

### 35. The Anomalous Behaviour of Geomagnetic Variations of Short Period in Japan and Its Relation to the Subterranean Structure. The 9th report.

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#### Summary

The possible cause of the anomaly of short-period geomagnetic variations as observed in Japan is examined in detail. The only way to account for such an anomaly seems to assume a high conducting circuit of roughly elliptical shape, about 200 km and 1000 km in width and length respectively. Both the ends of the circuit are to be connected with the conducting part of the earth's mantle, while a low conducting cut is also required between the connecting points. The model thus supposed seems to harmonize very well with the anomaly of another sort found in the cases of *Sq* and *Dst*.

A study on the electromagnetic induction within a spherical sheet having a non-conducting hole is also made in order to examine the influence of the sea. It turns out that the Japanese anomaly is hardly ascribed to the effect of the electric currents induced in the sea.

#### 1. Introduction

The anomaly of short-period geomagnetic variations, that have been frequently reported in this bulletin<sup>1),2),3),4)</sup> and elsewhere<sup>5),6)</sup>, is thought to be caused by induced electric currents flowing in a circuit beneath Japan. Although no close account has been taken of such a high conducting circuit so far, it seems unlikely that we may have any other

1) T. RIKITAKE, I. YOKOYAMA and Y. HISHIYAMA, *Bull. Earthq. Res. Inst.*, **30** (1952), 207, **31** (1953), 19, 89, 101 and 119.

2) T. RIKITAKE and I. YOKOYAMA, *Bull. Earthq. Res. Inst.*, **33** (1955), 297.

3) T. RIKITAKE, I. YOKOYAMA, S. UYEDA, T. YUKUTAKE and E. NAKAGAWA, *Bull. Earthq. Res. Inst.*, **36** (1958), 1.

4) T. RIKITAKE, S. UYEDA, T. YUKUTAKE, I. TANAOKA and E. NAKAGAWA, *Bull. Earthq. Res. Inst.*, **37** (1959), 1.

5) T. RIKITAKE and I. YOKOYAMA, *Journ. Geomagn. Geoelectr.*, **5** (1953), 59.

6) T. RIKITAKE and I. YOKOYAMA, *Naturwissenschaften*, **41** (1954), 420.

alternative by which the observed anomaly can be explained satisfactorily.

The main aim of this paper is to examine the possibility of the supposed circuit as closely as possible. In Section 2, first of all, the writer studies the magnetic effect of a conductor embedded in the non-conducting earth. It becomes clear that no such conductor can produce magnetic fields which account for the anomaly so long as the electromagnetic induction in an isolated conductor is concerned.

The next possibility, that is examined in Section 3, is the branch circuit which is connected to the conducting part of the mantle below the depth of about 400 *km*. If the circuit forms a suitable loop underneath Japan, it might be possible to expect strong electric currents flowing in the circuit provided a sufficient electric potential difference appears between the two connecting points by the electromagnetic induction within the conducting mantle. It turns out, however, that, corresponding to an inducing field of 100 *gammas*, an induced field amounting to only a few *gammas* would be possible by the procedure considered.

The easiest way to increase the induced field would be to suppose a non-conducting cut between the connecting points as is investigated in Section 4 on the basis of a two-dimensional model experiment. By adopting such a configuration of insulator, it is shown that an induced field as large as the inducing one can be expected at the central part of the circuit.

In Section 5 is studied the electromagnetic induction within a spherical sheet having a non-conducting hole. This is useful for studying the influence of the electric currents induced in a sea surrounding a land mass on geomagnetic variations. The view that the anomaly of geomagnetic variations in Japan seems unlikely to be caused by the effect of the sea is also supported by the investigation.

## 2. Magnetic fields produced by electric currents induced within a conductor embedded in the earth

In the previous papers, it has often been pointed out that the anomalously large amplitude of the vertical component, which is observed in the central part of Japan at the time of short-period geomagnetic variations, cannot be explained as the magnetic field produced by the electric currents induced within a circuit which is isolated from the conducting region below the depth of 400 *km*. On this point, however, some further examinations will be made in this section.

Let us suppose that a spherical conductor of radius  $a$  is embedded in the earth which is regarded as non-conducting. If we further suppose a uniform magnetic field changing with a time-factor  $e^{iat}$ , the magnetic potential of the field can be written as

$$W_e = rHP_1(\cos \theta), \quad (1)$$

where  $H$  is the intensity of the field. The time-factor is removed from the expression. It is assumed that the field is directed to the  $\theta=0$  direction. The spherical polar coordinates  $r$ ,  $\theta$  and  $\phi$  are taken here, the origin being taken at the centre of the sphere considered.

The magnetic potential of the field produced by the electric currents induced in the sphere by the uniform field is easily obtained as<sup>7)</sup>

$$W_i = a^3 r^{-2} I(i\alpha) HP_1(\cos \theta), \quad (2)$$

$$I(i\alpha) = \frac{1}{2} \left( 1 - \frac{F_1(ka)}{F_0(ka)} \right), \quad k^2 = 4\pi\sigma i\alpha, \quad (3)$$

where  $\sigma$  denotes the electrical conductivity of the sphere and the magnetic permeability is assumed as unity, while the functions  $F_0$  and  $F_1$  are given as

$$F_0(x) = \frac{\sinh x}{x}, \quad F_1(x) = \frac{3}{x^2} \left( \cosh x - \frac{\sinh x}{x} \right). \quad (4)$$

From (2), it is easily seen that the induced field can be expressed as that of a magnetic dipole, the axis of which being directed to the  $\theta=0$  direction and the magnetic moment being specified by  $M$  which is given as

$$M = a^3 I(i\alpha) H. \quad (5)$$

For a perfectly conducting sphere, it has been readily shown that  $I=1/2$ , so that

$$\lim_{\sigma \rightarrow \infty} M = \frac{1}{2} a^3 H. \quad (6)$$

Assuming various values for  $\sigma$ ,  $a$  and the period of variation, modulus of  $M$  is calculated as shown in Table 1, where  $H$  is always taken as 100 *gammas*.

7) S. CHAPMAN and J. BARTELS, *Geomagnetism* Ch. 12 (1940).

Table 1.

Table 1-1. $a=2\text{ km}$				Table 1-2. $a=20\text{ km}$			
$T$	$\sigma\text{ (e.m.u.)}$			$T$	$\sigma\text{ (e.m.u.)}$		
	$10^{-12}$	$10^{-10}$	$\infty$		$10^{-12}$	$10^{-10}$	$\infty$
1 min.	$1.4 \times 10^{10}$	$1.4 \times 10^{12}$	$4.0 \times 10^{12}$	1 min.	$1.4 \times 10^{15}$	$3.6 \times 10^{15}$	$4.0 \times 10^{15}$
1 hour	$2.3 \times 10^8$	$2.3 \times 10^{10}$	"	1 hour	$2.3 \times 10^{13}$	$1.7 \times 10^{15}$	"
2	$1.2 \times 10^8$	$1.2 \times 10^{10}$	"	2	$1.2 \times 10^{13}$	$1.1 \times 10^{15}$	"
6	$3.8 \times 10^7$	$3.8 \times 10^9$	"	6	$3.8 \times 10^{12}$	$3.7 \times 10^{14}$	"
12	$1.9 \times 10^7$	$1.9 \times 10^9$	"	12	$1.9 \times 10^{12}$	$1.5 \times 10^{14}$	"
24	$9.6 \times 10^6$	$9.6 \times 10^8$	"	24	$9.6 \times 10^{11}$	$1.5 \times 10^{14}$	"

Table 1-3. $a=100\text{ km}$				Table 1-4. $a=200\text{ km}$			
$T$	$\sigma\text{ (e.m.u.)}$			$T$	$\sigma\text{ (e.m.u.)}$		
	$10^{-12}$	$10^{-10}$	$\infty$		$10^{-12}$	$10^{-10}$	$\infty$
1 min.	$4.2 \times 10^{17}$	$4.9 \times 10^{17}$	$5.0 \times 10^{17}$	1 min.	$3.6 \times 10^{18}$	$4.0 \times 10^{18}$	$4.0 \times 10^{18}$
1 hour	$7.4 \times 10^{16}$	$4.3 \times 10^{17}$	"	1 hour	$1.7 \times 10^{18}$	$3.7 \times 10^{18}$	"
2	$3.9 \times 10^{16}$	$4.1 \times 10^{17}$	"	2	$1.1 \times 10^{18}$	$3.6 \times 10^{18}$	"
6	$1.8 \times 10^{16}$	$3.5 \times 10^{17}$	"	6	$3.7 \times 10^{17}$	$3.4 \times 10^{18}$	"
12	$5.0 \times 10^{15}$	$3.0 \times 10^{17}$	"	12	$1.5 \times 10^{17}$	$3.1 \times 10^{18}$	"
24	$3.2 \times 10^{15}$	$2.3 \times 10^{17}$	"	24	$1.5 \times 10^{17}$	$2.8 \times 10^{18}$	"

