s1} at the same L/L_{s1} where L_{s1} is the wave length of the incident SH wave. Furthermore, this result is ascertained by Fig. 6, and Fig. 7,

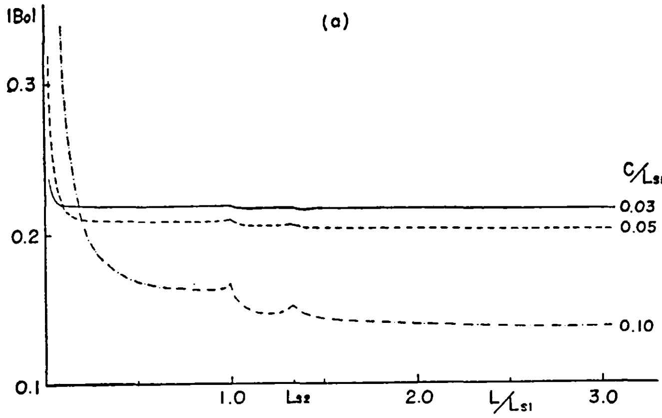

Fig. 2 (a). $|B_0|$.

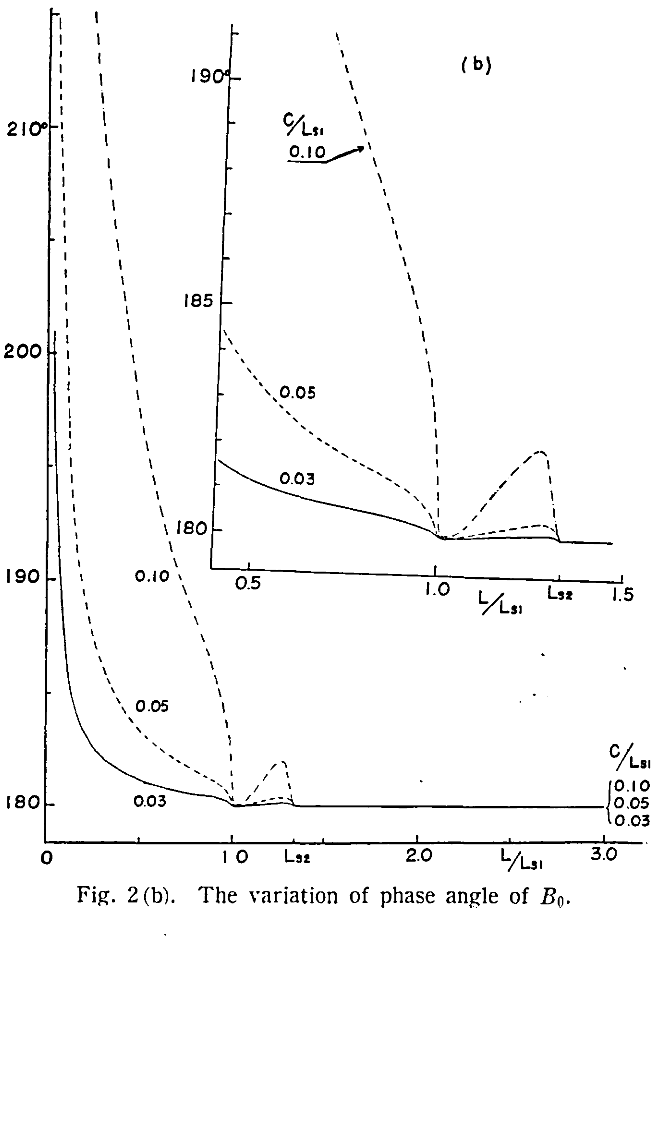

Fig. 2 (b). The variation of phase angle of B_0.

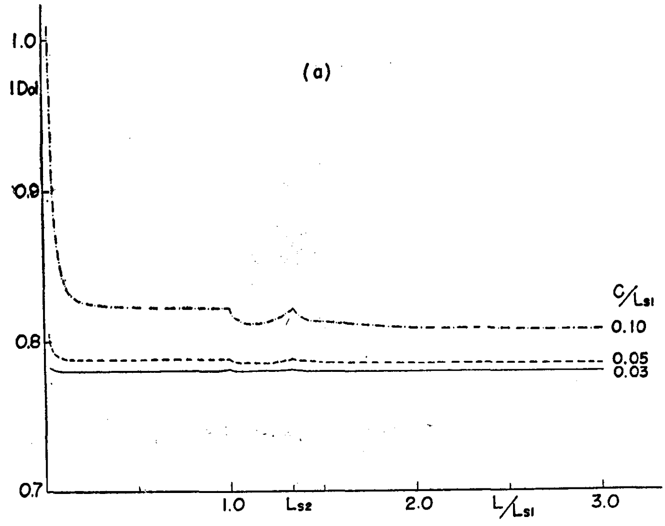

Fig. 3(a). $|D_0|$.

