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1. *Stability and Non-Steady State of Self-Exciting Dynamos. I.*

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Summary

Time-dependent behaviours of two conducting spheres rotating in an infinitely extended conductor are studied. Although it has been proved by Herzenberg that such a system works as a self-exciting dynamo, the steady state of it turns out to be unstable for small disturbances. In some special cases, a paradoxical result that the magnetic field continues its growth in spite of non-rotation of the spheres is obtained by the numerical integration of the non-linear simultaneous differential equations with the aid of a relay computer. This difficulty is caused by the crudeness of approximation for the electromagnetic coupling between the two spheres. If we improve this point, it seems likely that oscillatory fields and velocities could be found. It is intended to apply this study to investigations of the magneto-hydrodynamic actions which are supposed to be in the earth's core.

1. Introduction

The possibility of reversals of the earth's magnetic field is one of the most conspicuous suggestions from palaeomagnetic studies which have been flourishingly developing of late. In order to illustrate such reversals, a few theories have been put forward on the basis of homogeneous disk dynamo models. Bullard[1] studied a single homopolar dynamo and found particular oscillations of electric current and angular velocity but no reversal of the current. Rikitake[2] examined two disk dynamos that were coupled with one another and showed by numerical integration that the current could change its sign at the initial stage. Using a digital computer, Allan[3] confirmed such a reversal for a more extended

* Communicated by T. Rikitake.

1) E. C. BULLARD, *Proc. Camb. Phil. Soc.*, **51** (1955), 744.

2) T. RIKITAKE, *Proc. Camb. Phil. Soc.*, **54** (1958), 89.

3) D. W. ALLAN, *Nature*, **182** (1958), 469.

period of time. Since possible reversals of the earth's magnetic field have been demonstrated even by these simple systems of disk dynamos, it is desirable to study a more complicated model in order to have a better understanding of the dynamo action which is supposed to be prevailing in the earth's core.

Recently Herzenberg[4] has proved in a rigorous way that a system composed of two conducting spheres rotating in a large sphere, which is also conducting, works as a steady dynamo. Dr. Rikitake has suggested to the writer that it would be of importance and interest to study non-steady states of such a dynamo because Herzenberg's model would be more realistic than the disk dynamo models as far as its application to the magneto-hydrodynamic actions supposed in the earth's core is concerned.

In this paper the stability of a steady state of Herzenberg's model is examined and, in some particular cases, time-dependent behaviours of the system are traced by numerical calculation.

2. Equations

Suppose that two eddies, which are regarded as two conducting spheres of equal radius in this paper, are located in some favourable way, then it is possible for them to compose a system of self-exciting dynamo in an infinitely extended conductor which is assumed to stand still, the influence of the motion of eddies being ignored. It is intended here to study how these two eddies are connected electromagnetically with one another.

In the first place, induction due to the rotation of one sphere placed in a uniform magnetic field will be examined. The magnetic field induced by the rotation of a conducting sphere about an axis agreeing with the direction of the external uniform field (H) is of the toroidal type or T_2-type. The r, θ and φ components of such a field are written as

$$\boldsymbol{h}=\begin{bmatrix} 0 \\ 0 \\ -A(kr)^{-1/2}J_{5/2}(kr)\dfrac{dP_2(\theta)}{d\theta} \end{bmatrix}, \qquad r<a$$

$$=\begin{bmatrix} 0 \\ 0 \\ -B(kr)^{-1/2}Y_{5/2}(kr)\dfrac{dP_2(\theta)}{d\theta} \end{bmatrix}, \qquad r>a$$

4) A. HERZENBERG, *Phil. Trans. Roy. Soc. London*, A, **250** (1958), 543.

respectively for the inside and outside of the sphere, where $k^2=4\pi\sigma p$. The origin of the spherical coordinates (r, θ, φ) is taken at the centre of the sphere. σ denotes the electrical conductivity both inside and outside the sphere and p is the operational representation of $\partial/\partial t$.

