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11. *Studies of the Thermal State of the Earth. The Fifth Paper: Relation between Thermal Conductivity of Sedimentary Rocks and Water Content.*

By Ki-iti HÔRAI,
Graduate School, University of Tokyo
and
Seiya UYEDA,
Earthquake Research Institute.

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Summary

Relation between thermal conductivity of soft sediments and water content was studied. It was ascertained experimentally that, starting from a dry state, the thermal conductivity increases with water content until its maximum value is attained when water saturates the pore of the sample, and decreases gradually in the oversaturated region. Observed variation of thermal conductivity in the over-saturated region was found to be in agreement with Bullard's model. The model was extended to account for the features in the under-saturated region as well.

1. Introduction

The dependence of thermal conductivity of rocks on water content has been studied by several investigators. Clark's unpublished work, as referred to in "Handbook of Physical Constants"[1], reports that the thermal conductivity of sedimentary rocks increases considerably after soaking in water, while Bullard et al.[2] find that it decreases with water content in a water-rich region. To the authors' knowledge, however, no result covering the whole range of water content has been reported. In the course of determination of the terrestrial heat flow at the Innai Oil Field, Akita Prefecture, Japan[3], the writers happened to estimate the thermal conductivity of some soft porous shale which is

1) *Handbook of Physical Constants* edited by F. Birch, *Special Papers* Number 36, Geological Society of America, (1954).

2) E. C. BULLARD, A. E. MAXWELL and R. REVELLE, *Adv. Geophys.*, *III* (1956), 153.

3) To be published in the next issue of this Bulletin.

considered to be saturated with water *in situ*. The results presented below describe the variation with water content of the thermal conductivity of the shale obtained as a byproduct of the authors' geothermal studies.

2. Experiments

Two samples used in the present study were cores of shale collected by the Teikoku Oil Company in a bore-hole at the Innai Oil Field, Akita Prefecture, Japan. They belong to the lower Sasaoka-bed and the Tentokuzi-bed at depths of 635 m and 827 m below the surface respectively.[3] Quick X-ray examination by a "Norelco" diffractometer showed that their principal constituent minerals are quartz, feldspar and clay minerals, such as kaoline and montmorillonite.

The samples were easily powdered and could be mixed with water in various proportions. The thermal conductivity was measured by the divided-bar device[4),5)]. For water-rich specimens a thin ring of perspex fitted between the bars was used as a container. The thickness of the ring was 1.0 mm and its height, i.e. the thickness of the specimen, was 6.0 mm. In order to prevent evaporation and leakage of water during measurements, ordinary cellophane tape was pasted around the contacts between the ring and the brass bars. Thermal contact between the specimen and the bars was found to be perfect for the water-rich specimen, so that only one specimen was sufficient for one value of water content. For the water-poor specimens, pressure of 150-180 kg/cm^2 was applied to consolidate the specimen in a disc-shape and no perspex ring was used. In this case, measurement on three or four discs with different thickness and with identical water content was necessary for the elimination of the interfacial temperature drop. Specimens were weighed immediately after the conductivity measurement and after completely getting rid of water contained for the determination of water content. Necessary corrections were made when the perspex ring was used.

3. Results and Interpretation

Results obtained are shown in Figs. 1 and 2. The outstanding feature to be observed in these figures is that the thermal conductivity

4) S. UYEDA, T. YUKUTAKE and I. TANAOKA, *Bull. Earthq. Res. Inst.*, **36** (1958) 251.
5) K. HÔRAI, *Bull. Earthq. Res. Inst.*, **37** (1959), 571.

has a maximum value at approximately 16 and 14 percent of water content (weight percentage), realizing the two effects of water on thermal conductivity mentioned in the Introduction. The whole situation may be interpreted as follows: in the water-rich region, the thermal conductivity decreases with the *increase* of water content because the solid particles having a greater thermal conductivity than water are