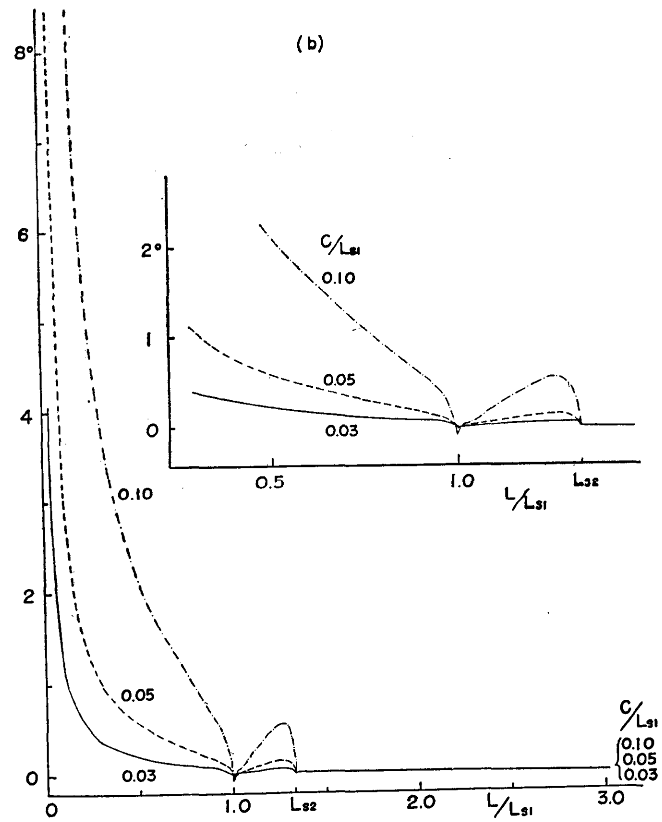

Fig. 3(b). The variation of phase angle of D_0.

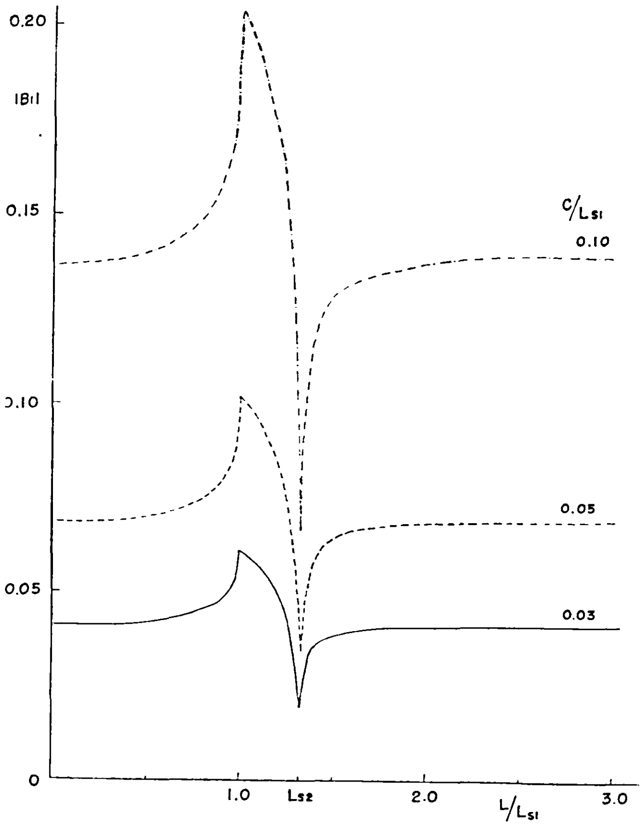

Fig. 4.

for the values of $|B_1|/|B_0|$ are larger than those of $|D_1|/|D_0|$. (3) The order of magnitude of B_0 and D_0 with respect to c/L_{s1} is opposite each other. That is, the smaller c/L_{s1} is, the larger B_0, while the smaller c/L_{s1} is, the smaller D_0. (4) Both $|B_0|$ and $|D_0|$ become very large for $L/L_{s1}<0.3$ and diverge at $L/L_{s1}=0$. This is because this method of calculation is not applicable for small L/L_{s1}. This is also shown in Fig. 8 and Fig. 9 and mentioned in (10). (5) The phase angles of B_0 are about π for $L/L_{s1}>1$, and those of D_0, about 0 for $L/L_{s1}>1$. For $L/L_{s1}<1$, the phase angles of both B_0 and D_0 have an increasing tendency with L/L_{s1} decreasing.

From Figs. 4 and 5, we can see the following trends as to B_1 and D_1.

(6) The variations of $|B_1|$ and $|D_1|$ with L/L_{s1} are opposite in general. $|B_1|$ becomes minimum at $L=L_{s2}$, while $|D_1|$ becomes maximum at L

