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26. *Maximum Amplitude and Epicentral Distance. —Proposed a Theoretical Elucidation of Empirical Formulas and Some Development.—*

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1. Introduction and some remarks.

Various empirical formulas of the maximum amplitude of an earthquake wave as a function of the epicentral distance are presented by many authors, but a generally accepted one has not yet been found. Difficult points in these studies originate clearly in the fact that the seismograph is not sufficiently completed for the analysis of earthquake waves with complicated feature. Moreover, something has been left unstudied to give to these various formulas a lucid elucidation by the well known theory of wave attenuation: and, in this paper, this point is studied. This is, in short, a new, but perhaps already tacitly understood in general, interpretation of the "observations of maximum amplitude." The writer assumes therein that the observations give the amplitude of the maximum component of the spectrum of what is recorded in the seismogram, whereas the seismogram itself shows the resultant of waves of various period and phase differences. Therefore, it is to be noted that the above stated interpretation does not refer to the "maximum amplitude" itself, but to the "observations or measurements of maximum amplitude," obtained by a certain process.

From the seismometrical stand point, it seems to the writer such an interpretation should have already been accepted as a matter of course, at least in the first approximation, because, after a manual of seismometry published from the Central Meteorological Observatory of Japan, maximum amplitude (A) and its period (T) are measured, either smoothing out the higher harmonics, or picking up a short period oscillation on which is superposed long period and small amplitude oscillation, and a half of total amplitude is measured without reference to the zero line: this method of observation is clearly a rough spectral analysis.

And it is to be pointed out also that, even though the "maximum amplitude" in the seismogram is corrected by the response curve of the seismograph to get a corresponding "displacement amplitude," it is doubtful if it is a true maximum displacement amplitude, because the maximum amplitude is assigned in the seismogram previous to the correction. So that the results of study of the "maximum amplitude" versus epicentral distances based on such observations are naturally affected by the characteristics of the seismograph; practically, if we emphasize the "maximum," the maximum amplitude "A" in the results of study in some cases naturally stands none the less, for example, for the "maximum acceleration amplitude" according to the characteristics of the seismograph, in spite of the procedure to get a displacement amplitude by the response curve of the seismograph. The question as to which interpretation of the two or any other is more suitable will be utterly determined by the characteristics of the seismograph. And, in some cases, the procedure of the correction by the response curve after the "rough spectral analysis" on the seismogram, generally adopted as it is, gives but an ambiguous quantity in the study of the "maximum amplitude" vs. "epicentral distances" relation.

2. Some discussions of various formulas.

Various expressions of the empirical formula of $A \sim \Delta$ relation hitherto presented[1] are classified as follows;

$$\log A = a - m \log \Delta \, , \tag{1}$$

$$\log A = a - m \log \Delta - k\Delta \, , \tag{2}$$

$$\log A = a - m \log \Delta - f(T)\Delta \, , \tag{3}$$

(A: obs. of max. amplitude, Δ: epicentral distance)

where a, m and k are certain constants, m and k being positive, T period of waves of the "maximum amplitude" concerned. Of the three the first, being free from any theoretical foresight, perhaps appears most empirical-formula-like, and is adopted by Richter, by Tsuboi and later by several authors in Japan. Richter in his studies of earthquakes in America obtained $m=3$, whereas Tsuboi $m=1.73$ based on the observations of earthquakes in Japan after a careful computation. As shown later, according to the writer's interpretation of the "observations of maximum amplitude," the difference of the m-value by the two authors

1) C. TSUBOI, "On the Magnitudes of Earthquakes." (*Reviews*), *Zisin* **10** (1957), 6-23.

is caused by the different response of the seismographs used in the two countries.

As to the expression of (2), the latest results of Kawasumi's studies of 80 earthquakes in Japan are as follows:

$$m=1/2, \quad k=0.00305\,\mathrm{km}^{-1}, \quad 100\,\mathrm{km}<\Delta<750\,\mathrm{km},$$

$$m=1/2, \quad k=0.00183\,\mathrm{km}^{-1}, \quad 750\,\mathrm{km}<\Delta<2000\,\mathrm{km}.$$

In his earliest paper of the seismic intensity, if "A" represents "maximum acceleration," m is as large as 2.3, while $k=0.002\,\mathrm{km}^{-1}$. Each of these results by the two authors is based on the observations of many earthquakes in Japan extending over many years. So that the understanding seems reasonable that these formulas respectively give us the observations of some, if not general, earthquakes in Japan, though there are some different opinions against it and also against the procedure by which these formulas were derived, and though even Tsuboi himself seems to acknowledge little significance in the functional form of his formula, perhaps because there was no practical method to presume the most probable function for the empirical formula.

The expression (3), $n=1/2$ for surface wave and $n=1$ for bodily wave, is familiar in the theory of wave attenuation, and was used by Wadati and later by the present writer in a theoretical calculation, in which the period of the "maximum amplitude" is presumed to be fixed by the magnitude of the earthquake and constant for all epicentral distance. As regards the study of the writer, statistical computation to deduce any empirical formula was not a project of his. The results of study of those presumptions are not yet conclusive in Japan, chiefly because, according to an informal opinion of several authors, of a response defect of the seismograph. Since the results are closely related to the present study, observations of the period vs. Δ are illustrated in this paper concerning several remarkable earthquakes in Japan.