On the other hand, we have the electric field induced by this field as follows,

$$\frac{1}{\sigma}\mathrm{rot}\boldsymbol{h}=\frac{1}{\sigma}\begin{bmatrix} 6Ar^{-1}(kr)^{-1/2}J_{5/2}(kr)P_2(\theta) \\ A\frac{1}{r}\frac{d}{dr}[r(kr)^{-1/2}J_{5/2}(kr)]\frac{dP_2(\theta)}{d\theta} \\ 0 \end{bmatrix}, \qquad r<a$$

$$=\frac{1}{\sigma}\begin{bmatrix} 6Br^{-1}(kr)^{-1/2}Y_{5/2}(kr)P_2(\theta) \\ A\frac{1}{r}\frac{d}{dr}[r(kr)^{-1/2}Y_{5/2}(kr)]\frac{dP_2(\theta)}{d\theta} \\ 0 \end{bmatrix}. \qquad r>a$$

At the boundary-surface of the sphere or $r=a$, the normal component of the magnetic flux density and the tangential component of the electric field should be continuous. Thus we have

$$AJ_{5/2}(ka)=BY_{5/2}(ka)\ ,$$

$$\left[-3A\frac{1}{r}\frac{d}{dr}\{r(kr)^{-1/2}J_{5/2}(kr)\}\right]_{r=a}-4\pi\sigma\Omega Ha$$
$$=\left[-3B\frac{1}{r}\frac{d}{dr}\{r(kr)^{-1/2}Y_{5/2}(kr)\}\right]_{r=a},$$

where Ω represents the angular velocity of the sphere.

From these equations, we can determine the free constants A and B, so that the maximum component of the magnetic field (h_e) outside the sphere is easily obtained as

$$h_e=-\frac{2}{3}\pi^2\sigma\Omega Ha^2(ka)^{1/2}(kr)^{-1/2}J_{5/2}(ka)Y_{5/2}(kr)\ . \qquad (1)$$

In the case of a relatively slow change, ka and kr take small values, so that the Bessel functions involved can be replaced by a few terms of their ascending power series expressions. If we assume a sphere, 1000 *km* in radius, with a conductivity amounting to 10^{-6} *e.m.u.*, for example, the following may approximate (1) well so long as changes having a period larger than 2×10^4 *years* are considered.

$$h_e=\frac{2}{5}\pi\sigma a^5r^{-3}\left[1-\frac{4\pi\sigma a^2}{14}p+\frac{4\pi\sigma r^2}{6}p\right]\Omega H\ . \qquad (2)$$

The magnetic field thus produced is the strongest on the circle of $\theta=\pi/4$ on the sphere and penetrates into the outside of the sphere with its intensity weakening.

Let us next study the coupling between two spheres. Far away from one sphere (I), where the magnetic field (H_1) produced by its rotation may be regarded as uniform enough, the other sphere (II) is revolving as can be seen in Fig. 1. Its axis of rotation, which is nearly parallel to the field H_1, can be directed so as to let the direction of the field H_2 induced alike by sphere (II) almost coincide with the axis of sphere (I) and reinforce the original field assigned parallel to its rotational axis. If mechanical energy that makes the spheres rotate is supplied by some external force against the dissipation of energy due to Joule heat, the magnetic fields of each sphere are maintained by the above process. It should be noted, however, that if the field H_1 is completely parallel to the axis of sphere II, sphere I comes to be placed on the equatorial plane of sphere II where no field of H_2 can be induced, and *vice versa*. Therefore the axis of rotation should be slightly inclined against the direction of the field. In that case, some other components of magnetic field necessarily come out. According to the studies[5)6)] of electromagnetic induction by a rotating sphere, it has been proved that the fields induced in an asymmetric field do not grow infinitely however large the rotation speed is. It is then obvious that, by making the distance between the two spheres fairly large, only the fields induced in the field symmetric about the rotation axes become important, the other fields being possible to be ignored in an approximate study.

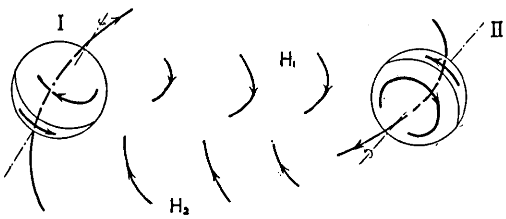

Fig. 1. A system of self-exciting dynamo.