In an analysis<sup>1)</sup> of the polar magnetic storm that occurred on June 19, 1936, it was shown by T. Rikitake and others that, the anomalous distribution of the induced field can be approximated roughly by the magnetic field of a dipole situated at a depth of 150 km in the central part of Japan, the magnetic moment having been estimated at  $8.6 \times 10^{18}$  e.m.u. at the maximum of the change. On the basis of this study, H. J. Lippmann<sup>8)</sup> concluded that the anomaly of short-period geomagnetic variations as observed in Japan can be interpreted as the effect of the induced field within a spherical conductor having a radius of 130 km and a conductivity of  $1\Omega^{-1}m^{-1}$  ( $=10^{-11}$  e.m.u.). He assumed a uniform inducing field of 100 gammas with a period of 1 hour. In that case, the moment of the induced dipole amounts to  $10^{18}$  e.m.u. as can be also seen in Table 1.

It seems to the writer, however, that Lippmann's conclusion is hardly acceptable from various considerations which will be examined one by one in the following. First of all, the inducing field at the time of

8) H. J. LIPPMANN, *Zeits. f. Geophys.*, **24** (1958), 113.

the polar magnetic storm concerned amounts to only 30 *gammas* in the vicinity of Japan. It is therefore required to suppose a spherical conductor larger than Lippmann's one. In order to account for the magnetic moment amounting to  $8.6 \times 10^{18}$  *e.m.u.*, a spherical conductor of nearly 400 *km* in radius must exist under Japan, while its conductivity is of the order of  $10^{-12}$  *e.m.u.* or higher. It seems unlikely that such a large conducting region really exists beneath Japan. If it existed, that region would be of high temperature and might be probably detected by some other geophysical methods such as by investigation of seismic waves.

On carefully examining Table 1, we should also pay attention to the fact that the moments of the induced dipole for long-period variations do not differ much from those for short-period ones in case the conductivity is high and the radius is large. If  $\sigma = 10^{-10}$  *e.m.u.* and  $a = 100$  *km*, for example, the magnetic moment for a 24-hour variation amounts to a half of that for a 1-hour variation. The fact suggests that, if we have a conductor of the size and conductivity underneath Japan, there should be observed an anomaly of similar sort even in the case of *Sq* field. However, close examination of *Sq*<sup>9)</sup> in Japan as well as *Dst*<sup>1)</sup> proved that these slower variations behave quite differently, so that it is not conceivable to suppose such an underground conducting mass beneath Japan.

One more objection arises from the fact that the anomaly is observed mainly in the vertical component. In the middle latitude, the magnetic vector for geomagnetic bays or polar magnetic storms lies in a nearly horizontal plane. If there is an underground spherical conductor, the apparent magnetic dipole induced should be also directed to an approximately horizontal direction, so that the induced magnetic field right up the sphere should predominate in the horizontal direction. In the central part of the anomalous area, however, the vertical amplitude of geomagnetic variations is usually equal to the horizontal one. Hence, the spherical conductor model does not account for the observed fact.

Since the spherical conductor model failed, we shall then consider a ring-shape conductor as an alternative. If a circular circuit is placed in a uniform periodic magnetic field which is normal to the circuit, the complex intensity of the electric current induced in the circuit is given by

$$RJ = -i\alpha\pi A^2 H - i\alpha LJ, \quad (7)$$

where  $R$ ,  $J$ ,  $\alpha$ ,  $A$ ,  $H$  and  $L$  denote respectively electric resistance of

9) T. RIKITAKE, I. YOKOYAMA and S. SATO, *Bull. Earthq. Res. Inst.*, **34** (1956), 197.

the circuit, intensity of the induced electric current, angular frequency, radius of the circuit, intensity of the inducing magnetic field and self-inductance of the circuit. If the circuit is composed of a circular wire and its radius is  $\rho$ , the self-inductance is given by

$$L = 4\pi A \left( \log_e \frac{A}{\rho} + 0.33 \right). \quad (8)$$

Suppose  $\rho = 10 \text{ km}$ ,  $L$  is estimated as  $5.5 \times 10^9$  and  $5.5 \times 10^{11} \text{ cm}$  respectively for  $A = 100$  and  $1000 \text{ km}$ . If we take the electrical conductivity ( $\sigma_1$ ) of the material composed of the circuit to be  $10^{-12} \text{ e.m.u.}$  (that is the conductivity below the level of  $400 \text{ km}$  for the normal part of the earth),  $R$  is obtained as  $2.0 \times 10^9$  and  $2.0 \times 10^{11} \text{ e.m.u.}$  respectively. If we further assume a period of 1 hour,  $L/R$  is estimated as  $4.8 \times 10^{-3}$  both for  $A = 100$  and  $1000 \text{ km}$ . In that case, the second term of the righthand-side of (7) may be ignored, so that, assuming  $H = 100 \text{ gammas}$  as before, we obtain

$$J = 0.27 \text{ e.m.u.} = 2.7 \text{ amp.} \quad (9)$$

for both the cases.

At the centre of the circular circuit, the magnetic field produced by the induced current is given by

$$H_i = \frac{2\pi J}{A}. \quad (10)$$

Adopting the value of  $J$  in (9),  $H_i$  becomes  $0.017$  and  $0.0017 \text{ gammas}$  for  $A = 100$  and  $1000 \text{ km}$ . Even if we take  $\sigma_1 = 10^{-10} \text{ e.m.u.}$ ,  $H_i$  does not exceed  $2 \text{ gammas}$ . It then turns out that the induced current produces extremely small magnetic fields, while the observed anomaly of the induced field amounts to the same order as the inducing field. It is then impossible to ascribe the anomaly to the induced current in a ring-shape circuit supposed in the above.

### 3. Branch circuit connected to the conducting part of the earth's mantle

The quantitative study on the magnetic effect of a spherical or ring-shape conductor embedded in the earth confirms the previous view that the electric currents which are responsible for the magnetic anomaly

observed must come from farther depths where the conductivity is believed to be high. One of the most straightforward ways of accounting for the anomaly would be to suppose a branch circuit, both the ends of which are connected to the conducting part of the earth's mantle. If the circuit forms a loop in a suitable way underneath Japan, a part of the electric currents induced in the mantle might flow along the loop and produces a fairly strong magnetic field near the centre of the loop. It has been the only way of explaining the anomaly to suppose such a hypothetical circuit though no exact account has been so far taken of the circuit.

It is the purpose of this section to examine quantitatively to what extent the anomaly may be explained by the branch circuit hypothesis. Let us suppose that the ends of the circuit are connected to the conducting mantle at two points 1000 *km* apart. Although this figure is taken arbitrarily, it would be unlikely to suppose that it is larger or smaller than 1000 *km* by a factor 5 judging from the area of anomaly. Now we are in a position to estimate the electric potential difference between the two points when electric currents are induced in the conducting mantle by geomagnetic variations arising from outside the earth.

For the present study the earth may be regarded as a uniform conductor covered by a non-conducting layer of 400 *km* in thickness. If a uniform conducting sphere of radius *a* is placed in a uniform time-dependent magnetic field directed to the  $\theta=0$  axis, the electric field induced in the sphere has only the azimuthal component which is given as<sup>7)</sup>

$$E = \frac{i\alpha}{2} \frac{F_1(ka)}{F_0(ka)} r H \frac{dP_1}{d\theta}, \quad (11)$$

where  $\alpha$  is derived from the time factor  $e^{iat}$  and  $F_0$  and  $F_1$  are defined by (4).

Table 2.