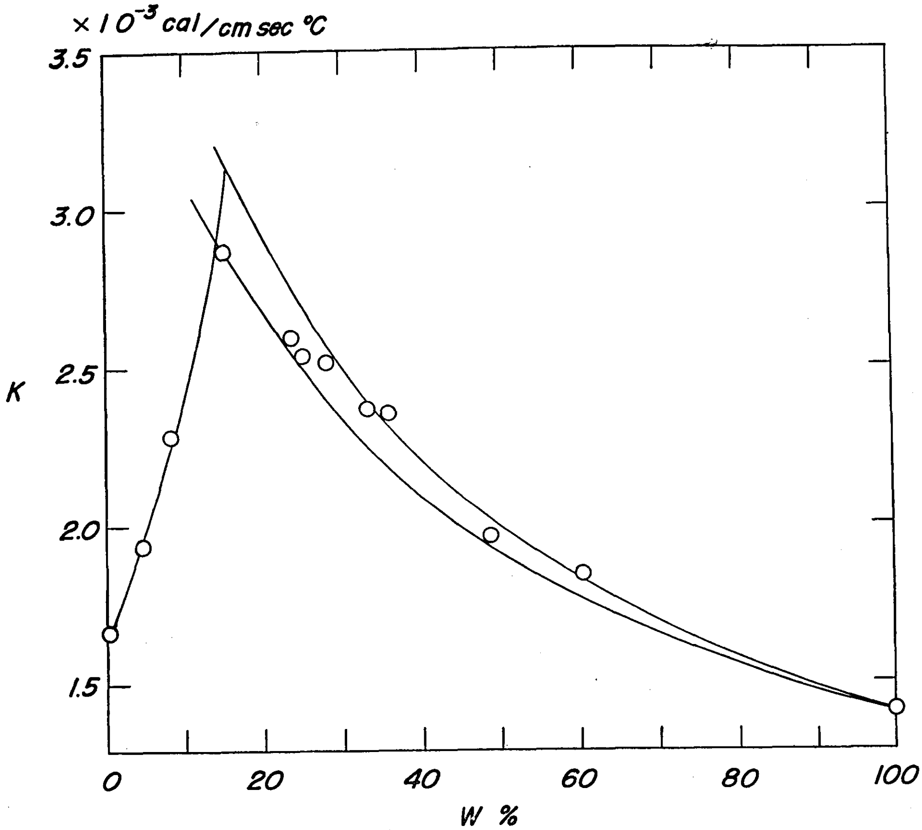


Fig. 1. Relation between thermal conductivity and water content (weight percentage): (Specimen IN VIII, Lower Sasaoka Bed).

more and more replaced by water by increment of water content, while in the water-poor region, the decreases of conductivity with *decrease* of water content is caused by substitution of water by the air, a poorer conductor of heat, because dense packing of solid particles no longer permits subtraction of water without forming air-cavities. These two regions, thus, may properly be called the oversaturated region and the undersaturated region.

The above interpretation may be formulated in the following way. For the oversaturated region, Bullard et al.[6] showed that the following relation based on the model that the solid particles of spherical shape are dispersed in water without mutual interaction[7] :

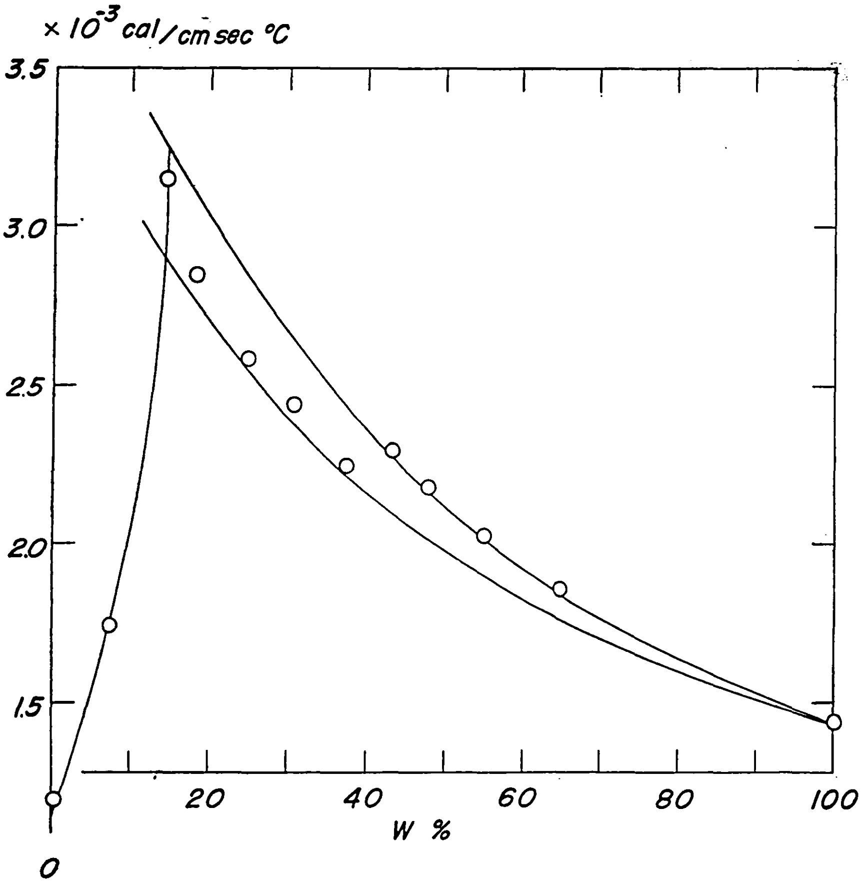


Fig. 2. Relation between thermal conductivity and water content (weight percentage): (Specimen IN X, Tentokuzi Bed).