In the light of the above discussion, it may be allowed to describe the behaviour of this system, using equation (2), as follows,

$$H_2(t)=-\frac{2}{5}\pi\sigma a^5 r^{-3}\left[\Omega_1(t)H_1(t)+4\pi\sigma\left(\frac{r^2}{6}-\frac{a^2}{14}\right)\frac{d}{dt}\{\Omega_1(t)H_1(t)\}\right], \quad (3)$$

$$H_1(t)=-\frac{2}{5}\pi\sigma a^5 r^{-3}\left[\Omega_2(t)H_2(t)+4\pi\sigma\left(\frac{r^2}{6}-\frac{a^2}{14}\right)\frac{d}{dt}\{\Omega_2(t)H_2(t)\}\right]. \quad (4)$$

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5) E. C. BULLARD, *Proc. Roy. Soc.*, A, **199** (1949), 413.

6) A. HERZENBERG & F. J. LOWES, *Phil. Trans. Roy. Soc. London*, A, **249** (1957), 507.

Attention should be paid to the fact, however, that (3) and (4) are somewhat idealized. In an actual case, the righthand members should be multiplied by some factors less than unity according to the geometrical arrangement. We may therefore consider (3) and (4) as the relationship for the extreme case only.

When external torques G_1 and G_2 are being given to the spheres, we have the following equations of motion for this system,

$$C_1\frac{d\Omega_1}{dt}=G_1-\frac{8\pi}{75}\sigma a^5\Omega_1 H_1^2\ , \tag{5}$$

$$C_2\frac{d\Omega_2}{dt}=G_2-\frac{8\pi}{75}\sigma a^5\Omega_2 H_2^2\ , \tag{6}$$

where C_1 and C_2 represent the moments of inertia of the spheres I and II respectively. The second terms of the righthand side are the torques due to electro-magnetic coupling. Solving the above four equations (3), (4), (5) and (6) simultaneously, we may thoroughly observe how this system behaves.

Since it is too difficult to solve these non-linear simultaneous equations analytically, the only way to tackle them seems to make use of numerical calculation. In order to make variables dimensionless the following transformations are carried out.

$$\Omega=\mathfrak{O}\Omega'\ ,\qquad t=\mathfrak{T}t'\ ,\qquad H=\mathfrak{H}H'\ ,$$

where $\mathfrak{O}=(\pi\sigma a^2)^{-1}$, $\mathfrak{T}=\pi\sigma a^2$, $\mathfrak{H}^2=G/a^3=C/\pi^2\sigma^2 a^7$. Substituting these into the equations (3), (4), (5) and (6), and dropping primes, we have

$$\frac{dH_1}{dt}=-\left(\frac{3}{2}\varepsilon^2+\frac{1}{\Omega_1}\frac{d\Omega_1}{dt}\right)H_1-\frac{1}{\varepsilon}\frac{15}{4}\frac{H_2}{\Omega_1}\ , \tag{7}$$

$$\frac{dH_2}{dt}=-\frac{1}{\varepsilon}\frac{15}{4}\frac{H_1}{\Omega_2}-\left(\frac{3}{2}\varepsilon^2+\frac{1}{\Omega_2}\frac{d\Omega_2}{dt}\right)H_2\ , \tag{8}$$

$$\frac{d\Omega_1}{dt}=1-\frac{8}{75}H_1^2\Omega_1\ , \tag{9}$$

$$\frac{d\Omega_2}{dt}=1-\frac{8}{75}H_2^2\Omega_2\ , \tag{10}$$

where $\varepsilon=a/r$. For simplicity, it has been assumed that $C_1=C_2$ and $G_1=G_2$ in the above calculation. A set of solutions in the steady state is easily obtained as follows,

$$\left.\begin{aligned} \Omega_{10}&=-\frac{5}{2}\frac{1}{\varepsilon^3 q}\,, \qquad H_{10}^2=-\frac{15}{4}\varepsilon^3 q\,, \\ \Omega_{20}&=-\frac{5}{2}\frac{q}{\varepsilon^3}\,, \qquad H_{20}^2=-\frac{15}{4}\frac{\varepsilon^3}{q}\,, \end{aligned}\right\} \tag{11}$$

where q is defined by the equation

$$\left(\frac{H_1}{H_2}\right)^2=\frac{\Omega_2}{\Omega_1}=q^2\,.$$

3. Stability of the steady state

If the initial state is taken to be $H_2=-H_1$ and $\Omega_1=\Omega_2$, that is $q=-1$, the behaviours of the system I and II are quite the same, and therefore the steady state is given by

$$\Omega_0=\Omega_{10}=\Omega_{20}=\frac{5}{2}\frac{1}{\varepsilon^3}\,, \qquad H_0^2=H_{10}^2=H_{20}^2=\frac{15}{4}\varepsilon^3\,. \tag{12}$$