Period	Electric field intensity	Potential difference between the two points 1000 <i>km</i> apart
1 <i>h</i>	12.4 <i>e.m.u./cm</i>	12.4 <i>volt</i>
2	8.74	8.74
6	5.02	5.02
12	3.56	3.56
24	2.48	2.48

At  $r=a$  ( $a=6000$  km), modulus of the coefficient of  $dP_1/d\theta$  is calculated for various periods as shown in Table 2, while the electrical conductivity is assumed as  $2 \times 10^{-12}$  e.m.u. as has been obtained by the present writer<sup>10)</sup> previously and the intensity of the inducing field is taken as 100 *gammas*.

The voltage between the two points 1000 km apart on the same latitude circle is given in the third column of the table. The figures, however, should be multiplied by  $dP_1/d\theta$  in order to get the actual voltage. Since  $dP_1/d\theta$  is not largely different from unity in middle and low latitudes, we may assume that the induced voltage will be of the order as given in the table.

On the basis of the voltage thus estimated, we are going to estimate what amount of electric current will flow in the branch circuit considered. Strictly speaking, the problem should be solved by applying the electromagnetic induction theory to a spherical conductor having the branch circuit. Unfortunately, however, no exact study of such a case is possible in the framework of existing electromagnetic induction theory. We are, therefore, obliged to proceed with an approximate method as will be developed in the following.

The branch circuit forms a closed circuit together with the superficial part of the conductor. We may, however, ignore the voltage which is induced in the circuit by the varying magnetic field, because what we have been dealing with in the final part of the last section suggests that the voltage induced in such a circuit would be extremely small for possible dimensions. Although the actual configuration of the circuit is not known, it does not seem likely that the above presumption changes largely.

From the above consideration, we may treat the matter as if quasi-steady flow of electric currents in the circuit driven by the potential difference between both the ends. The potential difference has been given in Table 2 for respective periods. It is impossible to estimate exactly the electric resistance of the circuit. But we may presume from the discussion of the loop, which consists of the upper part of the circuit, that the resistance would be larger than 2 and 200 *ohms* respectively for  $A=100$  and 1000 km provided  $\sigma_1=10^{-12}$  e.m.u. is assumed. Even if we take 2 *ohms*, which may be the smallest of the possible cases, the current flowing in the circuit amounts to only 6 and 1 *amperes* respectively for 1 and 24-hour period variations of 100 *gammas* in am-

10) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **28** (1950), 45, 219, 263, **29** (1951), 61.



plitude. If we take a higher conductivity,  $\sigma_1=10^{-10}$  *e.m.u.* say, which seems to be possible as will be discussed in the next section, the minimum values of the resistance of the circuit would be 0.02 and 2 *ohms* respectively for  $A=100$  and 1000 *km*. Corresponding to 0.02 *ohms*, the current for 1-hour period variation amounts to 600 *amperes*. Since such small amounts of current produce magnetic fields of only a few *gammas* at the centre of the loop and cannot obviously account for the observed anomaly, the hypothesis of branch circuit must be abandoned unless some other means to increase the potential difference can be found.

#### 4. Branch circuit and insulating wedge in the conducting mantle

Since it turns out in the last section that a simple branch circuit cannot account for the anomaly, we have to seek some other mechanism by which we may expect much larger potential difference between both the ends of the circuit. Let us consider a one-dimensional steady flow of electric current. If the conductivity of one part of the conductor is lower than that of the remaining parts, the potential drop per unit length becomes larger in the low conducting part than that in the high conducting part. In a limiting case in which the high conducting part can be regarded as a perfect conductor, the potential difference between the two ends of the low conductor becomes equal to the voltage given to the system. From the analogy of the steady flow in the above, we might be able to expect a potential difference higher than that discussed in the last section by supposing that low conducting material wedges into the conducting mantle and that the branch circuit is connected to the conducting mantle at two points just outside the wedge.

It would be extremely difficult to study electromagnetic induction in such a system of conductor. However, an approximate study, though very crude, will be made in the following. If the conducting mantle is uniform everywhere, the flow of electric current in the neighbourhood of the connecting points of the circuit is the same as that driven by the potential difference between two hypothetical plane-electrodes suitably placed in the conductor with a great distance. In the theory of electromagnetic induction within a uniform sphere, it has been well known that the induced currents flow perpendicularly to all radii and decrease gradually as the depth from the surface increases. Actually, for the conductivity considered in the last section, the current density becomes

one tenth of the surface values approximately at depths of 240 and 500 km respectively for 1 and 24-hour period variations. If we imagine that the earth's curvature can be neglected for the study of the currents around the branch circuit, the currents induced in the conductor would be approximated by the ones which are caused by the potential difference due to an imposed electromotive force given two vertical electrodes of wide contact plane, the distance between the electrodes being larger than the size of the circuit and the distribution of electric potential on the electrodes being assumed to fit the ones for the spherical case.

Even if there is an irregularity of conductivity around the connecting points of the branch circuit, the above analogy that we may take the hypothetical electrode approximation would be approximately held as long as the size of the irregularity is smaller than the wave-length of the varying magnetic field. Such a consideration would be acceptable because the induced currents are driven by electromotive forces which are excited all through the conductor and the electromotive force set up within the irregularity is to be small.

In the light of the above discussion, the influence of the insulating wedge on the distribution of electric voltage is examined by a model experiment. The experiment is conducted by use of a sheet of facsimile paper for recording by electric discharge. The paper is covered by a fine carbon film on which a very thin screen of wax is spread. If we have an electric discharge between the carbon layer and a sharp point close to the paper, a black spot is marked on the paper. Since

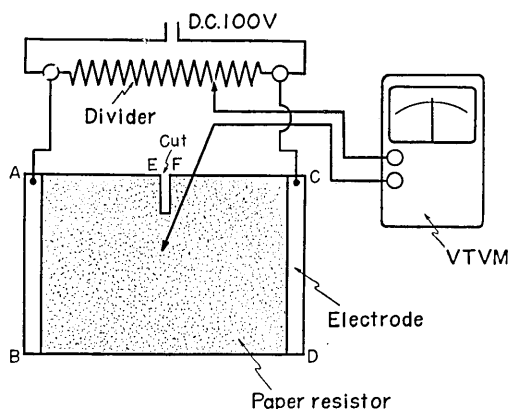


Fig. 1. The arrangement for the experiment.

the carbon layer is fairly uniform, the paper is a good medium for an experiment of two-dimensional steady electric current<sup>11)</sup> though such usage of the paper is far from the original purpose.

We take a rectangular sheet of 35 cm × 25.3 cm. Two brass electrodes are placed on the 25 cm sides. The electric contact between the paper and the electrodes are made by

11) K. KASAHARA, *Bull. Earthq. Res. Inst.*, **37** (1959), 39.

putting carbon powder on the paper with a pencil. Along the central line of the paper, a cut is made as can be seen in Fig. 1. Applying 100 volt between  $AB$  and  $CD$ , the potential at  $E$  and  $F$ , the edge points of the cut, is measured by means of null method with a valve voltmeter as the detector. Since the input impedance of the meter is very

Table 3. The voltage between  $E$  and  $F$  as measured by a valve-voltmeter.

Depth (cm)	Voltage (volt)	Depth (cm)	Voltage (volt)
0.0	0.95	13.0	62.5
1.0	7.1	14.0	66.0
2.2	13.9	15.0	68.0
3.0	18.4	16.0	72.0
4.0	24.2	17.0	74.0
5.0	29.6	18.0	75.1
6.0	34.8	19.0	77.5
7.0	38.2	20.0	79.0
8.0	44.0	21.0	81.0
9.0	48.5	22.0	82.5
10.0	53.5	23.0	85.0
11.0	57.0	24.0	86.5
12.0	60.0		

high, the voltage measured is highly reliable. The voltage between  $E$  and  $F$  is 0.95 volt when there is no cut. With the increase of the depth of the cut, which is 3 mm in width in this case, the voltage increases as shown in Table 3 and Fig. 2.

