BULLETIN OF THE EARTHQUAKE
RESEARCH INSTITUTE
Vol. 38 (1960), pp. 29-39

3. *Theoretical and Experimental Approach to the Designs and Calibrations of Electro-magnetic Seismograph: I. Voltage Sensitivity of the Moving-coil Type Seismometer.*

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(Read Sept. 22, 1959.—Received Dec. 1, 1959).

1. Introduction

Generally speaking, three types of electro-magnetic vibration-meters have been introduced, at least theoretically. They are the moving-coil type, the changing-flux type and the magneto-striction type. But from the practical point of view in earthquake measurement, no seismometers of the last type have hitherto been used, while the second type of instruments, which have boasted the highest sensitivity, are used only in the observation of minute earthquakes and distant earthquakes because of their non-linear response to stronger earth movements. On the other hand, the first type of instruments, which were introduced prior to the other types, have a wider range of linear response to the ground movement. Moreover, recent development in the material for the permanent magnet has made it possible to design smaller seismometers of higher sensitivity with much smaller permanent magnets.

On account of these advantages, almost all the seismometers of recent designs belong to the moving-coil type (with a fixed magnet) or to the moving-magnet type (with a fixed coil or coils), both of these types being the same theoretically: (essential difference of the above two types lies in the possibility of obtaining higher sensititity of the moving-magnet type without increasing the total weight of the seismometer when the magnet and the coil are given.) We may, therefore, say that seismometers of the moving-coil (or magnet) type are representative of the electro-magnetic seismometers at present. The writer is now engaged in theoretical and experimental studies of the design and calibration of this type of instrument based on the electro-acoustical theory. A part of these studies is hereby reported. In the following, an equivalent four-terminal network of this type of seismometer is deduced, and is also given a simple method of determining the voltage sensitivity which is one of the most important parameters (instrumental constants) of this equivalent network.

2. Equivalent circuit of a moving-coil type seismometer

From the electro-acoustical point of view, an electro-magnetic seismometer can be seen as a kind of electro-mechanical transducer. The input of this transducer is the ground motion while the output of the same is the electric current. If we make use of the current concept of the mechanical impedance as used in the electro-acoustic theory, the mechanical quantities pertaining to the dynamics of the transducer are converted into the equivalent electro-magnetic quantities, and the equation of motion of the transducer is derived and solved by the analogous circuit theory. For the present purpose, the usual electrical mechanical analogy of mass-inductance is not relevant, but the mass-capacitance analogy is very apposite. The mechanical entities and their electrical equivalents to the purpose are listed in Table 1.

Table 1.

Mechanical element	Symbol used	Electrical element	Symbol used
Force	F	Current	I
Velocity	v	Voltage	E
Mass	M	Capacitance	C
Compliance	$1/k$	Inductance	L
Mechanical Resistance	r_m	Conductance	$1/R$

Now let us deduce the equations of motion of the seismometer in the electro-acoustic manner. The electro-magnetic seismometer converts the input ground motion into the output electric current. Therefore, this electro-mechanical transducer is to be considered as a four-terminal network. Let us denote the electric quantities in the mechanical input side by a suffix 1 and those in the output side by a suffix 2. Let the air-gap flux density of the magnet be B, and the total effective length of the wire forming the coil be l. Responding, then, to a current I_2 entering into the coil through the output terminal, a vibro-motive force BlI_2 is generated at the coil. Under additional external force F, the total force exerted on the mass of the seismometer (which is here considered, for simplicity's sake, to be a one mass system) is $F+BlI_2$. This resultant force has to balance with the mechanical reaction consisting of the inertia force, the mechanical resistance and the spring force of the seismometer. We may think of this mechanical reaction as proportional to the velocity v of the seismometer mass. If we denote the proportionality constant by z_m the equation of motion governing the mechanical side is obtained as follows.

$$z_m v = F + AI_2 \tag{1}$$

where $A \equiv Bl$. And A, which is the same as force factor in the treatment of electro-acoustic problems, is a voltage sensitivity[1] of the seismometer.