According to Tsuboi, differences between the two values of "A" computed by the two formulas, one by Tsuboi himself and the other by Kawasumi, are practically negligible considering the accuracy of observations. It is true, and what is worse, we have so far scarcely any theoretical reason or method to assign *a priori* a certain function to the empirical formula. But the study of those differences will give some means to solve these ambiguous circumstances; some means to find out the most reasonable expression of the empirical formula for the improvement of the theory of earthquake waves in future. The writer, therefore, first tries to reveal the differences among the formulas by pure mathematics, and, at the same time, presents the observations of several destructive earthquakes in Japan for an illustration of his procedure of mathematical treatment and, also, for an examination of those formulas.

3. Mathematical study of the empirical formulas already presented.

So far as $\log A \sim \Delta$ relation is concerned, the difference between those various formulas derived from (1) or (2) may be apparently indistinct, because, as regards both of the expressions (1) and (2), not only $\log A$ but also $\partial \log A/\partial \Delta$ are decreasing functions of Δ. However, $\log \Delta^n A \sim \Delta$ relation, when n is sufficiently large, is quite different. So long as n is small and $n<m$, $\log \Delta^n A$ remains a decreasing function of Δ, but if $n>m$ and $k\neq 0$, $\log \Delta^n A$ has a maximum value at a certain epicentral distance, Δ_m. And the epicentral distance Δ_m increases with the assigned value of n. If $k=0$, i.e. the empirical formula is expressed by (1), used by Tsuboi and Richter, $\log \Delta^n A$ is always monotonous for any value of n, or constant particularly when $n=m$; ever decreasing when $n<m$, and ever increasing when $n>m$. Mathematically speaking, it follows that

$$\Delta_m=\frac{n-m}{k}, \qquad \frac{\partial \Delta_m}{\partial n}=\frac{1}{k}. \tag{4}$$

By these formulas, the most probable

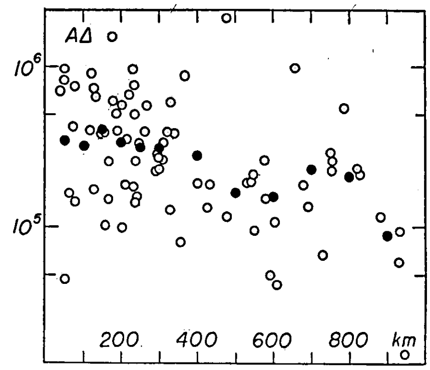

Fig. 1. $\log \Delta A \sim \Delta$ relation of the Shizuoka earthquake.

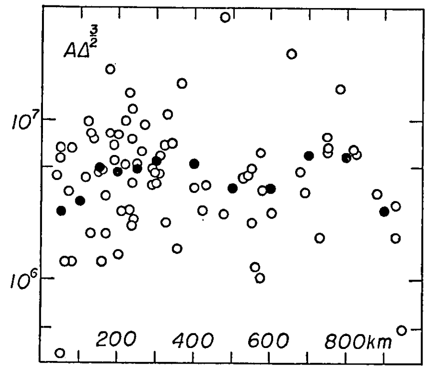

Fig. 2. $\log \Delta^{3/2} A \sim \Delta$ relation of the Shizuoka earthquake.

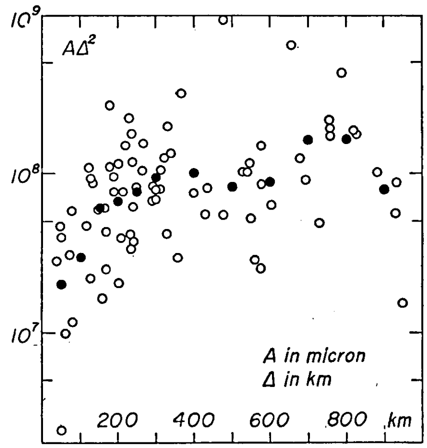

Fig. 3. $\log \Delta^2 A \sim \Delta$ relation of the Shizuoka earthquake.

value of m and k, when the expression (2) is adopted, will be computed graphically from the observations, though as yet in the present accuracy of observation the practical location of Δ_m is difficult for a large value of n, since as Δ increases the accuracy of observations of A and therefore $\log \Delta^n A$ falls. Propriety of an empirical formula to the observations of an earthquakes is also examined closely by these formulas: in the case of Kawasumi's formula, Δ_m is 163 km, 326 km and 488 km when n is equal to 1, 3/2 and 2 respectively. And, in the following, examples are computed with the observations of some destructive earthquakes in Japan.

Figs. 1–3 are the relation of $\log \Delta^n A$ vs. Δ of the Shizuoka earthquake in 1935, putting $n=1$, 3/2 and 2 respectively. Logarithmic mean for every 50 km or for 100 km are given by filled-up circles in each figures. Perhaps, without an objection, nothing conclusive will be obtained from these mean values, because the observations are extremely dispersive. However, for the range of epicentral distance less than 600 km or so, a maximum is likely near the distance expected from Kawasumi's formula in each of those figures. Therefore, in this case, Kawasumi's formula is probably appropriate; at least $n<1.5$ is reasonable for the most probable empirical formula, rejecting Tsuboi's, because we can see clearly a maximum already in Fig. 2 of $\log \Delta^{3/2} A$ vs. Δ.

Table 1.