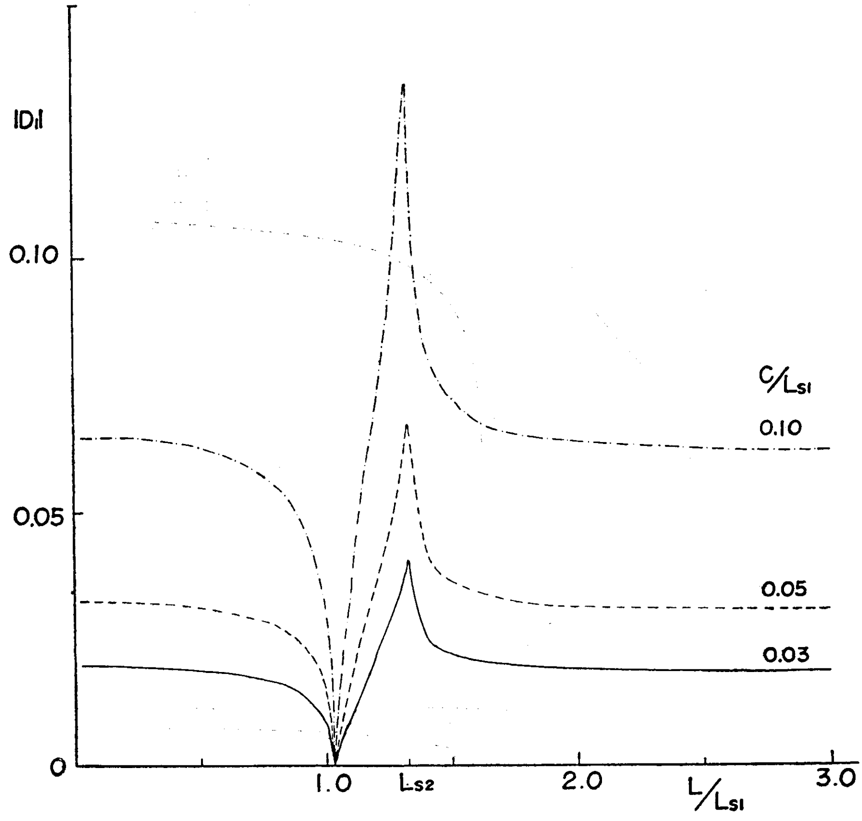

Fig. 5.

$=L_{s2}$ where L_{s2} is the wave length of the refracted wave. $|B_1|$ reaches maximum at $L=L_{s1}$. $|D_1|$ is 0 at $L=1.025L_{s1}$ and becomes minimum. The value of L which gives zero of $|D_1|$ depends on μ_2/μ_1 and velocity ratio and in general is larger than L_{s1}.

Because of small variations of B_0 and D_0 with L/L_{s1}, the curves of $|B_1|/|B_0|$ and $|D_1|/|D_0|$ given in Figs. 6 and 7 resemble a great deal those of $|B_1|$ and $|D_1|$ in Figs. 4 and 5 respectively.

(7) From Tables 1, 2 and 3, the values of $|B_1|/|B_0|$ are about several times larger than those of $|D_1|/|D_0|$. (8) For $c/L_{s1}=0.10$, there is a range where $|B_1|/|B_0|$ is larger than 1. This may be because of too large value of c/L_{s1} for the approximation or the neglection of phase angles of B_0 and B_1. (9) The value of L/L_{s1} which gives maximum of $|B_1|/|B_0|$ for $c/L_{s1}=0.10$ is not 1, but larger than 1, while that of L/L_{s1} for $c/L_{s1}=0.03$ or 0.05 is 1.

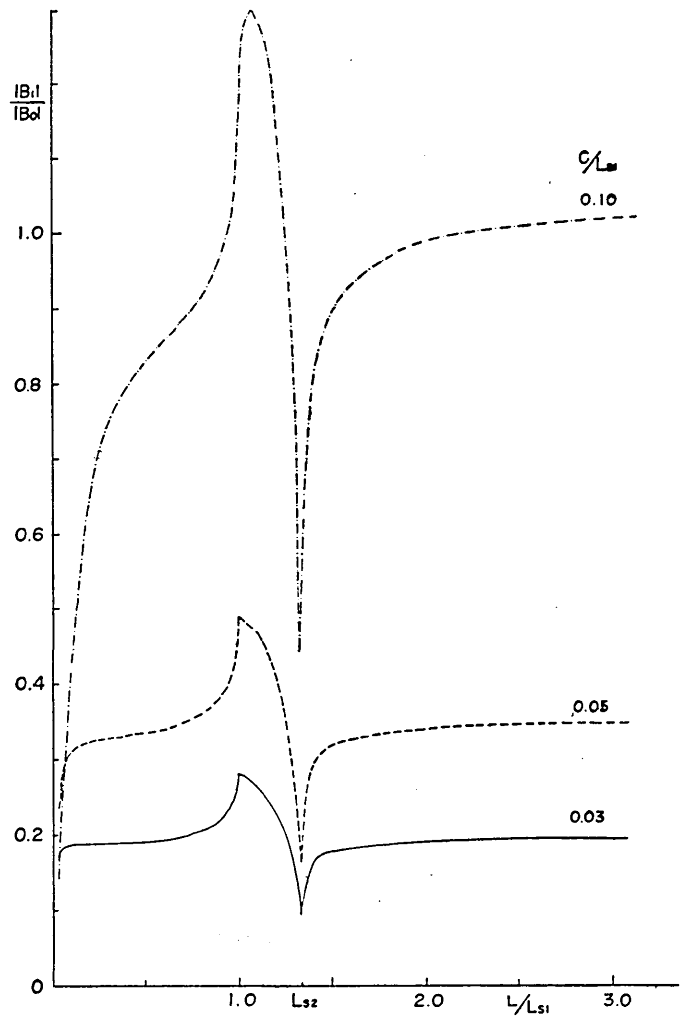

Fig. 6.