Since the coil moves in the magnetic field, electro-motive force $-Blv$ is generated in the moving coil. If we also take an exciting voltage E_2 into account, the total electro-motive force becomes E_2-Blv. Denote now the electric impedance by Z, then the current I_2 flowing through the coil is given by Kirchhoff's theorem as follows.

$$ZI_2 = E_2 - Blv = E_2 - Av \tag{2}$$

Making use of the mechanical electrical analogy shown in Table 1, we introduce here the following transformations.

$$F = AI_1 \tag{3}$$

$$v = E_1/A \tag{4}$$

Then (1) and (2) reduce to

$$ZI_2 = -E_1 + E_2 \tag{5}$$

$$I_1 + I_2 = z_m E_1/A^2 \tag{6}$$

To see clearly the meaning of the quantities in these equations, we shall examine the case when the external force F is absent, that is

$$F = AI_1 = 0 \tag{7}$$

Then we have

$$E_1 = A^2 I_2/z_m \tag{8}$$

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[1] K. TAZIME, *Zisin*, **6** (1953), 43–44; **7** (1954), 96–115 (in Japanese).

$$E_2=(Z+A^2/z_m)I_2 \tag{9}$$

From the equations (8) and (9) we can see that A^2/z_m has the dimension of electrical impedance, and is interpreted as the increment of the impedance due to the motion of seismometer. Thus A^2/z_m is called the motional impedance.[2] Introducing, therefore, the notation Z_m for A^2/z_m and solving I_1 and I_2 from (5) and (6), we have

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix}=\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}\begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \tag{10}$$

where Y_{ij}'s are

$$\left.\begin{aligned} &Y_{11}=1/Z_m+1/Z \\ &Y_{12}=Y_{21}=-Y_{22}=-1/Z \end{aligned}\right\} \tag{11}$$

respectively. Equations (10) represent the characteristic relations of a physically realizable four-terminal network, because the matrix of Y_{ij} which may be called admittance matrix is symmetrical. Y_{ij}'s are admittance parameters.

Now let us examine Z_m and z_m in a little more detail. As we have assumed that the seismometer is one mass system, the equation of motion of the system when no current I_2 enters into the coil from the outside, is given by

$$Mpv+r_mv+kv/p=F \tag{12}$$

in which $p=d/dt$ and M, r_m, $1/k$ denotes the mass, damping coefficient and compliance respectively. By the definitions already stated,

$$Z_m=A^2/z_m\,, \quad \text{and} \quad z_m=F/v=Mp+r_m+k/p$$

so that

$$Z_m=A^2/(Mp+r_m+k/p) \tag{13}$$

If we transform M, r_m and k by the electrical equivalents shown in Table 1, we have

$$\begin{aligned} Z_m&=A^2/(Cp+1/R+1/Lp) \\ &=1/(Cp/A^2+1/A^2R+1/A^2Lp) \end{aligned} \tag{14}$$

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[2] A. E. KENNELLY and G. W. PIERCE, *Proc. Am. Ac. Arts and Sc.*, **48** (1912).

Hence the equivalent circuits which give rise to the same impedance with Z_m are the parallel connection of L, C and R, magnitudes of which being A^2/k, M/A^2 and A^2/r_m. The wiring diagram of the L, C and R is shown in Fig. 1. The same circuit has first been suggested by Scherbatskoy and Neufeld[3] in 1937.

It is also to be remarked here that impedance Z, usually believed to be a series connection of a resistance r and a inductance L, is not so simple because of the existence of a iron core along the central axis of the moving coil of the seismometer. Effects of the hysteresis and eddy current losses are to be taken into consideration. But no adequate theoretical treatment of them is yet possible. The only practical approach to it is made on an experimental basis by the introduction of additional equivalent impedance in parallel as shown in Fig. 3a. If frequency f is known, Fig. 3a can be expressed as Fig. 3b, where $R'=R/\{1+(R/2\pi fL)^2\}$ and $L'=L/\{1+(2\pi fL/R)^2\}$. In the case when f is sufficiently small, then R and L in Fig. 3a or R' and L' in Fig. 3b become so small that these additional impedances may be neglected.

3. A simple method for determining the voltage sensitivity

Through these discussions we came to the conclusion that, if the voltage sensitivity A is known, the constants of the seismometer may be determined by measuring the electrical impedances. As A is the factor of transformations between mechanical and electrical elements, we cannot determine it by electrical measurements only. And so, to study the characteristics of a seismometer the measurement of A is inevitable.