As can be seen in the table and figure, it is noticeable that the potential difference between  $E$  and  $F$  increases immensely as the cut goes deeper. The ratio of the voltage to that of the case for no cut becomes 31, 56, 73 and 83 respectively for  $l$  (the depth of the cut)=5, 10, 15 and 20 cm. In order to see the distribution of electric voltage, equipotential lines

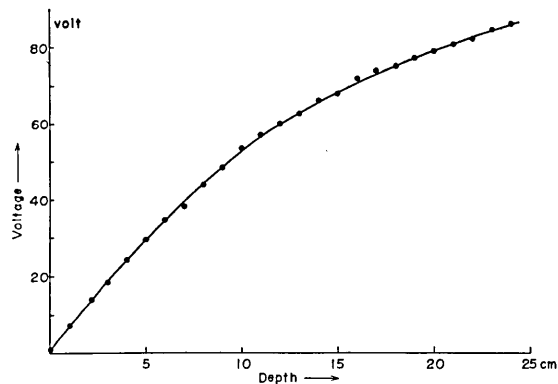


Fig. 2. The changes in the voltage between  $E$  and  $F$  as the depth of the cut increases.

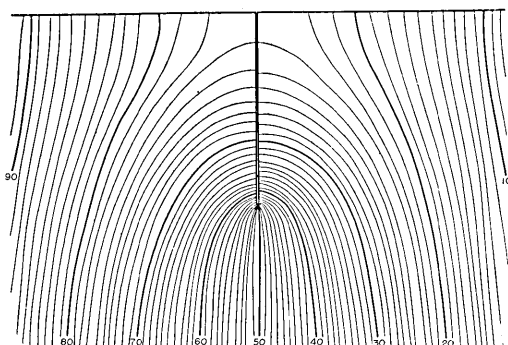


Fig. 3. The equipotentials for  $l=10$  cm.

it is not impossible to expect a potential difference 50 times larger than the values given in Table 2 just outside the insulating wedge. In that case, the electric current which will flow in the branch circuit connected to the outside of the wedge becomes 50 times as large as the figures obtained in the last section, amounting to 300 and 30,000 *amperes* respectively for  $\sigma_1=10^{-12}$  and  $10^{-10}$  *e.m.u.*, 1-hour period inducing field of 100 *gammas* in amplitude and  $A=100$  km being assumed as before. There is a possibility that the current intensity would be larger than the figure cited above, because, in the case of a deep cut, the amount of electric currents flowing in the mantle would become small, while that flowing in the circuit would increase in turn. Although no accurate study is possible on this point, the above tendency may be approved from the fact that the electric resistance between  $AB$  and  $CD$  as shown in Fig. 1 increases as the cut goes deeper. One example of the experimental results is shown in Fig. 4 where the relation between the total resistance and the depth of the cut is given, the width of the cut being 3 cm in this case. If we take the latter value of the current intensity, the magnetic field at the centre of the loop amounts to 190 *gammas*, so that a magnetic field larger than the inducing

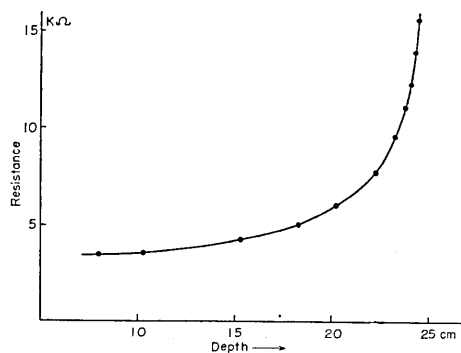


Fig. 4. The changes in the electric resistance between the electrodes as the depth of the cut increases.

12) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **34** (1956), 291.

one may be expected in this case. We therefore see that, in order to get a magnetic field equal to the inducing one,  $\sigma_1=10^{-11}\sim 10^{-10}$  e.m.u. should be taken as long as the above branch circuit model with an insulating wedge is assumed.

Now we shall examine whether or not  $\sigma_1=10^{-11}\sim 10^{-10}$  e.m.u. is possible at depths of 100~200 km. Analyses of geomagnetic variations of various periods<sup>10)</sup> suggest that the mean conductivity of the earth is as low as  $10^{-15}$  e.m.u. at that level. We must therefore take it either that there is a special material which is different from usual composition and is of high conductivity or that the temperature is high enough in the circuit though the composition is the same.

According to experimental studies on some rocks<sup>13),14)</sup>, it has been known that the conductivity of some rocks becomes higher than  $10^{-10}$  e.m.u. at 1200°C. Although the pressure-effect is not well known, it is likely that the conductivity is reduced to less than  $10^{-11}$  e.m.u. at depths of 100~200 km as has been shown by the theory<sup>14),15)</sup> and experiment<sup>16)</sup> even though the temperature is the same.

In order to make the conductivity take a value of  $10^{-11}\sim 10^{-10}$  e.m.u. at the 100~200 km depth, therefore, we are obliged to assume a temperature higher than 1200°C in the circuit as long as the composition of the material is something not greatly different from peridotite, gabbro or basalt. The temperature at that depth obtained by T. Rikitake<sup>15)</sup> for the averaged state of the earth amounts to 900~1100°C. Although some others gave slightly higher temperature there, the most conceivable way of interpretation of the branch circuit would be that the temperature inside the circuit is slightly higher than that around the circuit. Since the conductivity at that temperature is very sensitive to slight changes in temperature, the temperature difference would not be large, probably less than a few hundred degrees in centigrade.

It is not possible to determine accurately the temperature of the circuit. The essential point of the present discussion is that the conductivity as high as  $10^{-11}\sim 10^{-10}$  e.m.u. would not be utterly impossible. We may well expect such a conductivity for a usual substance which is thought to exist at that depth provided the temperature within the circuit is slightly higher than that in its environment. However, nothing is known about the reason why there exists such a high temperature circuit.

13) T. NAGATA, *Bull. Earthq. Res. Inst.*, **15** (1937), 663.

14) H. P. COSTER, *M. N. R. A. S. Geophys. Suppl.*, **5** (1949), 193.

15) T. RIKITAKE, *Bull. Earthq. Res. Inst.*, **30** (1952), 13.

16) H. HUGHES, *Journ. Geophys. Res.*, **60** (1955), 187.

### 5. Electromagnetic induction in a spherical conducting sheet with a hole

As the writer has often pointed out, the anomalous behaviour of geomagnetic variations in Japan seems unlikely to be ascribed to the effect of the magnetic field produced by the electric currents induced in the sea surrounding Japan. Since the question whether the anomaly might be caused by the influence of the sea is still sometimes raised by some geophysicists, however, the writer would here like to add one more study on the nature of such an effect upon the previous discussion.

Let us take a spherical conducting sheet with a circular hole. By the model, it would be possible to study approximately the effect of the sea which surrounds a non-conducting land mass. The non-conducting land occupies the inside area of the circular spherical sheet defined by  $\theta = \theta_0$ , the  $\theta = 0$  axis crosses the spherical surface at the centre of the area. The inducing magnetic field may take any type. For the sake of mathematical simplicity, however, it is first assumed that the field is uniform and parallel (*Case 1*) or perpendicular (*Case 2*) to the  $\theta = 0$  axis.

#### Case 1.

The arrangement of the model for this case is shown in Fig. 5.

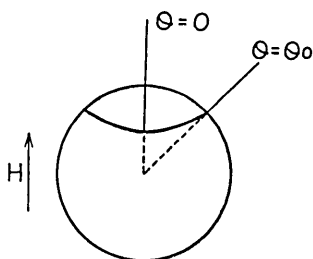


Fig. 5. Case 1.

Following the procedure developed in the 6th report<sup>2)</sup>, the magnetic potential of the inducing field is expressed as

$$W_e = -rHP_1(\cos \theta). \quad (12)$$

The current function of the induced electric currents can be expressed as

$$\Psi = \sum_n a_n P_n(\cos \theta), \quad (13)$$

the expression being effective not only on the conducting part of the sheet but on the whole surface of the sphere of which the radius is denoted by  $a$ .

The boundary condition at  $r=a$  becomes

$$\Psi = 0 \quad \text{on} \quad 0 < \theta < \theta_0, \quad (14)$$

$$\frac{1}{a^2} \left\{ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) \right\} \Psi = K \frac{\partial H_r}{\partial t} \quad \text{on} \quad \theta_0 < \theta < \pi, \quad (15)$$

in which  $K$  and  $H_r$  denote respectively the integrated conductivity and the radial component of the magnetic field. The magnetic permeability is assumed as unity in electromagnetic unit.

For a sudden change, (15) becomes

$$H_r = 0. \quad (16)$$

Even for a periodic inducing field, this will be a good approximation of (15) in so far as the induction in a sheet of the earth's size by changes having periods of a few *minutes* is concerned.