Earthquake	Date (J.S.T.)	Epicentre		M
1. North-Izu	Nov. 26, 1930	35.1°N	139.0°E	7.0
2. Saitama	Sept. 21, 1931	36.1 N	139.2 E	7.0
3. Shizuoka	July 11, 1935	35.1 N	138.4 E	6.3
4. Kawachi-Yamato	Feb. 21, 1936	34.5 N	135.7 E	6.4
5. Shionomisaki	April 18, 1948	33.1 N	135.6 E	7.2
6. Fukui	June 28, 1948	36.1 N	136.2 E	7.3
7. Imaichi	Dec. 26, 1949	36.7 N	139.7 E	6.7

From Seism. Bull. J. M. A. Suppl. Vol. (1958).

Observations of six more earthquakes in Table 1 are taken up: mean maximum amplitudes (A_m) are calculated for every 50 km ($\Delta \leq 300$ km) or for every 100 km ($\Delta \geq 400$ km): and, in Figs. 4-6, $\log \Delta^n A_m$ are illustrated together with the two curves, K—K and T—T expected respectively from Kawasumi's and Tsuboi's formulas, ordinates of which are arbitrary and to be determined according to the magnitude of each earthquake. As regards the Imaichi earthquake, two shocks occured only about eight minutes apart: maximum amplitude of the second shock (A_2) is estimated

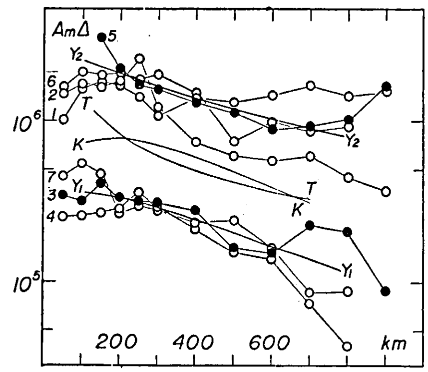

Fig. 4. Mean $\log \Delta A \sim \Delta$ relation of seven earthquakes in the table.

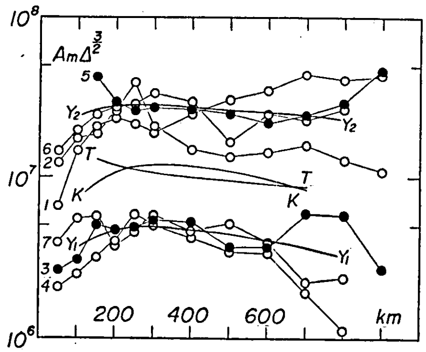

Fig. 5. Mean $\log \Delta^{3/2}A \sim \Delta$ relation of seven earthquakes in the table.

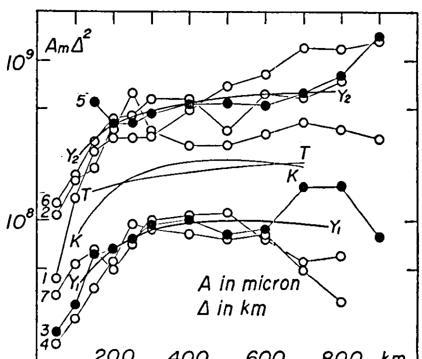

Fig. 6. Mean $\log \Delta^2 A \sim \Delta$ relation of seven earthquakes in the table.

everywhere as large as 1.3 times that of the first one (A_1) by a correlation diagram: to reduce the observational error, mean maximum amplitudes (A) of the two shocks are calculated by $\log A = (1/2)[\log A_1 + \log (A_1 \times 1.3)]$ and are used for the maximum amplitudes of the "Imaichi earthquake."

To discuss the propriety of each formula for these earthquakes is not the main subject of study in this paper; it is sufficient to see that in some cases Kawasumi's formula is applicable and in other cases Tsuboi's or some others: and expression of the formula by a function different from those already given seems necessary, too. Characteristic aspect of $A \sim \Delta$ relation of each earthquake will come out still more clearly in future observations of high accuracy: then it must be one of the important problems to distinguish among those formulas the most appropriate expression for each earthquake by the method above studied.

In the following, an idea is first presented to deduce those various formulas from a presumed fundamental law of wave attenuation on a certain assumption, the appropriateness of which will be examined by observations in future. Though, in the course of the studies, it will be evident that it does not matter if the presumed fundamental law above stated is any particular one or not the classical one is adopted, for the present, to explain the idea as concretely as possible.

4. Fundamental assumptions and illustration of the procedure.

The assumptions were made in three respects as follows:

1) Earthquake waves are built up by a superposition of monochromatic waves of amplitude A_i and period T_i. Theoretically the period T_i remains constant on the way of propagation of the wave. In some cases the spectrum may be of course continuous and unbounded. However, the idea is explained for simplicity by waves of several line spectrum.