$$k=k_w\frac{3-2v(1-k_w/k_s)}{3k_w/k_s+v(1-k_w/k_s)} \qquad (1)$$

where the thermal conductivity of the specimen k is expressed as a function of the thermal conductivities of water k_w, and of the solid

6) E. C. BULLARD et al., *loc. cit.* (2).

7) J. C. MAXWELL, "*A Treatise on Electricity and Magnetism*", 3rd ed. [ii], **1** (1904), Dover.

particles k_s and the water content by volume v. v is related to the water content by weight w as,

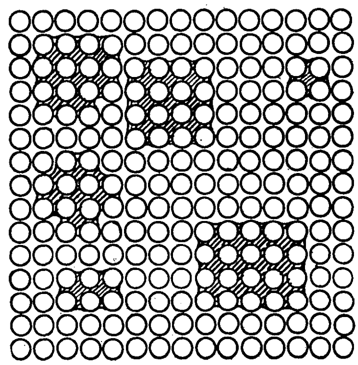
$$v = w/[w+(1-w)\rho_w/\rho_s] \tag{2}$$

where ρ_w and ρ_s denote the density of water and the solid particles.

For the undersaturated region, we deduced a similar formula to (1) by slightly modifying it. From the above expression (1), the thermal conductivity at the saturation point is given as, (Fig. 3, b),

$$K_2 = k_w \frac{3-2v_p(1-k_w/k_s)}{3k_w/k_s+v_p(1-k_w/k_s)} \tag{3}$$

where v_p stands for the porosity of the specimen. Substituting k_w in (3) by the thermal conductivity of the air k_a, the



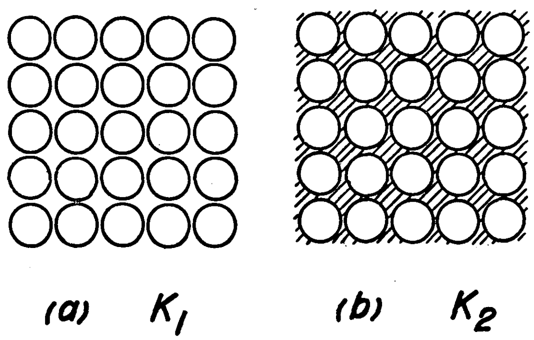


Fig. 3. Schematic representation of the model.
(a) Completely dry state, giving K_1 for thermal conductivity. Circles represent the solid particles.
(b) Saturated state, giving K_2 for thermal conductivity. Hatched part represents the water-filled space.
(c) Intermediate state in the undersaturated region.

conductivity value in the completely dry state may be expressed as, (Fig. 3, a),

$$K_1 = k_a \frac{3-2v_p(1-k_a/k_s)}{3k_a/k_s+v_p(1-k_a/k_s)} \,. \tag{4}$$

These quantities are illustrated in Fig. 4. In the intermediate states in the undersaturated region, the water contained in the specimen would be in such a state as shown in Fig. 3, c, i.e. as if patches of the apparent thermal conductivity K_2 are dispersed in the medium of apparent thermal conductivity K_1. Application of the same idea as in (1) to such a situation gives the thermal conductivity of the specimen in bulk as,

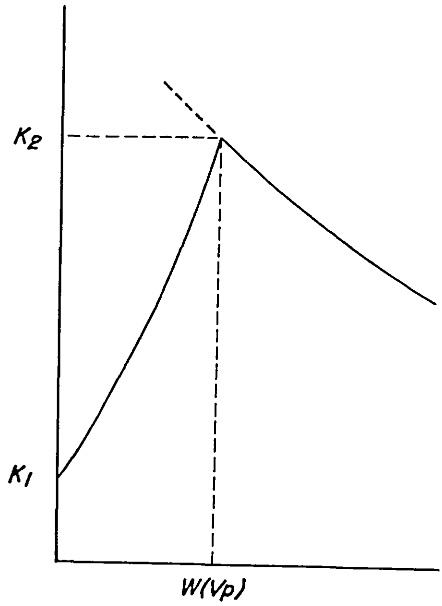


Fig. 4. Schematic representation of thermal conductivity (ordinate) vs. water content (abscissa) relation.