(16) can be written as

$$\frac{aH_0}{4\pi}P_1 + \sum_n \frac{n(n+1)}{2n+1}a_nP_n = 0, \quad (17)$$

where  $H_0$  is the amplitude of the uniform magnetic field suddenly applied from outside. From (14) and (17), the boundary condition at  $r=a$  can be expressed as

$$\left. \begin{aligned} \sum_n a_n P_n &= 0 \quad \text{for } 0 < \theta < \theta_0, \\ \sum_n a_n P_n &= -\sum_n \left\{ \frac{n(n+1)}{2n+1} - 1 \right\} a_n P_n - \frac{aH_0}{4\pi} P_1, \quad \text{for } \theta_0 < \theta < \pi. \end{aligned} \right\} \quad (18)$$

After multiplying by  $P_N \sin \theta$ , we shall integrate (18) with respect to  $\theta$  from  $\theta=0$  to  $\pi$ . In that case, the integral should be divided into two parts, the first being from  $\theta=0$  to  $\theta=\theta_0$  and the second from  $\theta=\theta_0$  to  $\theta=\pi$ .

We thus obtain

$$\begin{aligned} a_N K_{NN} + \sum_n \left\{ \frac{n(n+1)}{2n+1} - 1 \right\} a_n \left( K_{nN} - \int_0^{\theta_0} P_n P_N \sin \theta d\theta \right) \\ = -\frac{aH_0}{4\pi} \left( K_{1N} - \int_0^{\theta_0} P_1 P_N \sin \theta d\theta \right), \end{aligned} \quad (19)$$

where

$$K_{nN} = \int_0^\pi P_n P_N \sin \theta d\theta. \quad (20)$$

It is readily seen that

$$\int_0^{\theta_0} P_n P_N \sin \theta d\theta = \frac{\sin \theta_0 \left( \frac{dP_N}{d\theta} P_n - \frac{dP_n}{d\theta} P_N \right)_{\theta=\theta_0}}{(n-N)(n+N+1)} \quad \text{for } n \neq N, \quad (21)$$

Table 4.  $K_{nN} - \int_0^{\theta_0} P_n P_N \sin \theta d\theta$  ( $\theta_0 = 20^\circ$ ).

$n$	$N$											
	1	2	3	4	5	6	7	8	9	10	11	12
1	0.60992	-0.05336	-0.04853	-0.04255	-0.03576	-0.02853	-0.02126	-0.01448	-0.00802	-0.00264	0.00165	0.00475
2	-0.05336	0.34975	-0.04581	-0.04030	-0.03404	-0.02737	-0.02064	-0.01434	-0.00833	-0.00328	0.00077	0.00375
3	-0.04853	-0.04581	0.24380	-0.03708	-0.03157	-0.02568	-0.01972	-0.01412	-0.00874	-0.00417	-0.00046	0.00233
4	-0.04255	-0.04030	-0.03708	0.18915	-0.02848	-0.02356	-0.01855	-0.01380	-0.00919	-0.00524	-0.00194	0.00061
5	-0.03576	-0.03404	-0.03157	-0.02848	0.15689	-0.02110	-0.01716	-0.01336	-0.00964	-0.00638	-0.00357	-0.00131
6	-0.02853	-0.02737	-0.02568	-0.02356	-0.02110	0.13545	-0.01558	-0.01278	-0.01002	-0.00749	-0.00522	-0.00327
7	-0.02126	-0.02064	-0.01972	-0.01855	-0.01716	-0.01558	0.11968	-0.01201	-0.01027	-0.00848	-0.00675	-0.00515
8	-0.01448	-0.01434	-0.01412	-0.01380	-0.01336	-0.01278	-0.01201	0.10676	-0.01056	-0.00937	-0.00815	-0.00686
9	-0.00802	-0.00833	-0.00874	-0.00919	-0.00964	-0.01002	-0.01027	-0.01056	0.09545	-0.00974	-0.00907	-0.00817
10	-0.00264	-0.00328	-0.00417	-0.00524	-0.00638	-0.00749	-0.00848	-0.00937	-0.00974	0.085600	-0.00968	-0.00912
11	0.00165	0.00077	-0.00046	-0.00194	-0.00357	-0.00522	-0.00675	-0.00815	-0.00907	-0.00968	0.073746	-0.00963
12	0.00475	0.00375	0.00233	-0.00061	-0.00131	-0.00327	-0.00515	-0.00686	-0.00817	-0.00912	-0.00963	0.070582

Table 5. The coefficients and the righthand members of the simultaneous equations (19).

1	2	3	4	5	6	7	8	9	10	11	12	righthand member*
1	-0.02270	-0.07537	-0.11471	-0.13641	-0.13956	-0.12620	-0.10118	-0.06488	-0.02442	0.01807	0.05533	1.31627
0.03839	1	-0.07115	-0.10865	-0.12986	-0.13390	-0.12253	-0.10018	-0.06739	-0.03033	0.00843	0.04368	-0.11354
0.03492	-0.01949	1	-0.09996	-0.12043	-0.12563	-0.11706	-0.09865	-0.07070	-0.03856	-0.00504	0.02714	-0.10553
0.03060	-0.01715	-0.05761	1	-0.10863	-0.11526	-0.11011	-0.09643	-0.07434	-0.04846	-0.02123	0.00711	-0.09385
0.02572	-0.01449	-0.04904	-0.07678	1	-0.10322	-0.10186	-0.09334	-0.07798	-0.05900	-0.03909	-0.01525	-0.07897
0.02052	-0.01221	-0.03988	-0.06352	-0.08050	1	-0.09250	-0.08930	-0.08105	-0.06926	-0.05716	-0.03808	-0.06256
0.01530	-0.00879	-0.03064	-0.05000	-0.06546	-0.07623	1	-0.08392	-0.08308	-0.07842	-0.07391	-0.06000	-0.04617
0.01042	-0.00611	-0.02194	-0.03721	-0.05097	-0.06252	-0.07130	1	-0.08542	-0.08665	-0.08922	-0.07992	-0.03127
0.00576	-0.00355	-0.01357	-0.02477	-0.03677	-0.04901	-0.06096	-0.07377	1	-0.09008	-0.09930	-0.09517	-0.01736
0.00190	-0.00138	-0.00648	-0.01412	-0.02434	-0.03664	-0.05034	-0.06546	-0.07880	1	-0.10597	-0.10624	-0.00576
-0.00119	0.00032	-0.00072	-0.00523	-0.01363	-0.02553	-0.04007	-0.05695	-0.07336	-0.08951	1	-0.11217	0.00381
-0.00341	0.00160	0.00361	0.00165	-0.00499	-0.01599	-0.03058	-0.04792	-0.06609	-0.08434	-0.10544	1	0.01056

\* The righthand members should be multiplied by  $-(aH_0/4\pi)$ .



and

$$\begin{aligned}
 & (2N+3) \int_0^{\theta_0} (P_{N+1})^2 \sin \theta d\theta \\
 & = (2N+1) \int_0^{\theta_0} (P_N)^2 \sin \theta d\theta - \cos \theta_0 \{ (P_{N+1})^2 + (P_N)^2 \}_{\theta=\theta_0} + 2(P_N P_{N+1})_{\theta=\theta_0} \\
 & \qquad \qquad \qquad \text{for } n=N. \quad (22)
 \end{aligned}$$

With the aid of (21) and (22),  $K_{nN} - \int_0^{\theta_0} P_n P_N \sin \theta d\theta$  can be calculated as given in Table 4 for  $\theta_0 = 20^\circ$ . With these values, (19) can be regarded as the simultaneous equations for  $a_1, a_2, \dots, a_{12}$ . The coefficients of  $a_n$ 's and the righthand members are given in Table 5. Since the coefficients of the diagonal terms are relatively large, we can easily solve the equations though no account is taken of the rigorous proof concerning the convergency. The solutions that are solved by making use of the relaxation method are given in Table 6.

Table 6. The solutions of (19) in units of  $-aH_0/4\pi$ .