2) Amplitude A_i of the monochromatic wave decays with Δ according to the above said fundamental law of wave attenuation, and it is assumed, as can readily be understood theoretically,

$$A_i = \frac{B_i(T_i)}{\Delta^{n_0}} \exp\{-f(T_i)\Delta\} . \tag{5}$$

By the well-known classical theory, $n_0=1$ for bodily wave or $n_0=1/2$ for surface wave, and $f(T) \propto 1/T^2$. If the last postulate is not the case, $f(T)$ will be at least a function such that $f(T_1) > f(T_2)$ when $T_1 < T_2$. Now, it makes no difference in our discussion whether the above said amplitude stands for displacement or acceleration or something else, if it is only used for a definite one throughout the studies. And we can neglect the effect of the characteristics of the seismograph in our discussion for the present, because the response curve of the seismograph in general is flat over a considerable range of the period of wave at least for some one among the displacement, the velocity and the acceleration. If not, it will be of no consequence as to the point of the discussion: it is only hoped to simplify the mathematical expression at present. Then the amplitude, $\boldsymbol{A}$, in the seismogram will be expressed as a function of time and epicentral distance as follows:

$$\boldsymbol{A}(t, \Delta) = \sum_i \boldsymbol{A}_i , \quad \boldsymbol{A}_i = A_i \sin\left\{\frac{2\pi}{T_i}\left(\frac{\Delta}{V_i} - t\right) + \varepsilon_i\right\} , \tag{6}$$

where t is time coordinate, ε_i phase angle, T_i ($i=1, 2, 3, \cdots$ and $T_1 < T_2 < T_3 < \cdots$) period of the component wave, and V_i velocity of propagation, possibly a function of T; A_i is given by (5) and $B_i(T_i)$ is determined from the spectrum of the earthquake wave at its origin.

It is quite natural to consider that the maximum amplitude in the seismogram at some station where $\Delta = \Delta_0$ is given by the maximum of

$A(t, \Delta_0)$. However, since A is affected by the phase angle ε_i, functional form of $f(T)$ and $V(T)$, we can never expect a simple relation between the maximum of A and Δ.

3) The third assumption is that A, the "observations of the maximum amplitude" is not the maximum of A, but is equal to the amplitude, A_i, of one of component waves which is the largest at $\Delta=\Delta_0$. Practically, the observations are not the results of a complete harmonic analysis; the wave whose period and amplitude are measured in the seismogram apparently as a monochromatic wave is in some cases composed of many monochromatic waves. Therefore, in a future study of higher accuracy than that in this paper, an application of a theory of propagation of a wave packet will be necessary, too. From the theoretical standpoint, it is desirable that the observations would be given by the results of a complete harmonic analysis to avoid an ambiguity in the course of reasoning.

Then the $A\sim\Delta$ relation, especially when $\log \Delta^{n_0}A\sim\Delta$ relation is considered, is much simplified as follows: $\log \Delta^{n_0}A_i\sim\Delta$ relation is linear, its gradient $\partial \log \Delta^{n_0}A_i/\partial\Delta=-f(T_i)$, and magnitude of $\log \Delta^{n_0}A_i$ at $\Delta=0$ is equal to $\log B_i$. In the diagram to show the $\log \Delta^{n_0}A\sim\Delta$ relation, we have first straight lines, which give $\log \Delta^{n_0}A_i\sim\Delta$ relation, as many as the number of A_i. And then by the third assumption, we can trace the curve of $\log \Delta^{n_0}A\sim\Delta$ relation from those straight lines; it is, in some cases, a bent line built up by several segments of these lines; the bent line tends to a smooth curve of envelope when the spectrum of the earthquake wave is continuous; it is, in some other cases, built up by a few segments. An example is illustrated in Figs. 7–9 when the number of A_i is only three, A_1, A_2 and A_3, explaining none the less satisfactorily the important point of the process of the reasoning. The relative position of the three lines, A_1, A_2 and A_3, depends on the relative magnitude of B_1 B_2 and B_3; the slope of the lines are $f(T_1)$, $f(T_2)$ and $f(T_3)$, where $T_1<T_2<T_3$ and therefore $f(T_1)>f(T_2)>f(T_3)$.

If $B_1<B_2<B_3$, the three lines A_1, A_2 and A_3 will be such as shown in Fig. 7, and then by the third assumption the $\log \Delta^{n_0}A\sim\Delta$ relation is clearly given by the A_3-line itself. When B_1 is largest and B_2 is almost equal to B_3, the lines will be such as shown in Fig. 8, and the relation will be given by the two segments of the two lines, A_1 and A_3, i.e. by PQR. When $B_1>B_2>B_3$, the relation will be given by the three segments of the three lines, A_1, A_2 and A_3, as shown in Fig. 9 by $PQRS$.

Perhaps we can easily associate Fig. 7 with Kawasumi's formula, and Fig. 8 or 9 with Tsuboi's or with Richter's. Moreover, it is to be noted that $|\partial \boldsymbol{A}_i/\partial t| = 2\pi|\boldsymbol{A}_i|/T_i$. So that, even if the $\log \varDelta^{n_0}A \sim \varDelta$ relation of an earthquake is such as shown in Fig. 7 when it is observed by displacement seismograph, the same relation may be such as shown in Fig. 8 or 9 when it is observed by velocity seismograph or acceleration seismograph. Of course, the different appearance of the $A \sim \varDelta$ relation when the seismographs used are different is originally quite natural, but, to give a reasonable explanation of a presumed remarkable difference, the third assumption here mentioned will be most effective. And from the difference of the characteristics of the seismographs used in two countries, it seems quite natural that Richter's formula was obtained on the observations in America, and Tsuboi's on those in Japan. Another important point is that $B_i(T_i)$, spectrum of the earthquake wave at its origin, is estimated by extending back the segments of each line, A_1, A_2, A_3, observable in Fig. 9, to the ordinate axis $\varDelta = 0$. We can not estimate B_2 and B_3 from the $\log \varDelta^{n_0}A \sim \varDelta$ relation such as shown in Fig. 7, because two lines A_2 and A_3 are not observable there. And in some cases, the estimation of all B_i is possible only when the $\log \varDelta^{n_0}A \sim \varDelta$ relation such as shown in Fig. 9 is obtained by the observation with seismograph which records velocity or acceleration or still higher time-derivatives of the displacement.