From Figs. 8 and 9, the following tendency is seen as to $|B_2|$ and $|D_2|$.
(10) Both $|B_2|$ and $|D_2|$ are very small in comparison with $|B_0|$ and $|D_0|$ except for $L/L_{s1}\approx 0$. In the neighbourhood of $L/L_{s1}=0$, both $|B_2|$ and $|D_2|$ increase very rapidly with L/L_{s1} tending to 0 until they reach infinity at just $L/L_{s1}=0$. This shows that this method of calculation is not fit for $L/L_{s1}<0.5$, since $d\zeta/dx$ is very large. (11) Both $|B_2|$ and $|D_2|$ have two maxima and two minima. (12) There exists a certain

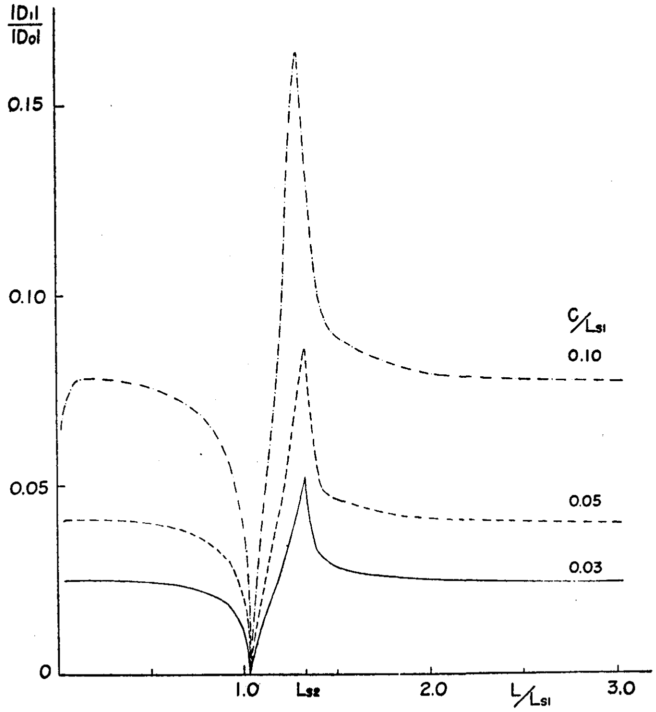

Fig. 7.

kind of boundary waves. That is, in the range of $L/L_{s2}>2$ all waves are bodily waves, while for $2L_{s2}>L$ some of irregular waves seem to be a kind of boundary waves amplitudes of which decrease with $|z|$ increasing. In the range of $2L_{s2}>L>2L_{s1}$, only the irregularly refracted wave with spectrum of the second order which is represented by D_2 is like the boundary wave. In the range of $2L_{s1}>L>L_{s2}$, in addition to the irregularly refracted wave with spectrum of the second order, the irregularly reflected wave with spectrum of the second order represented by B_2 becomes like the boundary wave. Furthermore, in the range of $L_{s2}>L>L_{s1}$, only the irregularly reflected wave with spectrum of the first order represented by B_1 is a bodily wave, and other waves are like boundary waves. For $L_{s1}>L$, all waves are like boundary waves. The amplitude of these waves decreases in such a way that the higher is the order of spectrum of irregular waves, and the smaller, the wave length of the corrugation, and the larger, the wave length of the incident SH wave, the larger, the decrease of the amplitude.

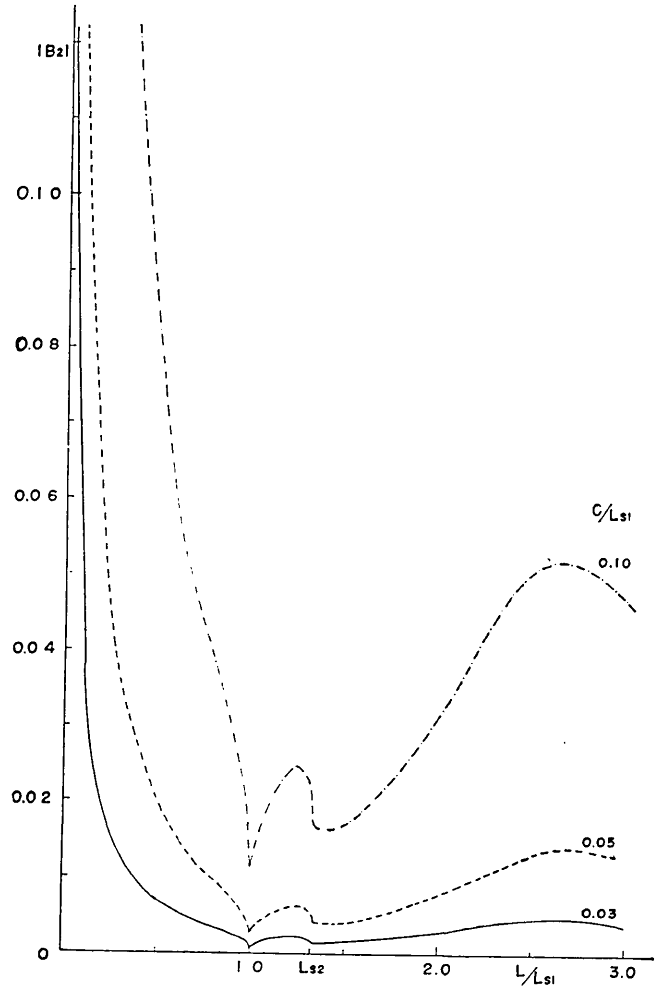

Fig. 8.