In this special case, equations (7), (8), (9) and (10) are reduced to the following,

$$\Omega\frac{dH}{dt}=-\left\{\frac{3}{2}\varepsilon^2(\Omega-\Omega_0)+\frac{d\Omega}{dt}\right\}H\,, \tag{13}$$

$$\frac{d\Omega}{dt}=1-\frac{H^2\Omega}{H_0^2\Omega_0}\,. \tag{14}$$

On the assumption that the magnetic field and the angular velocity deviates a little from the steady state, let us first examine whether those deviations will grow or decay. Let the magnetic field and angular velocity be

$$H=H_0+h\,, \qquad \Omega=\Omega_0+\omega\,,$$

where h and ω are small departures from the steady state. Substituting these into equations (13) and (14) and ignoring the products of these small quantities, we have,

$$\frac{dh}{dt}=\frac{2}{\Omega_0}h-\frac{H_0}{\Omega_0}\left(\frac{3}{2}\varepsilon^2-\frac{1}{\Omega_0}\right)\omega\,,$$

$$\frac{d\omega}{dt}=-\frac{2}{H_0}h-\frac{1}{\Omega_0}\omega\,.$$

Provided h and ω are proportional to $e^{\lambda t}$, two roots of λ are easily obtained from the above equations,

$$\lambda=\frac{1}{2\Omega_0}\{1\pm\sqrt{1+12\varepsilon^2\Omega_0}\}\ , \tag{15}$$

of which one is positive real and the other negative real. The existence of a positive real root indicates that the steady state expressed by (12) is unstable for small disturbances. It is therefore doubtful that the steady state of the dynamo investigated by Herzenberg[4] lasts over a long period.

When a departure from the steady state is considerable, we cannot examine the behaviour of this system by the above small perturbation method. It may readily be understood that the initial state of the system will play an important role in the determination of the variation in the magnetic field and the change in the angular velocity. Let the deviations from the steady state, regarded as appreciable at the initial stage, be H_i and Ω_i, then the magnetic field and the angular velocity are written as

$$H=H_0+H_i+h\ , \qquad \Omega=\Omega_0+\Omega_i+\omega\ ,$$

where h and ω are small quantities of the first order which vary with time. Substituting these into equations (13) and (14), we have the following equations by ignoring the products of h and ω,

$$\begin{aligned}\frac{dh}{dt}&=A+Bh+C\omega\ ,\\ \frac{d\omega}{dt}&=D+Eh+F\omega\ ,\end{aligned} \tag{16}$$

where

$$A=-\frac{H_0+H_i}{\Omega_0}\left\{\frac{3}{2}\varepsilon^2\Omega_0(1-y)+y-x^2\right\}\ ,$$

$$B=-\frac{1}{\Omega_0}\left\{\frac{3}{2}\varepsilon^2\Omega_0(1-y)+y-3x^2\right\}\ ,$$

$$C=-\frac{H_0+H_i}{\Omega_0}\left(\frac{3}{2}\varepsilon^2-\frac{1}{\Omega_0}\right)y^2\ ,$$

$$D=\frac{1}{y}(y-x^2)\ ,$$

$$E=-\frac{2}{H_0+H_i}\frac{x^2}{y}\ ,$$

$$F=-\frac{x^2}{\Omega_0}\ ,$$

$$x^2=\frac{(H_0+H_i)^2}{H_0^2}\ ,$$

$$y=\frac{\Omega_0}{\Omega_0+\Omega_i}\ .$$

From these the following equations are obtained.

$$\begin{aligned}&\frac{d^2h}{dt^2}-(B+F)\frac{dh}{dt}+(BF-CE)h+AF-CD=0\ ,\\&\frac{d^2\omega}{dt^2}-(B+F)\frac{d\omega}{dt}+(BF-CE)\omega+BD-AE=0\ ,\end{aligned}\tag{17}$$

where

$$B+F=-\left(\frac{3}{2}\varepsilon^2-\frac{2x^2}{\Omega_0}\right)+\left(\frac{3}{2}\varepsilon^2-\frac{1}{\Omega_0}\right)y\ ,$$

$$BF-CE=\frac{3x^2}{\Omega_0}\left\{\left(\frac{3}{2}\varepsilon^2-\frac{3x^2}{\Omega_0}\right)-\left(\frac{3}{2}\varepsilon^2-\frac{1}{\Omega_0}\right)y\right\}\ ,$$

$$AF-CD=\frac{H_0+H_i}{\Omega_0}\left\{\left(\frac{3}{2}\varepsilon^2-\frac{1}{\Omega_0}\right)(y-x^2)^2-\frac{3}{2}\varepsilon^2x^2(x^2-1)\right\}\ ,$$

$$BD-AE=-\frac{3}{2}\varepsilon^2(1-y)\left(1+\frac{x^2}{y}\right)-\frac{y}{\Omega_0}\left(1-\frac{x^2}{y}\right)^2\ .$$