Now, let us state a simple method for determining the voltage sensitivity A. When the current I passes through a coil, force AI is generated. So, if we can measure the force and the current, the voltage sensitivity will be determined. In Fig. 4 a simple apparatus for this purpose is shown. This apparatus consists of a lamp-and-scale, a balance and an ammeter. The balance has a mirror at the end of its rider-beam as shown in the figure, and it reflects the light-spot from the lamp to the scale. By this apparatus we can easily measure the small deflection of the balance from its equilibrium position.

Now, the measurement of the voltage sensitivity is carried out in the following sequence:

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[3] S. A. SHERBATSKOY and J. NEUFELD, *Geophysics*, **2** (1937), 213–242.

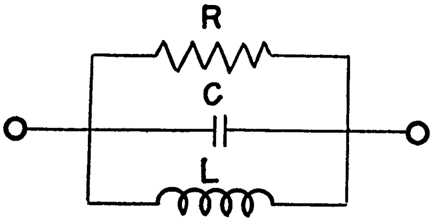


Fig. 1. Equivalent circuit of the motional Impedance.

$C=M/A^2$, $R=A^2/r_m$, $L=A^2/k$.

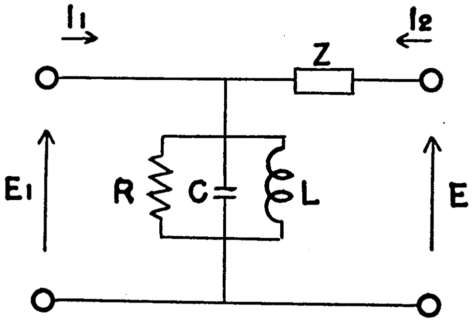
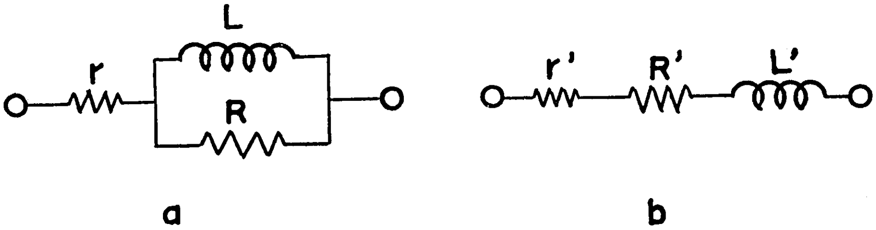


Fig 2. Equivalent circuit of the moving coil type electro-mechanical transducer.

$I_1=F/A$, $E_1=Av$



a

b

Fig. 3. Equivalent circuit of the impedance of moving coil. r: DC resistance.