$a_1$	$a_2$	$a_3$	$a_4$	$a_5$	$a_6$
1.186	-0.279	-0.250	-0.221	-0.191	-0.161
$a_7$	$a_8$	$a_9$	$a_{10}$	$a_{11}$	$a_{12}$
-0.131	-0.101	-0.072	-0.046	-0.024	-0.007

Since we have obtained  $a_n$ 's, the current function can be calculated from (13) as shown in Fig. 6. In the figure, the current function that should have been obtained if the sheet has no hole is also shown. We see that the condition that the current function should be zero for  $0 < \theta < 20^\circ$  is not exactly satisfied. The fact is due to the procedure of calculation in which we only take harmonics for  $n=1, 2, \dots, 12$ . In order to conduct a more accurate

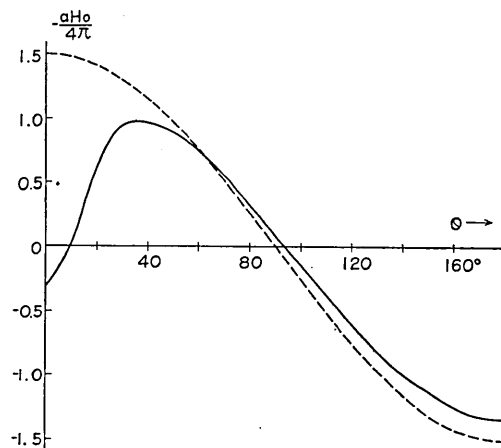


Fig. 6. The distribution of the current function for Case 1 (full line) and that for a sheet without hole (broken line).

study, some more harmonics should be taken into account though such an analysis would be tedious. The writer is of the opinion, however,

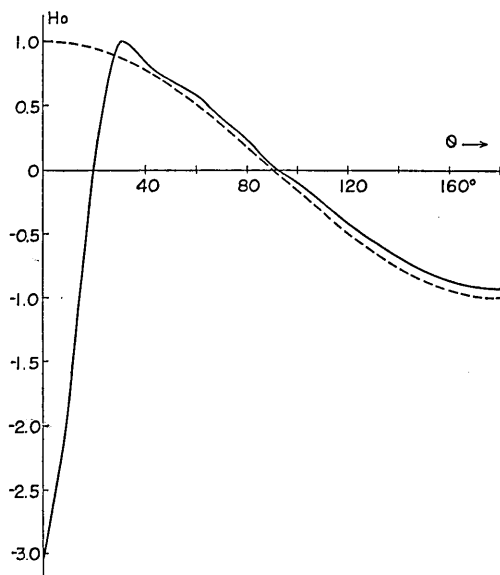


Fig. 7. The distribution of the normal component of the induced magnetic field for *Case 1* (full line) and that for a sheet without hole (broken line).

that the general tendency of the effect of the hole could be investigated by the present study.

The normal component of the magnetic field produced by the induced electric currents are also easily calculated with the coefficients given in Table 6 as can be seen in Fig. 7 in which the reversed external field or the induced field for the sheet with no hole is also shown. It is clearly seen that the induced field nearly cancels the inducing one on the conducting sheet. On the other hand, the direction of the induced field is the same as that of the inducing one

over the non-conducting hole, so that the magnetic field suddenly applied from outside is enhanced a few times near the centre of the hole.

#### *Case 2.*

In the next place, we shall study the case in which the external field, which is also assumed to be uniform, is applied to a direction perpendicular to the  $\theta=0$  axis as shown in Fig.

8. In that case the magnetic potential of the inducing field is given as

$$W_e = -rHP_1^1(\cos \theta) \cos \phi. \quad (23)$$

where

$$P_n^1(\cos \theta) = -\sqrt{\frac{2}{n(n+1)}} \frac{dP_n(\cos \theta)}{d\theta}.$$

The current function of the induced field is to be of the form such as

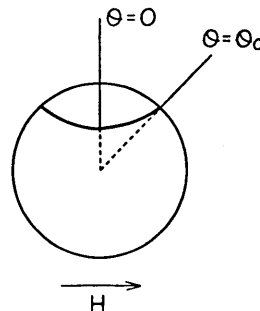


Fig. 8. *Case 2*

$$\mathcal{W} = \sum_n b_n P_n^1 \cos \phi. \quad (24)$$

From the boundary condition that the current function is zero on  $0 < \theta < \theta_0$  and that the normal component of magnetic field vanishes on  $\theta_0 < \theta < \pi$  as we have dealt with before, we obtain

$$\left. \begin{aligned} \sum_n b_n P_n^1 &= 0 && \text{for } 0 < \theta < \theta_0, \\ \sum_n b_n P_n^1 &= -\sum_n \left\{ \frac{n(n+1)}{2n+1} - 1 \right\} b_n P_n^1 - \frac{aH_0}{4\pi} P_1^1 && \text{for } \theta_0 < \theta < \pi. \end{aligned} \right\} \quad (25)$$

After multiplying by  $P_n^1 \sin \theta$ , we shall integrate (25) with respect to  $\theta$  from  $\theta=0$  to  $\theta=\pi$  in a fashion similar to the former case. We then have

$$\begin{aligned} b_N M_{NN} + \sum_n \left\{ \frac{n(n+1)}{2n+1} - 1 \right\} b_n \left( M_{nN} - \int_0^{\theta_0} P_n^1 P_N^1 \sin \theta d\theta \right) \\ = -\frac{aH_0}{4\pi} \left( M_{1N} - \int_0^{\theta_0} P_1^1 P_N^1 \sin \theta d\theta \right), \end{aligned} \quad (26)$$

where

$$M_{nN} = \int_0^\pi P_n^1 P_N^1 \sin \theta d\theta. \quad (27)$$

Since we have the relations such as

$$\int_0^{\theta_0} P_n^1 P_N^1 \sin \theta d\theta = \frac{\sin \theta_0 \left( \frac{dP_N^1}{d\theta} P_n^1 - \frac{dP_n^1}{d\theta} P_N^1 \right)_{\theta=\theta_0}}{(n-N)(n+N+1)} \quad \text{for } n \neq N, \quad (28)$$

and

$$\begin{aligned} \int_0^{\theta_0} (P_N^1)^2 \sin \theta d\theta &= 2 \int_0^{\theta_0} (P_N^1)^2 \sin \theta d\theta - \sqrt{\frac{2(N-1)!}{(N+1)!}} (P_N^1 P_N^1 \sin \theta)_{\theta=\theta_0} \\ &\quad \text{for } n=N, \end{aligned} \quad (29)$$

we can obtain  $M_{nN} - \int_0^{\theta_0} P_n^1 P_N^1 \sin \theta d\theta$  as given in Table 7 for  $\theta_0 = 20^\circ$ . With these values, (26) can be regarded as the simultaneous equations for  $b_1, b_2, \dots, b_{12}$ . The coefficients of  $b_n$ 's and the righthand members are given in Table 8. Solving the equations in a way similar to the previous case, we can determine  $b_n$ 's as given in Table 9.

The current function and the normal component of the induced field

Table 7.  $M_{nN} - \int_0^{\theta_0} P_n^i P_N^i \sin \theta d\theta$  ( $\theta_0 = 20^\circ$ ).

$n$	$N$											
	1	2	3	4	5	6	7	8	9	10	11	12
1	1.32976	-0.00593	-0.00787	-0.00934	-0.01027	-0.01062	-0.01027	-0.00992	-0.00863	-0.00704	-0.00535	-0.00360
2	-0.00593	0.79014	-0.01310	-0.01555	-0.01710	-0.01770	-0.01713	-0.01658	-0.01446	-0.01184	-0.00905	-0.00614
3	-0.00787	-0.01310	0.55400	-0.02070	-0.02278	-0.02362	-0.02288	-0.02224	-0.01945	-0.01599	-0.01232	-0.00848
4	-0.00934	-0.01555	-0.02070	0.41983	-0.02713	-0.02818	-0.02730	-0.02672	-0.02348	-0.01942	-0.01513	-0.01061
5	-0.01027	-0.01710	-0.02278	-0.02713	0.33368	-0.03121	-0.03019	-0.02989	-0.02639	-0.02202	-0.01738	-0.01248
6	-0.01062	-0.01770	-0.02362	-0.02818	-0.03121	0.27509	-0.03122	-0.03169	-0.02814	-0.02370	-0.01904	-0.01406
7	-0.01027	-0.01713	-0.02288	-0.02730	-0.03019	-0.03122	0.23474	-0.03298	-0.02890	-0.02450	-0.02002	-0.01523
8	-0.00992	-0.01658	-0.02224	-0.02672	-0.02989	-0.03169	-0.03298	0.20888	-0.02817	-0.02452	-0.02069	-0.01642
9	-0.00863	-0.01446	-0.01945	-0.02348	-0.02639	-0.02814	-0.02890	-0.02817	0.18537	-0.02319	-0.02024	-0.01674
10	-0.00704	-0.01184	-0.01599	-0.02070	-0.02278	-0.02362	-0.02288	-0.02244	-0.02348	0.16947	-0.01936	-0.01669
11	-0.00535	-0.00905	-0.01232	-0.01513	-0.01738	-0.01904	-0.02002	-0.02069	-0.02024	-0.01936	0.15642	-0.01635
12	-0.00360	-0.00614	-0.00848	-0.01061	-0.01248	-0.01406	-0.01523	-0.01642	-0.01674	-0.01669	-0.01635	0.14489

Table 8. The coefficients and the righthand members of the simultaneous equations (26).