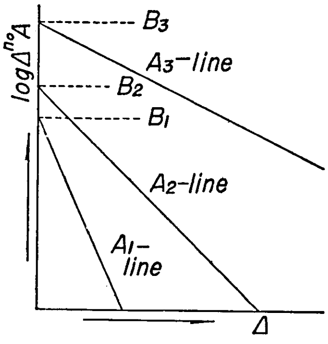

Fig. 7.

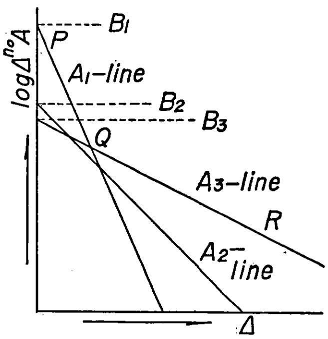

Fig. 8.

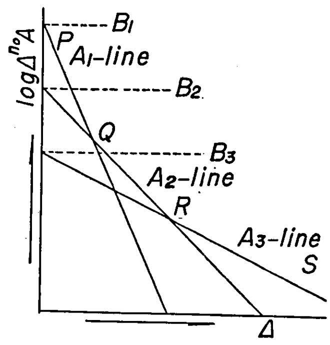

Fig. 9.

How to determine the n_0-value and functional form of $f(T)$ from the observations is a difficult problem. However, clearly, for the application of the present idea, n_0 should not be larger than the smallest value of n determined by the formulas (4) from the observed $A \sim \Delta$ relation. So that, considering the result of studies by Kawasumi and the theory of elastic waves, $n_0 = 1/2$ seems most probable at present.

5. Continuous spectrum and envelope.

When the spectrum of the wave is continuous, the $\log \Delta^{n_0} A \sim \Delta$ relation such as shown in Fig. 9 tends to an envelope of A_t-lines, functional form of which depends on those of $B(T)$ and $f(T)$, and is obtained by eliminating T from the following two equations,

$$\frac{1}{B}\frac{dB}{dT} = \Delta \frac{df}{dT} , \tag{7}$$

$$\log \Delta^{n_0} A = \log B(T) - f(T)\Delta . \tag{8}$$

For example, if we assume that

$$f(T) = \frac{k}{T^2} , \tag{9}$$

$$B(T) = C \exp(-sT^2) , \tag{10}$$

we get from (7),

$$T^4 = \frac{k}{s}\Delta , \tag{11}$$

for the relation between the epicentral distance Δ and the "period of the maximum amplitude" to be observed there. Substituting (9), (10) and (11) into (8), we obtain

$$\log \Delta^{n_0} A = \log C - 2\sqrt{ks}\sqrt{\Delta} , \tag{12}$$

or

$$A = \frac{C}{\Delta^{n_0}} \exp(-2\sqrt{ks}\sqrt{\Delta}) , \tag{13}$$

(13) is the expected $A \sim \Delta$ relation, in which A is, of course, the "observations of maximum amplitude" of what is assigned by (10) to the spectrum $B(T)$; of acceleration, of displacement or of some others. Now following the result of a laboratory experiment if we assume $f(T) = k/T$ instead of (9),

$$T^3 = \frac{k}{2s}\Delta , \tag{14}$$

$$A = \frac{C}{\Delta^{n_0}} \exp\left\{ -3s\left(\frac{k}{2s}\Delta\right)^{2/3} \right\} . \tag{15}$$

Suppose instead of (10) the spectrum is given by

$$B(T)=CT^2 \exp(-sT^2)\ , \tag{16}$$

which, if (10) is an acceleration amplitude spectrum, will be a reasonable assumption as a corresponding displacement amplitude spectrum. Then we have, putting $f(T)=k/T^2$,

$$T^2=\frac{1+\sqrt{1+4ks\Delta}}{2s}\ , \tag{17}$$

$$A=\frac{C}{\Delta^{n_0}}\frac{1+\sqrt{1+4ks\Delta}}{2s}\exp(-\sqrt{1+4ks\Delta})\ , \tag{18}$$

in place of (11) and (13) respectively. "A" in (18), formula of the expected $A\sim\Delta$ relation, represents the "observations of maximum displacement amplitude," corresponding to the "observations of maximum acceleration amplitude A" in (13). There may be more reasonable and plausible expressions of $B(T)$ than those above presented; studies on that point are left for the future.

The formulas (13), (15) and (18) are quite different in appearance from what we know well, but as it will be easily seen if we consider $d \log \Delta^{n_0}A/d\Delta$, these formulas fill up conveniently the gap between the two formulas, (1) and (2), or say, Kawasumi's type and Richter's or Tsuboi's type. Therefore, in future studies when observations of high accuracy are obtained these formulas will do much. As regards (11), (13), (17) and (18), the following points are to be noted especially.