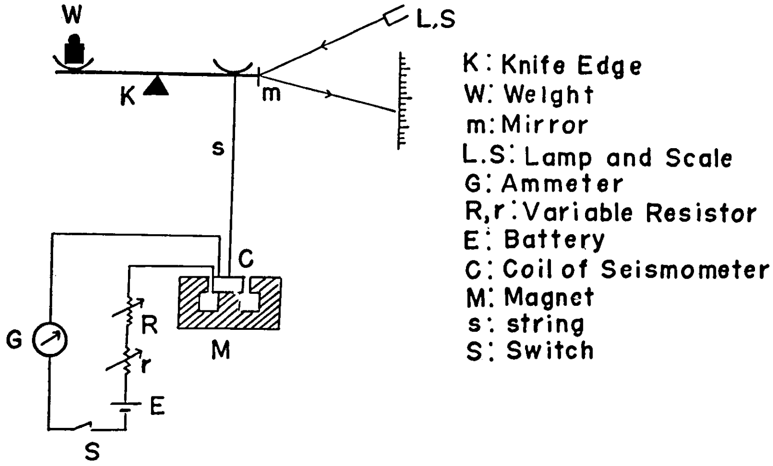
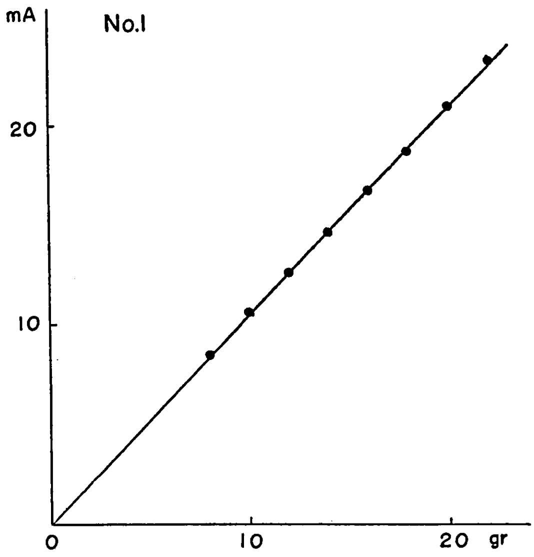
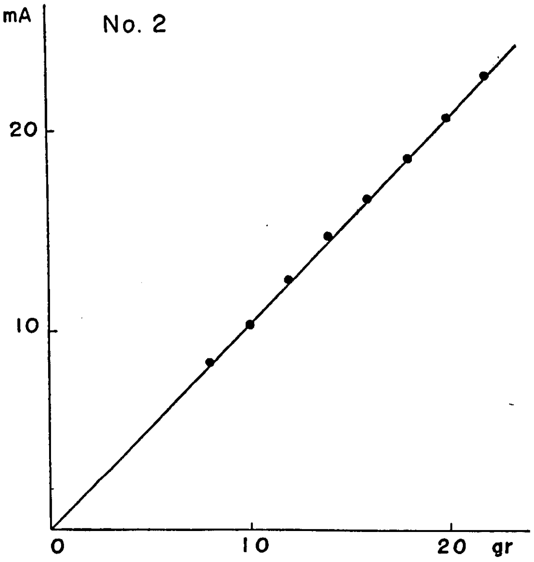


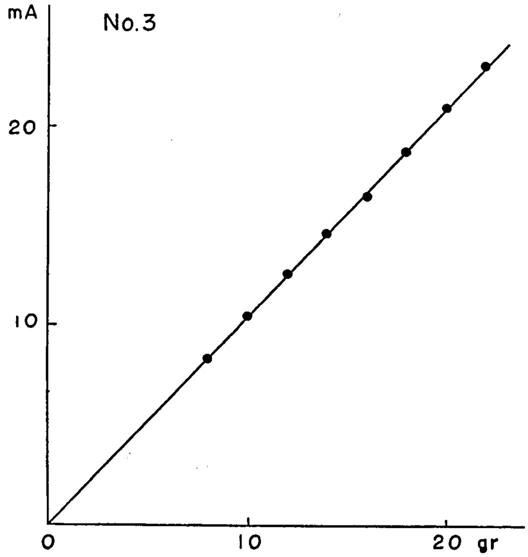
Fig. 4.

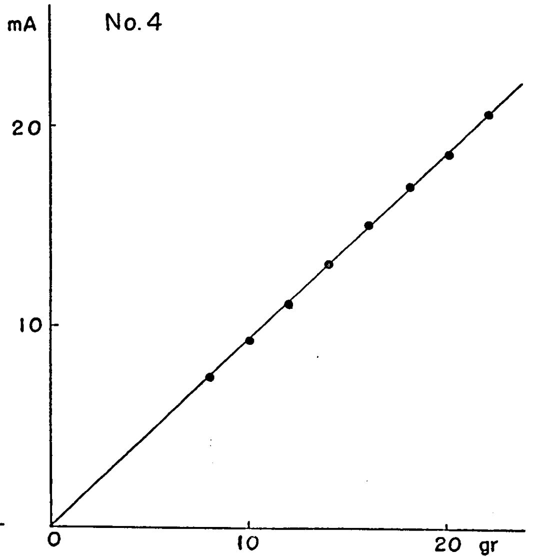
i) Connect the centre of the coil and the rider-beam with a string, and adjust the relative position of the balance and the coil so as the string is streched vertically.

ii) Appropriate weights are placed on the balance to bring the coil into the normal position of equilibrium when it is used actually. Remember the position of the light-spot on the scale at this equilibrium