It should be noticed that the last terms in the equations (17) do not appear in the case of the previous small perturbation method. These depend only on the initial state of the system and have effects of a kind of an external force exerting the system as in the case of a forced oscillation. This may be understood when we consider that at such a state the couples produced electromagnetically by the magnetic field do not balance themselves with the external torque accelerating the sphere, which consequences to reinforce or diminish the magnetic field and the angular velocity.

The solutions of the equations (17) are,

$$h=h_s+Me^{\xi t}+Ne^{\eta t}\ ,$$

$$\omega=\omega_s+Pe^{\xi t}+Qe^{\eta t}\ ,$$

where ξ and η denote the solution of homogeneous equations of (17) and

$$h_s=\frac{H_0+H_i}{\Omega_0}\left\{\frac{3}{2}\varepsilon^2x^2(x^2-1)-\left(\frac{3}{2}\varepsilon^2-\frac{1}{\Omega_0}\right)(y-x^2)^2\right\}\ ,$$

$$\omega_s=\frac{3}{2}\varepsilon^2(1-y)\left(1+\frac{x^2}{y}\right)+\frac{y}{\Omega_0}\left(1-\frac{x^2}{y}\right)^2\ .$$

When both of ξ and η have negative real part, h and ω approach the values h_s and ω_s respectively. Therefore if h_s and ω_s are equal to zero, the state in question is one of the stable steady states. Or otherwise, it may be regarded as unstable, so long as h_s and ω_s take some definite values even if real parts of ξ and η are negative.

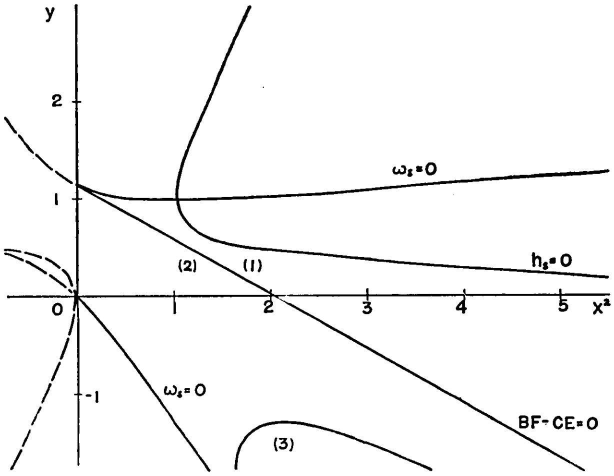

Fig. 2. Ordinate: $y=\dfrac{\Omega_0}{\Omega_0+\Omega_l}$, Abscissa: $x^2=\left(\dfrac{H_0+H_l}{H_0}\right)^2$

$\varepsilon=1/10^{\frac{1}{3}}$,

Region 1: $\xi>0$, $\eta<0$.

Region 2: $\xi<0$, $\eta<0$.

Region 3: ξ and η are complex.

In Fig. 2 curves of zero lines of h_s and ω_s are shown. The steady states are represented by the intersection points of these curves, *i.e.*, $x^2=1$, $y=1$ and $x^2=0$, $y=0$.

Provided that

$$\left\{\left(\frac{3}{2}\varepsilon^2-\frac{4x^2}{\Omega_0}\right)-\left(\frac{3}{2}\varepsilon^2-\frac{1}{\Omega_0}\right)y\right\}^2+\frac{8x^2}{\Omega_0}\left(\frac{3}{2}\varepsilon^2-\frac{1}{\Omega_0}\right)y<0\,,$$

which is shown in Fig. 2 as the inner range of a hyperbola (3), ξ and η become complex. In this range h and ω are oscillatory with diminishing amplitudes and tend to h_s and ω_s. When

$$\left\{\left(\frac{3}{2}\varepsilon^2-\frac{4x^2}{\Omega_0}\right)-\left(\frac{3}{2}\varepsilon^2-\frac{1}{\Omega_0}\right)y\right\}^2+\frac{8x^2}{\Omega_0}\left(\frac{3}{2}\varepsilon^2-\frac{1}{\Omega_0}\right)y>0\,,$$