8. Conclusion

In order to see the effect of a corrugated boundary surface on reflection and refraction of elastic waves, the case of incidence of SH wave on the corrugated boundary surface which can be represented by Fourier's series is treated in the present paper. For the simplicity's sake, numerical calculation was carried out for the case of normal incidence on the boundary surface given by $\zeta = c \cos px$. The results are given in Tables 1, 2 and 3, and Figs. 2–9. From these figures, it will

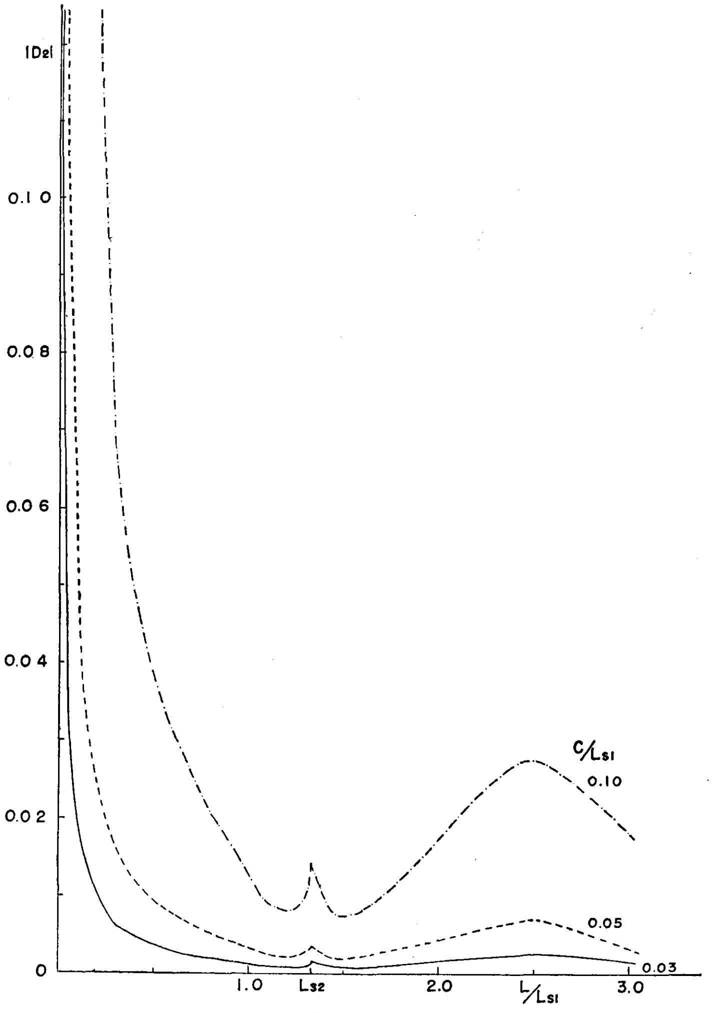

Fig. 9.

be seen that at a certain value of L/L_{s1}, for example, $L=L_{s1}$ or $L=L_{s2}$, irregularly reflected or refracted waves become of comparable order with that of regular waves, or they become very small in comparison with regular waves.

Furthermore, the effect of amplitude of corrugation on the amplitude of a regularly reflected wave is opposite to and larger than its effect on a regularly refracted wave. That is, the larger the amplitude of corrugation is, the smaller, the amplitude of a regularly reflected wave, while the larger the amplitude of corrugation is, the larger, the amplitude of a regularly refracted wave.

Table 1. $|B_0|$, $|D_0|$, etc. in the case of $c/L_{s1}=0.03$.