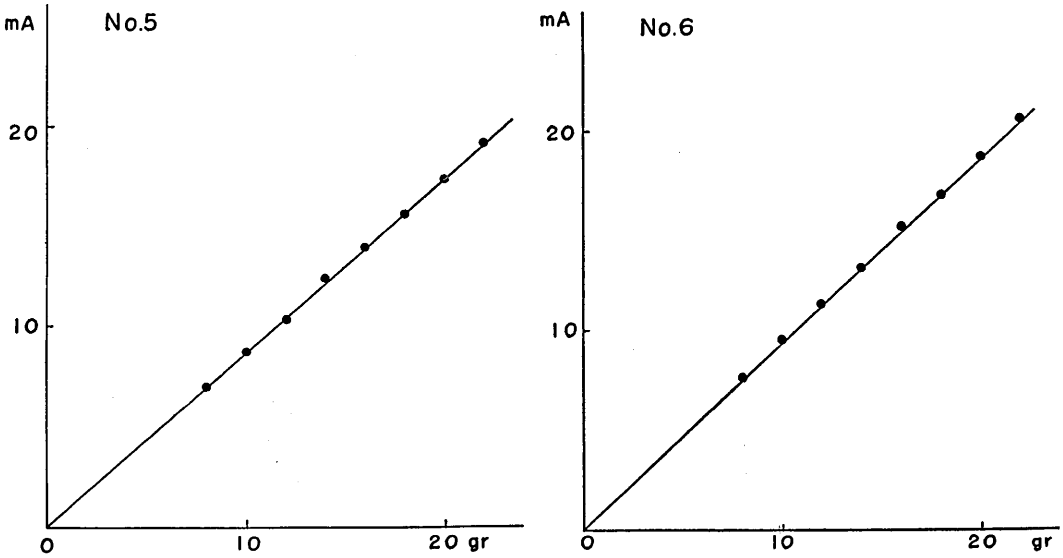


Fig. 5.

state. This position is to be referred to as zero position.

iii) Add a weight, say M gram, on a pan of the balance. Now switch the current into the circuit, and adjust the current by the variable resistances R and r, so that the balance regains its equilibrium and the light-spot comes to the abovementioned zero position of the stage ii). Read then the value of the current i, which is flowing in the circuit.

Then the voltage sensitivity can be calculated by the following formula,

$$A=Mg/i \tag{15}$$

where g is the acceleration due to gravity. In the iii) stage, the electric force pulls downwards to equiponderate with the weight. Two large and small variable resistances R and r are introduced in the apparatus in order to facilitate rough and fine adjustments.

Examples of actual measurements are shown in Fig. 5. From these figures we worked out the values of A, and tabulated them in Table 2. Comparing these values with the same calculated on the assumption of magnetic paths, we found a very good agreement between them.

Table 2.

Seismometer	Measured A in volt/kine	Calculated A in volt/kine
1	0.093	
2	0.095	
3	0.094	0.101
4	0.104	
5	0.112	
6	0.105	

4. Discussions

We have obtained an equivalent circuit of a seismometer in section 2. The case of a galvanometer can also be inferred by the abovementioned circuit by interchanging the direction of the input and the output. Namely, entities of the inputs are electrical and those of the outputs are mechanical in this problem.

It is convenient to express an electro-magnetic seismometer by a four-terminal network, because the characteristics of a galvanometer directly coupled to a seismometer are at once deduced by mere multiplications of the admittance matrices, as the given problem is the series connection of the four-terminal networks. And the above consideration can be extended to any case which contains an amplifier or some other circuits in the network of seismic apparatus. Doubtless to say, this is one of the merits of our equivalent circuit.

The determinations of the voltage sensitivity have, so far, been made by observing the deflections of a mass from the equilibrium position and the currents simultaneously. But in carring out these measurements, we have inevitably experienced that the larger the current flowing through the coil, the smaller is the apparent voltage sensitivity that results. This is due to the fact that the flux density decreases its intensity the more, the further the coil protrudes from the region of uniform flux density. As the voltage sensitivity is linearly proportional to the flux density, the reduction of the flux density turns into the reduction of the voltage sensitivity. In our measurement, which is a zero-method, the coil stays in its optimum position, and the proportionality of the force and the current intensity holds in a wider range of their magnitudes than the case when the coil moves. This fact enables us to extend our measurement to a wider range, so that the accuracy of measurement may appreciably be improved.