ξ and η are real. According as $BF-CE \lesseqgtr 0$, that is,

$$y \gtrless -\frac{3x^2/\Omega_0-(3/2)\varepsilon^2}{(3/2)\varepsilon^2-1/\Omega_0},$$

one of the roots has the positive sign and the other negative, or both of them are negative. To the former case does the first steady state (1,1) belong, and therefore it is looked upon as an unstable saddle point. To the latter belongs the other steady state (0, 0), and it is the only stable node.

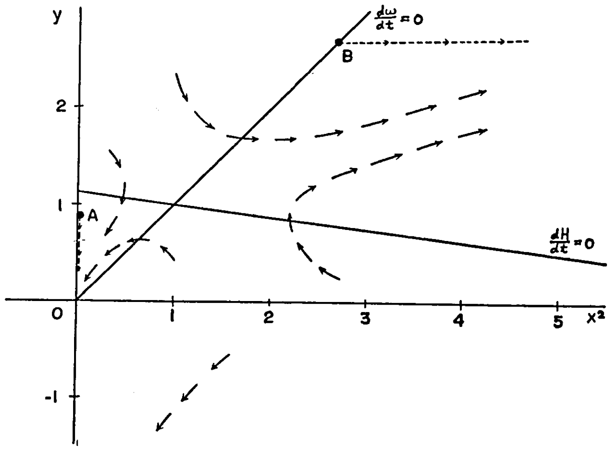

Fig. 3. Trajectories of representative points
$\varepsilon=1/10^{\frac{1}{3}}$,
A: $H_1=0.1$, $H_2=-0.1$, $\Omega_1=\Omega_2=25$.
B: $H_1=1.0$, $H_2=-1.0$, $\Omega_1=\Omega_2=9.375$.

Taking equations (13) and (14) into account too, the trajectories of the representative points on x^2-y plane are schematically drawn in Fig. 3. As for some particular cases, equations (7), (8), (9) and (10) are integrated numerically, using the relay computer FACOM-128B. For examples, three of them are shown in Fig. 4, trajectories of which are also depicted in Fig. 3 with dotted lines. From these we see that, if the magnetic field and the angular velocity whose values belong region (2) are given initially x^2 and y approach themselves to zero, namely the angular velocity increases indefinitely while the magnetic field decreases monotonously. The sphere will be infinitely accelerated by the external force that has nothing to do with the generation of magnetic field while the magnetic field dissipates its energy as Joule heat. Perplexity occurs when values of the magnetic field and the angular velocity belong to region (1). The growth of the magnetic field is accompanied by the decrease in the angular velocity because the electromagnetic coupling is

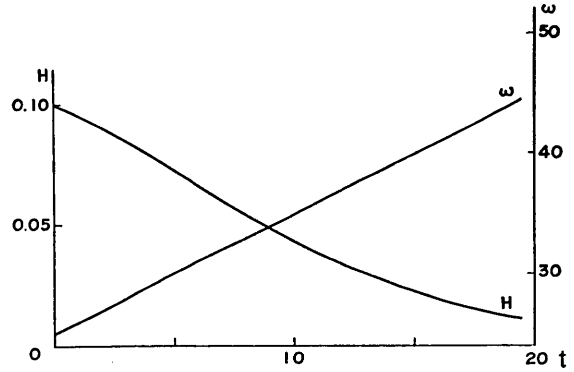

Fig. 4-1. $\varepsilon=1/10^{\frac{1}{3}}$,
Initial values: $H_1=0.1$, $H_2=-0.1$, $\Omega_1=\Omega_2=25$.

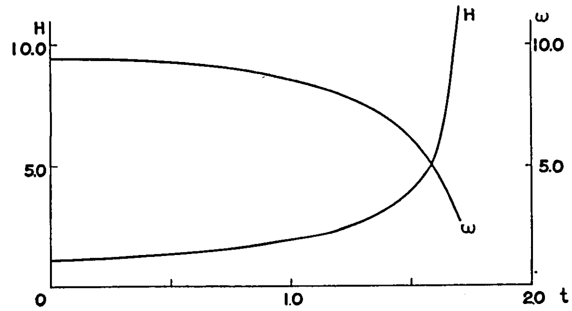

Fig. 4-2. $\varepsilon=1/10^{\frac{1}{3}}$,
Initial values: $H_1=1.0$, $H_2=-1.0$, $\Omega_1=\Omega_2=9.375$.