L/L_{s1}	$\lvert B_0\rvert$	$\lvert D_0\rvert$	Phase angle of B_0	Phase angle of D_0	$\lvert B_1\rvert$	$\lvert D_1\rvert$	$\lvert B_1\rvert/\lvert B_0\rvert$	$\lvert D_1\rvert/\lvert D_0\rvert$	$\lvert B_2\rvert$	$\lvert D_3\rvert$
3.0	0.215522	0.779392	180°00′	0°00′	0.041838	0.018554	0.194124	0.023806	0.004193	0.001650
2.5	0.215595	0.779442	180°00′	0°00′	0.041679	0.018713	0.193321	0.024008	0.004599	0.002484
2.0	0.215739	0.779542	180°00′	0°00′	0.041258	0.019134	0.191240	0.024545	0.002834	0.001570
1.5	0.216119	0.779838	180°00′	0°00′	0.038751	0.021641	0.179304	0.027751	0.001495	0.000638
1.4	0.216382	0.780024	180°00′	0°00′	0.035819	0.024573	0.165536	0.031502	0.001459	0.000828
1.333333	0.216816	0.780683	180°00′	0°00′	0.020060	0.040333	0.092521	0.051664	0.001516	0.001282
1.31	0.216641	0.780411	180°06′	0°03′	0.031459	0.036174	0.145213	0.046352	0.002038	0.000961
1.28	0.216495	0.780156	180°07′	0°03′	0.039678	0.031522	0.183274	0.040405	0.002187	0.000820
1.24	0.216399	0.779935	180°07′	0°03′	0.046434	0.026140	0.214576	0.033516	0.002223	0.000735
1.2	0.216387	0.779809	180°06′	0°03′	0.050890	0.021354	0.235180	0.027384	0.002175	0.000716
1.1	0.216676	0.779801	180°02′	0°01′	0.057494	0.010281	0.265345	0.013184	0.001818	0.000833
1.07	0.216871	0.779885	180°01′	0°01′	0.058792	0.006714	0.271092	0.008609	0.001642	0.000898
1.04	0.217152	0.780034	180°00′	0°00′	0.059899	0.002567	0.275839	0.003291	0.001374	0.000966
1.015	0.217528	0.780255	180°00′	−0°00′	0.060704	0.002162	0.279063	0.002771	0.001213	0.001045
1.0	0.218151	0.780658	180°02′	0°01′	0.061143	0.009552	0.280278	0.012236	0.001016	0.001131
0.98	0.217956	0.780693	180°12′	0°01′	0.053589	0.010910	0.245871	0.013975	0.001702	0.001257
0.96	0.217895	0.780705	180°15′	0°02′	0.051187	0.012185	0.234916	0.015608	0.001987	0.001328
0.93	0.217841	0.780715	180°20′	0°03′	0.048982	0.013564	0.224852	0.017374	0.002316	0.001431
0.92	0.217834	0.780716	180°20′	0°03′	0.048690	0.013721	0.223519	0.017575		
0.91	0.217815	0.780720	180°22′	0°03′	0.047950	0.014259	0.220141	0.018264		
0.905	0.217811	0.780720	180°22′	0°03′	0.047727	0.014412	0.219121	0.018460		
0.9	0.217805	0.780721	180°23′	0°04′	0.047514	0.014559	0.218149	0.018648	0.002600	0.001526
0.85	0.217764	0.780730	180°28′	0°04′	0.045861	0.015737	0.210600	0.020157		
0.8	0.217738	0.780736	180°33′	0°05′	0.044743	0.016561	0.205490	0.021212	0.003472	0.001860
0.5	0.217541	0.780752	181°10′	0°12′	0.041865	0.018743	0.192446	0.024006	0.006975	0.003415
0.3	0.217787	0.780767	182°05′	0°22′	0.041242	0.019217	0.189369	0.024613	0.012427	0.005964
0.25	0.217854	0.780774	182°32′	0°27′	0.041115	0.019323	0.188727	0.024749		
0.20	0.217975	0.780792	183°12′	0°34′	0.041039	0.019381	0.188274	0.024822		
0.15	0.218246	0.780819	184°17′	0°46′	0.040977	0.019426	0.187756	0.024879		
0.1	0.219022	0.780905	186°29′	1°09′	0.040942	0.019478	0.186931	0.024943	0.038418	0.018310
0.05	0.223161	0.781382	192°47′	2°18′	0.040918	0.019475	0.183356	0.024924	0.077045	0.036696
0.03	0.232691	0.782499	200°43′	3°50′	0.040913	0.019478	0.175825	0.024892	0.121708	0.061185

Table 2. $|B_0|$, $|D_0|$, etc. in the case of $c/L_{s1}=0.05$.