As already stated, two kinds of amplitude spectrum of the same earthquake wave at its origin are expressed by (10) and (16), the one is of acceleration and the other of displacement. In accordance with that, the two formulas, (13) and (18), are the two kinds of $A\sim\Delta$ relation of the same earthquake wave, the former by the acceleration seismograph and the latter by the displacement seismograph. The two formulas are so different that perhaps we can never presume the one from the other, to say nothing of that the one can not be obtained from the other by mere amplitude correction, following the response curve.

For a small epicentral distance, mathematical behaviour of (13) is somewhat similar to Tsuboi's or Richter's formula, while that of (18) to Kawasumi's. It is also remarkable that the smallest value of T given by (11) is zero, while that by (17) is $\sqrt{1/s}$. The reason from which these circumstances arise is illustrated already in the last paragraph: in this case we can not obtain the complete spectrum from the $A\sim\Delta$ relation based on the observations by the displacement seismograph, because the component waves of short period do not join in the relation.

6. Calculation from the observations.

On the other hand, by the following procedure, from the observed $A \sim \Delta$ relation, if we assume a fundamental law of attenuation, for which (5) being adopted for the present, we can determine $B(T)$, the spectrum of the earthquake wave at its origin, and also predict the "period T of the maximum amplitude," which will be examined comparing with the observations.

From the observations of A, $\log \Delta^{n_0} A \sim \Delta$ relation is first calculated, and put $\log \Delta^{n_0} A = F(\Delta)$. Then,

$$f(T) = -\frac{dF}{d\Delta}, \qquad (19)$$

which predicts $T \sim \Delta$ relation if $f(T)$ is known theoretically. And, if $A \sim \Delta$ and $T \sim \Delta$ relation are given, we can determine the function $f(T)$.

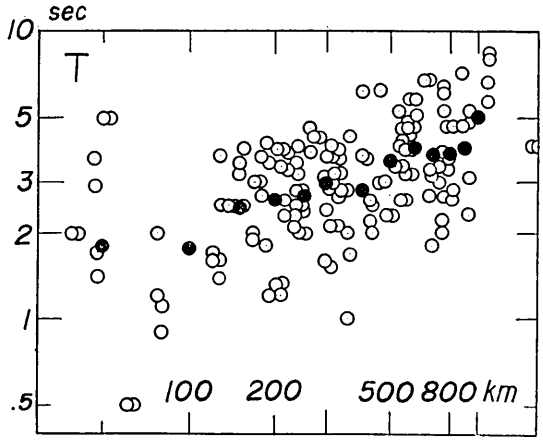

Fig. 10. $T \sim \Delta$ relation of the Shizuoka earthquake, in logarithmic scale.

From (8),

$$\log B(T) = F(\Delta) + f(T)\Delta \qquad (20)$$

in which, Δ of the right hand member being substituted by T from the $T \sim \Delta$ relation above obtained. For example, suppose the observed $A \sim \Delta$ relation is Richter's type,

$$A = C/\Delta^n,$$

$$F(\Delta) = \log C - (n - n_0)\log \Delta;$$

then, if

$$f(T) = k/T^2,$$

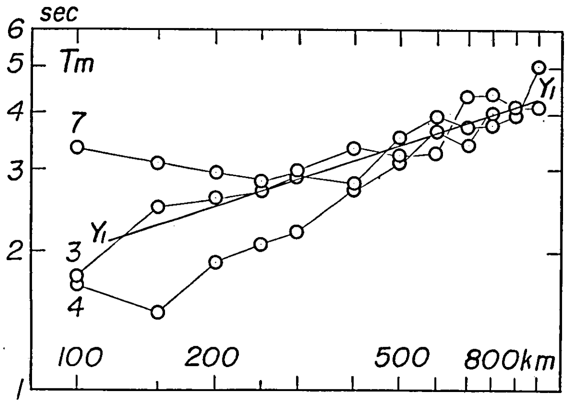

Fig. 11. Mean $T \sim \Delta$ relation of three earthquakes, (3) Shizuoka, (4) Kawachi-Yamato and (7) Imaichi earthquake.

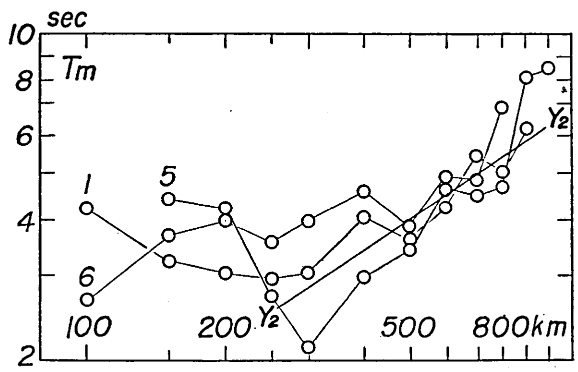

Fig. 12. Mean $T\sim\varDelta$ relation of three earthquakes, (1) North-Izu, (5) Shionomisaki and (6) Fukui earthquake.