1	2	3	4	5	6	7	8	9	10	11	12	righthand member*
1	-0.00124	-0.00581	-0.01193	-0.01887	-0.02571	-0.03090	-0.03522	-0.03571	-0.03284	-0.02770	-0.02052	1.49397
-0.00222	1	-0.00968	-0.01985	-0.03143	-0.04285	-0.05155	-0.05888	-0.05982	-0.05522	-0.04686	-0.03500	-0.00619
-0.00294	-0.00273	1	-0.02642	-0.04186	-0.05719	-0.06885	-0.07897	-0.08047	-0.07458	-0.06380	-0.04835	-0.00814
-0.00349	-0.00325	-0.01529	1	-0.04985	-0.06823	-0.08215	-0.09489	-0.09714	-0.09057	-0.07834	-0.06049	-0.00975
-0.00384	-0.00357	-0.01682	-0.03463	1	-0.07556	-0.09085	-0.10614	-0.10919	-0.10269	-0.09000	-0.07115	-0.01093
-0.00398	-0.00370	-0.01744	-0.03597	-0.05735	1	-0.09395	-0.11254	-0.11642	-0.11053	-0.09859	-0.08014	-0.01153
-0.00384	-0.00358	-0.01690	-0.03485	-0.05548	-0.07558	1	-0.11711	-0.11956	-0.11426	-0.10367	-0.08682	-0.01131
-0.00372	-0.00347	-0.01643	-0.03411	-0.05493	-0.07672	-0.09925	1	-0.11655	-0.11436	-0.10713	-0.09360	-0.01089
-0.00324	-0.00302	-0.01436	-0.02997	-0.04849	-0.06813	-0.08697	-0.10004	1	-0.10815	-0.10481	-0.09543	-0.00955
-0.00264	-0.00247	-0.01181	-0.02479	-0.04046	-0.05738	-0.07373	-0.08707	-0.09594	1	-0.10025	-0.09515	-0.00775
-0.00200	-0.00189	-0.00910	-0.01931	-0.03194	-0.04610	-0.06025	-0.07347	-0.08373	-0.09029	1	-0.09320	-0.00585
-0.00135	-0.00128	-0.00627	-0.01354	-0.02294	-0.03404	-0.04583	-0.05830	-0.06925	-0.07784	-0.08466	1	-0.00392

\* The righthand members should be multiplied by  $-(aH_0/4\pi)$ .

Table 9. The solutions of (26) in units of  $-aH_0/4\pi$ .

$b_1$	$b_2$	$b_3$	$b_4$	$b_5$	$b_6$
1.491	-0.008	-0.010	-0.012	-0.013	-0.014
$b_7$	$b_8$	$b_9$	$b_{10}$	$b_{11}$	$b_{12}$
-0.014	-0.013	-0.012	-0.010	-0.009	-0.007

on  $\phi=0$  are calculated with  $b_n$ 's in Table 9 as can be seen in Figs. 9 and 10 where the curves for a sheet without hole are also illustrated. Although the approximation with the twelve harmonics does not seem to be satisfactory as we have seen in the previous case, we see that the differences between the curves

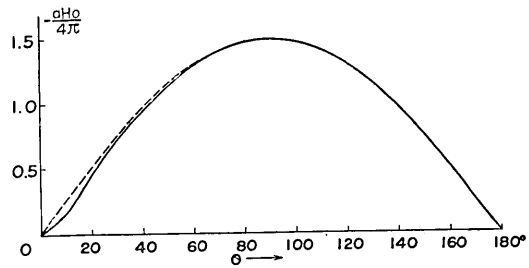


Fig. 9. The distribution of the current function on  $\phi=0$  for *Case 2* (full line) and that for a sheet without hole (broken line).

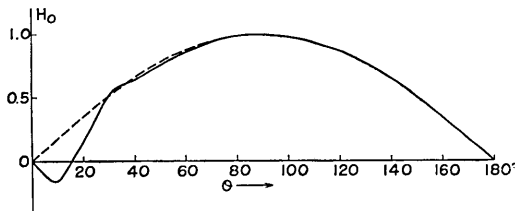


Fig. 10. The distribution of the normal component of the induced magnetic field on  $\phi=0$  for *Case 2* (full line) and that for a sheet without hole (broken line).

for the sheet with and without the hole are not as large as those in *Case 1*. The influence of the non-conducting hole can be seen only in the neighbourhood of the hole.

### Case 3.

If we combine the results for *Case 1* with those for *Case 2*, the current function

as well as the induced magnetic field can be obtained for a uniform inducing field of any direction. Suppose the inducing field has an inclination  $\alpha$  to the  $\theta=0$  axis as shown in Fig. 11, the current function becomes

$$\begin{aligned} \Psi = & \cos \alpha \sum_n a_n P_n(\cos \theta) \\ & + \sin \alpha \sum_n b_n P_n^1(\cos \theta) \cos \phi. \end{aligned} \quad (30)$$

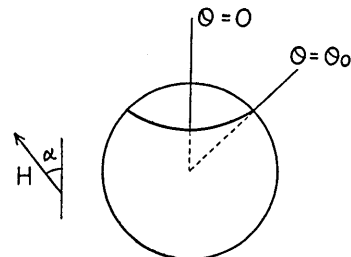


Fig. 11. *Case 3*.

The distribution of the normal component of the induced magnetic field over the spherical surface is shown in Fig. 12 for  $\alpha=45^\circ$ . Even

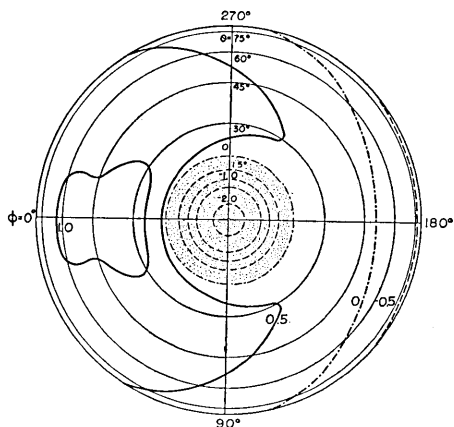


Fig. 12a. The distribution of the normal component of the induced magnetic field for Case 3 ( $\alpha=45^\circ$ ) viewed from a point right above the hole. The hole is shown by the shadow area.

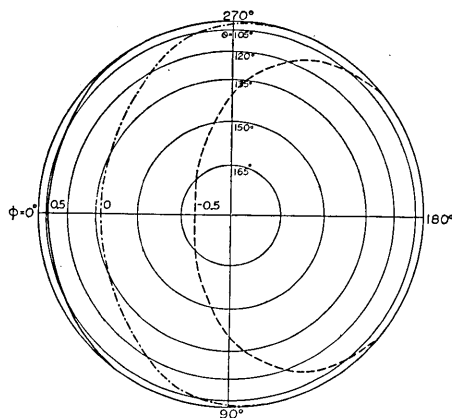


Fig. 12b. The distribution of the same component over another hemisphere.

in this case, we see an enhancement of the external field over the non-conducting area. On the hemisphere on which the inducing field is directed outwards, there is a zone over which the normal component of the inducing field is cancelled by that of the induced one. Inside that zone, the induced field overcomes the inducing one, so that we observe a normal component in opposite direction to the inducing field there. But the resultant field is not as remarkable as that over the non-conducting area.