L/L_{s1}	$\|B_0\|$	$\|D_0\|$	Phase angle of B_0	Phase angle of D_0	$\|B_1\|$	$\|D_1\|$	$\|B_1\|/\|B_0\|$	$\|D_1\|/\|D_0\|$	$\|B_2\|$	$\|D_2\|$
3.0	0.201695	0.784178	180°00′	0°00′	0.069730	0.030923	0.345720	0.039434	0.012899	0.003293
2.5	0.201899	0.784316	180°00′	0°00′	0.069464	0.031189	0.344053	0.039766	0.013162	0.006900
2.0	0.202297	0.784595	180°00′	0°00′	0.068763	0.031890	0.339911	0.040646	0.007872	0.004361
1.5	0.203354	0.785416	180°00′	0°00′	0.064584	0.036068	0.317594	0.045922	0.004156	0.001773
1.4	0.203886	0.785931	180°00′	0°00′	0.059699	0.040954	0.292806	0.052109	0.004052	0.002302
1.333333	0.205286	0.787767	180°00′	0°00′	0.033432	0.067222	0.162856	0.085332	0.004210	0.003500
1.31	0.204802	0.787011	180°18′	0°07′	0.052432	0.060291	0.256013	0.076608	0.005670	0.002670
1.28	0.204402	0.786305	180°21′	0°08′	0.066131	0.052535	0.323534	0.066813	0.006072	0.002277
1.24	0.204132	0.785691	180°20′	0°08′	0.077389	0.043566	0.379113	0.055449	0.006174	0.002041
1.2	0.204100	0.785340	180°16′	0°07′	0.084816	0.035588	0.415561	0.045315	0.006043	0.001988
1.1	0.204900	0.785315	180°05′	0°03′	0.095823	0.017136	0.467657	0,021821	0.005053	0.002314
1.07	0.205438	0.785551	180°02′	0°02′	0.097988	0.011190	0.476971	0.014245	0.004560	0.002493
1.04	0.206219	0.785963	180°00′	0°01′	0.099832	0.004279	0.484107	0.005444	0.003942	0.002700
1.015	0.207264	0.786577	180°00′	0°01′	0.101173	0.003603	0.488131	0.004581	0.003352	0.002903
1.0	0.208997	0.787695	180°06′	−0°02′	0.101904	0.015920	0.487586	0.019958	0.002822	0.003143
0.98	0.208463	0.787794	180°35′	0°04′	0.089315	0.018183	0.428445	0.023081	0.004729	0.003493
0.96	0.208297	0.787827	180°45′	0°06′	0.085311	0.020309	0.409564	0.025779	0.005517	0.003698
0.93	0.208153	0.787856	180°57′	0°08′	0.081637	0.022606	0.392197	0.028693	0.006436	0.003974
0.92	0.208135	0.787860	180°59′	0°08′	0.081149	0.022868	0.389886	0.029025		
0.91	0.208086	0.787871	181°04′	0°09′	0.079916	0.023766	0.384053	0.030165		
0.905	0.208073	0.787872	181°05′	0°09′	0.079544	0.024022	0.382289	0.030490		
0.9	0.208060	0.787875	181°07′	0°10′	0.079191	0.024267	0.380616	0.030801	0.007221	0.004241
0.85	0.207962	0.787899	181°22′	0°12′	0.076435	0.026229	0.367543	0.033290		
0.8	0.207906	0.787915	181°35′	0°15′	0.074570	0.027601	0.358672	0.035030	0.009642	0.005166
0.5	0.207988	0.787982	183°22′	0°33′	0.069774	0.031238	0.335471	0.039643	0.019374	0.009487
0.3	0.208751	0.788075	186°04′	1°01′	0.068736	0.032028	0.329273	0.040641	0.034520	0.016565
0.25	0.209292	0.788135	187°21′	1°14′	0.068524	0.032205	0.327409	0.040862		
0.20	0.210295	0.788246	189°14′	1°33′	0.068398	0.032302	0.325248	0.040980		
0.15	0.212458	0.788480	192°19′	2°05′	0.068302	0.032377	0.321485	0.041063		
0.1	0.218527	0.789150	198°14′	3°09′	0.068237	0.032428	0.312259	0.041092	0.106717	0.050862
0.05	0.248761	0.792746	213°27′	6°18′	0.068197	0.032459	0.274147	0.040945	0.214014	0.101935
0.03	0.308828	0.801226	227°46′	10°26′	0.068189	0.032467	0.220799	0.040522	0.356898	0.169962

Table 3. $|B_0|$, $|D_0|$, etc. in the case of $c/L_s=0.10$.