$$K=K_1\frac{3-2(1-v')(1-K_1/K_2)}{3K_1/K_2+(1-v')(1-K_1/K_2)} \quad (5)$$

where $1-v'$ is the volume ratio of the air contained to the interstitial space in the specimen. v' is connected to w in terms of the apparent density ρ' and the porosity p_v of the specimen as follows:

$$v'=\frac{\rho'}{p_v\cdot\rho_w(1/w-1)}\,. \quad (6)$$

Curves in Figs. 1 and 2 are the plot of the formulas (1) and (5). The constants chosen are listed in Table 1.

Table 1.

Specimen	IN VIII* (Lower Sasaoka Bed)	IN X (Tentokuzi Bed)
k_w (10^{-3} *cal/cm sec deg*)	1.41 (22°C)	1.44 (30°C)
k_s (〃)	upper curve in Fig. 1 3.86 lower curve in Fig. 1 4.45	upper curve in Fig. 2 3.76 lower curve in Fig. 2 4.69
K_1 (〃)	1.65	1.15
K_2 (〃)	2.95	3.20
ρ' (apparent) (*gr/cc*)	1.67	1.48
$\rho(=\rho_s)$ (true) (*gr/cc*)	2.39	2.19
p_v from conductivity measurement	0.16	0.14
p_v from density measurement	0.15	0.18

* "IN" stands for the Innai oil field.

4. Discussion

The values of k_s in Table 1 were chosen by trial and error in order to fit the formula (1) to the experimental results. In fact, the two curves in Figs. 1 and 2 seem to indicate the limits of the possible k_s values: they are 3.86×10^{-3} *cal/cm sec deg* and 4.45×10^{-3} *cal/cm sec deg* for the lower Sasaoka bed sample (IN VIII) and 3.76×10^{-3} *cal/cm sec deg* and 4.69×10^{-3} *cal/cm sec deg* for the Tentokuzi bed sample (IN X) respectively. These values are quite reasonable for the thermal conductivity of the minerals concerned in order of magnitude.

Porosity of the samples was estimated from the measured values of apparent density and of true density as listed in Table 1. These values of porosity again seem to be in a good agreement with the porosity values deduced from the thermal conductivity measurement, i.e. the porosity value corresponding to the water content which gives the maximum conductivity, (Table 1).

For the undersaturated region, the formula (5) was fitted to the experimental values by taking the empirical values for K_2 and K_1 as listed in Table 1. The values of K_1, namely the thermal conductivity in the dry state were found, however, to be somewhat different from that which are expected from the formula (4): the formula (4) does not give the empirical K_1-values unless one assumes two or three times greater values for the thermal conductivity of the air than the actual one. This discrepancy probably originates from the assumption that the dispersed solid particles have no mutual interaction.

As already been mentioned, a pressure of 150–180 kg/cm^2 was applied for consolidating the specimens. This pressure may correspond to the rock-pressure prevailing at the depth of some 500–600 m in the earth, so that the soft sediments, such as shales studied here, will have the same porosity *in situ* as the compressed samples in our experiments. In the actual state, water fills up the pore to saturation. As the thermal conductivity relevant to the geothermal studies, therefore, one may adopt the values at the saturation condition, i.e. the maximum values in Figs. 1 and 2.

Acknowledgment

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11. 地球熱学 (第 5 報)
堆積岩における熱伝導度と含水量の関係

東京大学大学院 地球物理学専門課程 宝来帰一
地震研究所 上田誠也

岩石の熱伝導度と含水量の関係については，水のきわめてすくない場合，および水の多い場合について Clark, Bullard 等の研究があるが，今回一つの試料について乾燥状態から含水量 100% (試料全体が水) の場合までを測定することによつて，その最大値が，含水量約 20% 弱のところに存在することが明らかになつた (第一図，第二図)．熱伝導度の最大値をあたえる含水量はわれわれが用いた秋田県院内油田の shale においては，水が丁度試料の空孔を満した場合に相当するが，上記の事情は一般的なものと解される．この最大値を境にして含水量が増減するときは熱伝導度は減少するが，この性質は簡単なモデルによつて定量的に説明し得ることを示した．なお，この研究は，院内油田での地殻内熱流量の推定に用いられる (続報)．