The above study has been made on the assumption that the area between  $\theta=0$  to  $\theta=\theta_0$  is absolutely non-conducting. In an actual case, however, the conductivity over that area must be finite. It is known that, in the case of an ideally sudden change, the normal component of the induced magnetic field cancels that of the inducing one at the instant of the sudden change, no matter how small the conductivity is. We see therefore that the pattern of the induced electric currents as well as magnetic fields are different from the ones obtained above at the very beginning of sudden changes. However, the induced currents in the non-conducting area decay very quickly reaching the state discussed above.

The currents induced in the conducting area decay more slowly.

Since it is tedious to study such a decay quantitatively, no account is taken of it here. But as can be supposed from what we have discussed in the 6th report<sup>2)</sup>, it is unlikely that the general tendency of the distribution of the induced currents and fields for a sudden change would be seriously modified so long as we consider a uniform inducing field together with a sea having a depth of hundreds or thousands of meters.

The model treated in the above seems to fit well for studying the influence of the land masses such as the Antarctica or Australia provided the continent is regarded as non-conducting. For the latter continent, W.D. Parkinson<sup>17)</sup> recently reported the tendency of the directions of rapid geomagnetic fluctuations as observed at a number of Australian observatories. It is interesting that he reported that the magnetic field vectors are almost confined in the horizontal plane at Alice Springs which is situated approximately in the centre of the continent. According to the mathematical study developed in this paper, however, the vertical magnetic field should be large at the centre of the non-conducting land mass.

The disagreement between the theory and the observational result may suggest that the continent cannot be treated as an ideal insulator surrounded by a conductor of large extent. The possible way to account for such disagreement would be to assume either that the conductivity contrast between land and sea is not so large or that no strong electric currents are induced in the surrounding owing to the irregular distribution of lands. At the moment, therefore, nothing definite can be said about the actual influence of the sea on geomagnetic variations of short period.

It is more difficult to investigate the effect of the sea surrounding Japan because the shape of the land is greatly different from that of the model considered. It may be said, however, that we should observe upward magnetic fields in Japan on occasions of sudden commencement of magnetic storm provided the land area is regarded as non-conducting. This is not the case for the Japanese geomagnetic variations. We observe, on the contrary, downward magnetic fields in the anomalous area. Although it is not possible to examine the influence of the sea surrounding Japan quite accurately, it is hardly likely to suppose that the anomalous features of short-period variations are caused by such an influence.

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17) W. D. PARKINSON, *Geophys. Journ. Roy. Astro. Soc.*, **2** (1959), 1.

In the light of the above discussion together with that was studied in the 6th report<sup>2)</sup>, the anomaly of geomagnetic variations found in Japan is not of the type which might be caused by the electric currents induced in the sea. One might further raise a question, however, that the anomalous underground structure, which has been suggested in the last section, would be responsible for the anomaly. The non-conducting wedge penetrating into the conducting mantle as supposed in the last section gives a configuration somewhat alike the model considered in this section. It seems also unlikely that such a particular structure can give rise to the anomaly in Japan.

## 6. Concluding remarks

The possible cause of the anomaly of short-period geomagnetic variations as observed in Japan is discussed as closely as possible in this paper. Since it turns out that an isolated conductor model embedded in the upper part of the earth's mantle as well as a simple branch circuit model which is connected to the conducting part of the mantle cannot account for the anomaly, the only possible way to give rise to such an anomaly seems to be effected by assuming a branch circuit together with a non-conducting wedge penetrating into the mantle. Large parts of the electric currents induced in the mantle by geomagnetic variations can flow in the circuit both the ends of which are connected to the outside of the wedge. In this case, it is not impossible to observe a large amplitude of vertical component as found by actual observations provided the circuit takes a suitable form underneath Japan.

The penetrating wedge supposed harmonizes well with the low conductivity as has been suggested from the analyses of  $Sq$  and  $Dst$  in Japan.

The underground structure thus presumed is an extraordinarily complicated one. Although the writer<sup>12)</sup> speculated its significance in relation to the occurrence of deep focus earthquakes, orogenic activity and so forth, nothing definite has been known about the nature of the branch circuit or wedge-shape penetration supposed here. Since it seems likely that the underground conditions brought out by geomagnetic method has an important bearing on geophysical and geological phenomena in Japan, it is highly desirable to conduct some further observations together with some more elaborate analyses.

From the investigation on the electromagnetic induction within a spherical sheet having a non-conducting hole, it is concluded that the



anomaly is not of the type which might be caused by the influence of the sea surrounding Japan. But it seems important that no large vertical fluctuations of geomagnetic field are actually observed in the central area of Australia which seems to fit well the model treated, while the theory suggests that the vertical component should be large there. One of the ways of compromise would be to suppose that the conductivity contrast between land and sea is not so large as that considered in the model from some unknown reason. Although this topic is not directly related to the anomaly of geomagnetic variations in Japan, much attention should be paid to a better understanding of the influence of sea on geomagnetic variations in the future.

In conclusion, the writer thanks to Messers K. Kasahara and I. Tanaoka for their help in the experiment. He also wishes to express his sincere thanks to Miss E. Nakagawa for her assistance in the numerical work. The writer would like to extend his thanks to the Ministry of Education who provided financial aid for this study through a Research Grant.

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## 35. 日本における地磁気短周期変化の異常と地下構造

## (第 9 報)

地震研究所 力 武 常 次

日本中部において、地磁気変化がきわめていちじるしい異常を示すことは、第 8 報までの各報告に詳しく述べたところである。

本報においては、その異常の原因として推測されてきた地下の特殊構造に対し、できる限りの量的吟味を加えることを目的とする。

まず第一に、地下に導体が独立して存在する場合、地磁気変化によつて導体中に誘起される電流のつくる磁場の影響を調べた。球状導体を仮定する場合には、半径  $400\text{ km}$ 、電気伝導度  $10^{-12}\text{ e.m.u.}$  以上ということになり、このように巨大な異常領域の存在は考えにくいし、当然  $Sq$  および  $Dst$  のようなおそい変化に対してもその影響が顕著にあらわれることになり、観測事実と相反する。つきに、輪状導体の存在を仮定する場合にも、可能と考えられる大きさおよび電気的性質に対し、周期 1 時間程度の外部磁場変化に際して、その  $1/50$  以上の誘導磁場を期待することは困難である。

したがつて、独立導体モデルが無理であるからには、日本の地磁気変化異常を発生する電流は地球中間層の深さ  $400\text{ km}$  より下の部分（この部分は導電的であることが知られている）より上昇している回路に沿つて流れるものと考えてみよう。異常地域のひろがりから、分岐回路が導電領域に連結する 2 点間の距離を  $1000\text{ km}$  程度と推定する。地磁気変化の際にこの 2 点間に生じる電位差をみつかり、回路の抵抗を計算してみると、非常に好都合の場合を採用しても、周期 1 時間、振幅  $100\text{ G}$  の変化に対し  $600\text{ A}$  程度の電流が期待されることになるが、これは異常磁場の数パーセントを説明するにすぎない。したがつて、上記 2 点間の電位差を増加させることが可能とならない限り、分岐回路モデルによる説明は困難である。

$Sq$  および  $Dst$  の解析より、日本地下数  $100\text{ km}$  の電気伝導度が地球全体の平均にくらべて小さいことが示唆されているので、上述モデルの 2 点間に高抵抗のくさび状領域を仮想する。この場合には、2 点間の電位差はいちじるしく増加し、くさびのない場合の 50 倍の電位差を得ることも無理でないことが、カーボン記録紙を使用した模型実験により察知される。このようにして推定された日本地下の特殊構造がユニークであるという保証はないが、現在の段階においては、各種地磁気変化の異常をほぼ統一的に説明できるモデルであるということができよう。

なお本報においては、海中に誘導される電流の影響に関し、新しい研究をつけ加えた。陸を非電導性と考え、導電球殻の一部に円孔がある場合の一樣磁場による電磁誘導を調べた。その結果、円孔つまり陸の中心部では外部磁場と同方向に磁場が発生することがわかつた。したがつて、日本において観測される異常磁場は全く反対の符号を有することとなり、日本をとりまく海洋の影響と考えてこの異常磁場を説明することはできない。

最近の報告によれば、オーストラリア大陸中心部の地磁気観測所において、鉛直分力変化がほとんどないことが報告されている。ここに取扱つたモデルは同大陸によく適合するものと考えられるにもかかわらず、理論と実測との間にいちじるしい相違があるということは、陸と海との電気伝導度のコントラストについて、従来のような簡単な考え方が成立たないことを示しているのかもしれない。