$$T=\sqrt{k\varDelta/(n-n_0)}$$

and

$$B(T)=C'/T^{2(n-n_0)}\ .$$

If $T\sim\varDelta$ relation from the observations and the functional form of $f(T)$ are given, we can compare the prediction curve of $A\sim\varDelta$ relation with its observation: in that case, the following formula is used, being obtained from (7) and (8),

$$\log \varDelta^{n_0}A=\text{const.}-\int f(T)d\varDelta\ . \qquad (21)$$

The writer does not know what is the result of study of the $T\sim\varDelta$ relation with the observations from which Richter and Tsuboi derived their formulas. So that observations of T versus $\varDelta$ of the Shizuoka earthquake are shown in Fig. 10, and mean values of the observations of the six earthquakes in the Table 1 in Figs. 11 and 12. For the Shizuoka earthquake, the exponent 1/2 in the above obtained result, which gives $T=a\sqrt{\varDelta}$, seems a little too large. Of course, propriety of the present idea will be examined in future from such points of discordancy between the prediction and the observations. However, as far as this earthquake is concerned, the discordancy above stated seems, to have some correlation with the fact that the observations of $A\sim\varDelta$ relation of this earthquake are not satisfactorily represented by their formulas. There is also an opinion by some authors, including Kawasumi, that observations of T at almost all observatories in Japan are utterly affected by undesirable resonance of seismograph, Wiechert type seismograph being used, and are found constant over whole epicentral distance: Kawasumi's opinion in this point is concordant with his formula of $A\sim\varDelta$ relation, and, at present, will not be a check on the way of thinking in this paper. However, to the writer, it seems not the case, too, at least in several destructive earthquakes; in any case observations of high accuracy in the immediate future are desired.

For an example of calculation, taking up the Shizuoka earthquake, and putting in the formulas (14) and (15) $n_0=1/2$, $k/2s=0.08\ \text{sec}^3\text{km}^{-1}$, $s=0.06\ \text{sec}^{-2}$, so that $k=0.0096\ \text{sec km}^{-1}$, and $C=7.25\cdot10^4\ \mu\ \text{km}^{1/2}$, we have curves of $T \sim \Delta$ relation and $\log \Delta^n A \sim \Delta$ relation, which are shown respectively in Figs. 4~6, and 11 by $Y_1 - Y_1$. The epicentral distance $\Delta_m(n)$, where $\log \Delta^n A$ attains its maximum value, are calculated from these constants 105 km, 300 km and 550 km when $n=1$, 3/2 and 2 respectively. The reason why $f(T)=k/T$ seems preferable in this case to $f(T)=k'/T^2$ is as follows: If we can take it for granted that $T=\alpha\Delta^{1/3}$ from Fig. 10 and put $f(T)=k'/T^2$, we have $B(T)=C\exp(-s'T)$, instead of (10), from (7), and $\Delta_m(n)$ is proportional to $(n-1/2)^3$, so that if $\Delta_m(3/2)=300$ km, that is plausible from Figs. 2 and 5, $\Delta_m(2)=1000$ km that is perhaps not the case from Figs. 3 and 6. However, accuracies of observations are not sufficient for any conclusion. If accuracies of observations are sufficient, k-value is to be compared with those obtained from experimental studies; s-value gives the spectrum of this earthquake waves, which also is to be compared with the observations.

The k-value, where $f(T)=k/T$, is related to a dimensionless and generally used quantity $1/Q$ by $1/Q=kc/\pi$, where c is the velocity of the wave; for some reference, putting $c=2\sim3$ km/sec, we have $1/Q=0.007\sim 0.009$, which is not so much different from the result of laboratory experiment by Born. Born's result[2)] is, though it is for compressional waves, $kc=0.02\sim0.03$.

In the next place, let us see if the above obtained k-value is applicable to the observations of another earthquakes. We will take up the largest earthquakes in the Table 1, the Fukui- and Shionomisaki earthquake, $A\sim\Delta$ relations of which are similar to each other for $\Delta>200$ km. As regards $T\sim\Delta$ relation, some differences are apparent between the two, and that of the North-Izu earthquake gives a mean tendency of the two. $A\sim\Delta$ and $T\sim\Delta$ relation of the Saitama earthquake are utterly different from those of the three, and are rather similar to those of minor earthquakes, but for its large amplitudes.

Putting $n_0=1/2$, $f(T)=k/T$, where $k=0.0096$ sec km^{-1}, and $T=\alpha\Delta^\beta$ in (21), we get,

$$\log \Delta^n A = C + (n-1/2)\log \Delta - k\Delta^{1-\beta}/(1-\beta)\alpha\ . \qquad (22)$$

From the observations, it is assumed that $T=4$ sec at $\Delta=500$ km, $\Delta_m(3/2)=250\sim300$ km, and $\Delta_m(2)\nless 800$ km. Then the following constants are obtained for one of possible solutions; $\beta=2/3$, $\alpha=4/63$ sec km$^{-2/3}$, $C=2\cdot10^6\ \mu$ km$^{-1/2}$, $\sqrt{\Delta}\cdot A=C\exp(-3k\Delta^{1/3}/\alpha)$, $B(T)=C\exp(-2k\sqrt{T}/\alpha^{3/2})$, $\log_{10}\Delta^n A=6.290+(n-1/2)\log_{10}\Delta-0.197\Delta^{1/3}$; Δ in km, A in μ. It seems

2) W. T. BORN, "The attenuation constant of earth materials," *Geophysics*, **6** (1941), 132–148.

preferable to put β a little larger than 2/3 for $300\,\text{km} < \Delta < 1000\,\text{km}$; observations for $\Delta < 200\,\text{km}$ are not clear, because of too large amplitudes. The above obtained constants will be used for $\Delta > 200\,\text{km}$, and, therefore, $T > 2.5\,\text{sec}$. Calculated $\log \Delta^n A \sim \Delta$ and $T \sim \Delta$ curves are shown in the respective diagrams by $Y_2 - Y_2$. It will be a problem of some interest in future to extend those curves to a large distance and examine the propriety of the above obtained constants, especially, of $B(T)$, when the k-value is re-examined on the observations of, at least, a little higher accuracy than herein presented.