Hitherto we have considered the voltage sensitivity as a constant. But, if we consider the influences of eddy current and the hysteresis losses of an apparatus, the voltage sensitivity does not remain a constant, and shows frequency characteristics. The same is true, also, for the impedance of the coil, and its magnitude increases when the driving frequency becomes higher.

When the seismometer inclines from the vertical—we can consider that such a circumstance may occur when we set a bore-hole-seismometer[4] at the bottom of a bore-hole—the coil may protrude out from the uniform magnetic field. And the apparent voltage sensitivity becomes smaller.

An example is given in Fig. 6, showing how the voltage sensitivity of a bore-hole seismometer changes its magnitude according to the deflection of the coil from the centre of its equilibrium. So, if we want precision in measurements, we have to consider these influences according to their actual conditions.

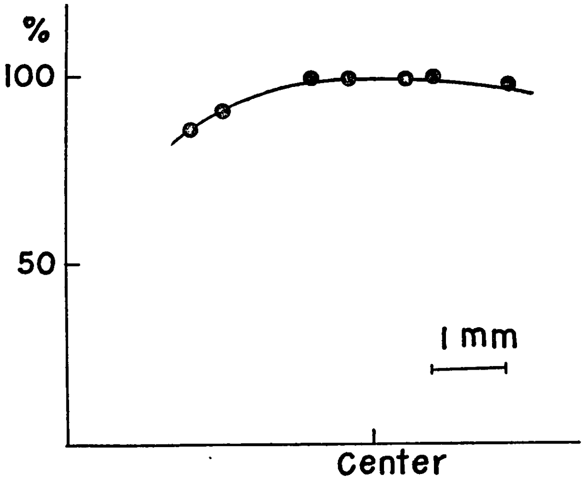


Fig. 6.

5. Concluding remarks

We have studied the moving coil type electro-magnetic seismometer as a four-terminal transducer, and deduced an equivalent circuit of the

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[4] H. KAWASUMI, E. SHIMA and M. SIBANO, *Annual Meeting of the Seismological Society of Japan* (April, 1955).

seismometer. This equivalent circuit agrees with the one which was first suggested by Sherbatskoy and Neufeld. By the use of this equivalent circuit, we can easily determine the instrumental constants of a seismometer simply by measuring the motional impedance electrically, when the voltage sensitivity is known. We can also use this equivalent circuit in the consideration of a galvanometer by reversing the input and the output. Thus, it has become possible to obtain the expressions of seismic apparata by the four-terminal networks conveniently and easily to infer the over-all characteristics of the apparata, and the characteristics are calculated by the mere multiplications of the admittance matrices.

We then designed a simple device for the measurement of the voltage sensitivity. By this device we could measure the voltage sensitivity more precisely than previously possible. Motional aspects of the voltage sensitivity are not dealt with in this paper, but we shall study them as the next problem.

In conclusion, author expresses his hearty thanks to Professor H. Kawasumi who gave him valuable discussions and perpetual encouragement.

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3. 電磁式地震計の理論的実験的研究

その 1　電圧感度について

地震研究所　嶋　　悦　三

Moving coil type の電磁式地震計を，動電型変換器という立場から見れば，四端子の等価回路で，その働きを表現することが出来る．そして，この等価回路から，もし電圧感度が知れている場合には，電気端子側から，電気的 Impedance を測定すれば，地震計の常数はすべて決定し得ることがわかる．

この等価回路は，Galvanometer についても，地震計と全く同様な考え方から，入力端子と出力端子を入れかえることにより，そのまま利用出来るのである．なぜならば，電気端子側から見れば，地震計と Galvanometer は全く同等であると考えられるからである．

電圧感度を決めるためには，電気的測定のみからは出来ず，同時に機械的量を測定しなければならない．そして，これはなるべく手軽に行うことが望ましい．そのために，電圧感度を簡単に測定する一つの方法を考案した．装置は，coil に流れる電流のために生ずる力を，天秤をつかつて直接はかろうとするものである．この方法は，zero-method であるために，従前の方法と較べると，比較的大きな電流を coil に流すことが出来るので，精度をあげることが出来る．

電圧感度の動的測定は，今回は行わなかつたが，振動数が高くなれば，当然その影響を考えなければいけない，いいかえれば，電圧感度はもはや constant でなく周波数特性をもつだろうということが予想される．