L/L_{s1}	$\lvert B_0\rvert$	$\lvert D_0\rvert$	Phase angle of B_0	Phase angle of D_0	$\lvert B_1\rvert$	$\lvert D_1\rvert$	$\lvert B_1\rvert/\lvert B_0\rvert$	$\lvert D_1\rvert/\lvert D_0\rvert$	$\lvert B_2\rvert$	$\lvert D_2\rvert$
3.0	0.136883	0.806611	180°00′	0°00′	0.139460	0.061846	1.018826	0.076674	0.046592	0.018339
2.5	0.137696	0.807165	180°00′	0°00′	0.138928	0.062377	1.008947	0.077279	0.051101	0.027594
2.0	0.139290	0.808279	180°00′	0°00′	0.137526	0.063780	0.987336	0.078908	0.031490	0.017446
1.5	0.143515	0.811566	180°00′	0°00′	0.129169	0.072137	0.900038	0.088886	0.016625	0.007091
1.4	0.145641	0.813624	180°00′	0°00′	0.119398	0.081908	0.819810	0.100671	0.016256	0.009291
1.333333	0.151239	0.820973	180°00′	0°00′	0.066863	0.134442	0.442102	0.163759	0.016840	0.014240
1.31	0.149312	0.817963	180°38′	0°27′	0.104864	0.120583	0.702315	0.147419	0.022679	0.010676
1.28	0.147770	0.815150	181°58′	0°33′	0.132260	0.105069	0.895040	0.128895	0.023544	0.009109
1.24	0.146693	0.812690	181°51′	0°32′	0.154776	0.087129	1.055101	0.107211	0.024704	0.008163
1.2	0.146537	0.811282	181°31′	0°28′	0.169632	0.071175	1.157605	0.087732	0.024171	0.007956
1.1	0.149700	0.811171	180°29′	0°13′	0.191647	0.034273	1.280207	0.042251	0.020209	0.009257
1.07	0.151847	0.812113	180°13′	0°08′	0.195973	0.022381	1.290595	0.027559	0.018238	0.009973
1.04	0.154977	0.813757	180°02′	0°03′	0.199663	0.008557	1.288340	0.010515	0.001574	0.010585
1.015	0.159157	0.816230	180°02′	0°02′	0.202345	0.007206	1.271027	0.008828	0.001341	0.011613
1.0	0.166093	0.820682	180°29′	−0°06′	0.203808	0.031840	1.227072	0.038797	0.011285	0.012571
0.98	0.164119	0.821086	182°56′	0°14′	0.178631	0.036366	1.088424	0.044290	0.018912	0.013971
0.96	0.163588	0.821221	183°51′	0°22′	0.170621	0.040618	1.042992	0.049460	0.022070	0.014793
0.93	0.163180	0.821350	184°52′	0°30′	0.163273	0.045212	1.000570	0.055046	0.025725	0.015993
0.92	0.163147	0.821368	185°04′	0°32′	0.162298	0.045735	0.994796	0.055681		
0.91	0.163033	0.821415	185°26′	0°35′	0.159832	0.047531	0.980366	0.057865		
0.905	0.163009	0.821427	185°34′	0°36′	0.159088	0.048043	0.975946	0.058487		
0.9	0.162988	0·821440	185°42′	0°37′	0.158382	0.048533	0.971740	0.059083	0.028886	0.016963
0.85	0.162912	0.821556	186°57′	0°47′	0.152869	0.052459	0.938353	0.063853		
0.8	0.163043	0.821647	188°08′	0°57′	0.149141	0.055201	0.914734	0.067183	0.038572	0.020667
0.5	0.167893	0.822247	196°57′	2°08′	0.139548	0.062477	0.831172	0.075983	0.077500	0.037948
0.3	0.183057	0.823593	208°47′	3°52′	0.137472	0.064057	0.750979	0.077777	0.138076	0.066262
0.25	0.192878	0.824400	213°41′	4°49′	0.137048	0.064410	0.710542	0.078130		
0.20	0.209631	0.826160	220°05′	5°56′	0.136796	0.064606	0.652556	0.078200		
0.15	0.242067	0.829729	228°31′	7°58′	0.136606	0.064753	0.564331	0.078041		
0.1	0.316955	0.839837	239°37′	11°55′	0.136474	0.064857	0.430578	0.077226	0.426865	0.203450
0.05	0.571421	0.892456	253°42′	22°58′	0.136394	0.064918	0.238693	0.072741	0.856053	0.407740
0.03	0.928658	1.006230	260°03′	35°15′	0.136378	0.064930	0.146855	0.064528	1.427560	0.679857

The method adopted in this paper is that used by Lord Rayleigh in the theory of gratings and is not appropriate in the neighbourhood of $L/L_{s1}=0$ because of the assumption that ζ and $d\zeta/dx$ are small in comparison with wave length of incident SH wave. Concretely speaking, this method seems to be invalid for $L/L_{s1}<0.3$ or $L/L_{s1}<0.5$ depending on the amplitude of corrugation as seen from Fig. 2(a), Fig. 3(a), Fig. 8 and Fig. 9. In this range of wave length of corrugation, other appropriate methods had better be adopted. The consideration of other methods and the study of the cases of incidence of P and SV waves will be made in the near future.

In addition to these results, it was found that depending on the ratio of the wave length of corrugation to that of the incident SH wave, there exist waves which appear to propagate along the boundary surface because their amplitude decreases exponentially with the distance from the boundary surface increasing. The amplitude of these waves decreases depending on the order of spectrum, the wave length of corrugation, etc.

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10. 褶曲境界面における弾性波の反射，屈折
第 1 報 SH 波入射の場合

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地球内部では，層の境界面が平面でない場合がかなりあり，原因の一部は褶曲の影響と考えられる現象もあるので，弾性波の反射屈折におよぼす褶曲の影響を調べてみた．方法はかつて，Lord Rayleigh が格子による音の回折を取り扱つた方法で，最近，佐藤良輔が褶曲した自由表面における弾性波の反射に応用したものを採用した．

本報では，平面 SH 波が，Fourier 級数に展開可能な境界面に入射する場合を計算した．褶曲の程度 ζ が小さいとして，第 2 近似，すなわち，ζ^2 の程度まで求めた．実際の数値計算は，簡単のために，$\zeta=c\cos px$で与えられる境界面に平面 SH 波が鉛直に入射する場合について行なつた．結果は，Tables1–3, Figs. 2–9 に与えられているが，取り扱つた程度の褶曲では，概して影響は小さいが，褶曲の波長が上下の層の S 波の波長に等しい付近では，必ずしも無視し得ない影響を与えること，褶曲の影響の仕方は，屈折波よりも，反射波の方に大きく，きき方は逆であること，褶曲の波長と入射 SH 波の波長の比によるが，境界面から遠ざかるにしたがつて，振巾が指数関数で減少する波が，境界面に沿つて伝播すること等が判つた．さらに褶曲の波長が入射 SH 波の波長の 3 割あるいは 5 割以下になると，各種反射波，屈折波共，急に大きくなるので本報で用いた近似方法では十分でないことを知り得る．