We can see also a reasonable difference in the expression of $B(T)$ of the two earthquakes, the Shizuoka- and the Shionomisaki-, in accordance with the difference of their magnitudes: for the former $B(T) = C \exp(-sT^2)$, while for the latter $B(T) = C' \exp(-s'\sqrt{T})$.

7. Concluding remarks and acknowledgement.

Though partial success is apparent, it is impossible to deduce a conclusive finding, for or against the writer's idea, from those scanty and dispersive observations so far obtained. At present, they are presented as data for future studies, expecting still more observations of high accuracy. The dispersive appearance of observations seems at present, apart from the geological condition of observatory and instrumental defects in observation, partly due to a want of clear definition of the "maximum amplitude" and a shortness of a guiding principle for observation. It is hoped that the way of thinking proposed here will give some indications in that respect.

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26. 最大振巾と震央距離との関係について
—実験式の波動論的考察—

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所謂最大振巾と震央距離との関係について種々の実験式が提出されている．それ等の間の理論的結び付きとその実験式を地震波の理論のためどのように使つていくかについて考察を試みた．このような理論的考察は複雑な地震記象を与えるいわゆる破壊的地震の波動論的解明に必要である．まず“最大振巾”の定義，すなわちその計測方法が確立していなければならないのであるが，諸家の実験式の資料となつている過去の観測報告はこの点において必ずしも満足すべきものとは思われない．しかし気象庁の“地震観測法”によつて判断するに，いつの頃からか振巾の計測には高調波を均らして測定し，全振巾を測つてその半をとり，また“最大振巾”の“周期”もあわせ測定する習慣が生れ，最近はこの方法に一定されたようである．この方法に落ちついたことから考えれば過去の資料も大体の傾向としてはこの方法によつたものと考えてよいであろう．このような観測方法の基調をなすものは調和解析の考え方である．したがつて“最大振巾とその周期”の“報告値”の本質はどちらかといえば最大振巾をもつ成分波の振巾とその周期を与えるものと考えてよい．今後この計測方針を一層明確にすることが“振巾”といい“周期”といい理論的にもすつきりするし，地震波理論の基礎資料としても有効である．厳密にいえば現在の目の子的調和解析による“最大振巾”の処理には波のむれの理論を必要とするが，それを使うことは未知量を増し数学形式を複雑にして，数学的興味を別とすれば，実効がないように思う．将来若し“最大振巾”の厳密な定義が問題となるようなことがあるならば，調和解析を完全にして複雑な波のむれの理論を必要としないような方向に観測資料の整理を考えることが理論的には望ましい．“最大振巾”の“報告値”をこのように成分波の観測値と解釈すれば“最大振巾”と“震央距離”との関係についてすでに提出された，あるいは将来提出されるであろう多くの実験式も唯一つの基本則から比較的簡単に誘導される．その過程が簡単であるから逆に多くの関係式から一つの基本則を帰納することもできる．この理論の裏付けにはいままで兎角注目されなかつた“最大振巾の周期”と震央距離との関係を調査することが必要である．日本の過去に数多い破壊的地震の観測資料もこのような観点からすればはなはだ数少いものとなつてしまう．計算例のため比較的観測精度が高いと思われる地震7個をとつたが，偶然大きさによつて二分される結果となつた．特にその中からそれぞれの大きさを代表して静岡地震(昭和10年)と汐岬地震(昭和23年)について数値計算を行なつた．“最大振巾”と“震央距離”の関係から震央におけるスペクトルを推定し数学式で表わすことができる．静岡地震では $\exp(-sT^2)$，汐岬地震では $\exp(-s'\sqrt{T})$ となり地震の大きさによる違いが予想通り現われたことは筆者の考え方が将来理論的に役立つことを示すものではないかと思う．地震波に対する地殻の内部摩擦係数も $1/Q=0.007\sim0.009$ と計算され，Born が実験的に，Ewing, Press 等が表面波の観測から推定しているものとほぼ同程度の値を得たが，少しく大きすぎると思われるので今後さらに検討の予定である．なおここにいう“最大振巾”は“変位振巾”だけに限らず“速度振巾，”“加速度振巾”あるいはもつと高次の微係数の振巾であることも，または逆に時間的積分の振巾であることも考えられる．いずれであるかはその観測網に用いられた地震計の特性で定まることで，この点についても筆者の解釈は従来のものと少しく異るのではないかと思う．すなわち地震記象であらかじめ“最大振巾”を測定してから地震計の特性による補正を施しては“最大”の意味が曖昧となり，理論的研究の資料としては混迷を招く．この見地からすれば同一観測網に属する地震計の特性を揃えることが必要であり，かつスペクトルを完全に求めるためには種々特性の異つた地震計を同一観測点にあわせ置くことが必要である．