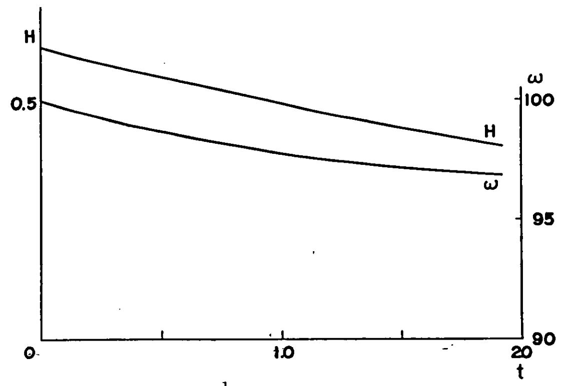

Fig. 4-3. $\varepsilon=1/10^{\frac{1}{3}}$,
Initial values: $H_1=\sqrt{3/8}$, $H_2=-\sqrt{3/8}$, $\Omega_1=\Omega_2=100$

strengthened by the increase in the field and the decelerating torque will come to overwhelm the driving force. It is incomprehensible, however, from the physical point of view, that the diminution of the speed of rotation continues endlessly, as is shown in Fig. 3 and 4. It is probable that this difficulty might be originated in the crude approximation such as (2), where the second order terms of p are ignored. A better approximation of the magnetic field might exclude this unreasonableness.

4. Concluding remarks

It is clarified in this paper that a steady state of the dynamo with two rotating spheres in an infinitely extending conductor cannot be stable for small disturbances. If the magnetic field decreases by chance below a certain value, the coupling torque is weakened and the spheres come to rotate with infinitely rapid speed. If the field is strengthened, the speed of rotation diminishes owing to the increase in the decelerating force. It is also anticipated that the magnetic field would reach its maximum value and begin to decrease, though an approximation of higher degree is needed to describe the electromagnetic coupling between two spheres in order to attain such an oscillatory field. It is not known whether or not we observe reversals of magnetic field in this system until we perform such a study.

In conclusion, the writer expresses his sincere gratitude to Dr. Rikitake who gave him kind direction and advice.

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1. 非定常ダイナモ (1)

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最近，Herzenberg によつて，導体球の中で回転する 2ケの小導体球が，self-exciting dynamo 系として，磁場を維持できることが証明された．ここでは，それと類似の model を用い，その安定性と時間的変化を調べた．従来，非定常ダイナモの研究は，円盤ダイナモを使つてなされてきた．力武は，2ケの円盤ダイナモを結合させて，電流や角速度の向きが，逆転することを示している．電流あるいは磁場の逆転の可能性がこのような簡単な model に対して示された現在では，近似の度をたかめ，より現実的な model を用いて，非定常ダイナモを研究することが望まれる．

ここでは，地球核内の流体運動を極端に理想化して，無限に拡がる導体中で，2ケの導体球が回転している場合を考え，そこに誘導される磁場や，角速度の時間的変化を調べた．この model に対して，定常状態は容易に求まるが，そのような定常状態は，小擾乱に対して，きわめて不安定であることが示される．電動計算機を用いて，数値的にも確められたことであるが，この系は初期値の与え方によつて，2 通りの変動をする．

1) 磁場は単調に減少し，角速度が一様に増加する場合．これは磁場が減少して，運動を抑制しようとする電磁的結合力が弱まると，球の回転が早くなり，外部から供給されたエネルギーが，全部運動エネルギーに転換される場合である．

2) 角速度が減少し，磁場が単調に増加する場合．磁場がある値より大きくなると，回転を抑制しようとする電磁的結合力が，運動を促進する外力に打ち勝つため，球の回転は遅められることになる．しかし，角速度は次第に減少し，磁場が無限に増大するような現象は，物理的に説明困難である．おそらく，角速度の減少と共に，やがては磁場の成長停止，減少という過程が起ると予測されるが，ここではそのような結果は得られなかつた．誘導磁場の近似が十分でなかつたためであろう．近似の度を高めることによつて，振動性の磁場が得られるものと期待される．
